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The Marginal Cost of Risk in a Multi-Period Risk Model

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This presentation is based on a research project on "Allocation of Costs of Holding Capital" sponsored by the Casualty Actuarial Society (CAS). Special thanks to Richard Derrig for his continued support.

Motivation and Overview

- Financial institutions use risk measure gradients to allocate capital to risks for purposes of pricing and performance measurement
- Typical example from insurance: Return on Risk-Adjusted Capital (RORAC)
  - RORAC for line k = (expected return in line k) / (capital allocated to line k)
  - expected return=u/w profit margin
    capital allocated based on risk measure gradient
  - Assess performance by comparing RORAC for line k to a target ROE
- *Criticism*: Risk measure is arbitrary and may not connect to the underlying economics of the business. Using this technique may boil down to pricing while avoiding the "rigors of the pricing project" (Venter, 2010)
- Previous work (Bauer and Zanjani, forthcoming) shows that "economically rigorous" pricing in a one period model is consistent with existing practice, but implies unfamiliar risk measures

Motivation and Overview

- We build RORAC from the ground up by calculating the marginal cost of risk in a multi-period economic model of the firm
- This changes both the notion of expected return and capital allocation. RORAC can be used, but only if its components are redefined:
  - *Redefine return*: Expected return calculations must consider non-actuarial sources of costs
  - *Redefine capital*: "Capital" has to be conceived more broadly—to include contingent sources of financing
  - *Redefine the benchmark*: The cost of capital has to be adjusted similarly: A target ROE on book equity is no longer appropriate

Return-on-Risk-Adjusted-Capital (RORAC)

$$\text{RORAC}_i = \frac{[\text{(Marginal) Return on Line } i\,]}{\frac{\partial\rho}{\partial q_i}} \geq [\text{Cost-of-Capital}]$$

$$\Leftrightarrow \quad [\text{(Marginal) Return on Line } i\,] \geq \underbrace{[\text{Cost-of-Capital}] \times \frac{\partial\rho}{\partial q_i}}_{\text{Marginal Cost of Risk}}$$

$$\Rightarrow \quad \underbrace{\sum_i q_i\,[\text{(Marginal) Return on Line } i\,]}_{\text{Tot. Return}} \geq [\text{Cost-of-Capital}] \times a$$

- **Economic motivation**: Optimization of company's profits Details

$$\underbrace{[\text{Premiums}]}_{\sum_i p_i} - \underbrace{\mathbb{E}[\text{Payoff}]}_{\sum_i R_i} - [\text{CoC}]\,a$$

subject to a risk measure constraint $\rho(q_1\,L_1 + \ldots + q_N\,L_N) \leq a$ yields:

$$\text{RORAC}_i : \left\{ \begin{array}{l} \text{Actuarial Profit} \\ \text{Allocated Capital} \end{array} \right. \begin{array}{l} \longleftarrow \\ \longleftarrow \end{array} \frac{\left(\frac{\partial p_i}{\partial q_i} - \frac{\partial R_i}{\partial q_i}\right)\, q_i}{\frac{\partial\rho}{\partial q_i}\, q_i} = [\text{CoC}]$$

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- How to choose the risk measure ρ?
  - Axiomatic approaches – e.g., coherence, convexity
  - Bauer & Zanjani, forthcoming:
    - Simple single period economic model of insurer
    - No risk measure, risk-taking constrained by **policyholder risk aversion**
    - Can be reconciled with risk measure constrained optimization
      ...BUT the correct risk measure is neither coherent nor convex
  - But what about in a richer model? With multiple periods? External financing? Does RORAC still make sense?

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- How to choose the risk measure ρ? **(Bauer & Zanjani, forthcoming)**
- What is the Cost-of-Capital? **(this paper)**
  - Different "costs of capital":
    - Cost of *internal* capital
    - Cost of raising *external* capital
    - Cost of raising *emergency* capital
  - What is the "correct" cost of capital to compare returns to?

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- What is the Cost-of-Capital? **(this paper)**
- What is the (marginal) Return on a Line? **(this paper)**
  - Actuarial return: Premiums minus actuarial costs?
  - Could there be other cost-components when expanding operations?

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- What is the (marginal) Return on a Line? **(this paper)**
- What is the capital a? **(this paper)**
  - Statutory equity capital in balance sheet?
  - What about untapped resources (letters of credit, access to emergency capital)?

Approach and Results

- We build a **(more) complex model** of an insurer
  - **Multiple** periods, **default** is possible
  - **Various external financing** opportunities (internal vs. external vs. emergency capital)
  - Policyholder risk aversion modeled through **premium function** relating margins to default risk and scale (later estimated based on NAIC data)

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- Optimal RORAC (or, rather RARAC) calculation:

Adjust for add. components: cont. value, scale, distress $\searrow$

All financial resources, contingent sources $\nearrow$

$$\overbrace{\frac{\cancel{\text{Actuarial}}\ \text{Profit}}{\text{Allocated}\ \cancel{\text{Capital}}}}^{\text{RAROC}} \overset{?}{\geq} \tau \rightarrow$$

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- We **implement** / (numerically) **solve** the model in the context of data from a catastrophe reinsurer, and **compare** "optimal" and conventional RAROC calculations
  - While capital costs are still most important cost component in non-extreme cases, **ignoring additional components can lead to inefficient decisions**
  - In extreme cases, other cost components gain importance

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  - $\rightarrow$ Resulting indemnity $I_t^{(i)} = I_t^{(i)}(L_t^{(i)}, q_t^{(i)}) = L_t^{(i)} \times q_t^{(i)}$ (proportional, generalizations possible)

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  - The company can raise or shed capital at the beginning of the period (R^b) at cost c_1, with $c_1(x) = 0,\ x < 0$
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⇒ **Law of motion**:

$$a_t = \left[a_{t-1}\,(1-\tau) + R_t^b - c_1(R_t^b) + \sum_i p_t^i \right] e^r - \sum_i I_t^{(i)} + R_t^e - c_2(R_t^e)$$

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- In case of **default**, remaining assets are paid to dollar at the same rate per dollar of coverage

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- $R^e \in \{0, R^e_*\}$, where R^e_* "just saves" company:
  - If ([*Fin. Resources*] $\geq I$), no need to raise (can raise cheaper at beginning of next period)
  - If ([*Fin. Resources*] $< I$), just raise enough to survive (can raise cheaper at beginning of next period)
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- **Bellman Equation:** (s.t. several constraints)

$$V(a) = \max_{\{p^{(j)}\},\{q^{(j)}\},R^b} \left\{ \begin{array}{l} \mathbb{E}\Big[1_{\{S \geq I\}} \times \Big(\sum_j p^{(j)} - e^{-r} I - \tau\, a - c_1(R^b) + e^{-r}\, V(a_{\text{new}}) \Big) \\ \quad + 1_{\{S < I \leq D\}} \times \Big(\frac{1}{1-\xi} \Big[[\sum_j p^{(j)} + a(1-\tau) + R^b - c_1(R^b)] - e^{-r} I \Big] \\ \qquad\qquad + e^{-r} V(0) - [a + R^b] \Big) \\ \quad + 1_{\{I > D\}} \left(-(a + R^b) \right) \Big] \end{array} \right\}$$

$\rightarrow$ Three regions: $I \leq S$ – no issues; $S < I \leq D$ – save the company; $I > D$ – default

Connecting Risk and Return

- **Premium**: For empirical tractability, we assume policyholders assess company quality via the default probability but demand gets saturated:

$$p_t^{(i)} = e^{-r}\mathbb{E}[I_t^{(i)}] \times \underbrace{\exp\{\alpha - \beta\mathbb{P}(I_t > D) - \gamma\mathbb{E}[I_t]\}}_{\text{Mark-up over actuarial price}}$$

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  - Generalizations possible...
- Recall from the **basic one-period model**:

The Marginal Cost of Risk

$$\text{RAROC}_i = \frac{\left(\frac{\partial p_i}{\partial q_i} - \frac{\partial R_i}{\partial q_i}\right) q_i}{\frac{\partial \rho}{\partial q_i} q_i} = [\text{CoC}] \Leftrightarrow \frac{\partial p_i}{\partial q_i} = \frac{\partial R_i}{\partial q_i} + [\text{CoC}] \times \frac{\partial \rho}{\partial q_i}$$

The Marginal Cost of Risk

We have for the marginal cost for risk $i \in \{1, 2, \ldots, N\}$:

$$\frac{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\right] \exp\{\alpha - \beta \mathbb{P}(I \geq D) - \gamma \mathbb{E}[I]\}}{(1 - c_1'^{b})}$$

$$= \underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}} 1_{\{I \leq D\}}\right]}_{(i)} + \underbrace{\gamma \, \mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\right] \exp\{\alpha - \beta \mathbb{P}(I > D) - \gamma \, \mathbb{E}[I]\}}_{(ii)} + \underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}} V'(S - I) \; 1_{\{I \leq S\}}\right]}_{(iii)}$$

$$+ \underbrace{\mathbb{E}\left[\frac{\xi}{1 - \xi} \frac{\partial I^{(i)}}{\partial q^{(i)}} 1_{\{S < I \leq D\}}\right]}_{(iv)}$$

$$+ \underbrace{\mathbb{E}\left[\left.\frac{\partial I^{(i)}}{\partial q^{(i)}}\right| I = D\right] \times \left\{\mathbb{P}(I \geq D) + \tau^*\right\}}_{(v)},$$

where the "shadow cost of capital" τ^* is defined as:

$$\tau^* = \frac{c_1'(R_b)}{1 - c_1'(R_b)} - \frac{\xi}{1 - \xi} \mathbb{P}(S \leq I \leq D) - \mathbb{E}\left[V'\left(S - I\right) 1_{\{S \geq I\}}\right]\Big\}$$

Marginal Cost of Risk – Interpretation

(i) actuarial value of the liability in solvent states

(ii) "scale costs": increased supply will yield a decrease in the price of insurance

(iii) impact on continuation value of the company: higher exposure will lead to a change in the capitalization at the end of the period, which will affect the value of the company

(iv) increase in costs to save the company (larger operations)

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- **RAROC** (RARAC?):

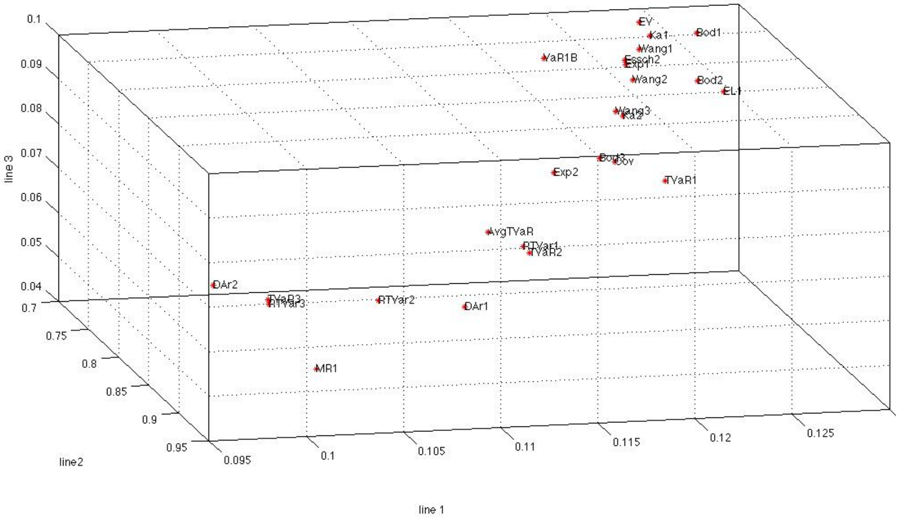
$$\frac{[\text{Marginal Revenue}] - \text{(i)} - \text{(ii)} - \text{(iii)} - \text{(iv)}}{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}} \middle| I = D \right]} \geq \mathbb{P}(I \geq D) + \tau^*$$

Alternative Interpretation: Risk-adjustment of cash flows

Case Study: Data from Cat Reinsurer

Line	Statistics			Aggs		
	Premiums	Expected Loss	Standard Deviation	Agg1	Agg2	Agg3
N American EQ East	6,824,790.67	4,175,221.76	26,321,685.65	1	1	1
N American EQ West	31,222,440.54	13,927,357.33	47,198,747.52	2	2	1
S American EQ	471,810.50	215,642.22	915,540.16	3	2	1
Australia EQ	1,861,157.54	1,712,765.11	13,637,692.79	4	3	1
Europe EQ	2,198,888.30	1,729,224.02	5,947,164.14	5	3	1
Israel EQ	642,476.65	270,557.81	3,234,795.57	6	3	1
NZ EQ	2,901,010.54	1,111,430.78	9,860,005.28	7	3	1
Turkey EQ	214,089.04	203,495.77	1,505,019.84	8	3	1
N Amer. Severe Storm	16,988,195.98	13,879,861.84	15,742,997.51	9	4	2
US Hurricane	186,124,742.31	94,652,100.36	131,791,737.41	10	4	2
US Winterstorm	2,144,034.55	1,967,700.56	2,611,669.54	11	4	2
Australia Storm	124,632.81	88,108.80	622,194.10	12	5	2
Europe Flood	536,507.77	598,660.08	2,092,739.85	13	5	2
ExTropical Cyclone	37,033,667.38	23,602,490.43	65,121,405.35	14	5	2
UK Flood	377,922.95	252,833.64	2,221,965.76	15	5	2
US Brushfire	12,526,132.95	8,772,497.86	24,016,196.20	16	6	3
Australian Terror	2,945,767.58	1,729,874.98	11,829,262.37	17	7	4
CBNR Only	1,995,606.55	891,617.77	2,453,327.70	18	7	4
Cert. Terrorism xCBNR	3,961,059.67	2,099,602.62	2,975,452.18	19	7	4
Domestic Macro TR	648,938.81	374,808.73	1,316,650.55	20	7	4
Europe Terror	4,512,221.99	2,431,694.65	8,859,402.41	21	7	4
Non Certified Terror	2,669,239.84	624,652.88	1,138,937.44	22	7	4
Casualty	5,745,278.75	2,622,161.64	1,651,774.25	23	8	4
N American Crop	21,467,194.16	9,885,636.27	18,869,901.33	24	9	3

Difference in Conventional Allocations



Premium Function

- **Specification**: For company *i* in year *t*

$$\log\{p_{it}\} = \alpha + \alpha_t - \beta\, d_{it} - \gamma\, E_{it} + \varepsilon_{it}$$

  where:
  - d_{it} is the default rate according to the letter rating (fitted based on AM Best Ratings)
  - E_{it} is the expected loss (based on average net loss and loss adjustment expense ratio)
- Estimated from NAIC data for Reinsurance Companies according to Reinsurance Association of America's annual review
- **Results**:

Variable	Coefficient	Std. Error	*t*-value
Intercept (α)	.65897	0.0614	10.73
Default rate (β)	3.92958	0.5090	-7.72
Expected Loss (γ)	1.48 E-10	2.24 E-11	-6.57

Year dummies are omitted. Observations: 288. Adj. $R^2 = 26\%$

Parametrizations

Parameter	1 (“base case")	2 (“profitable company")	3 (“empty company")
τ	3.00%	5.00%	5.00%
$c_1^{(1)}$	7.50%	7.50%	7.50%
$c_1^{(2)}$	1.00E-010	5.00E-011	1.00E-010
ξ	50.00%	75.00%	20.00%
r	3.00%	6.00%	3.00%
α	0.3156	0.9730	0.9730
β	392.96	550.20	550.20
γ	1.48E-010	1.61E-010	1.61E-010

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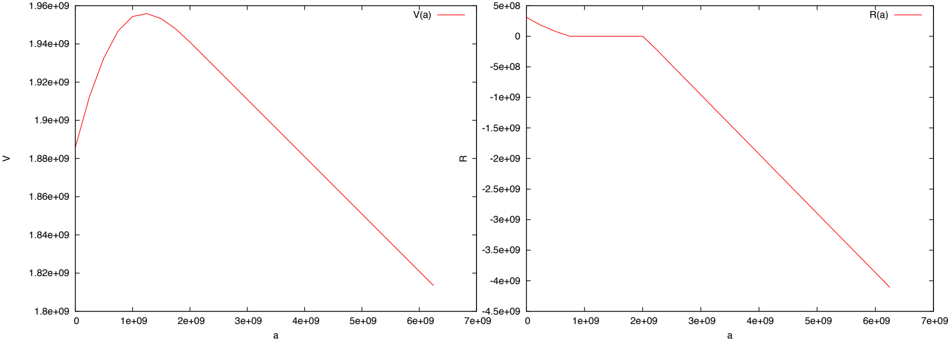
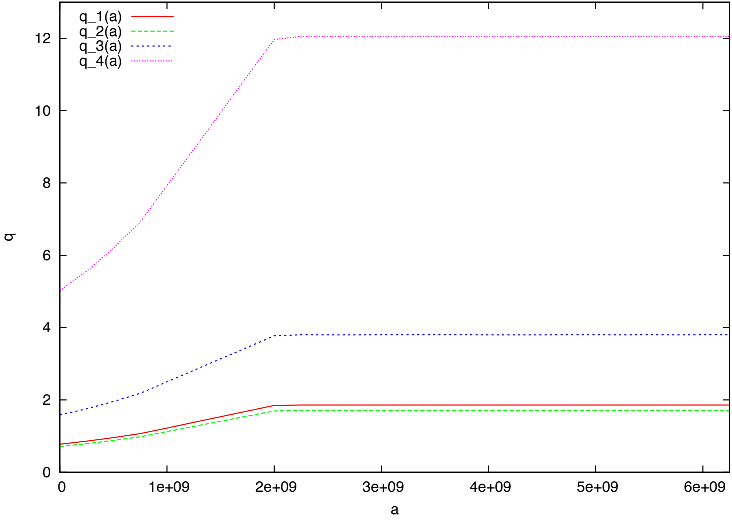
Base Case Results (I)



Value function and optimal raising decision for catastrophe reinsurer

Base Case Results (II)



Optimal portfolio for catastrophe reinsurer

Base Case Results (III)

	zero capital	optimal capital	high capital
a	0	1,000,000,000	4,000,000,000
$V(a)$	1,885,787,820	1,954,359,481	1,880,954,936
$R(a)$	311,998,061	0	-1,926,420,812
$q_1(a)$	0.78	1.23	1.86
$q_2(a)$	0.72	1.13	1.71
$q_3(a)$	1.60	2.51	3.80
$q_4(a)$	5.06	7.96	12.06
S	550,597,000	1,406,761,416	2,615,202,661
D	1,493,490,910	2,349,655,327	3,558,096,571
$\mathbb{E}[I]$	199,297,482	313,561,933	474,841,815
$\sum p^{(i)}/\mathbb{E}[i]$	1.32	1.30	1.27
$\mathbb{P}(I > a)$	100.00%	2.66%	0.002%
$\mathbb{P}(I > S)$	4.54%	0.45%	0.13%
$\mathbb{P}(I > D)$	0.002%	0.002%	0.002%
$c_1'(R_b)$	13.74%	4.65%	0.00%
$\frac{\xi}{1-\xi}\,\mathbb{P}(S < I < D)$	4.54%	0.45%	0.12%
$\mathbb{E}[V'\,\mathbf{1}_{\{I<S\}}]$	8.03%	1.09%	-2.66%
τ^*	3.36%	3.34%	2.53%

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$q_2(a)$	0.72	1.13	1.71
$q_3(a)$	1.60	2.51	3.80
$q_4(a)$	5.06	7.96	12.06
S	550,597,000	1,406,761,416	2,615,202,661
D	1,493,490,910	2,349,655,327	3,558,096,571
$\mathbb{E}[I]$	199,297,482	313,561,933	474,841,815
$\sum p^{(i)}/\mathbb{E}[i]$	1.32	1.30	1.27
$\mathbb{P}(I > a)$	100.00%	2.66%	0.002%
$\mathbb{P}(I > S)$	4.54%	0.45%	0.13%
$\mathbb{P}(I > D)$	0.002%	0.002%	0.002%
$c_1'(R_b)$	13.74%	4.65%	0.00%
$\frac{\xi}{1-\xi}\,\mathbb{P}(S < I < D)$	4.54%	0.45%	0.12%
$\mathbb{E}[V'\,\mathbf{1}_{\{I<S\}}]$	8.03%	1.09%	2.66%
τ^*	3.36%	3.34%	2.53%

Cost Allocations in Base Case, $a = 1,000,000,000$

$a = 1,000,000,000$	Line 1	Line 2	Line 3	Line 4	Aggregate
	Line 1	Line 2	Line 3	Line 4	Aggregate
Solvent payments, (i)	23,345,530	135,002,000	18,657,049	10,772,967	313,502,671
$(\mathbb{E}[L^{(i)}\,\mathbf{1}_{\{I<D\}}])$	9.14%	48.55%	14.94%	27.37%	100.00%
Scale effect, (ii)	1,475,632	8,535,701	1,179,341	681,028	19,819,580
$(\frac{\gamma}{1-c_1'(R_b)}\,\mathbb{E}[L^{(i)}]\,\sum_k p^{(k)})$	9.14%	48.56%	14.94%	27.37%	100.00%
Continuation value, (iii)	557,566	3,411,166	442,838	281,845	7,886,781
$(\mathbb{E}[L^{(i)}\,\mathbf{1}_{\{I<S\}}\;V'])$	8.68%	48.76%	14.09%	28.46%	100.00%
Raising cost, (iv)	501,193	3,356,439	489,699	227,776	7,442,867
$(\frac{\xi}{1-\xi}\,\mathbb{E}[L^{(i)}\,\mathbf{1}_{\{S<I<D\}}])$	8.27%	50.84%	16.52%	24.37%	100.00%
Capital cost, (v)	5,917,572	33,625,361	4,643,976	2,711,437	78,428,268
$(\mathbb{E}[L^{(i)} \mid I = D] \times [\mathbb{P}(I > D) + \tau^*])$	9.27%	48.34%	14.86%	27.53%	100.00%
Cost, (iii)-(v)	6,976,331	40,392,966	5,576,513	3,221,059	93,757,915
	9.14%	48.57%	14.93%	27.36%	100.00%
Non payments, (ii)-(v)	8,451,962	48,928,667	6,755,854	3,902,086	113,577,496
	9.14%	48.57%	14.93%	27.36%	100.00%

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Cost Allocations in Base Case, $a = 0$

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Solvent payments, (i)	23,345,530	135,002,000	18,657,049	10,772,967	199,259,815
$(\mathbb{E}[L^{(i)}\, 1_{\{I<D\}}])$	9.14%	48.55%	14.94%	27.37%	100.00%
Scale effect, (ii)	1,054,415	6,099,197	842,700	486,629	9,001,325
$(\frac{\gamma}{1-c_1'(R^b)}\, \mathbb{E}[L^{(i)}]\, \sum_k p^{(k)})$	9.14%	48.56%	14.94%	27.37%	100.00%
Continuation value, (iii)	1,787,494	10,069,585	1,395,397	781,737	14,794,219
$(\mathbb{E}[L^{(i)}\, 1_{\{I<S\}}\, V'])$	9.43%	48.77%	15.05%	26.75%	100.00%
Raising cost, (iv)	3,136,921	21,340,216	2,782,890	1,924,195	31,920,536
$(\frac{\xi}{1-\xi}\, \mathbb{E}[L^{(i)}\, 1_{\{S<I<D\}}])$	7.67%	47.91%	13.91%	30.51%	100.00%
Capital cost, (v)	6,423,322	34,269,301	4,891,903	2,532,602	50,194,848
$(\mathbb{E}[L^{(i)} \mid I = D] \times [\mathbb{P}(I > D) + \tau^*])$	9.99%	48.92%	15.55%	25.54%	100.00%
Cost, (iii)-(v)	11,347,737	65,679,102	9,070,189	5,238,533	96,909,604
	9.14%	48.57%	14.93%	27.36%	100.00%
Non payments, (ii)-(v)	12,402,152	71,778,299	9,912,889	5,725,162	105,910,928
	9.14%	48.56%	14.93%	27.36%	100.00%

Implications

- The direct connection between the marginal cost of risk and capital cost associated with default **breaks down**:
  - Different cost components: "scale costs", "impact on continuation value", "cost of emergency raising"
  - Nonetheless: Capital cost important piece, similar form as before when considering...

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  - Nonetheless: Capital cost important piece, similar form as before when considering...

... **Correct "notion" of capital**: All available capital D (including untapped resources)

... **Correct "notion" of capital cost**: "Shadow cost of capital" τ^* originating from optimal capital policies

$$\frac{\partial p_i^*}{\partial q_i} = \frac{\partial e_i^Z}{\partial q_i} + \text{"other costs"} + \tilde{\phi}_i \left[\mathbb{P}_D + \tau^*\right] \Leftrightarrow \underbrace{\frac{\frac{\partial p_i^*}{\partial q_i} - \frac{\partial e_i^Z}{\partial q_i} - \text{"other costs"}}{\tilde{\phi}_i}}_{\text{RORAC}_i} = \mathbb{P}_D + \tau^*$$

$\rightarrow$ Important to consider these aspects in RORAC calculations...

RAROC calculations, Base Case, $a = 1,000,000,000$

	Alloc.	Cost considered	Line 1	Line 2	Line 3	Line 4
Correct Allocation	D	yes	3.34%	3.34%	3.34%	3.34%
VaR Allocation	D	yes	3.83%	3.38%	3.25%	3.21%
TVaR Allocation	D	yes	90.72%	2.13%	10.79%	4.73%
MyersRead	D	yes	-8.43%	1.83%	-447.25%	5.81%
VaR Allocation	D	act. only	5.46%	4.92%	4.73%	4.62%
VaR Allocation	S	act. only	9.89%	8.00%	8.00%	7.83%
VaR Allocation	a	act. only	16.06%	13.16%	12.23%	8.36%
VaR Allocation	D	act. and scale	4.51%	4.06%	3.90%	3.81%
VaR Allocation	S	act. and scale	8.17%	6.61%	6.60%	6.46%
VaR Allocation	a	act. and scale	13.26%	10.86%	10.09%	6.90%
TVaR Allocation	D	act. only	129.58%	3.10%	15.70%	6.81%
TVaR Allocation	S	act. only	8.99%	7.68%	7.34%	9.08%
TVaR Allocation	a	act. only	12.91%	11.13%	11.98%	11.00%
TVaR Allocation	D	act. and scale	106.96%	2.56%	12.96%	5.62%
TVaR Allocation	S	act. and scale	7.42%	6.34%	6.06%	7.49%
TVaR Allocation	a	act. and scale	10.65%	9.19%	9.89%	9.08%

RAROC calculations, Base Case, $a = 1,000,000,000$

	Alloc.	Cost considered	Line 1	Line 2	Line 3	Line 4
Correct Allocation	D	yes	3.34%	3.34%	3.34%	3.34%
VaR Allocation	D	yes	3.83%	3.38%	3.25%	3.21%
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RAROC calculations, Base Case, $a = 1,000,000,000$

	Alloc.	Cost considered	Line 1	Line 2	Line 3	Line 4
Correct Allocation	*D*	yes	3.34%	3.34%	3.34%	3.34%
VaR Allocation	*D*	yes	[illegible]	3.38%	3.25%	3.21%
TVaR Allocation	*D*	yes	90.72%	2.13%	10.79%	4.73%
MyersRead	*D*	yes	-8.43%	1.83%	-447.25%	5.81%
VaR Allocation	*D*	act. only	[illegible]	4.92%	4.73%	[illegible]
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VaR Allocation	*a*	act. only	[illegible]	13.16%	12.23%	8.36%
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VaR Allocation	*a*	act. and scale	13.26%	10.86%	10.09%	6.90%
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→ Use simple (Normal) version of the model and include state space

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→ **Current/future work**:

- Allocation for P&C companies, development matters

→ Use simple (Normal) version of the model and include state space

- Extension to "economic" connection between risk and return (participation constraint)

→ Theory: *S* and *D* matter for allocation; how to connect to practice?

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Thank you!

Cost Allocations in Base Case, $a = 0$

$a = 0$	Line 1	Line 2	Line 3	Line 4	Aggregate
	Line 1	Line 2	Line 3	Line 4	Aggregate
Solvent payments, (i)	23,345,530	135,002,000	18,657,049	10,772,967	199,259,815
$(\mathbb{E}[L^{(i)}\, \mathbf{1}_{\{I<D\}}])$	9.14%	48.55%	14.94%	27.37%	100.00%
Scale effect, (ii)	1,054,415	6,099,197	842,700	486,629	9,001,325
$(\frac{\gamma}{1-c_1'(R_b)}\, \mathbb{E}[L^{(i)}]\, \sum_k p^{(k)})$	9.14%	48.56%	14.94%	27.37%	100.00%
Continuation value, (iii)	1,787,494	10,069,585	1,395,397	781,737	14,794,219
$(\mathbb{E}[L^{(i)}\, \mathbf{1}_{\{I<S\}}\, V'])$	9.43%	48.77%	15.05%	26.75%	100.00%
Raising cost, (iv)	3,136,921	21,340,216	2,782,890	1,924,195	31,920,536
$(\frac{\xi}{1-\xi}\, \mathbb{E}[L^{(i)}\, \mathbf{1}_{\{S<I<D\}}])$	7.67%	47.91%	13.91%	30.51%	100.00%
Capital cost, (v)	6,423,322	34,269,301	4,891,903	2,532,602	50,194,848
$(\mathbb{E}[L^{(i)} \mid I = D] \times [\mathbb{P}(I > D) + \tau^*])$	9.99%	48.92%	15.55%	25.54%	100.00%
Cost, (iii)-(v)	11,347,737	65,679,102	9,070,189	5,238,533	96,909,604
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Non payments, (ii)-(v)	12,402,152	71,778,299	9,912,889	5,725,162	105,910,928
	9.14%	48.56%	14.93%	27.36%	100.00%

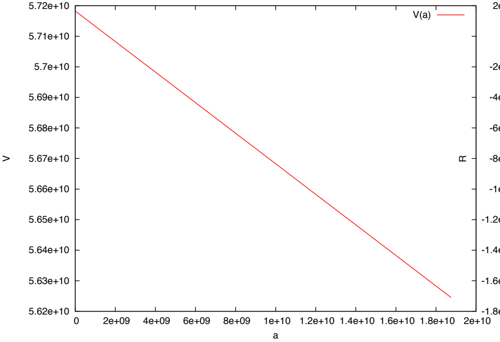
Results for Profitable Company

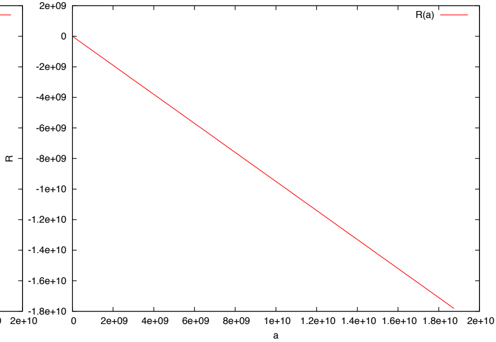
	zero capital	optimal capital	high capital
a	0	3,000,000,000	12,000,000,000
$V(a)$	22,164,966,957	22,404,142,801	22,018,805,587
$R(a)$	1,106,927,845	0	-6,102,498,331
$q_1(a)$	4.81	6.14	7.82
$q_2(a)$	4.42	5.64	7.18
$q_3(a)$	9.83	12.56	15.98
$q_4(a)$	31.19	39.85	50.69
S	3,659,208,135	6,215,949,417	9,412,766,805
D	9,200,449,874	11,757,191,157	14,954,008,545
$\mathbb{E}[I]$	1,227,901,222	1,569,126,466	1,995,776,907
$\sum p^{(i)}/\mathbb{E}[i]$	2.15	2.03	1.90
$\mathbb{P}(I > a)$	1.00%	10.70%	0.07%
$\mathbb{P}(I > S)$	3.65%	0.91%	0.34%
$\mathbb{P}(I > D)$	0.002%	0.002%	0.002%
$c_1'(R_b)$	18.57%	5.97%	0.00%
$\frac{\xi}{1-\xi}\,\mathbb{P}(S < I < D)$	10.94%	2.72%	1.00%
$\mathbb{E}[V'\,\mathbf{1}_{\{I<S\}}]$	2.93%	-2.99%	-4.58%
τ^*	8.94%	6.62%	3.58%

Cost Allocations in Profitable Company, $a = 3,000,000,000$

$a = 3,000,000,000$	Line 1	Line 2	Line 3	Line 4	Aggregate
Solvent payments, (i)	23,345,530	135,002,000	18,657,049	10,772,967	1,568,829,904
$(\mathbb{E}[L^{(i)}\, \mathbf{1}_{\{I<D\}}])$	9.14%	48.55%	14.94%	27.37%	100.00%
Scale effect, (ii)	12,749,807	73,750,484	10,189,785	5,884,241	856,948,543
$(\frac{\gamma}{1-c_1'(R_b)}\, \mathbb{E}[L^{(i)}]\, \sum_k p^{(k)})$	9.14%	48.56%	14.94%	27.37%	100.00%
Continuation value, (iii)	-235,099	-1,116,997	-202,680	-67,987	-13,002,123
$(\mathbb{E}[L^{(i)}\, \mathbf{1}_{\{I<S\}}\, V'])$	11.11%	48.47%	19.58%	20.84%	100.00%
Raising cost, (iv)	2,579,077	18,401,683	2,557,559	1,239,091	201,178,046
$(\frac{\xi}{1-\xi}\, \mathbb{E}[L^{(i)}\, \mathbf{1}_{\{S<I<D\}}])$	7.88%	51.61%	15.97%	24.55%	100.00%
Capital cost, (v)	12,029,113	65,894,390	9,133,206	5,463,678	778,163,393
$(\mathbb{E}[L^{(i)} \vert I = D] \times [\mathbb{P}(I > D) + \tau^*])$	9.50%	47.78%	14.74%	27.98%	100.00%
Cost, (iii)-(v)	14,373,091	83,179,077	11,488,085	6,634,782	966,339,316
	9.14%	48.56%	14.93%	27.36%	100.00%
Non payments, (ii)-(v)	27,122,898	156,929,561	21,677,870	12,519,023	1,823,287,859
	9.14%	48.56%	14.93%	27.36%	100.00%

Results for Empty Company





Value function and optimal raising decision for catastrophe reinsurer

Cost Allocations in Empty Company, $a = 0$

$a = 0$	Line 1	Line 2	Line 3	Line 4	Aggregate
Solvent payments, (i)	23,345,695	135,041,756	18,658,134	10,774,413	2,487,582,817
$(\mathbb{E}[L^{(i)}\ 1_{\{I<D\}}])$	9.86%	46.23%	17.75%	26.16%	100.00%
Scale effect, (ii)	17,919,327	103,653,262	14,321,322	8,270,057	1,909,380,339
$(\frac{\gamma}{1-c_1'(R_b)}\ \mathbb{E}[L^{(i)}]\ \sum_k p^{(k)})$	9.86%	46.23%	17.75%	26.16%	100.00%
Continuation value, (iii)	-782,453	-4,526,226	-625,396	-361,120	-83,376,677
$(\mathbb{E}[L^{(i)}\ 1_{\{I<S\}}\ V'])$	9.86%	46.23%	17.75%	26.16%	100.00%
Raising cost, (iv)	1,924,160	11,129,311	1,537,555	888,004	205,012,321
$(\frac{\xi}{1-\xi}\ \mathbb{E}[L^{(i)}\ 1_{\{S<I<D\}}])$	9.86%	46.23%	17.75%	26.16%	100.00%
Capital cost, (v)	na	na	na	na	na
$(\mathbb{E}[L^{(i)}\|I = D] \times [\mathbb{P}(I > D) + \tau^*])$	na	na	na	na	na
Cost, (iii)-(v)	1,141,707	6,603,085	912,159	526,885	121,635,644
	9.86%	46.23%	17.75%	26.16%	100.00%
Non payments, (ii)-(v)	19,061,034	110,256,347	15,233,481	8,796,942	2,031,015,983
	9.86%	46.23%	17.75%	26.16%	100.00%

Detailed One Period Model

- Insurer's problem:

$$\begin{cases} \max_{A,\{q^{(i)}\}} \sum p^{(j)} - \mathbb{E}[I\,\mathbf{1}_{\{I\le A\}}] - A\,\mathbb{P}(I > A) - c_1(A) \\ p^{(i)} = \mathbb{E}[I^{(i)}]\,\exp\{\alpha - \beta\,\mathbb{E}[I] - \gamma\mathbb{P}(I > A)\} \end{cases}$$

  - $I^{(i)} = q^{(i)} \times L^{(i)}$, $I = \sum I^{(j)}$, A assets, $c_1(\cdot)$ cost
  - Premium function: scale and risk effect (could be generalized, of course)

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  - $I^{(i)} = q^{(i)} \times L^{(i)}$, $I = \sum I^{(j)}$, A assets, $c_1(\cdot)$ cost
  - Premium function: scale and risk effect (could be generalized, of course)
- Two levers, exposure and capitalization. Can trade off:

$$\underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\right]\frac{p^{(i)}}{\mathbb{E}[I^{(i)}]}(1-\gamma\mathbb{E}[I])}_{\text{marginal premium}} = \underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\,\mathbf{1}_{\{I\leq A\}}\right]}_{\text{act. value}} + \underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\Big| I = A\right]}_{\text{cap. alloc}} \times \underbrace{\left(\mathbb{P}(I>A)+c_1'(A)\right)}_{\text{cap. cost}}$$

Detailed One Period Model

- Insurer's problem:

$$\begin{cases} \max_{A,\{q^{(i)}\}} \sum p^{(j)} - \mathbb{E}[I\,\mathbf{1}_{\{I\leq A\}}] - A\,\mathbb{P}(I > A) - c_1(A) \\ p^{(i)} = \mathbb{E}[I^{(i)}]\,\exp\{\alpha - \beta\,\mathbb{E}[I] - \gamma\mathbb{P}(I > A)\} \end{cases}$$

  - $I^{(i)} = q^{(i)} \times L^{(i)}$, $I = \sum I^{(j)}$, A assets, $c_1(\cdot)$ cost
  - Premium function: scale and risk effect (could be generalized, of course)
- Two levers, exposure and capitalization. Can trade off:

$$\underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\right]\frac{p^{(i)}}{\mathbb{E}[I^{(i)}]}(1-\gamma\mathbb{E}[I])}_{\text{marginal premium}} = \underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\,\mathbf{1}_{\{I\leq A\}}\right]}_{\text{act. value}} + \underbrace{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\Big|I = A\right]}_{\text{cap. alloc}} \times \underbrace{\left(\mathbb{P}(I > A) + c_1'(A)\right)}_{\text{cap. cost}}$$

- If company is risk-averse, need to think about "utility" U:

$$\begin{cases} \max_{A,\{q^{(i)}\}} \mathbb{E}\left[U\left(\sum p^{(j)} - I\,\mathbf{1}_{\{I\leq A\}} - A\,\mathbf{1}_{\{I>A\}} - c_1(A)\right)\right] \\ p^{(i)} = \mathbb{E}[I^{(i)}]\,\exp\{\alpha - \beta\,\mathbb{E}[I] - \gamma\mathbb{P}(I > A)\} \end{cases}$$

$$\Rightarrow \text{Marg. Prem} = \mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\,\mathbf{1}_{\{I\leq A\}}\underbrace{\frac{U'}{\mathbb{E}[U]}}_{w(I)}\right] + \mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\Big|I = A\right] \times \left(\mathbb{E}\left[\mathbf{1}_{\{I>A\}}\underbrace{\frac{U'}{\mathbb{E}[U]}}_{w(I)}\right] + c_1'(A)\right)$$

The Marginal Cost of Risk – Interpretation 2

We have for the marginal cost for risk $i \in \{1, 2, \ldots, N\}$:

$$\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\right] \exp\left\{\alpha - \beta \mathbb{P}(I \geq D) - \gamma \mathbb{E}[I]\right\} (1 - \gamma \mathbb{E}[I])$$

$$= \quad (1 - c_1')\,\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\, 1_{\{I \leq D\}}\, w(I)\right] + \mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}} \middle| I = D\right] \times \left\{c_1' + (1 - c_1')\,\mathbb{E}\left[\mathbf{1}_{\{I > D\}}\, w(I)\right]\right\},$$

where the weighting function w is defined as:

$$w(I) = \begin{cases} 1 + V'(S - I) & , I \leq S \\ 1 + {}^{\xi}/_{1-\xi} & , S < I \leq D \\ const & , I > D \end{cases}$$

In particular $\mathbb{E}[w(I)] = 1$.

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RAROC (RARAC?):

$$\frac{[\text{Marginal Revenue}] - (1 - c_1')\,\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\,1_{\{I \leq D\}}\,w(I)\right]}{\mathbb{E}\left[\frac{\partial I^{(i)}}{\partial q^{(i)}}\,\middle|\, I = D\right]} \geq c_1' + (1 - c_1')\,\mathbb{E}\left[\mathbf{1}_{\{I > D\}}\,w(I)\right]$$

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