Recursive Credibility:

Utilizing Credibility to Blend Reserve Assumptions

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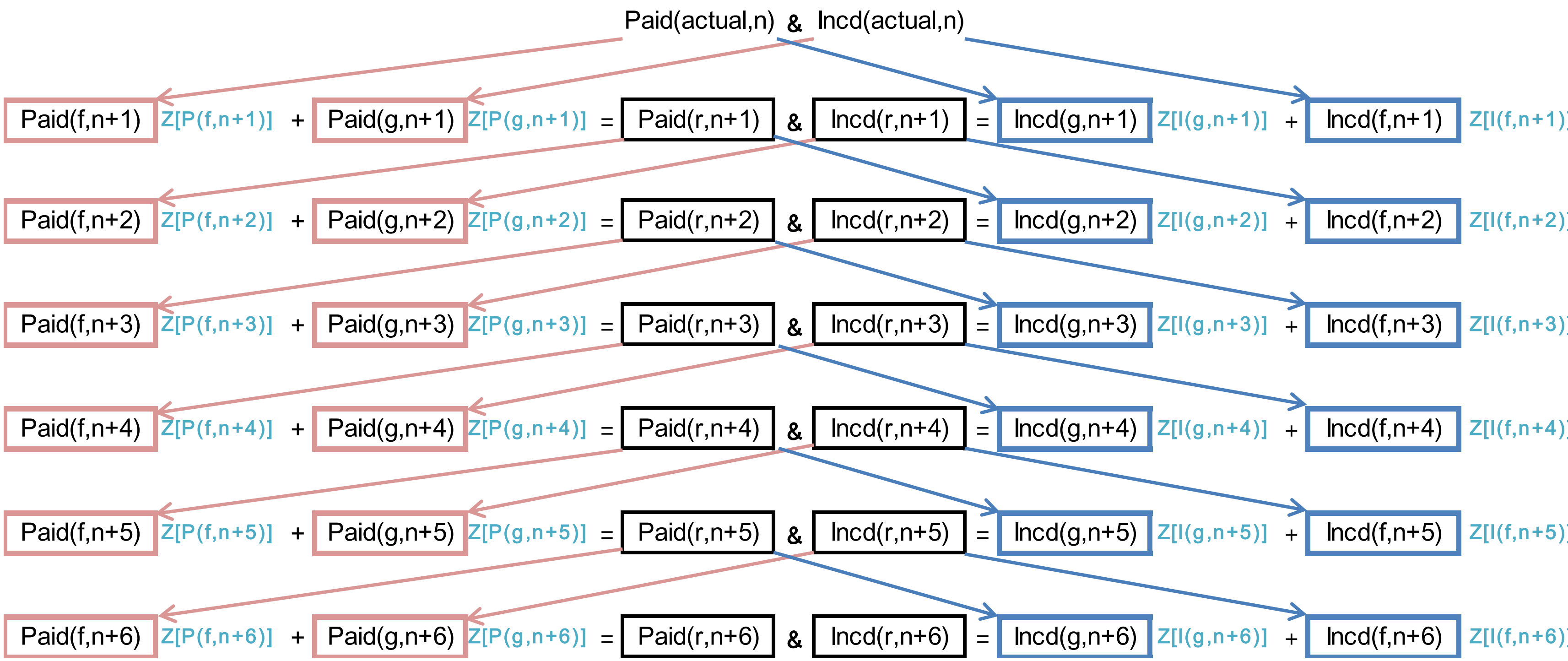
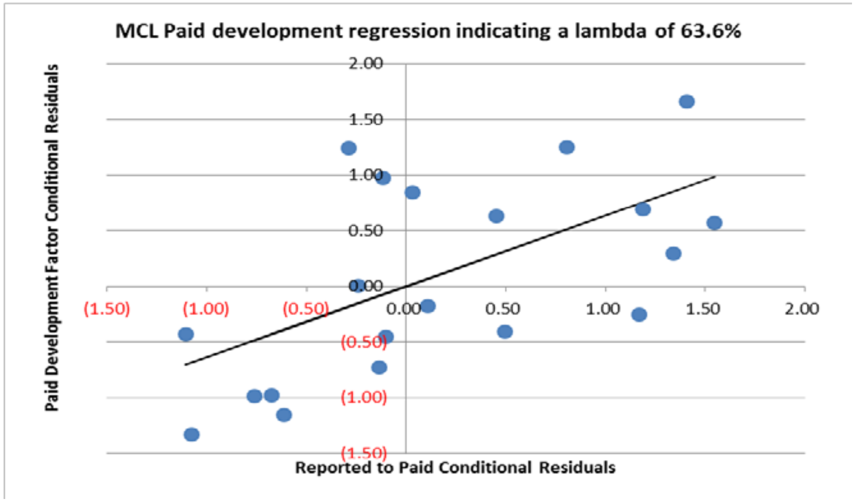
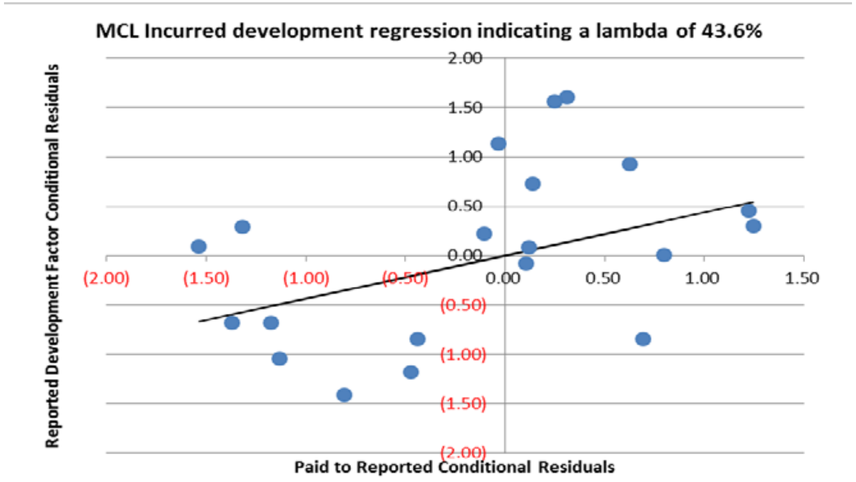
What is Recursive Credibility?



~ The application of credibility to weight indications at intermediate steps of a recursive process, rather than at the end of the process.

Background: Munich Chain Ladder





- MCL utilizes regression through the origin to determine expected development factor error.
- Error on development from time t to t+1 depends on information available at time t.
- Thus, based on this predictable error, development factors can be modified.

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Why Consider Recursive Credibility?

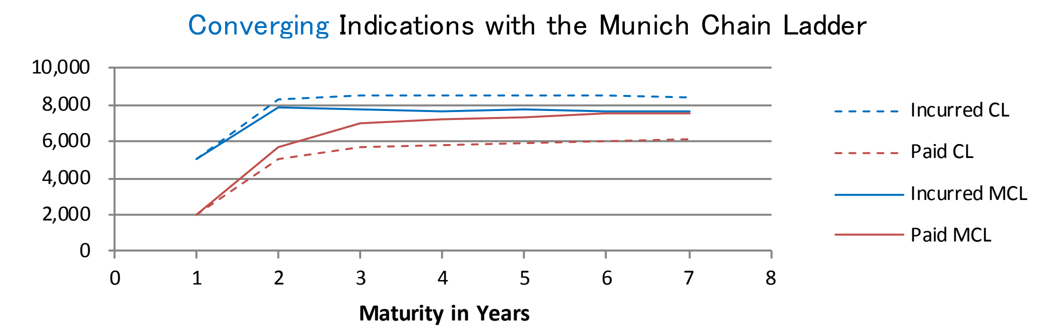
Ordinary LDF

I/P residual

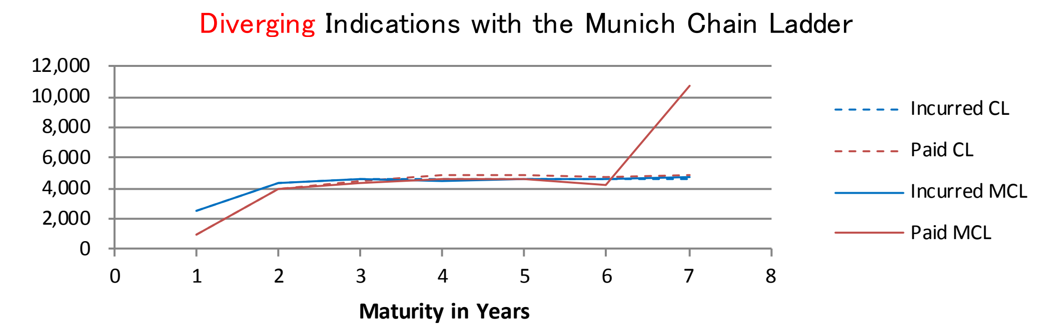
slope

Adjustment scaling term

$$MCL_{i,t}(Paid) = P_{i,s}\left(f^{P}_{s\to t} + \left(\frac{I_{i,s}}{P_{i,s}} - \frac{\sum_{i=1}^{n-s+1} I_{i,s}}{\sum_{i=1}^{n-s+1} P_{i,s}} \right) \lambda^{P} \frac{\sigma^{P}_{s\to t}}{\sigma^{Q^{-1}}_{s}} \right)$$



- MCL adjusts paid and incurred based on case reserves, resulting in converging paid and case incurred indications.



- However, MCL sometimes gives diverging paid and incurred indications,

Why Consider Recursive Credibility?

Simulation Results for Chain Ladder and Munich Chain Ladder

	Iterations	Average	Median	Standard Deviation	Maximum	Minimum
Paid CL	10,000	31,009	30,930	1,421	38,389	26,734
Incurred CL	10,000	33,055	33,050	841	36,285	29,649
I - P CL	10,000	2,046	2,140	1,101	5,199	(4,495)
Paid MCL	10,000	32,201	32,228	2,231	55,962	(99,621)
Incd MCL	10,000	33,104	33,076	1,573	133,405	14,789
I - P MCL	10,000	903	844	3,062	233,026	(16,352)

- Based on simulated "normally developing" triangles with the same final diagonal as the 7-year MCL triangle, the incurred minus paid results at year 7 are half of what they would be using the standard chain ladder.

- However, MCL also has some robustness problems, making its use with more volatile lines of business more problematic.

- Modifications may be necessary for use with certain triangles. But why?

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The Aha Moment!

$$MCL_{i,t}(Paid) = P_{i,s}\left(f^{P}_{s\to t} + \left(\frac{I_{i,s}}{P_{i,s}} - \frac{\sum_{i=1}^{n-s+1} I_{i,s}}{\sum_{i=1}^{n-s+1} P_{i,s}} \right) \lambda^{P} \frac{\sigma^{P}_{s\to t}}{\sigma^{Q^{-1}}_{s}} \right)$$

$$\approx P_{i,s}\left(f^{P}_{s\to t} + \left(\frac{I_{i,s}}{P_{i,s}} - \frac{\sum_{i=1}^{n-s} I_{i,s}}{\sum_{i=1}^{n-s} P_{i,s}} \right) \lambda^{P} \frac{\sigma^{P}_{s\to t}}{\sigma^{Q^{-1}}_{s}} \right)$$

$$= P_{i,s} f^{P}_{s\to t}\left(1 - \lambda^{P}\sigma^{P}_{s\to t} / (\sigma^{Q^{-1}}_{s} g^{P}_{s\to t})\right) + I_{i,s} g^{P}_{s\to t} \lambda^{P} \sigma^{P}_{s\to t} / (\sigma^{Q^{-1}}_{s} g^{P}_{s\to t})$$

$$= P_{i,s} f^{P}_{s\to t}\left(1 - W^{g^{P}}_{s\to t}\right) + I_{i,s} g^{P}_{s\to t} W^{g^{P}}_{s\to t} = WtdAvg(P^{f}_{i,s}, P^{g}_{i,s})$$

Chain Ladder Indication

Cross Link Indication

- The Munich Chain Ladder approximates the weighted average of two indications!!!

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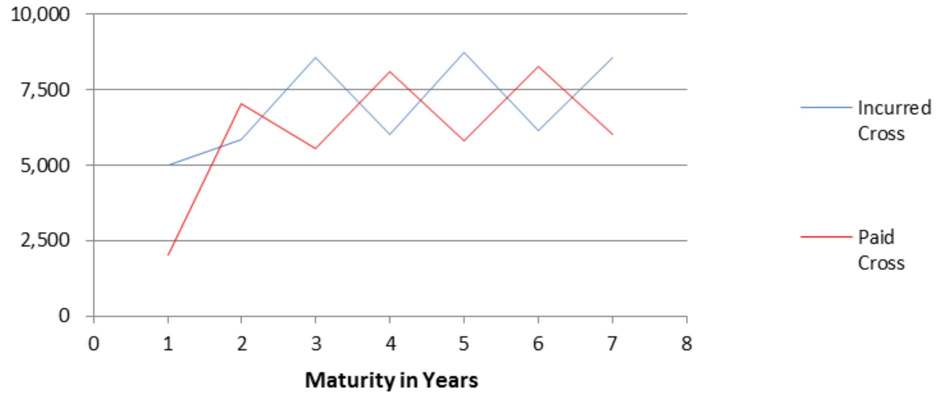
The Aha Moment and Robustness of the MCL

$$MCL_{i,t}(Paid) \approx P_{i,s}^{f}\left(1 - W_{s\to t}^{g^P}\right) + P_{i,s}^{g} W_{s\to t}^{g^P}$$

- MCL as a Weighted Average illustrates two sources of volatility beyond the Chain Ladder



- Cross Link is often a zig-zagging indication.

$$W_{s\to t}^{g^P} = \left(\frac{\lambda^P}{g_{s\to t}^P}\right)\left(\frac{\sigma_{s\to t}^P}{\sigma_s^{Q^{-1}}}\right)$$

- The definition of the weight illuminates the main source of parameter error.

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Credibility Framework Overview

- Find a credibility-weighted indication,

$$I_{i,t}^{R} = Z_{i,t}^{fI} I_{i,t}^{f} + Z_{i,t}^{gI} I_{i,t}^{g}$$

- With variance:

$$Var\left(Z_{i,t}^{fI} I_{i,t}^{f} + Z_{i,t}^{gI} I_{i,t}^{g}\right)$$

- **Minimizing variance** by setting the derivative equal to zero and **solving for *Z*.**

- Variance of each sub-indication is constructed in two parts
  - Variance given known prior losses (KPL) &
  - Variance given known development factors (KDF)

- Additionally, credibility weight parameter error is an element of Z.

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Recursive Credibility Sub-indication Variance Definitions

Recursive Credibility framework requires consistent sub-indication variance definitions.

We have based ours on Mack's Chain Ladder Variance Estimate.

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KPL Variance & KDF Variance

- Recursive deconstruction of Mack's Variance Estimate (Authored by Mack himself) shows his recursive variance of a single accident year to be the sum of two elements.

Paid sub-model f

Losses	2008	2009	2010	2011	2012	2013	2014				2015				2016
2008	576	1,804	1,970	2,024	2,074	2,102	2,131								
2009		866	1,948	2,162	2,232	2,284	2,348	x	1.01	=	2,380				
2010			1412	3,758	4,252	4,416	4,494	x	1.02	=	4,589	x	1.01	=	4,652
2011				2286	2,286	2,286	5,850	x	1.02	=	5,971	x	1.02	=	6,097
2012					1868	3,778	4,648	x	1.36	=	6,326	x	1.02	=	6,457
2013						1442	4,010	x	1.13	=	4,525	x	1.36	=	6,159
2014							2044	x	2.08	=	4,253	x	1.13	=	4,800

1. Variance from the development factors, (KPL)
2. Variance from the prior recursive estimate, (KDF)

Mack's recursive variance equation implies independence of development factors and prior recursive estimates.

$$Var\left(P_{i,t}^{f}\right)=\hat{P}_{i,s}^{R\,2}\left(\frac{\sigma_{s\to t}^{fP\;2}}{\hat{P}_{i,s}^{R}}+\frac{\sigma_{s\to t}^{fP\;2}}{\sum_{j=1}^{n-s}P_{j,s}}\right)+Var\left(P_{i,s}^{R}\right)f_{s\to t}^{P\;2}$$

KPL Variance & KDF Variance

- Chain Ladder Variance Estimates

$$Var\left(P_{i,t}^{f}\right)=\hat{P}_{i,s}^{R\,2}\left(\frac{\sigma_{s\to t}^{fP\;\;2}}{\hat{P}_{i,s}^{R}}+\frac{\sigma_{s\to t}^{fP\;\;2}}{\sum_{j=1}^{n-s}P_{j,s}}\right)+Var(P_{i,s}^{R})\,f_{s\to t}^{P\;\;2}$$

$$Var\left(I_{i,t}^{f}\right)=I_{i,s}^{R\,2}\left(\frac{\sigma_{s\to t}^{fI\;\;2}}{I_{i,s}^{R}}+\frac{\sigma_{s\to t}^{fI\;\;2}}{\sum_{j=1}^{n-s}I_{j,s}}\right)+Var(I_{i,s}^{R})\,f_{s\to t}^{I\;\;2}$$

1. KPL Variance
2. KDF Variance

- Cross Link Variance Estimates

$$Var\left(P_{i,t}^{g}\right)=P_{i,s}^{R\,2}\left(\frac{\sigma_{s\to t}^{gP\;\;2}}{P_{i,s}^{R}}+\frac{\sigma_{s\to t}^{gP\;\;2}}{\sum_{j=1}^{n-s}P_{j,s}}\right)+Var(I_{i,s}^{R})g_{s\to t}^{P}$$

$$Var\left(I_{i,t}^{g}\right)=I_{i,s}^{R\,2}\left(\frac{\sigma_{s\to t}^{gI\;\;2}}{I_{i,s}^{R}}+\frac{\sigma_{s\to t}^{gI\;\;2}}{\sum_{j=1}^{n-s}I_{j,s}}\right)+Var(P_{i,s}^{R})g_{s\to t}^{I\;\;2}$$

- The Cross Link variance estimate mirrors Mack's variance estimate.
- Case reserve development method may mirror the Mack estimate as well.
- Recursive Credibility's flexibility with other assumptions is not necessarily limited to this framework.

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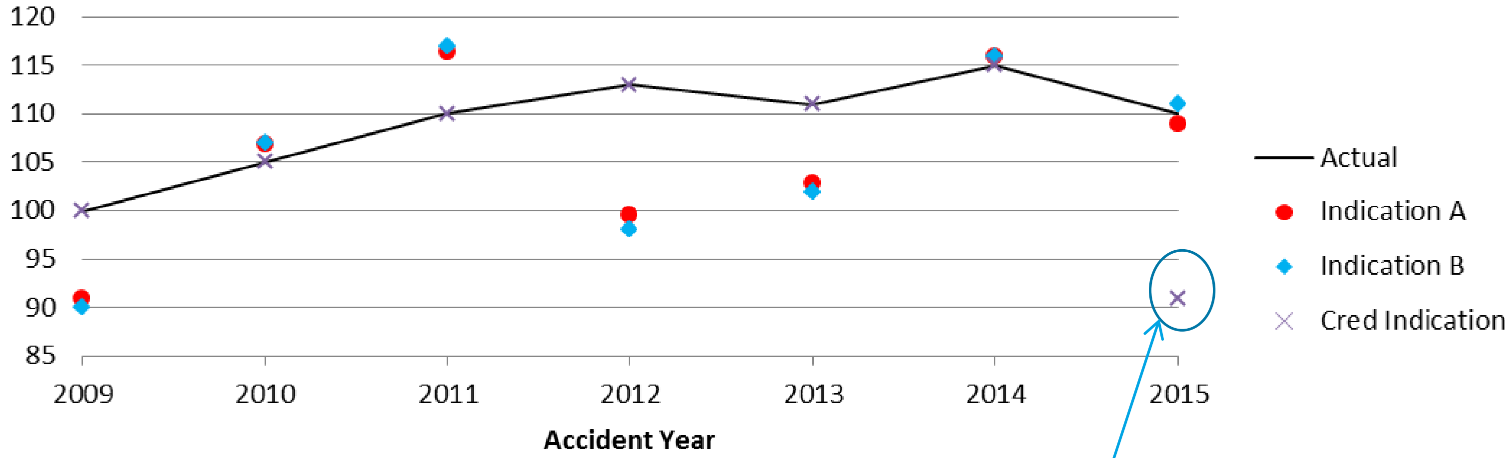
Credibility weight error: Why Consider?

Negative Credibility Weights
Can Cause Large Error!
&
That Error Varies Based
On Sub-indication Differences.

Credibility weight error: Potential

- One way that Least Squares Credibility differs from Classical or Bayesian credibility is that negative credibility weights are possible.

- Example: *Indication A* has historically deviated from *Actual* by 90% the deviation of *Indication B* with those deviations in the same direction. Then the credibility formula becomes *A* – *Actual* =0.9(*B* – *Actual*), and *Actual* = 10*A* - 9*B*.



- However, the parameter error contribution to total estimation error associated with credibility weights of 10 and -9 could be significant, and this error impacts the final indication, as well as the final variance estimate.

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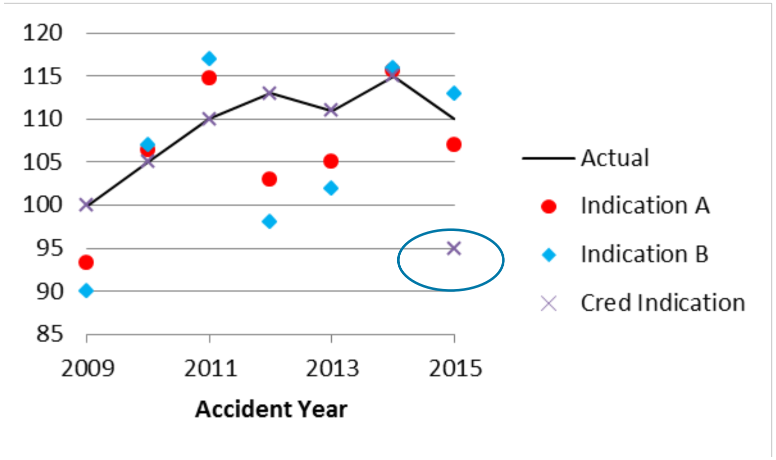
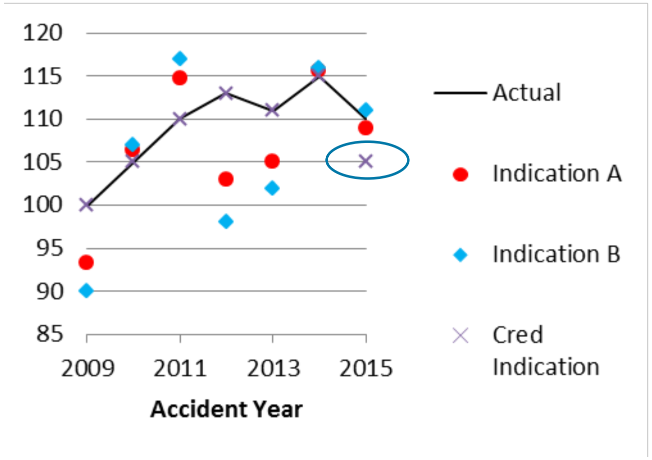
Credibility weight error: Relativity

- In these examples: *Indication A* has historically deviated from *Actual* by 2/3 the deviation of *Indication B* with those deviations in the same direction. Then *A* – *Actual* =2/3(*B* – *Actual*), and *Actual* = 3*A* - 2*B*.





- Compared to the right graph, the distance between the 2015 indications on the left is larger, and so is the error in the final estimate.

Credibility weight error: How Do We Measure It?

Recognize Heteroscedasticity and Scaling.

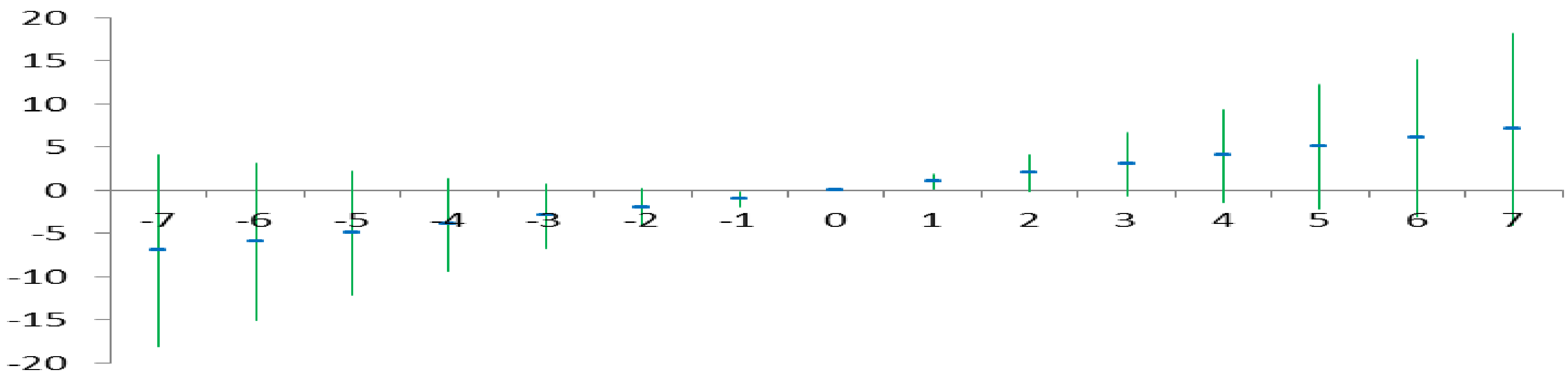
Zero-sum weights: What? & why?

- To simplify measurement of credibility weight error, credibility is redefined from "Z" weights summing to 1, to 0.5 + **zero-sum weights**, notated as "W":

$$I^R = Z_f I^f + Z_g I^g = \frac{I^f + I^g}{2} + W_f \left(I^f - I^g\right)$$

- The variance of the "Z" weights equals the variance of the "W" weights.

- Now we can specify the parameter error of zero-sum weights to **increase with their magnitude**.

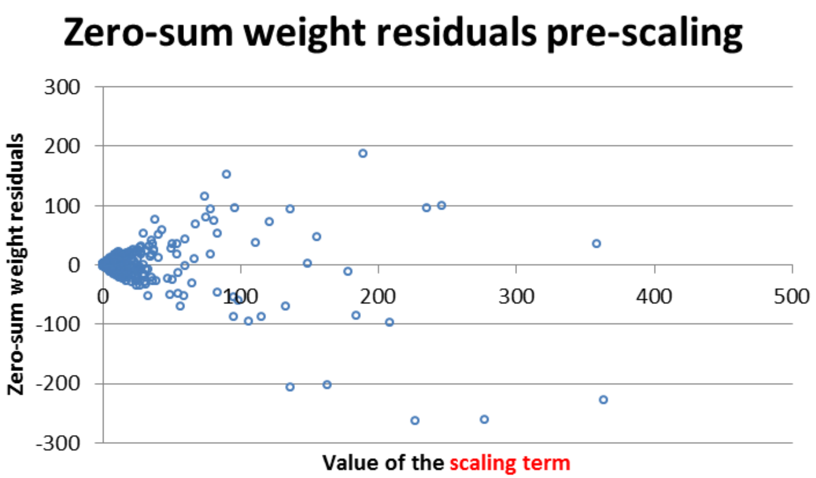


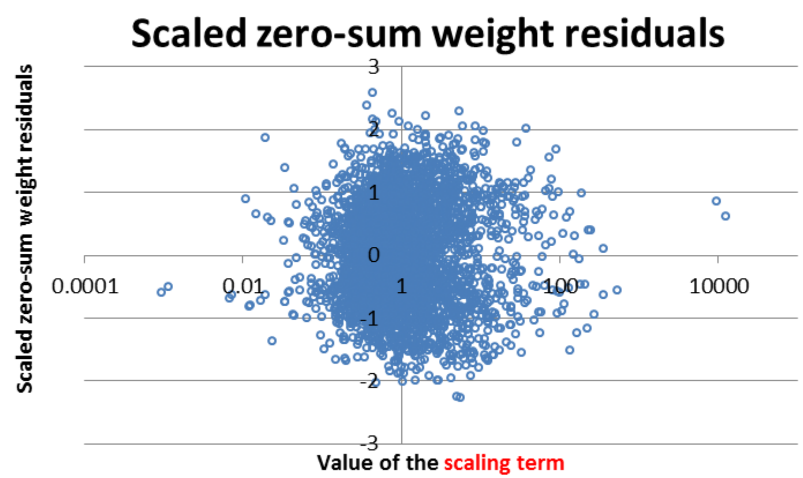
Zero-sum weight error scaling

- To “normalize” **zero-sum weight** residuals for each indication pair, we divide by the scaling term, where credibility is assumed to be known and constant:

$$\sqrt{\frac{Var(Z^{fI} I^f + Z^{gI} I^g)}{\left(I^f - I^g\right)^2}} = \sqrt{\frac{0.25 Var(I^f + I^g) + W^f \left(Var(I^f) - Var(I^g)\right) + {W^f}^2 Var(I^f - I^g)}{\left(I^f - I^g\right)^2}}$$

- The graphs below of credibility weight residuals based on private passenger auto industry data illustrate the effectiveness of this scaling factor.





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Credibility weight error: How Do We Measure It?

Using Scaled Residuals,
We Derive a Proportionality Constant
to Calculate
"Functional" Credibility Weight Error!

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What is “Functional” Error?

- As shown earlier (and validated empirically), error increases as <u>the difference between the sub-indications</u> decreases.

- The goal was an “unobtrusive” parameter error term, that grows with the weight in order to keep the credibility from becoming outrageously large, “shrinking” when the final weight is selected.

$$Var\left(W^{f}\right)=\sigma_{W}^{I\ 2}\frac{W^{f\ 2}Var(I^{f}-I^{g})}{\left(I^{f}-I^{g}\right)^{2}}$$

Variance(W)

$|W|$

Least Squares Credibility contemplating parameter error

versus

Least Squares Credibility <u>without</u> contemplating parameter error

$|W|$

Variance(W)

Sub-indication Variance

&

Functional Credibility Error

&

Voila!

Summary of Recursive Credibility Framework Details

- Each recursive credibility-weighted indication, can be restated using zero-sum weights.

$$I^R = Z_f I^f + Z_g I^g = \frac{I^f + I^g}{2} + W_f (I^f - I^g)$$

- Assuming no parameter error where $W = 0$, and with simplifying assumption that W is constant in covariance terms, the indication has variance,

$$Var\left(Z_{i,t}^{fI} I_{i,t}^f + Z_{i,t}^{gI} I_{i,t}^g\right) = 0.25 Var(I_{i,t}^f + I_{i,t}^g) - 0.5\left(Var(I_{i,t}^g) - Var(I_{i,t}^f)\right) W_{i,t}^{fI}$$

- …where the formula for the weight, W is

$$W_{i,t}^{fI} = \frac{0.5\left(Var(I_{i,t}^g) - Var(I_{i,t}^f)\right)(I_{i,t}^f - I_{i,t}^g)^2}{Var(I_{i,t}^f - I_{i,t}^g)\left((I_{i,t}^f - I_{i,t}^g)^2 + \sigma_W^{I\,2}(I_{i,t}^f - I_{i,t}^g)^2 + \sigma_W^{I\,2} Var(I_{i,t}^f - I_{i,t}^g)\right)}$$

- The variance of each sub-indication is constructed as KPL Variance + KDF Variance, and σ_W^I is the zero-sum parameter error constant.

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Wait a Minute! What About Covariance?

- Since we assumed that KPL variance is independent of KDF variance,.

$$Cov(P_{i,t}^{f},P_{i,t}^{g}) = Cov(P_{i,t}^{f},P_{i,t}^{g} \mid KPL) + Cov(P_{i,t}^{f},P_{i,t}^{g} \mid KDF)$$

- where

$$Cov(P_{i,t}^{f},P_{i,t}^{g} \mid KPL) = \sigma(P_{i,t}^{f} \mid KPL)\sigma(P_{i,t}^{g} \mid KPL)\rho\left(P^{f},P^{g} \mid KPL\right)$$

  - Conditional residuals of the type used for the MCL slope parameter are used to calculate KPL correlation.

$$\hat{\rho}(P^{f},P^{g} \mid KPL)) = \sum_{t=2}^{n-1}\sum_{i=1}^{n-t+1}\frac{Res\left(P_{i,t}^{f} \mid KPL\right)Res\left(P_{i,t}^{g} \mid KPL\right)}{(n-2)(n-1)/2}$$

- and where KDF covariance is defined,

$$Cov(P_{i,t}^{f},P_{i,t}^{g} \mid KDF) = f_{s\to t}^{P}g_{s\to t}^{P}\sigma(P_{i,s}^{R})\sigma(I_{i,s}^{R})\rho\left(P^{R},I^{R}\right)$$

  - Since paid and incurred RC indications move together, but not perfectly, we assume for simplicity:

$$\rho\left(P^{R},I^{R}\right) = 0.75$$

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Comparison Results:

Recursive Credibility Simulation Results:

Simulation Results of RC, MCL, CL and XL indications at age 7

	Iterations	Average	Median	Standard Deviation	Maximum	Minimum
Paid CL	10,000	31,009	30,930	1,421	38,389	26,734
Incurred CL	10,000	33,055	33,050	841	36,285	29,649
I - P CL	10,000	2,046	2,140	1,101	5,199	(4,495)
Paid MCL	10,000	32,201	32,228	2,231	55,962	(99,621)
Incd MCL	10,000	33,104	33,076	1,573	133,405	14,789
I - P MCL	10,000	903	844	3,062	233,026	(16,352)
Paid RC	10,000	31,907	31,908	1,082	37,085	27,794
Incd RC	10,000	32,599	32,597	1,063	36,692	28,473
I - P RC	10,000	692	671	153	1,670	(1,585)
Paid XL	10,000	30,802	30,766	1,003	34,978	27,557
Incurred XL	10,000	33,270	33,242	1,178	38,076	29,208
I - P XL	10,000	2,468	2,465	667	5,298	(443)

- RC has lower S.D. than MCL and smaller case reserve range.
- Paid and Incurred results are very comparable to CL and XL results, with more consistent case reserve estimates.

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Industry Results for RC, MCL, CL & XL

Total All Years Average Scaled Residuals at Age 7 (Measure of Bias)

	Grid Pairs	Paid RC(f, g)	Incurred RC(f, g)	Paid MCL	Incurred MCL	Paid Solo CL	Incurred Solo CL	Paid Solo XL	Incurred Solo XL
Private Passenger Auto	95	0.2	(0.1)	2.6	(4.2)	9.5	(4.1)	3.0	2.7
Commercial Auto	82	1.7	1.5	6.4	(3.9)	10.5	(0.8)	9.2	0.9
Workers Compensation	65	9.4	7.7	6.5	5.2	10.4	10.0	12.3	7.2
Other Liability	95	2.6	2.4	7.9	3.0	10.2	2.2	8.1	3.6

Total All Years Root Mean Squared Error at Age 7 (Measure of Uncertainty)

	Grid Pairs	Paid RC(f, g)	Incurred RC(f, g)	Paid MCL	Incurred MCL	Paid Solo CL	Incurred Solo CL	Paid Solo XL	Incurred Solo XL
Private Passenger Auto	95	13.1	17.5	15.5	46.2	23.7	27.0	20.0	15.6
Commercial Auto	82	21.6	20.3	37.4	50.3	40.5	22.0	48.2	18.9
Workers Compensation	65	24.0	26.9	22.2	28.4	46.8	26.7	31.8	24.7
Other Liability	95	20.6	23.3	31.5	24.2	34.4	27.4	26.8	20.9

**Based on CAS's "Loss Reserving Data Pulled from NAIC Schedule P" (Meyers 2011),*
***For each triangle, using accident Years 1990 to 1996, development lags 1 to 7.*
****Excluding grid pairs w/ any year losses <=0 or inadequate development pattern.*

- RC clearly minimizes both uncertainty and bias for Private Passenger Auto, Commercial Auto, and Other Liability.
- MCL reduces bias for Workers Comp (*unadjusted for benefit levels & trend*). RC works best with unbiased indications.

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Recursive Credibility Summary & Conclusions

- Recursive Application of Lease Squares Credibility
  - Using Mack Chain Ladder-type variance for each sub-indication
  - Transforming credibility to 0.5 + a zero-sum weight to contemplate variance

- Strengths
  - Different maturities / Different assumption weights
  - Volatile assumptions become useful

- Weakness
  - Invariant/unresponsive variance and correlation
  - Does not consider claims / exposures / premiums

- What else is in the paper?
  - Parameterization of all elements
  - Step by step numerical example

- Next Steps
  - Generalization to N sub-models

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¿¿¿Questions???

Appendices

Appendix 1: Parameter Steps

- We use the data in the upper triangular matrix to calculate parameters needed for the sub-indication models and for implementing recursive credibility.
  - Step P1: We calculate **sub-model parameters** and values associated with the upper triangular matrix.
    - Step P1a: Development factors
    - Step P1b: Indications
    - Step P1c: Proportionality constants
    - Step P1d: Variances
  - Step P2: We calculate **RC parameters** associated with the upper triangular matrix.
    - Step P2a: Correlations given known prior losses
    - Step P2b: Zero-sum constants

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Appendix 2: Development Steps

- Development Steps: We use the values calculated from the upper triangular matrix in Steps P1 and P2 to iteratively add diagonals to the development triangle, “squaring” the triangle.

  – Step D1: Calculate the incurred and paid sub-indications.

  – Step D2: Calculate total covariance of the underlying sub-indications.

  – Step D3: Calculate the zero-sum weights.

  – Step D4: Using the zero-sum weights, calculate the RC indication.

  – Step D5: Using zero-sum weights calculate variance of the RC indication.

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