A Cost of Capital Risk Margin Formula for Non-Life Insurance Liabilities

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Recent History

- Set up CAS Loss Reserve Database in 2011
  - Both upper and lower Schedule P triangles for hundreds of insurers.
  - Purpose was to enable "Aggressive Retrospective Testing" of stochastic loss reserve models.

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  - Both upper and lower Schedule P triangles for hundreds of insurers.
  - Purpose was to enable "Aggressive Retrospective Testing" of stochastic loss reserve models.
- Published the monograph "Stochastic Loss Reserving Using Bayesian MCMC Models" in 2015
  - Used retrospective testing to identify shortcomings in two currently popular stochastic loss reserve models.
  - Proposed new models to address these shortcomings.

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- Actuary presents a range to management.
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- The "best estimate" is defined as the probability weighted average of the discounted future loss payouts.
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- The "best estimate" is defined as the probability weighted average of the discounted future loss payouts.
- The loss reserve liability (a.k.a. "technical provision") is equal to the best estimate plus a risk margin.
  - Wider range increases the risk margin.
  - Longer payout increases the risk margin.
- A cost of capital risk margin has these properties.

The Risk Margin Environment When I Began Thinking of the Topic of This Paper.

- When I first joined ISO in 1988, they were using a risk load formula based on first the Buhlmann Variance Principle, and then the Buhlmann Standard Deviation Principle. Both approaches were deemed unacceptable (for good reason) by the insurer committees that met at ISO.
- My first assignment at ISO was to fix the risk load formula. My solution was a constrained optimization approach inspired by the CAPM. published in the 1991 CAS Proceedings.
- Interacted with Rodney Kreps who wrote the paper "Reinsurance Risk Loads from Marginal Capital Requirements" published in the 1990 CAS Proceedings.

Continuing

- When I tried my approach on catastrophes (1996), I saw the necessity to recognize how long capital needed to be held.
- In the late 1990s and early 2000s DFA and later ERM were hot topics in the CAS. My contribution was "The Cost of Financing Insurance" in the CAS e-Forum in 2001.
- Joined the IAA Insurance Solvency Assessment Working Party in 2002
  - Produced the IAA Blue Book.
  - I became familiar with the Swiss Solvency Test and the evolving Solvency II Risk Margin.
- Continued on the IAA Solvency Subcommittee and Insurance Regulation Committee until I then joined the ASTIN Committee as the IAA representative.

This Paper

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- Its roots are in my "The Cost of Financing Insurance" approach of 2001.

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- Provides an alternative solution to the same risk margin problem addressed by the Swiss Solvency Test (SST) and Solvency II (SII).
- Its roots are in my "The Cost of Financing Insurance" approach of 2001.
- A solution to the risk margin problem has been a goal for the greater part of my actuarial career. But first:
    - I needed a good stochastic loss reserve model. Meyers 2015.
    - I needed to address the dependencies issue. Accepted by *Variance* and presented here last Monday.

The Insurer Capital Cash Flow

- At the end of the current calendar year, $t = 0$:
  - The insurer posts the amount of capital, C_0, needed to contain the uncertainty in its loss estimate. It invests C_0 in a fund that earns income at the risk-free interest rate i.

The Insurer Capital Cash Flow

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  - The insurer posts the amount of capital, C_0, needed to contain the uncertainty in its loss estimate. It invests C_0 in a fund that earns income at the risk-free interest rate i.
- At $t = 1$, the insurer uses its next year of loss experience to reevaluate its liability.
  - The insurer posts the updated estimate of its capital, C_1, needed to contain the uncertainty its revised loss estimate.
  - C_0 is invested at the risk-free interest rate i.
  - The difference, $C_0 \cdot (1 + i) - C_1$, is returned to the investor. If that difference is negative, the investor is expected to contribute an amount to make up that difference.

The Insurer Capital Cash Flow - Continued

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The Insurer Capital Cash Flow - Continued

- The process continues for future calendar years, t, with the amount, $C_{t-1} \cdot (1+i) - C_t$, being returned to (or being contributed by) the investor.
- At some time $t = u$, the loss is deemed to at ultimate, i.e. no significant changes in the loss is anticipated. Then we set $C_t = 0$ for $t > u$. For the examples in this presentation, $u = 9$.

The Cost of Capital Risk Margin

- The present value of the capital returned, discounted at risky rate, r is equal to:

$$\sum_{t=1}^{u} \frac{C_{t-1} \cdot (1+i) - C_t}{(1+r)^t}$$

- The cost of capital risk margin is defined as the difference between the initial investment, C_0, and the present value of the capital returned.

$$R_{COC} \equiv C_0 - \sum_{t=1}^{u} \frac{C_{t-1} \cdot (1+i) - C_t}{(1+r)^t} = (r-i) \cdot \sum_{t=0}^{u} \frac{C_t}{(1+r)^t}$$

Compare R_{COC} with Solvency II Risk Margin

$$R_{COC} = (r - i) \cdot \sum_{t=0}^{u} \frac{C_t}{(1+r)^t}$$

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$$R_{COC} \equiv C_0 - \sum_{t=1}^{u} \frac{C_{t-1} \cdot (1+i) - C_t}{(1+r)^t}$$

- If the present value of the capital returned is very small, the risk margin should be C_0.

Bayesian MCMC with Risk Margins

- Bayesian MCMC provides a list of scenarios that represent the future.
  - Use a Changing Settlement Rate (CSR) Model.
- The set of parameters of possible lognormal distributions of loss, $X_{w,d}^{j}$, for each w and d is represented by

$$\{\mu_{w,d}^{j}, \sigma_{d}^{j}\}_{j=1}^{10,000}$$

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- Use the parameters to calculate statistics of interest.
- In particular - those statistics that are needed for risk margins.

The Best Estimate

- The "best estimate" is defined as the probability weighted average of the discounted future loss payouts.
- Recall - the mean of a lognormal distribution $= e^{\mu+\sigma^2/2}$.
- The mathematical expression for best estimate:

$$E_{Best} = \sum_{j=1}^{10{,}000} \sum_{w=2}^{10} \sum_{12-w}^{10} \frac{e^{\mu_{w,d}^{j}+(\sigma_{d}^{j})^2/2} - e^{\mu_{w,d-1}^{j}+(\sigma_{d-1}^{j})^2/2}}{10,000 \cdot (1+i)^{w+d-11.5}}$$

- Sum is over the expected payout in the lower triangle.
- Assumes that loss is paid out one half year before the end of calendar year $t = w + d - 11$.

Ultimate Losses

- Capital requirements are based on the variability of the *estimates of the ultimate loss*.
- If we know the scenario, j, a pretty good estimate of the ultimate loss is given by:

$$U_j \equiv \sum_{w=1}^{10} e^{\mu^j_{w,10} + (\sigma^j_{10})^2/2}$$

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- By "pretty good" I mean that after discounting, uncertainty in further loss development should have a small effect on the risk margin.

Updating the Scenario Probabilities

- We don't know the scenario, j, but we can update the scenario probabilities as more losses come in at the end of each calendar year.
- Initially, all scenarios are equally likely (=1/10,000).
- For future calendar year $t = 1, \ldots, 9$, define the loss trapezoid, T_t as the set of the top t diagonals in the lower triangle.
- Then using Bayes' theorem:

$$Pr[J = j | T_t] = \frac{\prod_{X_{wd} \in T_t} \phi(\log(X_{wd}) | \mu_{wd}^j, \sigma_d^j)}{\sum_{k=1}^{10{,}000} \prod_{X_{wd} \in T_t} \phi(\log(X_{wd}) | \mu_{wd}^k, \sigma_d^k)}$$

Calculating Statistics of Interest for Risk Margins

1. Calculate $P_t \equiv \left\{ Pr[J = j | T_t] \right\}_{j=1}^{10,000}$
2. Take a sample, S_t, of size 10,000 from $\left\{ U_j \right\}_{j=1}^{10,000}$ with sampling probabilities P_t. Easy to do in R.

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- $E_t = \text{mean}(S_t) =$ Expected ultimate loss at the end of future calendar year T_t.
- $A_t = TVaR@97\% = \text{mean}\left(sort(S_t)[9701 : 10000]\right) =$ Required assets at the end of future calendar year T_t.
- $C_t = A_t - E_t =$ Capital required at the end of future calendar year T_t.
- $R_t = C_t - C_{t-1} =$ Capital released at the end of future calendar year t.

Steps for Calculating the Risk Margin

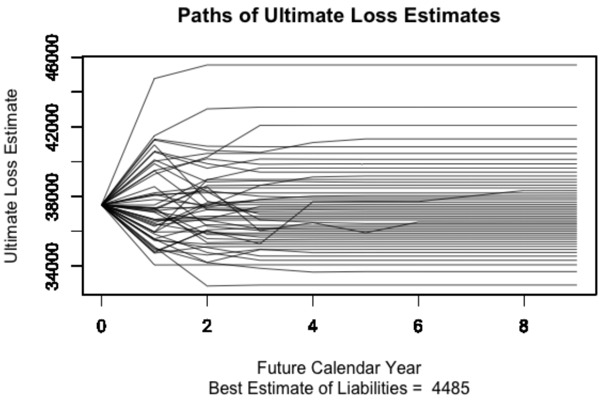
Do the following 10,000 times.

1. Select a parameter scenario at random. With that parameter scenario, simulate the loss trapezod, T_t for $t = 1, \ldots, 9$
2. Calculate each P_t and then the required capital, C_t, for future calendar years $t = 0, \ldots, 9$.
3. Calculate the risk margin using the following formula:

$$R_{COC} \equiv C_0 - \sum_{t=1}^{9} \frac{C_{t-1} \cdot (1+i) - C_t}{(1+r)^t}$$

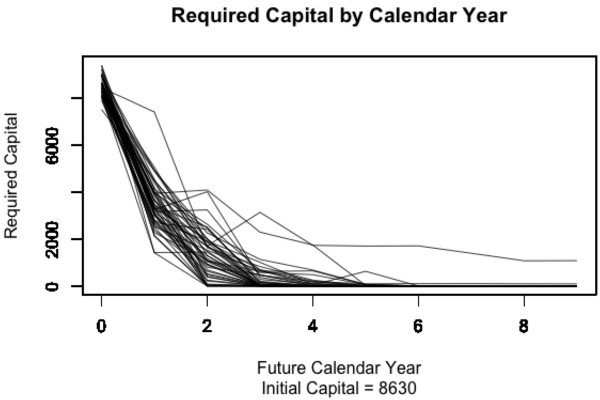
- Posted Risk Margin is equal the the average of the 10,000 risk margins calculated above.

Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$



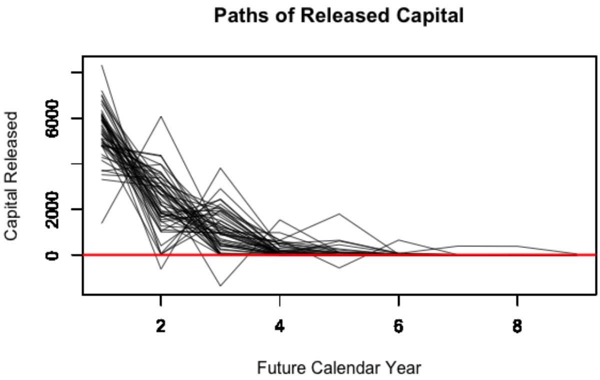
Note the leveling off of the paths.

Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$



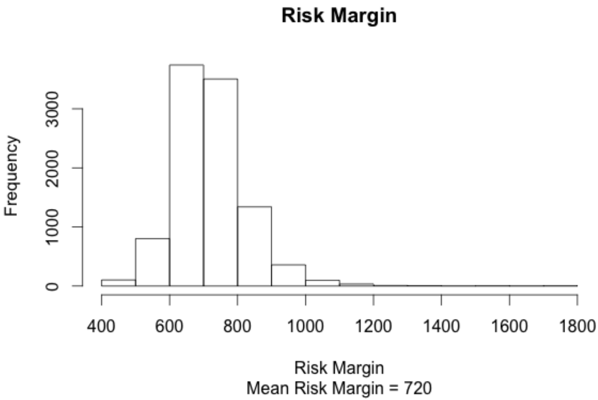
Note that capital requirement decrease to very near zero.

Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$

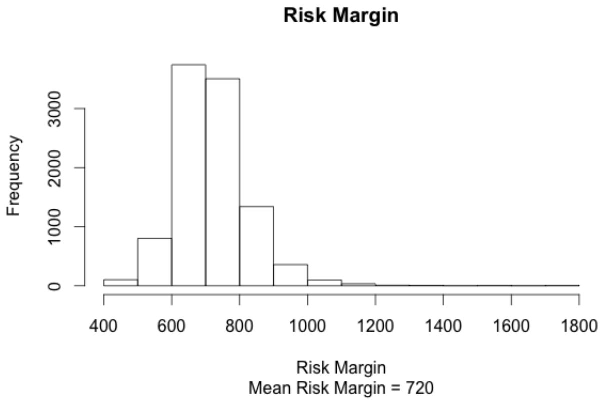


Occasionally the insurer must add capital.

Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$



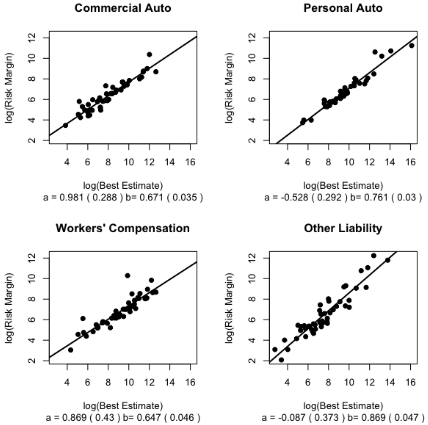
Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$



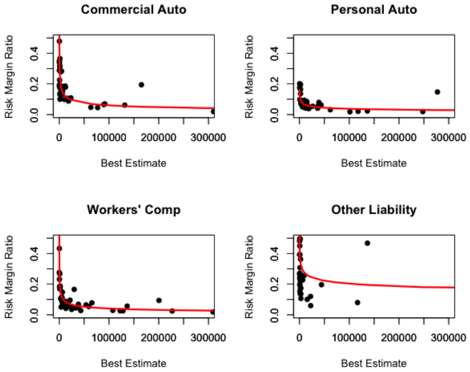
Should the variability of the risk margin influence r?

The Effect of Risk Size on the Risk Margin

Plot $\log(R_{COC})$ against $\log(E_{Best})$ for 200 Insurers



$$\frac{R_{COC}}{E_{Best}} = e^a \cdot E_{Best}^{b-1}$$



Many of the small lines are part of a large insurer group.

Correlations Between Lines of Business

- The problem with multiline models is quantifying correlations, or more generally dependencies.
- This paper assumes lines of business are *independent!*
- Avanzi, Taylor and Wong (ASTIN - 2016) conclude that:
  1. Illusory correlations may result from "rough modeling."
  2. "Such empirical evidence examined in the paper reveals cross-LOB that vary only in the range from zero to very modest."
- Meyers *Variance* - 2017 (to appear) shows that the CSR model fit without correlations fits better than a similar model that allows for correlations (after allowing for the additional parameter in the bivariate model.)

Diversification Credit

- For line of business l, let ${}_{l}S_t$ be the sample of $\{{}_{l}U\}_{j=1}^{10{,}000}$ taken with sampling probabilities ${}_{l}P_t$.
- Define ${}_{T}S_t \equiv_1 S_t + \ldots +_L S_t$
- Calculate total risk margin, ${}_{T}R_{COC}$ as done above.
- Define the Diversification Credit as:

$$1 - \frac{\text{Total Risk Margin}}{\text{Sum of Standalone Risk Margins}}$$

- Five insurers in my CAS Loss Reserve Database had triangles in all four lines. The diversification credits were:

  47.5%, 30.3%, 36.7%, 48.3% and 44.1%

Allocate the Total Risk Margin to Line of Business

1. Calculate ${}_T R_{COC}$.
2. For each line l calculate ${}_{(-l)}R_{COC}$ to be the total risk margin with the line l left out.
3. Define the marginal capital for line l as:

$$ {}_{(l)}R_{COC} \equiv {}_T R_{COC} - {}_{(-l)} R_{COC} $$

4. Then allocate the capital, ${}_T R_{COC}$, to line l in proportion to the marginal capital for each line.

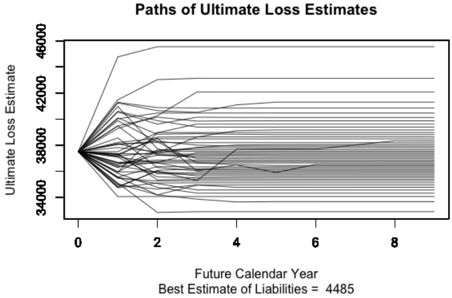
$$ {}_{(l)}R_{ACOC} \equiv {}_{(l)}R_{COC} \cdot \frac{{}_T R_{COC}}{{}_{(1)}R_{COC} + \cdots + {}_{(L)} R_{COC}} $$

Two Numerical Examples Illustrating Allocation

Grp./Line	Estimated Ult. Loss	Best Esitmate	Marginal Risk Margin	Allocated Risk Margin	Standalone Risk Margin	Diver. Credit
1528/CA	88,756	13,822	464	852	1,447	41.1%
PA	311,659	36,507	542	996	1,519	34.4%
WC	129,762	13,207	61	111	550	79.8%
OL	19,143	4,697	243	447	1,065	58.0%
Total	549,320	68,233	1,310	2,406	4,581	47.5%
1767/CA	2,205,897	310,203	108	171	5,963	97.1%
PA	90,312,996	9,921,107	20,620	32,566	76,527	57.4%
WC	1,677,179	227,010	175	276	5,637	95.1%
OL	2,443,660	956,344	76,495	120,812	132,428	8.8%
Total	96,639,732	11,414,664	97,398	153,825	220,555	30.3%

- Note the very large diversification effect for small lines.

From an Ultimate to a One-Year Time Horizon

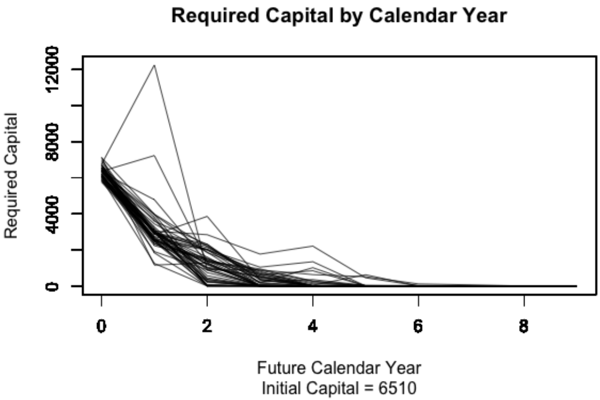


- A difference in the spread as t goes from 1 to 9.
- C_t calculated assuming the "ultimate" time horizon.
- Many regulators want to use a one-year time horizon.

Adopting the MCMC Strategy to a One Year Horizon

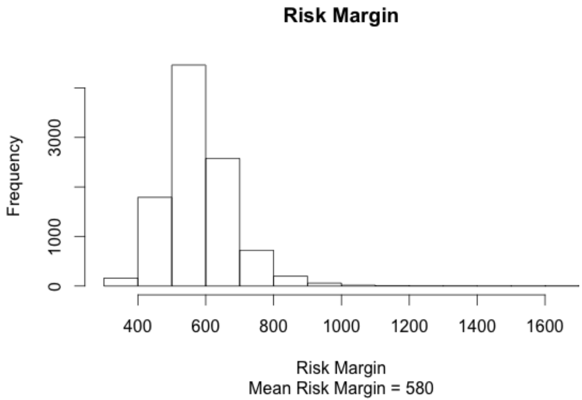
- Assigning the ultimate, U_j, to a scenario, j, is a good approximation for the "ultimate" time horizon.
- Not a good approximation for a one-year time horizon.
- We need to assign the expected value of the estimate given the scenario.
- Do the following 12 times for each scenario, j.
  1. Simulate the lower triangle with the scenario parameters.
  2. Calculate E_ts as described above.
- Set O_j equal to the average of the E_ts.
- Replace each U_j with O_j and proceed as above.

Our Example with a One-Year Time Horizon



Initial capital for an "ultimate" time horizon = 8,630.

Our Example with a One-Year Time Horizon



Risk margin for an "ultimate" time horizon = 720.

Which Time Horizon?

- Significantly longer run time for one-year - 2.5 minutes vs 20 minutes.
- Why not lower TVaR level on ultimate - say 97%?
- Solvency II Experience
  - Capital requirement based on one-year time horizon.

Which Time Horizon?

- Significantly longer run time for one-year - 2.5 minutes vs 20 minutes.
- Why not lower TVaR level on ultimate - say 97%?
- Solvency II Experience
  - Capital requirement based on one-year time horizon.
  - Need to validate internal model.
  - It takes a long time to validate estimates of ultimate.
  - But one can validate the posted outcomes after one year.

Concluding Remarks

- Bayesian MCMC models provide unprecedented flexibility in developing stochastic loss reserve models.
- The CSR model used in this paper validates well on lower triangle holdout data.
- Used this Bayesian MCMC model to provide a sample of parameters that represents the future loss development.
  - Can model future statistics of interest.
  - Did it for the statistics relevant to cost of capital risk margins.
- This paper provides numerical examples illustrating properties of risk margins - e.g. effect of risk size and diversification.
- R/Stan scripts implementing these examples are available on request.

Alternative Risk Margin Approaches

- The script can be easily modified to use R_{SII} instead of R_{COC}.
- The script can be easily modified to use the Value at Risk (VaR) instead of the the Tail Value at Risk (TVaR) for calculating the required capital.
- If dependencies are an issue, the paper shows how to account for dependencies given a copula.
- The paper shows how to calculate risk margins using a one-year time horizon for calculating the required capital.

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Done!

Link to the “accepted” version of the paper