

**Capital Allocation  
Concepts and  
Controversies**

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### Agenda

- Capital Basics**
  - Sources of Risk -Dual role of Capital
  - Total vs Allocation - Different Perspectives
- Measuring Adverse Outcomes**
  - Absolute vs Relative – One Year vs ULT
- Formulas**
  - Total – VaR, Capital Deficit Ratio
  - Allocation – Increm, co-TVaR , Euler, Bodoff, Myers-Read
- Polling on Controversies**

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### Sources of Risk

 Underwriting Premium Risk	 Cat Risk	 Reserve Risk
 Asset Investment Risk	 Operational Risk	 Business Risk

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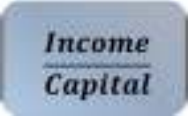
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### Dual Role of Risk Capital

 Safety Net	 ROE Denominator
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### Required Risk Capital vs Allocated Capital

<p><b>Required Risk Capital</b></p> <ul style="list-style-type: none"> <li>• Protects against adverse outcomes</li> <li>• Focused on solvency and capital management</li> </ul>	<p><b>Allocated Capital</b></p> <ul style="list-style-type: none"> <li>• Attempts to quantify risk drivers</li> <li>• Focused on profitability analysis and risk management</li> </ul>
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### Different Perspectives

Regulators

Rating Agencies

Customers

Investors

Managers

**Differing GOALS** → **Differing Capital Concerns**

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### Perspectives and Goals along a Line

<p><b>Regulators</b></p> <ul style="list-style-type: none"> <li>• Minimum Standard</li> </ul>	<p><b>Rating Agencies</b></p> <ul style="list-style-type: none"> <li>• Grades for Range of Standard</li> </ul>	<p><b>Customers</b></p> <ul style="list-style-type: none"> <li>• Security</li> <li>• Long-term Viability</li> </ul>	<p><b>Investors</b></p> <ul style="list-style-type: none"> <li>• Minimize downside</li> <li>• Reduce need to diversify</li> </ul>	<p><b>Managers</b></p> <ul style="list-style-type: none"> <li>• Allocation to measure risk adjusted return</li> </ul>
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← Total Capital      CAPITAL CONCERN      Precise Allocation →

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Absolute vs Relative  
 One-year vs Ultimate

### MEASURING ADVERSE OUTCOMES

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### Absolute vs Relative

**A RELATIVE** adverse result occurs when results are below expectations

An **ABSOLUTE** adverse result occurs when a loss occurs and/or capital falls below a specific level

	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Capital Ratio	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Absolute Adverse Result	0.00	0.25	0.50	0.75	1.00	1.25	1.50
Relative Adverse Result	0.00	0.25	0.50	0.75	1.00	1.25	1.50

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### One-year vs Ultimate

One year

*Adverse result recognized over the next calendar year*

Ultimate

*Adverse result recognized over the full horizon*

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VaR  
EDC Ratio  
Diversification and Correlation

## TOTAL RISK CAPITAL



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### Total Risk Capital Algorithms

**Risk Measures:  $C = \rho(T)$ . Examples:**

<p><b>VaR</b></p> <ul style="list-style-type: none"><li>Enough capital to cover rare adverse outcomes (e.g. 1/100 years)</li></ul> 	<p><b>Capital Deficit Ratio</b></p> <ul style="list-style-type: none"><li>Enough to keep average shortfall at a pre-selected small percent of capital.</li></ul> 
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Others: Standard Deviation, TVaR

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### Required Capital: VaR

- C = Required Capital
- $A_k$  = Adverse Outcome from  $k^{\text{th}}$  risk source
- $T = \sum A_k$
- N = Return period

$$C = VaR_{1-1/N}(T)$$

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**Robbin Capital Deficit Ratio Method**

- $T - C = \text{Total Loss} - \text{Capital} = \text{Deficit when } T > C$
- ECD ratio: Expected Deficit as % of Capital
- Adjust Capital to Get Below Target

$$ECD = E[D]$$

$$= \text{Prob}(T > C) \times E[T - C | T > C]$$

$$ECDR = \frac{ECD}{C}$$

How frequently capital is needed

Conditional Severity of capital used

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VaR  
EDC Ratio  
Diversification and Correlation

## RISK CAPITAL EXAMPLE

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Capital Example Trials: Two LOBs

Trial	A	B	T= A+B
1	(10)	0	(10)
2	(40)	(20)	(60)
3	35	15	50
4	60	30	90
5	(30)	10	(20)
6	(20)	40	20
7	10	(45)	(35)
8	(25)	(25)	(50)
9	0	(40)	(40)
10	20	(15)	5
Mean	0	(5)	(5)
Stnd Dev	30	27	45
Correl	100%	23%	
	23%	100%	

A and B amounts shown are the adverse results - negatives are "good guys" in this table

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### Capital Example: VaR

Capital based on VaR(80%)					
	Standalone	Standalone	Sum of	Capital	
	Pct	A	B	Standalone	for A+B
	80%	35	30	65	50

Rank	VaR Pct	A	B	Sum of Standalone	T=A+B
1	90%	60	40	100	90
2	80%	35	30	65	50
3	70%	20	15	35	20
4	60%	10	10	20	5
5	50%	0	0	0	(10)

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### Robbin Capital Deficit Ratio Example

Required Capital - ECD	
Capital: C	60.0
Prob(T>C)	10.0%
E[T-C T>C]	30.0
ECD	3.0
ECD Ratio	5.0%
ECD Target	5.0%

Solved for the capital needed to hit the target deficit ratio

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### Diversification and Correlation Example

Risk source	Standalone A	Standalone B	Sum of Standalone	Independent A+B	Independent A+B
Required Capital	35	30	65	50	40

Sum of Standalone Capital	65
Required Capital of Independent Sum	40
Diversification Benefit if Independent	(25)

Sum of Standalone Capital	65
Required Capital of Sum Reflecting Correlation	50
Net Diversification Benefit	(15)

Correlation Penalty	(10)
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Incremental  
Co-TVaR  
Euler  
Percentile(Bodoff)

**CAPITAL ALLOCATION**

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**Incremental Allocation**

- Incremental (Last-in) Allocation: Allocation is the increment in total capital caused by adding the risk tower  
*How much does total risk capital increase when a risk source is added*

$$C_{Increm}(A_k|A) = C(A) - C(A - A_k)$$

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**Co-Measure Allocation**

- Co-measure Allocation: Allocation is the contribution made to total required capital  
*How much does a risk source contribute to events driving capital?*

$$C_{CoMeasure}(A_k|A) = \sum \frac{a_k}{a} r(a) f(a)$$

where  $C(A) = \sum r(a) f(a)$

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### Incremental Allocation Example

Incremental Capital				
	A	B	Sum	T
Standalone capital	35	30	65	50
Incremental capital	20	15	35	
Incremental %s	57%	43%	100%	
Allocation of T with %	29	21	50	

Calculation of Incremental Capital for A	
Capital Required for Portfolio without A	30
Capital Required for Portfolio after A is added	50
Incremental Capital for A	20

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### Euler Allocation

- Euler Allocation: Allocation is the derivative with respect to total required capital

What is the rate at which capital increases with respect to an increase in a risk source ?

$$C_{Euler}(A_k|A) = \frac{dC(A + hA_k)}{dh}$$

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### Euler Calc Example- Adding h and ranking

Euler Calc- adding h=0.10				Euler Calc - Ranking			
Trial	T=A+B	T + h*A	T + h*B	Rank	T	T+ h*A	T + h*B
1	(10.00)	(11.00)	(10.00)	1	90.00	96.00	93.00
2	(60.00)	(64.00)	(62.00)	2	50.00	53.50	51.50
3	50.00	53.50	51.50	3	20.00	18.00	24.00
4	90.00	96.00	93.00	4	5.00	7.00	3.50
5	(20.00)	(23.00)	(19.00)	5	(10.00)	(11.00)	(10.00)
6	20.00	18.00	24.00	6	(20.00)	(23.00)	(19.00)
7	(35.00)	(34.00)	(39.50)	7	(35.00)	(34.00)	(39.50)
8	(50.00)	(52.50)	(52.50)	8	(40.00)	(40.00)	(44.00)
9	(40.00)	(40.00)	(44.00)	9	(50.00)	(52.50)	(52.50)
10	5.00	7.00	3.50	10	(60.00)	(64.00)	(62.00)

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### Euler Calc Example- Euler VaR allocation

**Euler Calc - Ranking**

Rank	T	T+ h*A	T + h*B
1	90.00	96.00	93.00
2	50.00	53.50	51.50
3	20.00	18.00	24.00
4	5.00	7.00	3.50
5	(10.00)	(11.00)	(10.00)
6	(20.00)	(23.00)	(19.00)
7	(35.00)	(34.00)	(39.50)
8	(40.00)	(40.00)	(44.00)
9	(50.00)	(52.50)	(52.50)
10	(60.00)	(64.00)	(62.00)

**Euler Allocation using VaR**

Var Pct	Δ for A	Δ for B	Sum
90%	60	30	90
80%	35	15	50
70%	(20)	40	20
60%	20	(15)	5
50%	(10)	0	(10)
40%	(30)	10	(20)
30%	10	(45)	(35)
20%	0	(40)	(40)
10%	(25)	(25)	(50)
0%	(40)	(20)	(60)

Euler: A ->  $(53.50 - 50)/0.1 = 35$

Euler adds up!

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### Euler Calc Example -Derivative of TVaR

**Euler Calculation - TVaR**

Rank	T=A+B	T + h*A	T + h*B
1	90.00	96.00	93.00
2	70.00	74.75	72.25
3	53.33	55.83	56.17
4	41.25	43.63	43.00
5	31.00	32.70	32.40
6	22.50	23.42	23.83
7	14.29	15.21	14.79
8	7.50	8.31	7.44
9	1.11	1.56	0.78
10	(5.00)	(5.00)	(5.50)

**Euler Δ of TVaR**

TVaR Pct	Δ for A	Δ for B	Sum
90%	60	30	90
80%	48	23	70
70%	25	28	53
60%	24	18	41
50%	17	14	31
40%	9	13	23
30%	9	5	14
20%	8	(1)	8
10%	4	(3)	1
0%	0	(5)	(5)

Euler TVaR: A ->  $(55.83 - 53.33)/0.1 = 25$

Euler adds up!

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### Euler Allocation Example \_ΔTVaR % allocation

Allocation via Euler Derivative of TVaR(70%)				
	A	B	Sum	T
<b>Total Capital</b>				50
<b>Euler Δ</b>	25	28	53	53
<b>Euler Δ %</b>	47%	53%	100%	
<b>Allocation</b>	23	27	50	

Euler allocation% using TVaR is the same as the co-TVaR allocation!

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Percentile Allocation

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- Percentile Allocation: For each dollar of capital allocate to scenarios that use it and allocate to the risk sources within scenario.  
*Assign each \$ of capital – not just the last one .*

$$C_{\text{Percentile}}(A_k|A) = \sum \frac{a_i}{a} \min(C, a) f(a)$$

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Myers-Read Allocation

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- Default risk measured by default value. Propose that “sensible” regulation will require companies to maintain the same default value per dollar of liabilities

- $E[D] = \text{Prob}(T > C) \times E[T - C | T > C]$
- $c = D/E[T]$
- $c = (dD(A+hA_k)/dh)/E[A_k]$

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Properties  
COHERENCE

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Coherence Properties

**Homogeneity**  
 $C(bA) = bC(A)$

**Translation invariant**  
 $C(A+b) = C(A)$

**Monotonicity**  
 $C(A_1) \leq C(A_2)$  for  $A_1 \leq A_2$

**Subadditivity**  
 $C(A_1 + A_2) \leq C(A_1) + C(A_2)$

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Short tail vs Long Tail  
One year vs Ultimate

UW AND RSV RISK - EXAMPLE

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Example – Absolute: Ult vs One-Year

Required Capital - Absolute Basis  
Ultimate vs One Year

	SWP Risk (short tail)	SWP Risk (long tail)	SWP Risk (total)	SWP Risk (short tail)	SWP Risk (long tail)	SWP Risk (total)	Total
One Year	437	896	873	236	648	883	1,756
Ultimate	100	800	1,800	800	700	1,800	3,800

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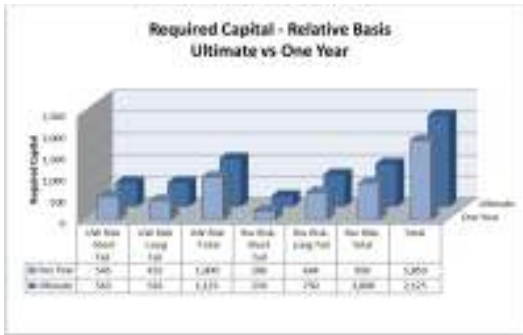
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Example – Relative: Ult vs One-Year



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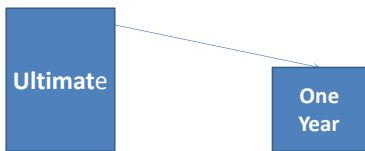
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One-year Basis Impact

- One-year basis lowers required capital, notably for reserve risk for long-tail lines
- Also considers the ability for companies to react to certain adverse outcomes before they fully emerge



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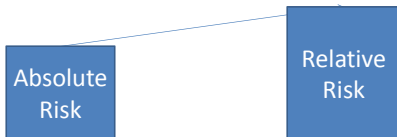
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Absolute vs Relative Impact

- Relative basis raises required capital as absolute basis has implicit credit for expected profit



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POLLING QUESTIONS

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**POLLING Q 1-4: ABSOLUTE VS RELATIVE**

For key stakeholders:  
Regulators; Rating Bureaus; Company Management

For Total Capital Requirements  
and Capital Allocation by Management

**ABSOLUTE** **RELATIVE**

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**POLLING Q 5-8: ONE-YR VS ULTIMATE**

For key stakeholders:  
Regulators; Rating Bureaus; Company Management

For Total Capital Requirements  
and Capital Allocation by Management

**ONE -YEAR** **ULTIMATE**

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**POLLING: COHERENCE?**

**9. TOTAL CAPITAL**

**A** Coherence is necessary.      **B** Coherence not necessary.

**10. ALLOCATION**

**A** Coherence is necessary.      **B** Coherence not necessary.

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**POLLING: NEGATIVES?**

**11. ALLOCATION**

**A** Negative Capital OK      **B** No Negative Capital unless true offset or negative correlation exists.

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**POLLING: TAIL-FOCUSED**

**12. TOTAL CAPITAL**

**A** Tail-focus      **B** Beyond tail

**13. ALLOCATION**

**A** Tail-focus      **B** Beyond tail

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### VaR and TVaR

<b>Value at Risk</b>	<ul style="list-style-type: none"> <li>• <math>VaR(p) = \sup\{x \mid F(x) \leq p\}</math></li> <li>• Best of the worst 1-p percent</li> </ul>
<b>Tail Value at Risk</b>	<ul style="list-style-type: none"> <li>• <math>TVaR(p) = \text{conditional mean of } x \text{ values in the tail, } 1 - p, \text{ of probability}</math></li> <li>• Average of the worst 1-p percent</li> </ul>

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### VaR and TVaR Ranking Definitions

- Ranking Definitions
  - Let  $X_1 \geq X_2 \dots \geq X_n$  be an ordering of n trials of X
  - Suppose  $k = (1 - p)n$ , then

$$VaR(p) = X_k$$

$$TVaR(p) = \frac{1}{k} \sum_{j=1}^k X_j$$

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### TVaR and CTE are Not the Same!

- CTE = Conditional Tail Expectation for points larger than the corresponding VaR
- $CTE(p) = E[X|X > VaR(p)]$ 
  - {or  $E[X|X \geq VaR(p)]$ }
  - When there are mass points, the CTE may not necessarily capture the exact (1 - p) tail of probability
- The “correct” TVaR is defined as the average of x values over the ( 1 - p ) tail of probability

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**Co-VaR and Co-TVaR- Example**

Rank of Total	Co-VaR Pct	Co-VaR(A)	Co-VaR(B)	VaR(T=A+B)
1	90%	60	30	90
2	80%	35	15	50
3	70%	(20)	40	20
4	60%	20	(15)	5
5	50%	(10)	0	(10)

Rank of Total	Co-TVaR Pct	Co-TVaR(A)	Co-TVaR(B)	TVaR(T=A+B)
1	90%	60	30	90
2	80%	48	23	70
3	70%	25	28	53
4	60%	24	18	41
5	50%	17	14	31

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**Co-TVaR- % Allocation Example**

Allocation via Co-TVaR(70%)				
	A	B	Sum	T
<b>Total Capital</b>				50
<b>Co-TVaR</b>	25	28	53	53
<b>Co-TVaR %s</b>	47%	53%	100%	
<b>Allocation</b>	23	27	50	

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