A Cost of Capital Formula for Non-Life Insurance Liabilities

A Historical Perspective

Glenn Meyers

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Introduction

- This Paper was accepted for publication in *Variance* in August 2017. I presented it at last year's CAS Annual Meeting.
- I also presented it at the 2017 ASTIN Colloquium. For that I received one of the Charles A. Hachemeister awards for 2018, and was invited to present it here.
- The ideas in this paper have an interesting history. I have been chasing this problem for the greater part of my actuarial career.
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- Rather than repeat last year's presentation I decided to present a personal account of that history.
- Note — I have included links to many of the documents that are relevant to this history.

First Encounter

- Began with a presentation (1984 CAS Annual Meeting) by the CAS Committee on the Theory of Risk (COTOR).
  - Gary Patrik, Steve Philbrick and Richard Woll
- Discounting of loss reserves was a "discussion item" at the time. The defense for not discounting was that it provided an implicit "safety margin."
- The presentation discussed the pro and cons of replacing the discount with an explicit risk margin based on risk theoretic principles.
- The presentation was very good on the "what to do" but rather short on the "how to do."

Joining COTOR

- Gary Patrik invited me to join COTOR — quite possibly because I had developed a good reputation for building models to quantify risk.
- COTOR wrote a white paper on their risk margin deliberations.
  - Recognized the need for further work to get the distribution of outcomes.
  - Did not address the evolving distribution over time.
  - Risk measures discussed — probability of ruin and utility theory.
  - Justification for discounting — longer tail, greater risk, bigger margin generated by the failure to discount.

Discounts vs Utility-Based Risk Margins

- Meyers (PCAS 1989) gives a simplified example on pension reserves.
  - Long Payout
  - Simple 3-parameter mortality model
  - Underlying math is well established.
- Features of the model
  - Used standard life contingengency formulas for process risk.
  - Used Bayesian analysis to quantify parameter risk and get the predictive distributions of pension reserves.
  - Used utility theory to calculate risk margin.
  - Used a market-based profit margin to determine parameters of the utility function.
- Discounts and risk margins were very different.

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- Discounts and risk margins were very different.
- My intent was to provide a template for calculating the risk margin. This template evolved over time.

CAPM Challenge to Utility Theory

- Presented the above paper at the Second International Conference on Insurer Solvency - Brighton England (1988)
- My use of utility functions to describe an insurer's pricing decision was challenged. CAPM was advocated as the alternative.
- The predominant interpretation at the time of CAPM was that there should be no recognition of diversifiable (i.e. firm specific) risk. I disagreed — pointing to the existence of a healthy reinsurance industry.
- Nonetheless, I thought that it was important to understand the opposition, so I took a deep dive into the math underlying the CAPM.

A Change of Emphasis — Joined ISO in 1988

- My first assignment was to fix their risk load formula for increased limits ratemaking.
- Emphasis was more on determining the price of risk rather than a safety margin for loss reserves.

The Incompleteness of Utility Theory

- Utility theory computes a minimum price for an insurer to take on a risk.
- Utility theory computes a maximum price for an insured to pay for risk reduction.
- Insurer will accept a price higher than the maximum, and the insured will accept a price lower than the minimum.
- Where the price ends up depends upon the insurance market.

Hence — Utility theory is incomplete!

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  Hence — Utility theory is incomplete!
- The CAPM attempts to take the "market" into account.

CAPM Behavioral Assumptions

- Investor chooses a portfolio of securities that will:
  1. Maximize total expected return.
  2. Be subject to a total variance constraint
- For a given set of securities, and their associated returns, an investor calculates how many shares to buy in each.
- The total number of shares bought may not match the total number of shares in the market.

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- The CAPM calculates the return of the security that forces a match of the total number of shares bought with the total number of shares in the market.
- This derivation works for **any number** (e.g. small) of investors and securities.

Securities vs Lines of Insurance

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- The variance of n_i shares of the i^{th} security takes the form:

$$n_i^2 \cdot \sigma_i^2$$

- The variance of the i^{th} line of business with exposure e_i takes the form:

$$e_i \cdot A_i + e_i^2 \cdot B_i$$

where A_i is large (say 1,000) and B_i is small (say 0.01.)

CAPM Idea Applied to Insurance

- Insurer chooses an insurance portfolio that will:
  1. Maximize total expected return.
  2. Be subject to a total variance constraint
- For a each line of business, the insurer calculates how much exposure to write in each line of insurance.
- The total insured exposure may not match the total amount of exposure desired by the market.
- The formula calculates the return of the policy that forces a match of the amount of exposure sold with the total exposure desired by the market.

The Implementation

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- As one looks at the various *Proceedings* issues around that time they will find papers with other competing proposals for risk loads.

A controversial topic. Spring 1992 e-Forum.

The Marginal Capital Approach

- One paper that attracted my attention was "Reinsurance Company Risk Loads from Marginal Surplus Requirements" by Rodney Kreps — *PCAS* 1990
- It also attracted the attention of Phil Heckman, who wrote "Some Unifying Remarks on Risk Loads" Spring 1992 e-Forum.
- Heckman showed that the Marginal Capital and Competitive Market Equilibrium (CME) approaches are equivalent.
- I followed up a few years later by deriving the CME by the marginal capital approach in "The Competitive Market Equilibrium Risk Load Formula for Catastrophe Ratemaking" *PCAS* 1996.

Behavioral Assumptions Underlying the Marginal Capital Approach — in 1996

- The insurer's required capital is given by

$$C[X] = T \cdot \sqrt{\text{Var}[X]}$$

  where X is the loss of the insurer's risk portfolio.

- The insurer will prefer to take on new Y_1 with return $R[Y_1]$ instead of new Y_2 with return $R[Y_2]$ if

$$\frac{R[Y_1]}{C[Y_1+X]-C[X]} > \frac{R[Y_2]}{C[Y_2+X]-C[X]}$$

- Over time, this behavior leads to

$$\frac{R[Y]}{C[X]-C[X_{-Y}]} = \lambda$$

  for all Y in the insurer's portfolio.

Solving the Optimization Problem

- The above behavior leads to a solution of the optimization problem — Maximize total return subject to a variance constraint.
- To see this (hueristically) suppose we have risks i and j with the same marginal capital. If $R[Y_i] > R[Y_j]$, then a portfolio with risk i will have a greater total return than the portfolio with the portfolio with risk j.

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- Proceeding as before, find the risk loads $\{R[Y_i]\}$ that match supply with demand.

Matching Supply and Demand

- Here is a formula that purports to match supply and demand. (Equation 6.1 in Meyers 1996)

$$R[Y] = \overline{\lambda} \cdot \left(Var[Y] + 2 \cdot \sum_{i=1}^{n} Cov[\overline{X}_i, Y] \right)$$

$$\text{where} \quad \overline{\lambda} = \frac{1}{\frac{1}{m}\sum_{j=1}^{m} \frac{1}{\lambda_j}} \quad \text{and} \quad \overline{X}_i = \frac{1}{m}\sum_{j=1}^{m} X_{ij}$$

- Y = random loss for the insurance contract under consideration.
- X_{ij}= random loss for insurer j in line i.
- λ_j is the return on marginal capital required by insurer j.

How Should an Insurer Use the CME Risk Load?

- The CME risk load is a prediction of market prices.
- $\overline{\lambda}$ will vary by line of business depending on who is competing for that business.
- It is hard to gather the data needed to use the CME to predict market prices.
- There are better ways to do this — ask your agents.
- *Ceteris paribus* — The insurer should accept a risk, Y, if:

  $$R[Y] \geq \lambda \cdot \text{Marginal Capital}[Y]$$

  where λ is chosen so that the insurer obtains an adequate return on its total capital.
- λ should be the same for all lines of insurance.

Do Insurers Actually Behave This Way?

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  - Costs that can be passed on in rate filings.

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  - Costs that can be passed on in rate filings.
  - Reinsurance brokers have enterprise risk management (ERM) models.
  - The gist of a quote I heard from a reinsurance broker —"ERM is a plot to discourage liability reinsurance."

Short vs Long-Tailed Lines of Business

- ISO included an estimate of $\overline{\lambda}$ in its commercial liability advisory increased limits factors with risk load.
- Applying that $\overline{\lambda}$ to catastrophe model output led to the conclusion that nobody should write catastrophe business.
- Yet catastrophe business is clearly being written.
  — What's wrong with this picture?

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- For commercial liability, an insurer has to hold capital as long as the ultimate payout is uncertain.
- That uncertainty, and therefore the required capital, will decrease over time.

The Cash Flow of Capital

- Let:
  - i be the risk-free rate of return in investments.
  - r be the risky rate of return that investors demand from the insurer.
  - C_t be the amount of capital required at the end of t years.
- At time $t = 0$ the investors provide C_0.
- At time $t = 1$ the insurer calculates C_1. Then $C_0 \cdot (1 + i) - C_1$ is returned to the investors.
- $\cdots$
- At time t the insurer calculates C_t. Then $C_{t-1} \cdot (1 + i) - C_t$ is returned to the investors.
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- At time t the insurer calculates C_t. Then $C_{t-1} \cdot (1 + i) - C_t$ is returned to the investors.
- ...
- The present value of this cash flow to investors is:

$$\sum_{t=1}^{\infty} \frac{C_{t-1} \cdot (1+i) - C_t}{(1+r)^t}$$

Risk Load and Long Tailed Lines

- The risk load, R, is the difference between the initial investment, C_0, and the actuarial present value (APV) of the returned capital.

$$R = C_0 - \sum_{t=1}^{\infty} \frac{C_{t-1} \cdot (1+i) - C_t}{(1+r)^t} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^t}$$

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- Long Tail $\Longrightarrow$ Small APV $\Longrightarrow$ Large Risk Load
- My proposal to take the cash flow of capital into account first appeared in my 1999 paper "Underwriting Risk" .
- Unspecified here — How do we get the C_ts?

Enterprise Risk Management (DFA/ERM)

- Around 1999 ISO decided to develop an ERM product.
- My 2001 paper, "The Cost of Financing Insurance" best captured our vision of an ERM product. Features include:
  - Deals with multiple (correlated) lines of insurance
  - Required capital determined by Tail Value-at-Risk (TVaR)
  - Deals with long-tailed lines of insurance — see above
  - Allocates the cost of capital to lines of business
  - Treats reinsurance as a substitute for capital
  - Final output — Target combined ratios by line of business
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- The product was parameterized with data from ISO.
- In the end, the ISO product was a commercial failure. Reinsurance brokers were providing "pro-bono" support.
- More importantly — ISO could not get access to internal insurer operations.

IAA and ASTIN Activities

- 2002 — Joined the IAA Insurance Solvency Assessment Working Party.
  - Produced the IAA Blue Book on Risk Based Capital
- 2004 to 2008 — Working Party $\Longrightarrow$ Solvency Subcommittee.
- 2008 to 2014 — Represented the IAA on the ASTIN Committee.
- Watched the developments of and has access to the actuarial leadership views on:
  - Solvency II
  - International Association of Insurance Supervisors (IAIS)
  - International Accounting Standards Board (IASB)
  - International Financial Reporting Standards (IFRS)

Risk Margins for Insurance Liabilities

- Two References
  - IAA-1 — Measurement of Liabilities for Insurance Contracts:Current Estimates and Risk Margins (2009)
  - IAA-2 — Risk Adjustments for Insurance Contracts Under IFRS 17 (2018)
- IAA publications are typically "consensus" documents that recognize the major points of view. They attempt to be non-judgmental with respect to the current practice.
- There is considerable overlap between the two documents. The big difference was that IAA-2 was finished after IFRS-17 came out.

Quotes from the Executive Summary of IAA-1

- The IAA, the IAIS, and the IASB have indicated that there are five key desirable characteristics of risk margins:
  1. The less that is known about the current estimate and its trend, the higher the risk margins should be.
  2. Risks with low frequency and high severity will have higher risk margins than risks with high frequency and low severity.
  3. For similar risks, contracts that persist over a longer timeframe will have higher risk margins than those of shorter duration.
  4. Risks with a wide probability distribution will have higher risk margins than those risks with a narrower distribution.
  5. To the extent that emerging experience reduces uncertainty, risk margins will decrease, and vice versa.

Quotes from the Executive Summary of IAA-1

- Approaches for determining risk margins have been grouped into the following four families of approaches that meet the IASB's current view to have an explicit risk margin:
  1. Quantile methods ···
  2. Cost of capital methods are based on the amount of return, in addition to the amount earned by the insurer from its investment of capital, that is required for the total return on the insurance enterprise to be adequate.
  3. Discount related methods ···
  4. Explicit assumptions ···
- In their conclusion — "The cost of capital method (without simplification) is the most risk sensitive and is the method most closely related to pricing risk in other industries. However, in part as a result, it is also more challenging to implement than the other methods."

What Was Happening Across the Pond

- The Swiss Solvency Test
  - Project started in 2003
  - Implemented in 2006
- Solvency II
  - Project started in 2001
  - Originally scheduled to be implemented in 2012
  - Actual implementation in 2016
- Reminder — The risk margin is a small part of this effort.

Comparing Risk Margin Formulas

- From "The Cost of Financing Insurance"

$$(r-i)\cdot\sum_{t=0}^{\infty}\frac{C_t}{(1+r)^t}$$

- From Solvency II and the Swiss Solvency Test

$$(r-i)\cdot\sum_{t=0}^{\infty}\frac{C_t}{(1+i)^{t+1}}$$

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- The remaining problem is to calculate the C_ts.

A Hierarchy of Simplifications for Calculating C_ts

- Taken from the instructions for Solvency II QIS 5, 2010 p.59
  1. Make a full calculation of all future C_ts without using simplifications.
  2. Approximate the individual risks or sub-risks within some or all modules and sub-modules to be used for the calculation of future C_ts.
  3. Approximate the whole C_ts for each future year, e.g. by using a proportional approach.
  4. Estimate all future C_ts "at once", e.g. by using an approximation based on the duration approach.
  5. Approximate the risk margin by calculating it as a percentage of the best estimate.
- The challenge — Item #1. For that we need a stochastic loss reserve model.

Weighting Scenarios with Bayes' Theorem

- My first attempt at a stochastic loss reserve model — "Estimating Predictive Distributions for Loss Reserve Models" *Variance* 2007(2).
- Features of the model include:
  - Schedule P data
  - Scenarios derived by maximum likelihood on a "Cape Cod" like model for 40 large insurers.
  - Used Bayes' theorem on Schedule P data to get weights for each scenario.
  - The final model was a probability weighed mixture of the scenarios.
  - Performed "Aggressive retrospective testing" on the stochastic model.

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  - Performed "Aggressive retrospective testing" on the stochastic model.
- Eventually moved to a more promising approach.

The COTOR Challenges

- Starting in 2004, the CAS Committee on the Theory of Risk, issued a series of competitions to fit claim severity distributions.
- The "data" for the computation was simulated from an unknown model created by Stuart Klugman.
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- Using Bayesian MCMC!

Stochastic Loss Reserving with Bayesian MCMC

- Potential of Bayesian MCMC for loss reserving
  - If you can code a model and specify the prior distribution, you can obtain an arbitrarily large sample from the posterior distribution.
  - No preset limit to the number of parameters in a model
- The CAS Loss Reserve Database
  - Worked with Jed Frees (University of Wisconsin) on a Correlation and Dependencies Task Force
  - NAIC allowed free access to their Schedule P data to the University of Wisconsin
  - We got permission to post hundreds of upper triangle training data, and lower triangle test data on the CAS website.
- IMPORTANT
  - Easy to create new stochastic loss reserve models.
  - Can compare the performance of new models with existing models.

The Changing Settlement Rate Model

- X_{wd} = cumulative paid loss for AY w and DY d
- Model — $X_{wd} \sim \text{lognormal}(\mu_{wd}, \sigma_d)$ where

$$\mu_{wd} = \alpha_w + \beta_d \cdot (1-\gamma)^{w-1}$$

- Prior distributions for the $\alpha_w, \beta_d, \gamma,$ and σ_d are fairly wide.
- The model is called the Changing Settlement Rate (CSR) model. It is described in my 2015 monograph. A 2nd edition of that monograph is under review.
- Note — Risk margins are covered in the 2nd edition. The models and the numbers that follow reflect the latest tweaks that are in the 2nd edition. The results are very close to those in the paper.

Samples from the Posterior Distribution.

- Data comes from a Schedule P upper triangle taken from the CAS Loss Reserve Database.
- Use Bayesian MCMC software to produce a random sample from the posterior distribution of the model.

$$\left\{\mu_{wd}^{j}, \sigma_{d}^{j}\right\}_{j=1}^{10{,}000}$$

given the upper triangle data.
- The sample can be used to simulate any desired statistic of the random output from the lower triangle distribution of outcomes. — This includes the C_ts.
- The statistics reflect both process and parameter risk.

Assumptions Underlying This COC Formula

- Given the upper triangle of data, the set of parameters

$$\left\{\mu^j_{wd}, \sigma^j_d\right\}_{j=1}^{10,000}$$

represents the possible states of the world for the insurer.

- For a given j the ultimate loss

$$U_j = \sum_{w=1}^{10} \exp\left(\mu^j_{w,10} + (\sigma^j_{10})^2/2\right)$$

- For future calendar year $t = 1, \ldots, 9$, define the loss trapezoid, T_t as the set of the top t diagonals in the lower triangle. Then the set of loss trapezoids $\left\{T^j_t\right\}_{j=1}^{10,000}$ simulated from $\left\{\mu^j_{wd}, \sigma^j_d\right\}_{j=1}^{10,000}$ represent the set of possible future developments of the losses.

Updating the Scenario Probabilities

- Updating scenarios given T_t^k for 10,000 ks with MCMC takes too long.
- Instead, like my first attempt, update the the probabilities of initial 10,000 scenarios with Bayes' theorem.

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- Instead, like my first attempt, update the the probabilities of initial 10,000 scenarios with Bayes' theorem.
- For each, $k = 1, \ldots, 10,000$, update the scenario probabilities as more losses come in at the end of each calendar year.
- Initially, all scenarios are equally likely (=1/10,000).
- Then using Bayes' theorem:

$$Pr[J = j | T_t^k] = \frac{\prod_{X_{wd} \in T_t^k} \phi(\log(X_{wd}) | \mu_{wd}^j, \sigma_d^j)}{\sum_{j=1}^{10,000} \prod_{X_{wd} \in T_t^k} \phi(\log(X_{wd}) | \mu_{wd}^j, \sigma_d^j)}$$

Calculating Statistics of Interest for Risk Margins

For each $k = 1, \ldots, 10,000$ do the following.

1. Calculate $P_t^k \equiv \left\{ Pr[J = j | T_t^k] \right\}_{j=1}^{10,000}$
2. Take a sample, S_t^k, of size 10,000 from $\left\{ U_j \right\}_{j=1}^{10,000}$ with sampling probabilities P_t^k. Easy to do in R.

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- $E_t^k = \text{mean}(S_t^k) =$ Expected ultimate loss at the end of future calendar year t.
- $A_t^k = TVaR@97\% = \text{mean}\left(sort(S_t^k)[9701 : 10000]\right) =$ Required assets at the end of future calendar year t.
- $C_t^k = A_t^k - E_t^k =$ Capital required at the end of future calendar year t.
- $Rel_t^k = C_{t-1}^k \cdot (1 + i) - C_t^k =$ Capital released at the end of future calendar year t.

Continuing the Calculation for $k = 1, \ldots, 10,000$

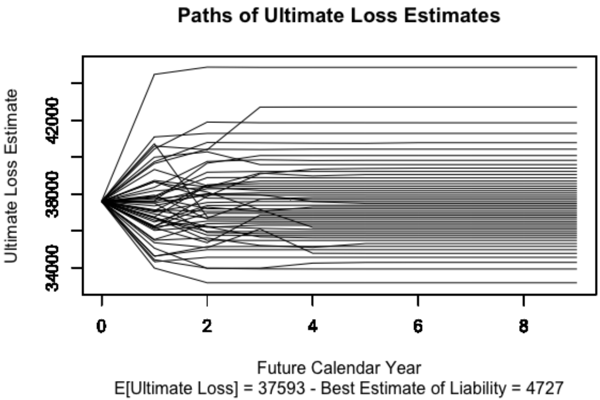
3. Then for each k the risk margin equals:

$$R^k = C_0 - \sum_{t=1}^{9} \frac{C_{t-1}^k \cdot (1+i) - C_t^k}{(1+r)^t} = (r-i) \cdot \sum_{t=0}^{9} \frac{C_t^k}{(1+r)^t}$$

- Then the indicated risk margin is given by:

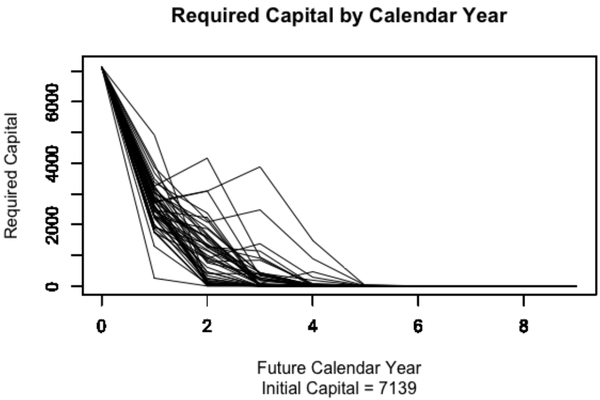
$$R = \text{mean}[R^k]$$

Example - Insurer 353 in Commercial Auto
$i = 4\%$ and $r = 10\%$



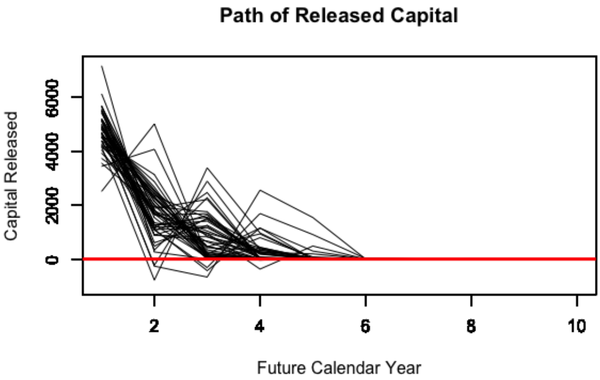
Note the leveling off of the paths.

Example - Insurer 353 in Commercial Auto
$i = 4\%$ and $r = 10\%$



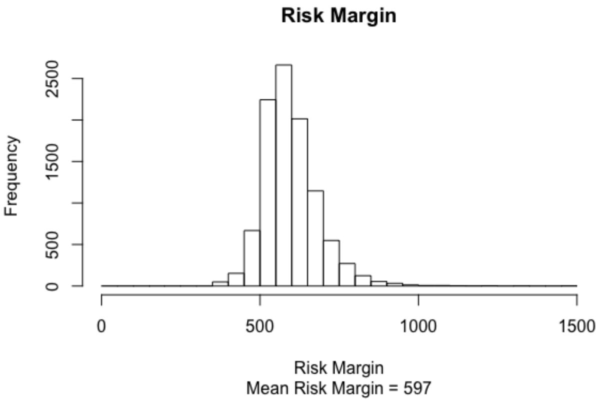
Note that the capital requirements decrease to very near zero.

Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$



Occasionally the insurer must add capital.

Example - Insurer 353 in Commercial Auto $i = 4\%$ and $r = 10\%$



Should the variability of the risk margin influence r?

Applied Risk Margin Calculation to 200 Triangles

- Calculated the risk margin for 50 triangles in each of four lines of insurance.

Summary of Results

- Observed a strong dependence of the risk size.
  - The effect is most pronounced for the very small risks, and eventually levels out.
- Calculated the effect of diversification for insurers with loss triangles in all four lines.
  - The effect of diversification can be significant — especially for small lines in the same insurer with a large line.

Alternative Risk Criteria

- The risk criteria used in the paper were my personal preferences.
  1. TVaR@xx.x% instead of VaR@99.5%
  2. "Ultimate" time horizon instead of one-year time horizon.
  3. Independence between lines of insurance. In Meyers 2018, I make the case that if you have dependence, you are missing something in your model.

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- The paper shows how one can adjust the calculation to use the other risk criteria.
  1. Change "9701:10000" to "9950:9950" in the R script.
  2. Brute force calculation of the one year expectation for each scenario.
  3. Use a copula to aggregate the lines of business.

The Objectives that Drove Me in this "Project"

- Initially — Risk margin for loss reserves
- Then moved to a risk load for insurance ratemaking
  - The CAPM idea
  - Marginal capital
  - How long is it necessary to hold capital?
  - Reinsurance as a substitute for capital
- This led to enterprise risk management. A weakness was the ability to estimate the capital requirements over time.
- This paper uses Bayesian MCMC to provide one solution to that problem.
- The risk margin for loss reserves, as posed at the IAA, can be addressed by one person with some data and a computer.
- The tools developed here should be useful to an insurer enterprise management **team**.

Acknowledgements

- In preparing this paper I have benefitted from of a lot of good work by many others, some of whom I mentioned during the course of this talk.
- Ideal research environment
  - CAS Committee on the Theory of Risk (COTOR)
  - Various CAS DFA/ERM committees
  - ARIA Risk Theory Society
  - Correlations and Dependencies Task Force
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- Strong support from ISO
  - Wide variety of stimulating internal projects
  - Flexibility to travel and participate in conferences and external joint research work