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Severity Curve Fitting for Long-Tailed Lines: An Application of Stochastic Processes and Bayesian Models

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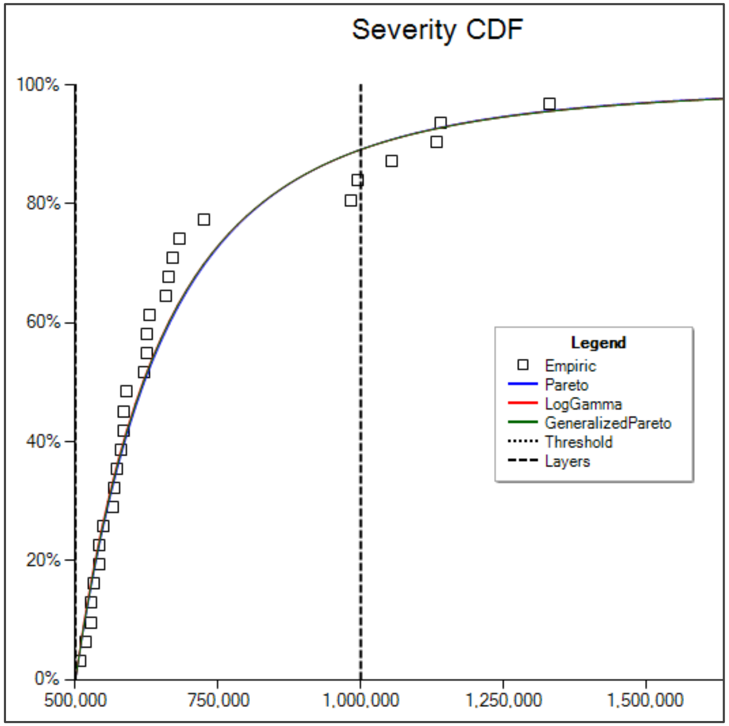
Statement of the Problem

- We like severity curves because:
  - They imply increased limit factors
  - They allow us to price excess (re)insurance
- Fitting a curve to property losses is straightforward
- Fitting a curve to liability losses, which develop slowly, is not straightforward
  - Claim values change and new claims emerge
  - Mixture of ages/reporting lags
- In reinsurance the dataset is typically small
  - Survey of current methods at the end of the paper shows a reliance on large, detailed datasets



Overview of Proposed Solution

- Fit claims within each accident year and age, now you have a triangle of e.g. μ's and σ's for a lognormal family
  - Now development does not need to reference individual claim amounts

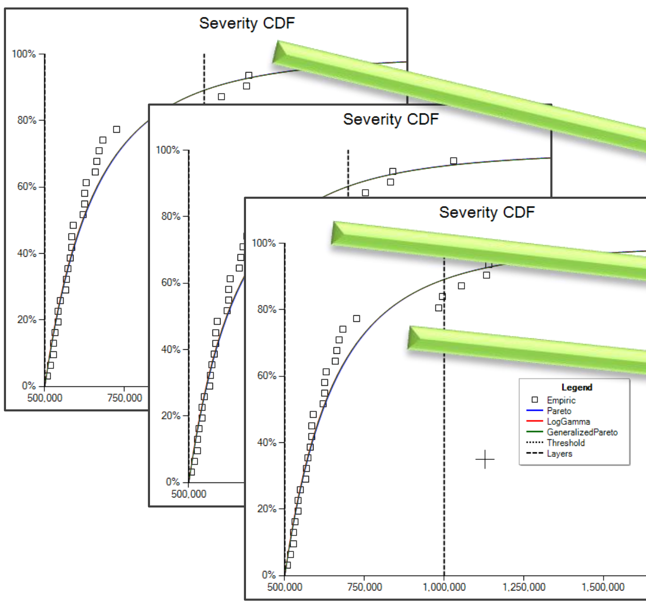
- μ's of a given accident year follow a stochastic process
  - This structure plus recognition of variance parameters lowers dimensionality (versus unconstrained link ratio analysis)

- Bayesian statistics allows low volume reinsurance data to be used with an informative prior distribution
  - Modern way of credibility weighting experience and exposure

Fit Claims within Accident Year and Age

- Instead of trying to trend and develop individual losses and fit a mixture of AY's and ages, fit a curve to claims of a single AY and age

- Then you have a triangle of μ's when fitting lognormal family (e.g.)



Column ▾
Sum of intercept

Row Labels ▾	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
[illegible]	7.536	8.033	8.360	8.493	8.545	8.569	8.578	8.575	8.585	8.592	8.596	8.597	8.598	8.609	8.608	8.609	8.611	8.611	8.614	8.614
1991	[illegible]	8.089	8.346	8.548	8.573	8.582	8.595	8.611	8.607	8.604	8.600	8.607	8.612	8.611	8.610	8.610	8.612	8.614	8.616	
1992	[illegible]	[illegible]	[illegible]	8.489	8.531	8.566	8.561	8.566	8.571	8.575	8.581	8.579	8.578	8.580	8.580	8.582	8.582	8.585		
1993	7.363	8.063	[illegible]	8.490	8.536	8.551	8.568	8.583	8.574	8.569	8.583	8.587	8.585	8.591	8.591	8.595	8.596			
1994	7.384	8.001	8.371	8.525	8.582	8.595	8.623	8.626	8.641	8.646	8.648	8.646	8.646	8.647	8.651	8.651				
1995	7.242	7.987	8.324	8.467	8.530	8.554	8.563	8.586	8.602	8.606	8.609	8.608	8.616	8.619	8.619					
[illegible]	7.179	7.935	8.349	8.509	8.540	8.570	8.579	8.596	8.603	8.613	8.622	8.620	8.620	8.620						
1997	[illegible]	[illegible]	[illegible]	[illegible]	8.609	8.665	8.665	8.682	8.686	8.695	8.692	8.698	8.697							
1998	7.343	[illegible]	[illegible]	[illegible]	8.683	8.725	8.746	8.749	8.760	8.757	8.756	8.761								
1999	7.402	8.054	8.421	[illegible]	8.789	8.813	8.802	8.814	8.816	8.817	8.822									
2000	7.524	8.222	8.646	8.875	8.958	8.984	8.983	8.986	8.982	8.980										
[illegible]	7.758	8.435	8.805	8.998	9.057	9.098	9.096	9.088	9.095											
2002	[illegible]	[illegible]	[illegible]	[illegible]	9.147	9.160	9.164	9.165												
2003	7.904	[illegible]	[illegible]	[illegible]	9.232	9.270	9.282													
2004	7.978	8.709	9.073	[illegible]	9.291	9.305														
2005	8.054	8.712	9.072	9.245	9.308															
2006	8.121	8.835	9.171	9.306																
2007	8.197	8.867	9.192																	
2008	8.238	8.892																		
2009	8.304																			

Graph the μ's to See the Stochastic Process

- Each line is a single accident year and appears to follow a smooth curve with error
  - Also an increase between each line (on average) representing trend/inflation

	Colun / Sum of intercept																			
Row Labels	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1990	7.536	8.033	8.360	8.493	8.545	8.569	8.578	8.575	8.585	8.592	8.596	8.597	8.598	8.609	8.608	8.609	8.611	8.611	8.614	8.614
1991	7.461	8.089	8.346	8.548	8.573	8.582	8.595	8.611	8.607	8.604	8.600	8.607	8.612	8.611	8.610	8.610	8.612	8.614	8.616	
1992	7.599	8.015	8.379	8.489	8.531	8.566	8.561	8.566	8.571	8.575	8.581	8.579	[illegible]							
1993	7.363	8.063	8.393	8.490	8.536	8.551	8.568	8.583	8.574	8.569	8.583	8.587	[illegible]							
1994	7.384	8.001	8.371	8.525	8.582	8.595	8.623	8.626	8.641	8.646	8.648	8.646	[illegible]							
1995	7.242	7.987	8.324	8.467	8.530	8.554	8.563	8.586	8.602	8.606	8.609	8.608	[illegible]							
1996	7.179	7.935	8.349	8.509	8.540	8.570	8.579	8.596	8.603	8.613	8.622	8.620	[illegible]							
1997	7.342	8.021	8.357	8.556	8.609	8.665	[illegible]	8.682	8.686	8.695	8.692	8.698	[illegible]							
1998	7.343	8.062	8.431	8.562	8.683	8.725	[illegible]	[illegible]	[illegible]	8.757	8.756	8.761								
1999	7.402	8.054	8.421	8.687	8.789	8.813	8.802	8.814	[illegible]	[illegible]	8.822									
2000	7.524	8.222	8.646	8.875	8.958	8.984	8.983	8.986	8.982	[illegible]										
2001	7.758	8.435	8.805	8.998	9.057	9.098	9.096	9.088	9.095											
2002	7.759	8.507	8.873	9.083	9.147	9.160	9.164	9.165												
2003	7.904	8.651	9.013	9.193	9.232	9.270	9.282													
2004	7.978	8.709	9.073	9.222	9.291	9.305														
2005	8.054	8.712	9.072	9.245	9.308															
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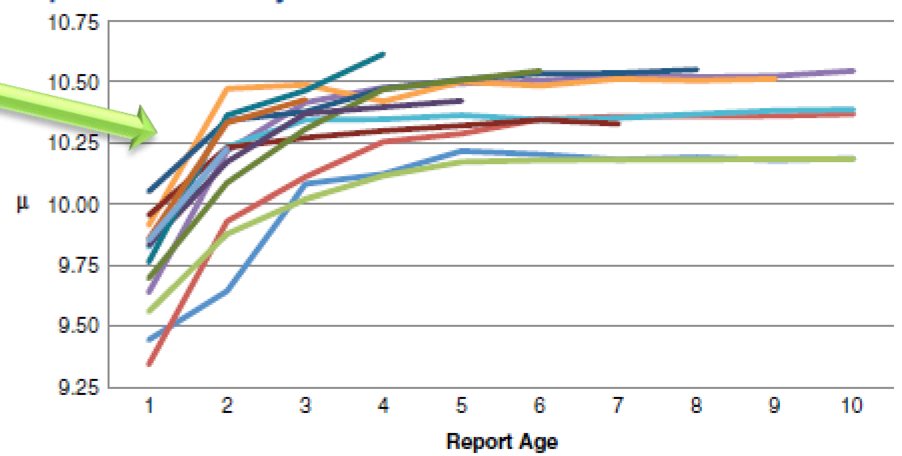
Figure 2. Fitted lognormal μ by AY and age from dataset of professional liability claims



Parameterize the Stochastic Process

- Two triangles with ~ N^2 values are transformed to pair of stochastic processes with 11 total parameters

- I assumed both μ and σ fit an exponential decay model
  - $\mu(i) = \mu(i-1) + a * e^{-b*i} + \epsilon$
  - Method can be extended to more general shape of the curve

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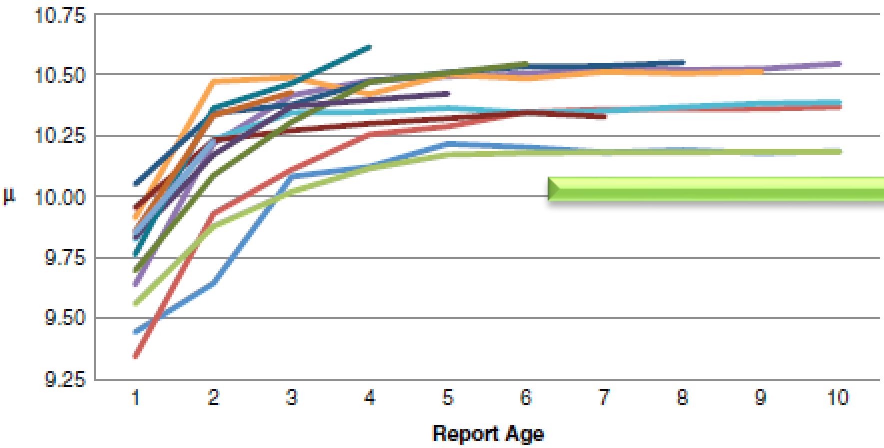


$$\mu(i,1) = \mu_{start} + trend * i + \epsilon_{\mu}(i,1)$$

$$\sigma(i,1) = \sigma_{start} + \epsilon_{\sigma}(i,1)$$

$$\mu(i,j) = \mu(i,j-1) + \mu_{incr} * e^{-(j-1)/\mu_{growth}} + \epsilon_{u}(i,j)$$

$$\sigma(i,j) = \sigma(i,j-1) + \sigma_{incr} * e^{-(j-1)/\sigma_{growth}} + \epsilon_{\sigma}(i,j)$$

$$\epsilon_{\mu}(i,j) \sim N\left(0, p_{\mu} * e^{-(j-1)*r_{\mu}}\right)$$

$$\epsilon_{\sigma}(i,j) \sim N\left(0, p_{\sigma} * e^{-(j-1)*r_{\sigma}}\right).$$

Estimate Posterior Distribution of Variable of Interest

- Data + Model + Prior Distributions + R + stan = Posterior Distributions
  - Design ensures that volume of data is balanced against certainty of prior estimates
  - Particularly helpful in reinsurance applications which have low data volume

Table 1. Assumed format of available data

Claim	Accident Year	Age 1 Incurred	Age 2 Incurred	Age 3 Incurred
1	1	500	1000	1100
2	2	100	500	
3	2	100	300	
4	3	200		

$$\mu(i,1) = \mu_{start} + trend * i + \epsilon_{\mu}(i,1)$$

$$\sigma(i,1) = \sigma_{start} + \epsilon_{\sigma}(i,1)$$

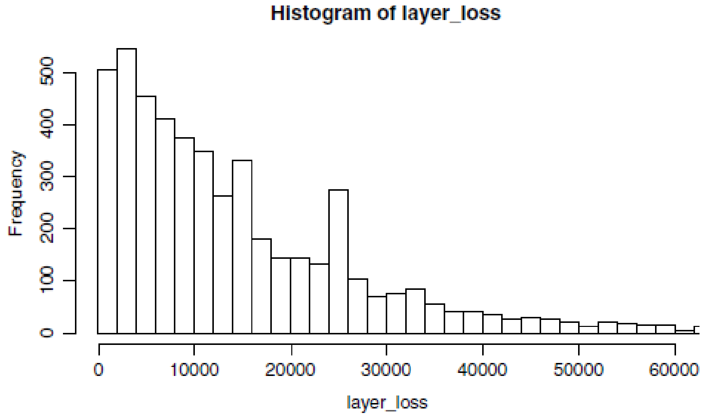
$$\mu(i,j) = \mu(i,j-1) + \mu_{incr} * e^{-(j-1)/\mu_{growth}} + \epsilon_{u}(\mathrm{i},\mathrm{j})$$

$$\sigma(i,j) = \sigma(i,j-1) + \sigma_{incr} * e^{-(j-1)/\sigma_{growth}} + \epsilon_{\sigma}(\mathrm{i},\mathrm{j})$$

$$\epsilon_{\mu}(i,j) \sim N\left(0, p_{\mu} * e^{-(j-1)*r_{\mu}}\right)$$

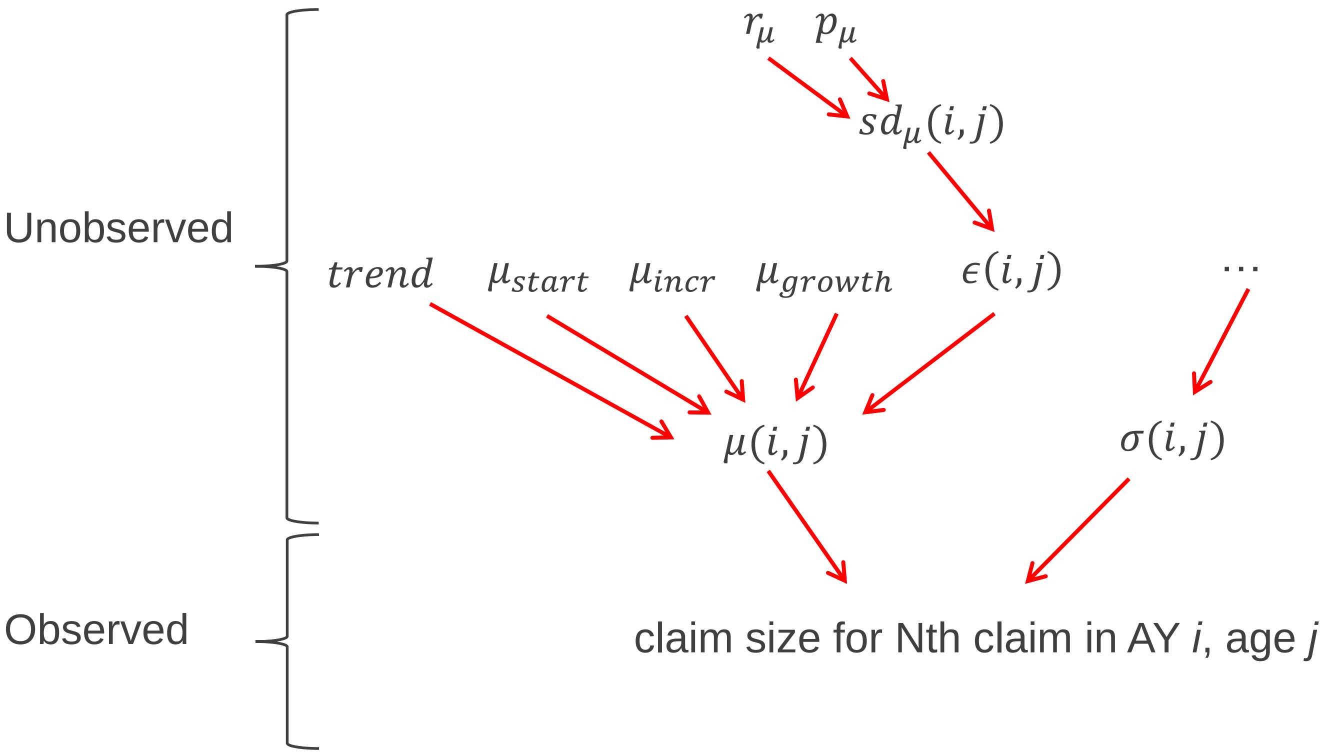
$$\epsilon_{\sigma}(i,j) \sim N\left(0, p_{\sigma} * e^{-(j-1)*r_{\sigma}}\right).$$

Figure 8. Histogram of posterior layer_loss



Bayesian Hierarchical Models

- This model is an example of a Bayesian hierarchical model, which just means there are multiple layers of unobserved variables in the model



Final Thoughts

- Paper provides full data and code to reproduce results and use for yourself
- Important to note that this model is severity only, but could be combined with a similar approach to frequency (Mildenhall, *Forum*, 2006) for a Bayesian reinsurance pricing model
- Does not replace actuarial judgement, just shifts analysis away from triangulating losses and determining credibility weight on experience vs. exposure to crafting prior distributions
  - Remember, credibility was an approximation to Bayesian posterior estimates all along
- We have done this analysis on real data maybe 10 times, each time μ has followed exponential decay, but σ was more complex
  - Still always followed a pattern, but may need 1 more parameter to model
- Questions? Thank you!