Excess Ratio Simulation and Variability of Observed Results

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Introduction

- Excess loss ratios are essential items of Workers Compensation ratemaking
  - Retrospective rating and deductible pricing
  - Excess ratios correspond with severity distributions. NCCI develops excess ratio curves by combining classes into hazard groups (HG) according to their excess loss potential
- NCCI is currently reviewing its mapping of classes into HGs
  - Four groups, pre-2007: I, II, III, and IV
  - Post 2007, seven groups: A-G
- Credible excess loss information requires more data than most classifications can provide. NCCI must estimate excess loss at a hazard group level

Introduction (Cont'd)

- *How distinct are the hazard groups*?
- The following thought experiment is our attempt to shed light on this question
  - Using the current excess loss information for the seven extant HGs, we simulated excess losses from loss portfolios of various sizes
  - How variable are the observed results?
  - How well can we infer a portfolio's HG from an observed excess ratio?
- This thought experiment will help NCCI set credibility standards as part of its hazard group review
- Since the experiment simulates only process risk, it provides a lower bound to real-life variability

Agenda

- Definitions and Countrywide Parameters
  - Excess Ratios and Hazard Groups
  - Countrywide Claim Counts, Severities, and Weights
- Illustrating the Effect of Noise on the Hazard Groups
  - Simulation Procedure
  - Observed Excess Ratios versus Expected for one HG
  - Observed Excess Ratios versus Expected for one Limit
  - Claim Count Distribution by Class
  - Distribution of Excess Ratios and HGs
  - Distribution of HGs by Excess Ratio at Nine Limits
- Concluding Comments
- Appendix

Excess Ratios and Hazard Groups

- Excess ratio at loss limit *L*: $R(L) = \int_L^\infty (x-L) f_X(x) dx \Big/ \int_0^\infty x f_X(x) dx$
  - Random variable *X* represents one claim; *R*(*L*) is a per-claim excess ratio
  - For classes at the same limit, the more hazardous class has the greater excess ratio
- The claims of many classes, even aggregated at a countrywide level, are too few to determine credible excess ratios
  - The dilemma of hazard grouping
    - Type I: mistaking noise for signal, thinking that the hazard is different when it's really the same
    - Type II: mistaking signal for noise, thinking that the hazard is the same when it's really different

Excess Ratios and Hazard Groups (Cont'd)

- NCCI has estimated countrywide excess ratio curves. They are in entry ratio form at a claim group (CG) level. The five claim groups are:
  - Fatal
  - Permanent Total
  - Permanent Partial and Temporary Total – Likely-to-develop
  - Permanent Partial and Temporary Total – Not-Likely-to-develop
  - Medical Only
- Scaling them by severities *E*[*X*] converts them into dollar curves

Excess Ratios and Hazard Groups (Cont'd)

- NCCI's latest ***Excess Loss Factor Calculations*** provided expected claim counts $E[N]$ and severities $E[X]$ by state, HG, and CG
  - Expected loss $E[L]$ equals $E[N] \times E[X]$
- We raised these expected values to a countrywide level and scaled the five excess ratio curves for all the HG and CG combinations
- The next two slides show the counts and severities, as well as the expected losses in excess of one limit, $100K

Countrywide Claim Counts

Distribution by Hazard Group and Claim Group

HG	Fatal	PT	PPTT(L)	PPTT(N)	MedOnly	Total	Claims
A	0.02%	0.03%	3.3%	16.2%	80.5%	100%	636,419
B	0.03%	0.04%	3.5%	17.6%	78.8%	100%	1,602,187
C	0.04%	0.06%	3.7%	18.5%	77.7%	100%	2,703,753
D	0.08%	0.08%	4.4%	21.6%	73.9%	100%	874,315
E	0.15%	0.12%	5.1%	25.2%	69.4%	100%	1,059,432
F	0.26%	0.15%	6.1%	30.2%	63.3%	100%	593,076
G	0.44%	0.21%	5.7%	28.3%	65.3%	100%	153,219
All	0.08%	0.07%	4.1%	20.5%	75.2%	100%	7,622,402

Fitted five-year expected claim counts aggregated by state from the most recent NCCI *Excess Loss Factor Calculations*

Countrywide Severities and Weights

HG		Fatal	PT	PPTT(L)	PPTT(N)	MedOnly
A	E[X]	268,273	1,117,513	82,887	26,351	1,325
	Excess Ratio at $100K	0.710	0.915	0.462	0.242	0.039
	Loss Weight	0.5%	3.4%	32.3%	51.0%	12.8%
B	E[X]	305,803	1,543,147	104,318	32,192	1,481
	Excess Ratio at $100K	0.739	0.937	0.519	0.279	0.041
	Loss Weight	0.7%	5.6%	32.8%	50.5%	10.4%
C	E[X]	317,569	1,687,465	113,131	34,572	1,503
	Excess Ratio at $100K	0.747	0.942	0.539	0.294	0.042
	Loss Weight	1.0%	7.9%	32.4%	49.6%	9.1%
D	E[X]	333,701	1,918,221	133,067	39,946	1,544
	Excess Ratio at $100K	0.757	0.949	0.576	0.321	0.043
	Loss Weight	1.6%	8.8%	33.5%	49.5%	6.6%
E	E[X]	372,044	2,389,996	157,832	45,391	1,705
	Excess Ratio at $100K	0.779	0.959	0.618	0.349	0.045
	Loss Weight	2.2%	12.3%	33.2%	47.3%	4.9%
F	E[X]	404,362	2,882,545	192,595	53,404	1,942
	Excess Ratio at $100K	0.794	0.966	0.662	0.385	0.049
	Loss Weight	3.1%	12.4%	34.2%	46.8%	3.6%
G	E[X]	436,802	3,377,770	215,517	57,159	1,878
	Excess Ratio at $100K	0.808	0.971	0.685	0.403	0.049
	Loss Weight	4.9%	18.4%	31.9%	41.7%	3.2%

Severities, excess ratios, and loss weights are weighted averages of the corresponding amounts by state taken from the NCCI ELF methodology
All forty limits, from $10K to $10M, processed; $100K limit chosen for illustration

Simulation Procedure

- Within a portfolio of losses we simulate each of the five claim groups according to the collective risk model:

$$S = X_1 + X_2 + \cdots + X_N$$

  - The claim count *N* is Poisson with a specified mean *E*[*N*]
  - The amounts *X* are independent and identically distributed
  - The size of a portfolio is the sum of the *E*[*N*] over CG, i.e., the expected claims for the entire HG
- The portfolio's loss in excess of limit *l* is the sum of its claims' losses in excess of *l*. Excess ratios are excess losses divided by the total loss

Simulation Procedure (Cont'd)

- Specify the HG size; e.g., E[*N*] = 5,000. Allocate it by CG:

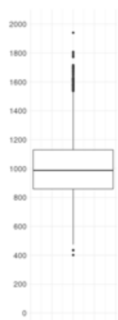
$$E[N] = E[N_{Fatal}] + E[N_{PT}] + E[N_{PPTT\,(L)}] + E[N_{PPTT\,(N)}] + E[N_{MedOnly}]$$

  - Example for HG C: 5,000 = 2 + 3 + 185 + 924 + 3,886
- For each iteration, sample $N_{HG\times CG}^{iter}$ claim counts from a Poisson distribution with mean $E[N_{HG\times CG}]$
- Simulate claim severities $X_{HG\times CG}^{iter}(1), X_{HG\times CG}^{iter}(2), \ldots, X_{HG\times CG}^{iter}(N_{HG\times CG}^{iter})$ from a discrete distribution based on the excess ratio curves
- Tabulate excess losses and excess ratios at each loss limit *l*:

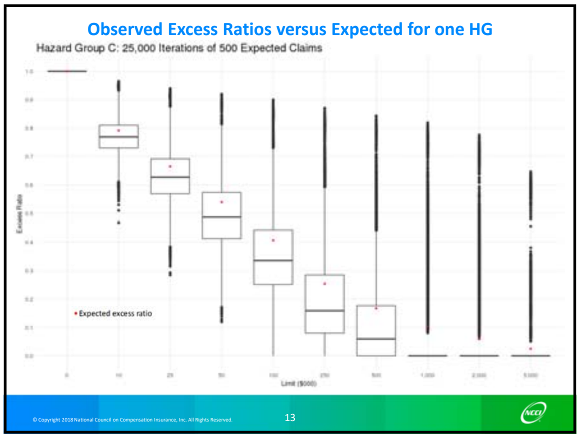
$$XsLoss_{HG}^{iter}(l) = \sum_{CG} \sum_{i=1}^{N_{HG\times CG}^{iter}} \max\left(0,\ X_{HG\times CG}^{iter}(i) - l\right)$$

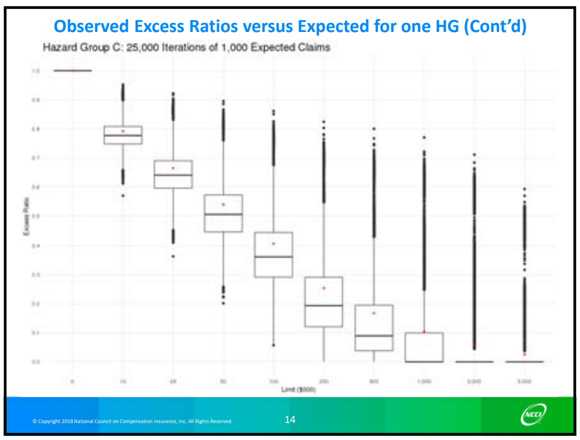
$$XsRatio_{HG}^{iter}(l) = XsLoss_{HG}^{iter}(l) / XsLoss_{HG}^{iter}(0)$$

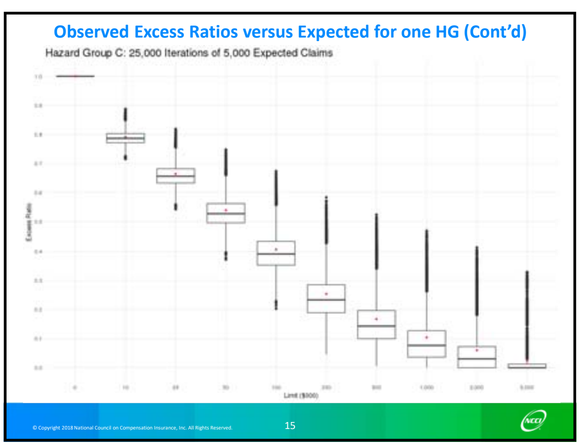
Key to the following Box-Whisker Plots



- 10,000 draws from a gamma distribution with mean 1,000 and standard deviation 200
- Sample median = 987
- Q25 = 859; Q75 = 1,130
- Interquartile range IQR = Q75 – Q25 = 271
- Whisker extremes:
  - Q25 – 1.5 × IQR = 452
  - Q75 + 1.5 × IQR = 1,536
- Dots beyond the whiskers represent outliers
  - three outliers < 452
  - ninety outliers > 1,536
- The box and whiskers of a normal distribution encompass 99.3% of the probability


Observed Excess Ratios versus Expected for one HG
Hazard Group C: 25,000 Iterations of 500 Expected Claims
Expected excess ratio
Limit ($000)
13


Observed Excess Ratios versus Expected for one HG (Cont'd)
Hazard Group C: 25,000 Iterations of 1,000 Expected Claims
Limit ($000)
14


Observed Excess Ratios versus Expected for one HG (Cont'd)
Hazard Group C: 25,000 Iterations of 5,000 Expected Claims
Limit ($000)
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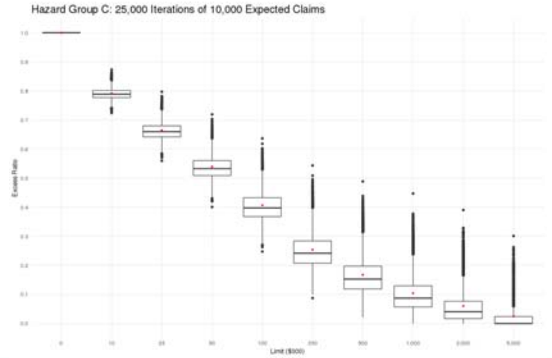
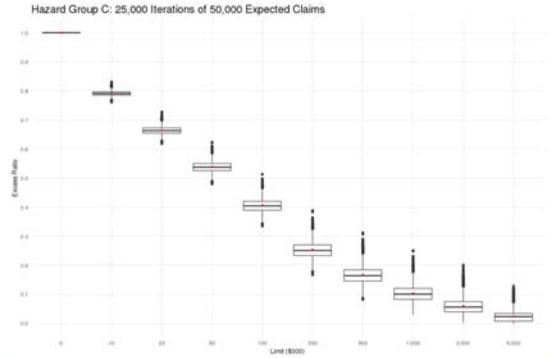
Observed Excess Ratios versus Expected for one HG (Cont'd)

Hazard Group C: 25,000 Iterations of 10,000 Expected Claims



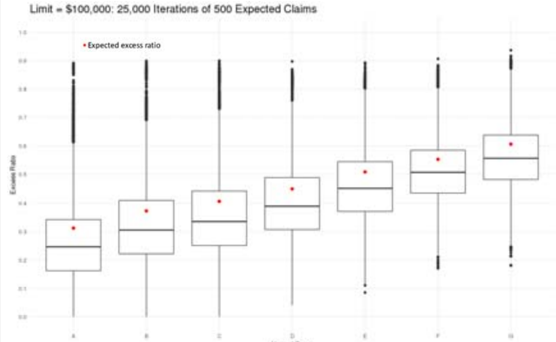
Observed Excess Ratios versus Expected for one HG (Cont'd)

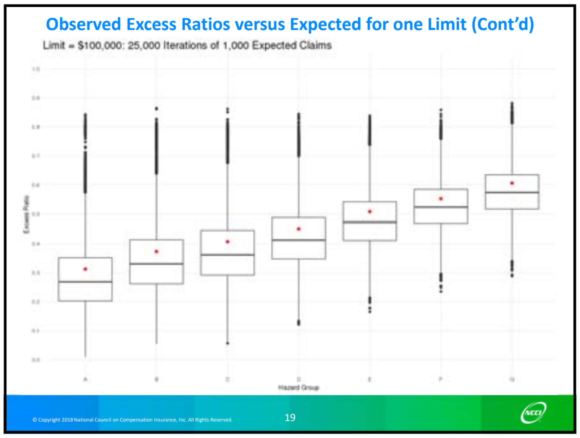
Hazard Group C: 25,000 Iterations of 50,000 Expected Claims



Observed Excess Ratios versus Expected for one Limit

Limit = $100,000: 25,000 Iterations of 500 Expected Claims




Observed Excess Ratios versus Expected for one Limit (Cont'd)
Limit = $100,000: 25,000 Iterations of 1,000 Expected Claims
Excess Ratio
Hazard Group
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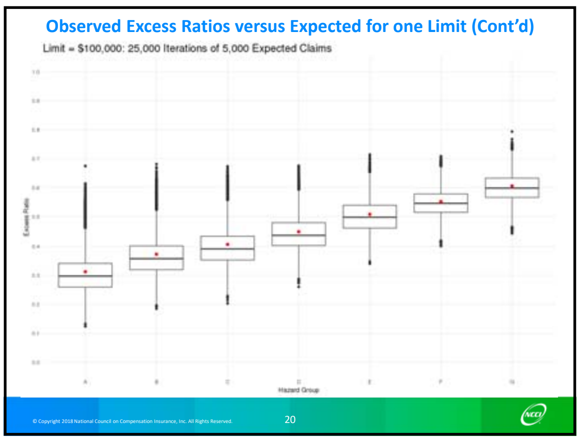
Observed Excess Ratios versus Expected for one Limit (Cont'd)
Limit = $100,000: 25,000 Iterations of 5,000 Expected Claims
Excess Ratio
Hazard Group
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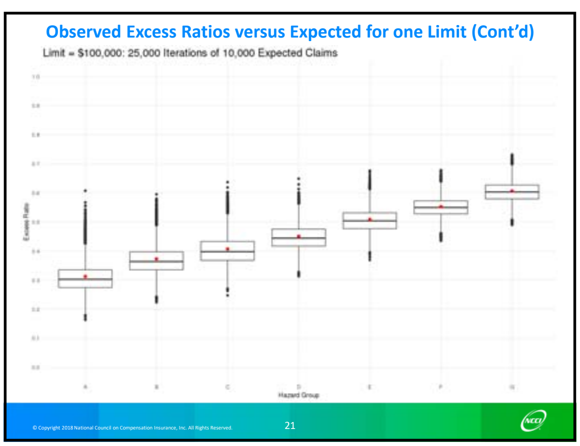
Observed Excess Ratios versus Expected for one Limit (Cont'd)
Limit = $100,000: 25,000 Iterations of 10,000 Expected Claims
Excess Ratio
Hazard Group
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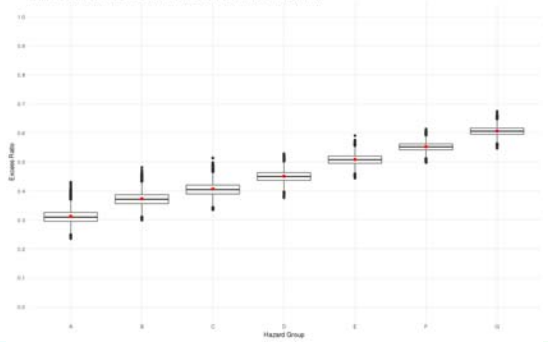
Observed Excess Ratios versus Expected for one Limit (Cont'd)

Limit = $100,000: 25,000 Iterations of 50,000 Expected Claims



Claim Count Distribution by Class

Total Claims (5 yrs)	Class Count	%	Cum %
Less than 100	59	8.9%	8.9%
100 to 200	42	6.3%	15.2%
200 to 300	18	2.7%	17.9%
300 to 400	25	3.8%	21.7%
400 to 500	17	2.6%	24.2%
500 to 1K	80	12.0%	36.2%
1K to 3K	130	19.5%	55.8%
3K to 5K	70	10.5%	66.3%
5K to 8,315	63	9.5%	75.8%
8,315 to 16,625	66	9.9%	85.7%
16,625 to 33,250	41	6.2%	91.9%
Over 33,250	54	8.1%	100.0%

- The expected five-year claim count of two thirds of the 665 classes is less than 5,000

Distribution of Excess Ratios and HGs

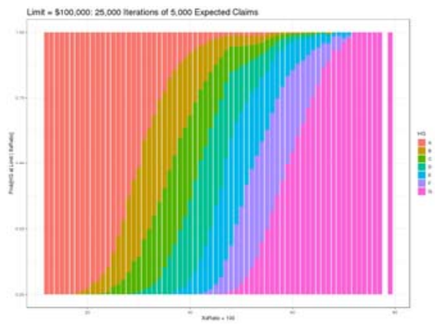
- How well can we infer the hazard group of an excess ratio?
  - The table below reshapes the simulated data graphed in slide 20, i.e.,
    Limit = $100,000: 25,000 Iterations of 5,000 Expected Claims
  - With what probability can we assign a stray excess ratio (here rounded to the nearest percent, XsRat00) to the proper hazard group?
  - Correct assignment is most probable for ratios near their HG means

HG	Mean XsRat00	Correct HG	Adjacent HGs	Others	Total Probability
A	31	41%	38%	21%	100%
B	37	32%	48%	19%	100%
C	40	32%	54%	14%	100%
D	45	35%	48%	16%	100%
E	50	36%	47%	17%	100%
F	55	38%	47%	15%	100%
G	60	54%	31%	15%	100%

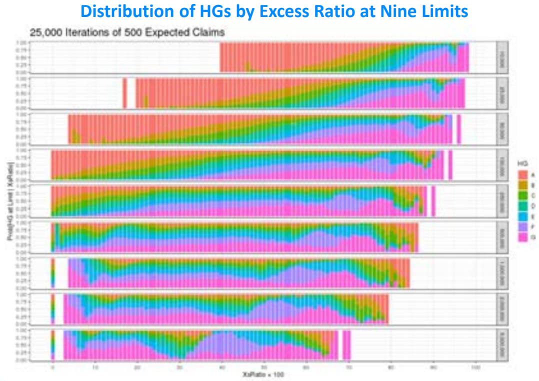
  - The Appendix provides details

Distribution of Excess Ratios and HGs (Cont'd)

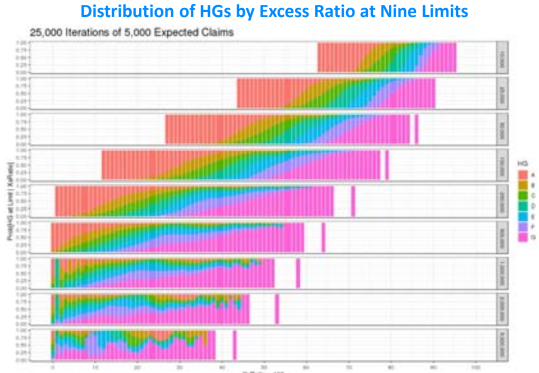
- HG regions as stacked probability bars on a horizontal excess ratio axis



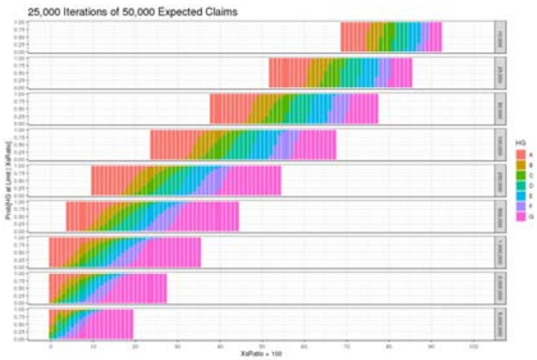
Distribution of HGs by Excess Ratio at Nine Limits



Distribution of HGs by Excess Ratio at Nine Limits



Distribution of HGs by Excess Ratio at Nine Limits



Concluding Comments

- We deemed as a benchmark a portfolio of 5,000 expected claims at a loss limit of $100,000
- The five-year *E*[*N*] of two-thirds of Workers Compensation classes is less than 5,000
  - The between variance (VHM/TotVar) of this benchmark portfolio is 74% (see the Appendix)
  - Hazard groups grow more distinct with more expected claims, and less distinct with higher loss limits (full list in the Appendix)
- By design, this simulation considers only process uncertainty. Parameter and model uncertainties would increase variability

Appendix

- Distribution of Excess Ratios and HGs (3 slides)
- Distribution of HGs by Excess Ratio at Nine Limits (5 slides)

Distribution of Excess Ratios and HGs

- Limit = $100,000: 25,000 Iterations of 5,000 Expected Claims

XsRat00	HG A	HG B	HG C	HG D	HG E	HG F	HG G	Total
12	1							1
13	2							2
14	11							11
15	16							16
16	32							32
17	69							69
18	142	1						143
19	208	3						211
20	378	7	1					386
21	501	22	1					524
22	769	38	2					809
23	914	71	5					990
24	1,194	127	12					1,333
25	1,335	205	49					1,589
26	1,553	366	76	1				1,996
27	1,604	509	123	1				2,237
28	1,638	752	211	5				2,606
29	1,737	952	379	11				3,079
30	1,692	1,120	441	28				3,281
31	1,498	1,388	698	65				3,649
32	1,477	1,526	879	112				3,994
33	1,299	1,582	1,069	171				4,121
34	1,119	1,705	1,254	268	2			4,348
35	1,023	1,692	1,363	466	6			4,550
36	852	1,681	1,550	695	17			4,795
37	705	1,566	1,627	873	48			4,819
38	603	1,405	1,746	1,106	110			4,970
39	487	1,269	1,641	1,286	142			4,825
40	402	1,183	1,607	1,488	301	3		4,984
41	325	1,008	1,491	1,718	452	6		5,000
42	245	842	1,362	1,882	660	20		5,011
43	213	679	1,171	1,785	840	40		4,728
44	148	586	1,026	1,846	1,103	91	1	4,801
45	115	461	854	1,624	1,379	177	3	4,613

XsRat00	HG A	HG B	HG C	HG D	HG E	HG F	HG G	Total
46	87	364	712	1,541	1,526	289	14	4,533
47	69	250	540	1,293	1,742	559	22	4,475
48	52	194	432	1,115	1,838	773	61	4,465
49	50	209	415	908	1,804	1,140	96	4,622
50	54	204	353	840	1,738	1,411	196	4,796
51	71	186	344	719	1,580	1,729	339	4,968
52	70	180	286	625	1,485	1,924	493	5,063
53	68	174	281	578	1,369	2,043	732	5,245
54	73	145	243	486	1,328	2,093	1,076	5,444
55	43	110	225	428	1,189	2,029	1,331	5,355
56	17	92	169	286	1,036	1,963	1,637	5,200
57	14	50	116	202	848	1,820	1,836	4,886
58	4	30	87	181	659	1,654	1,984	4,599
59	9	16	52	116	494	1,379	2,027	4,093
60	6	11	28	88	386	1,116	1,931	3,566
61	4	10	27	51	292	864	1,913	3,161
62		12	17	41	229	660	1,786	2,745
63		9	12	33	149	454	1,623	2,280
64		2	15	14	94	321	1,380	1,826
65		2	4	11	70	210	1,176	1,473
66		1	2	4	39	111	977	1,134
67	2	2	2	8	24	58	778	874
68		1		1	8	33	583	626
69					5	23	368	396
70					6	5	253	264
71					2	2	178	182
72							113	113
73							64	64
74							17	17
75							9	9
76							1	1
77							1	1
79							1	1

XsRat00	HG A	HG B	HG C	HG D	HG E	HG F	HG G	Total
Count	25,000	25,000	25,000	25,000	25,000	25,000	25,000	175,000
Expected	31.3	37.3	40.7	45.1	50.8	55.2	60.6	
Mean	30.5	36.5	39.8	44.4	50.2	54.8	60.0	45.2
Var	43.4	42.8	42.9	34.2	29.7	20.6	22.5	127.6

Distribution of Excess Ratios and HGs (Cont'd)

- The between variance (i.e., variance of the hypothetical means, or VHM) measures the distinctness of the HG distributions, because:

$$E\left[Var[X \mid HG]\right] + Var\left[E[X \mid HG]\right] = Var[X]$$

XsRat00	HG A	HG B	HG C	HG D	HG E	HG F	HG G	Total
Count	25,000	25,000	25,000	25,000	25,000	25,000	25,000	175,000
Mean	30.5	36.5	39.8	44.4	50.2	54.8	60.0	45.2
E2	973	1,377	1,630	2,003	2,549	3,019	3,625	2,168
Var	43.4	42.8	42.9	34.2	29.7	20.6	22.5	127.6
Var	43.4	42.8	42.9	34.2	29.7	20.6	22.5	
E[Var[ ]]								33.7
Dev	-14.7	-8.6	-5.3	-0.8	5.0	9.6	14.9	0.0
Dev Sq	215.5	74.7	28.5	0.6	25.3	91.9	220.6	
Var[E[ ]]								93.9
Sum of both conditionals								127.6

E[Var[XsRat | HG]]	Var[E[XsRat | HG]]	Var[XsRat]
EVPV	VHM	Tot Var
33.7	93.9	127.6
26%	74%	100%

Between Variance = 74% of overall variance

Distribution of Excess Ratios and HGs (Cont'd)

- How well can we infer the hazard group of an excess ratio?

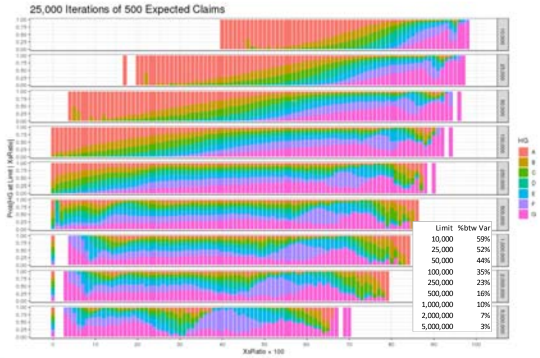
XsRat00	HG A	HG B	HG C	HG D	HG E	HG F	HG G
12	1.00						
13	1.00						
14	1.00						
15	1.00						
16	1.00						
17	1.00						
18	0.99	0.01					
19	0.99	0.01					
20	0.98	0.02	0.00				
21	0.96	0.04	0.00				
22	0.95	0.05	0.00				
23	0.92	0.07	0.01				
24	0.90	0.10	0.01				
25	0.84	0.13	0.03				
26	0.78	0.18	0.04	0.00			
27	0.72	0.23	0.05	0.00			
28	0.63	0.29	0.08	0.00			
29	0.56	0.31	0.12	0.00			
30	0.52	0.34	0.13	0.01			
31	0.41	0.38	0.19	0.02			
32	0.37	0.38	0.22	0.03			
33	0.32	0.38	0.26	0.04			
34	0.26	0.39	0.29	0.06	0.00		
35	0.22	0.37	0.30	0.10	0.00		
36	0.18	0.35	0.32	0.14	0.00		
37	0.15	0.32	0.34	0.18	0.01		
38	0.12	0.28	0.35	0.22	0.02		
39	0.10	0.26	0.34	0.27	0.03		
40	0.08	0.24	0.32	0.30	0.06	0.00	
41	0.07	0.20	0.30	0.34	0.09	0.00	
42	0.05	0.17	0.27	0.38	0.13	0.00	
43	0.05	0.14	0.25	0.38	0.18	0.01	
44	0.03	0.12	0.21	0.38	0.23	0.02	0.00
45	0.02	0.10	0.19	0.35	0.30	0.04	0.00

XsRat00	HG A	HG B	HG C	HG D	HG E	HG F	HG G
46	0.02	0.08	0.16	0.34	0.34	0.06	0.00
47	0.02	0.06	0.12	0.29	0.39	0.12	0.00
48	0.01	0.04	0.10	0.25	0.41	0.17	0.01
49	0.01	0.05	0.09	0.20	0.39	0.25	0.02
50	0.01	0.04	0.07	0.18	0.36	0.29	0.04
51	0.01	0.04	0.07	0.14	0.32	0.35	0.07
52	0.01	0.04	0.06	0.12	0.29	0.38	0.10
53	0.01	0.03	0.05	0.11	0.26	0.39	0.14
54	0.01	0.03	0.04	0.09	0.24	0.38	0.20
55	0.01	0.02	0.04	0.08	0.22	0.38	0.25
56	0.00	0.02	0.03	0.06	0.20	0.38	0.31
57	0.00	0.01	0.02	0.04	0.17	0.37	0.38
58	0.00	0.01	0.02	0.04	0.14	0.36	0.43
59	0.00	0.00	0.01	0.03	0.12	0.34	0.50
60	0.00	0.00	0.01	0.02	0.11	0.31	0.54
61	0.00	0.00	0.01	0.02	0.09	0.27	0.61
62		0.00	0.01	0.01	0.08	0.24	0.65
63		0.00	0.01	0.01	0.07	0.20	0.71
64		0.00	0.01	0.01	0.05	0.18	0.76
65		0.00	0.00	0.01	0.05	0.14	0.80
66		0.00	0.00	0.00	0.03	0.10	0.86
67	0.00	0.00	0.00	0.01	0.03	0.07	0.89
68		0.00		0.00	0.01	0.05	0.93
69					0.01	0.06	0.93
70					0.02	0.02	0.96
71					0.01	0.01	0.98
72							1.00
73							1.00
74							1.00
75							1.00
76							1.00
77							1.00
79							1.00

Limit = $100,000: 25,000 Iterations of 5,000 Expected Claims

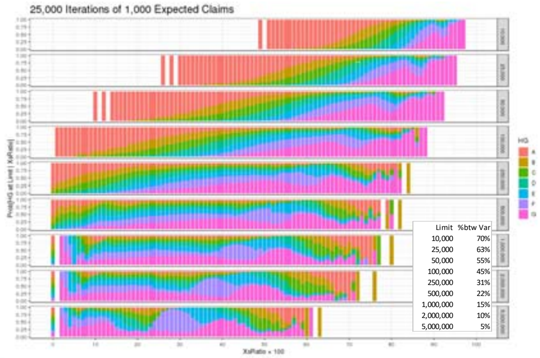
Distribution of HGs by Excess Ratio at Nine Limits

25,000 Iterations of 500 Expected Claims



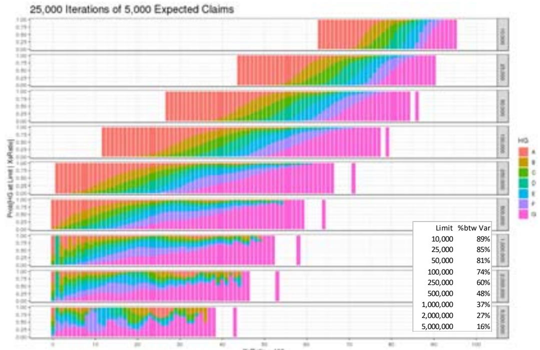
Distribution of HGs by Excess Ratio at Nine Limits

25,000 Iterations of 1,000 Expected Claims



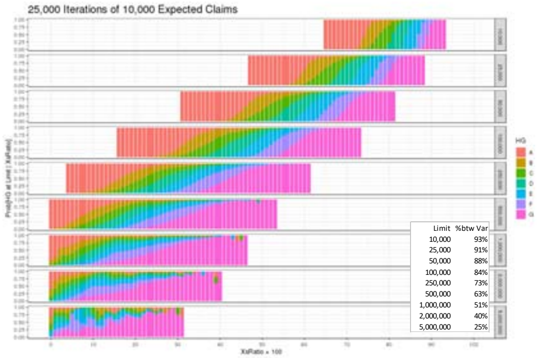
Distribution of HGs by Excess Ratio at Nine Limits

25,000 Iterations of 5,000 Expected Claims



Distribution of HGs by Excess Ratio at Nine Limits

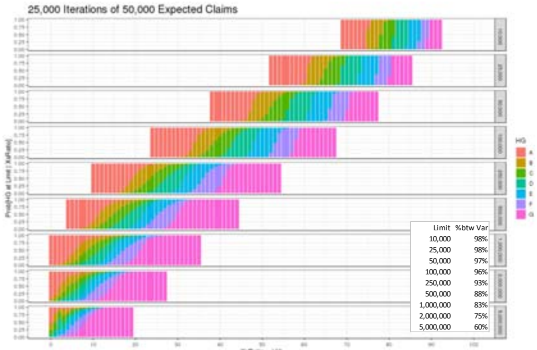
25,000 Iterations of 10,000 Expected Claims



Limit	%btw Var
10,000	93%
25,000	91%
50,000	88%
100,000	84%
250,000	73%
500,000	63%
1,000,000	51%
2,000,000	40%
5,000,000	25%

Distribution of HGs by Excess Ratio at Nine Limits

25,000 Iterations of 50,000 Expected Claims



Limit	%btw Var
10,000	98%
25,000	98%
50,000	97%
100,000	96%
250,000	93%
500,000	88%
1,000,000	83%
2,000,000	75%
5,000,000	60%