ODP Bootstrap Model

ODP Bootstrap Model

Bootstrapping

- Non-parametric bootstrapping is the same as simulating observations from an Empirical distribution
- Each observation in the sample has a chance of 1/*n* of being randomly selected
- There are also procedures called semi-parametric and parametric bootstrapping which we will cover later

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Age-to-Age Factors

- Treat each column of age-to-age factors as an empirical sample of size *n*, where *n* is the number of observations
- Randomly select an age-to-age factor from each column
- Compute age-to-ultimate factors
- Compute ultimates and unpaid amounts
- Do this many times

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ODP Bootstrap Model

Example with Mack Data

- Generate 9 uniform random variables
- Range value the random numbers
- Use percentile function "=percentile(column of age-to-age factors, random #)" to select random factor for each age except last (which has no variability)
- Compute age-to-ultimate factors
- Compute ultimates and unpaid (IBNR) amounts

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Example with Mack Data

Age	1	2	3	4	5	6	7	8	9
Random#	0.298	0.386	0.943	0.781	0.909	0.319	0.503	0.872	NA
Randomfactor	2.272	1.489	1.781	1.220	1.192	1.028	1.033	1.029	1.009
ATU	9.676	4.259	2.860	1.606	1.316	1.104	1.074	1.039	1.009
Incurred	2,063	5,395	13,112	12,314	15,852	26,180	27,067	23,466	16,704
Ultimate	19,962	22,977	37,500	19,776	20,861	28,903	29,070	24,381	16,858
IBNR	17,899	17,582	24,388	7,462	5,009	2,723	2,003	915	154
SumIBNR	78,136								

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Exercise

- Bootstrap 10-1000 realizations of age-to-age factors for ages 1-8 for Mack data.
- Assuming 9-10 factor is a constant, compute age-to-ultimate factors for Bootstrapped data
- Compute Ultimates and IBNR Amounts
- What is the mean, standard deviation and coefficient of variation by year and in total (all years combined) of the unpaid (IBNR) amounts?
- Extra Credit: Create a histogram of your results.
- Bonus: Run Exercise again using age-to-age factors by accident year for ages 1-8 (with graph).

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ODP Bootstrap Model

Example

Summary Non-Parametric Bootstrap

AY	Incurred to Date	Average Ultimate	Average IBNR	Std. Dev.	C.o.V.	Minimum	Maximum
1981	18,834	18,834	0	0	0.0%	0	0
1982	16,704	16,858	154	0	0.0%	154	154
1983	23,466	24,111	645	207	32.1%	285	999
1984	27,067	28,767	1,700	285	16.8%	1,054	2,365
1985	26,180	28,923	2,743	881	32.1%	1,055	5,224
1986	15,852	19,855	4,003	1,106	27.6%	1,188	6,749
1987	12,314	18,216	5,902	1,237	21.0%	2,804	9,972
1988	13,112	24,845	11,733	4,713	40.2%	4,715	29,759
1989	5,395	16,925	11,530	4,898	42.5%	4,085	33,104
1990	2,063	41,335	39,272	53,325	135.8%	4,838	528,560
Total	160,987	238,669	77,682	56,716	73.0%	25,903	599,037

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ODP Bootstrap Overview

- Non-parametric bootstrap involved sampling of age-to-age ratios.
- For semi-parametric bootstrap, we will use parameters to calculate residuals and sample the residuals.
- The residuals create new samples of the triangle.
- Then for each new triangle we can make a projection.
- And for each projection we can add random noise.
- Let's start with a simple example to review the algorithm… then review the theory.

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ODP Bootstrap Overview

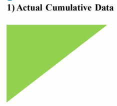
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Cumulative Data

	1	2	3	4	5	6
1	95	150	180	200	210	215
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	180			
5	130	170				
6	125					

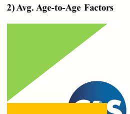
1) Actual Cumulative Data



Cumulative Data

	1	2	3	4	5	6
1	95	150	180	200	210	215
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	180			
5	130	170				
6	125					
Factors:		1.429	1.151	1.128	1.037	1.024

2) Avg. Age-to-Age Factors



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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Fitted Cumulative Data

	1	2	3	4	5	6
1	109.16	155.94	179.45	202.50	210.00	215.00
2	109.16	155.94	179.45	202.50	210.00	
3	113.20	161.71	186.10	210.00		
4	109.49	156.41	180.00			
5	119.00	170.00				
6	125.00					
Factors:		1.429	1.151	1.128	1.037	1.024

3) "Fit" Cumulative Data

Fitted Incremental Data

	1	2	3	4	5	6
1	109.16	46.78	23.51	23.05	7.50	5.00
2	109.16	46.78	23.51	23.05	7.50	
3	113.20	48.51	24.39	23.90		
4	109.49	46.92	23.59			
5	119.00	51.00				
6	125.00					

4) "Fitted" Incremental Data

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ODP Bootstrap Overview

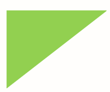
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

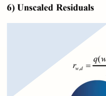
5) Actual Incremental Data



Unscaled Pearson Residuals

	1	2	3	4	5	6
1	-1.35	1.20	1.34	-0.64	0.91	0.00
2	0.08	0.47	-1.76	1.45	-0.91	
3	-0.77	1.65	0.12	-0.80		
4	1.00	-1.74	0.29			
5	1.01	-1.54				
6	0.00					

6) Unscaled Residuals



$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m^z_{w,d}}}$$

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ODP Bootstrap Overview

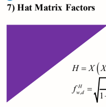
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Hat Matrix Factors

	1	2	3	4	5	6
1	1.65	1.27	1.23	1.29	1.44	0.00
2	1.65	1.27	1.23	1.29	1.44	
3	1.68	1.28	1.23	1.31		
4	1.80	1.30	1.24			
5	2.06	1.35				
6	0.00					

7) Hat Matrix Factors



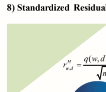
$$H = X\left(X^T W X\right)^{-1} X^T W$$

$$f^H_{w,d} = \sqrt{\frac{1}{1 - H_{i,i}}}$$

Standardized Pearson Residuals

	1	2	3	4	5	6
1	-2.24	1.53	1.64	-0.82	1.31	0.00
2	0.13	0.60	-2.15	1.87	-1.31	
3	-1.30	2.12	0.15	-1.04		
4	1.80	-2.26	0.36			
5	2.07	-2.07				
6	0.00					

8) Standardized Residuals



$$r^H_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m^z_{w,d}}} \times f^H_{w,d}$$

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ODP Bootstrap Model

ODP Bootstrap Overview

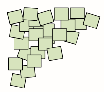
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Random Residuals

	1	2	3	4	5	6
1	-1.31	-2.07	0.36	-2.07	-2.07	1.64
2	-0.82	-0.82	-2.26	1.64	0.36	
3	2.07	1.87	1.87	-2.15		
4	-1.31	-0.82	1.80			
5	-0.82	1.80				
6	0.60					

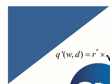
9) Sample Random Residuals



Sample Incremental Triangle

	1	2	3	4	5	6
1	95.42	32.59	25.26	13.09	1.82	8.66
2	100.56	41.16	12.55	30.92	8.49	
3	135.27	61.57	33.64	13.38		
4	95.73	41.29	32.34			
5	110.03	63.88				
6	131.70					

10) Sample Incremental Data



$$q'(w,d) = r^* \times \sqrt{m_{w,d}^z} + m_{w,d}$$

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ODP Bootstrap Overview

For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Sample Cumulative Triangle

	1	2	3	4	5	6
1	95.42	128.01	153.27	166.36	168.18	176.84
2	100.56	141.72	154.27	185.19	193.68	
3	135.27	196.84	230.49	243.87		
4	95.73	137.02	169.37			
5	110.03	173.91				
6	131.70					
Factors:	1.448	1.172	1.107	1.029	1.052	

11) Sample Age-to-Age Factors

Projected Cumulative Data

	1	2	3	4	5	6
1	95.42	128.01	153.27	166.36	168.18	176.84
2	100.56	141.72	154.27	185.19	193.68	203.65
3	135.27	196.84	230.49	243.87	251.02	263.95
4	95.73	137.02	169.37	187.43	192.93	202.87
5	110.03	173.91	203.81	225.55	232.16	244.13
6	131.70	190.67	223.46	247.30	254.55	267.66

12) Project Ultimate Values

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ODP Bootstrap Overview

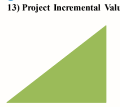
For the "ODP Bootstrap":

- Here's a simple example using a 6 x 6 triangle:

Projected Incremental Data

	1	2	3	4	5	6	Point Estimate
1							
2						9.98	9.98
3					7.15	12.93	20.08
4				18.07	5.49	9.94	33.50
5			29.91	21.74	6.61	11.96	70.22
6		58.98	32.79	23.84	7.25	13.11	135.97
							269.74

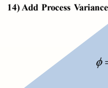
13) Project Incremental Values



Incremental Data w/ Process Variance

	1	2	3	4	5	6	Possible Outcome
1							
2						15.20	15.20
3					9.85	9.71	19.57
4				24.60	9.35	13.52	47.47
5			29.81	14.12	5.97	13.53	63.43
6		44.97	38.24	24.46	11.79	9.10	128.56
							274.23

14) Add Process Variance



$$\phi = \frac{\sum r_{w,d}^2}{N-p}$$

- Repeat steps 9 – 14, a large number of times!

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ODP Bootstrap Model

ODP Bootstrap Overview

- Start with a triangle of cumulative data:

w \ d	1	2	3	...	n-1	n
1	c(1,1)	c(1,2)	c(1,3)	...	c(1,n-1)	c(1,n)
2	c(2,1)	c(2,2)	c(2,3)	...	c(2,n-1)	
3	c(3,1)	c(3,2)	c(3,3)	...		
...	...	...				
n-1	c(n-1,1)	c(n-1,2)				
n	c(n,1)					

- For GLM, we will use the incremental data:

w \ d	1	2	3	...	n-1	n
1	q(1,1)	q(1,2)	q(1,3)	...	q(1,n-1)	q(1,n)
2	q(2,1)	q(2,2)	q(2,3)	...	q(2,n-1)	
3	q(3,1)	q(3,2)	q(3,3)	...		
...	...	...				
n-1	q(n-1,1)	q(n-1,2)				
n	q(n,1)					

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ODP Bootstrap Overview

- The GLM formulation is as follows:

$$E[q(w,d)] = m_{w,d}$$

$$Var[q(w,d)] = \phi E[q(w,d)] = \phi m_{w,d}^{z}$$

$$\ln[m_{w,d}] = \eta_{w,d}$$

$\eta_{w,d} = c + \alpha_w + \beta_d$, where: $w = 1, 2, \ldots, n$; $d = 1, 2, \ldots, n$; and $\alpha_1 = \beta_1 = 0$

$z = 0(Normal), 1(Poisson),$
$2(Gamma), \text{ or } 3(Inverse Gaussian)$

$\phi = Scale\ Parameter$

- Alternatively:

$\eta_{w,d} = \alpha_w + \beta_d$, where: $w = 1, 2, \ldots, n$ and $d = 2, 3, \ldots, n$

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ODP Bootstrap Overview

- Let's consider a simple example:

	1	2	3
1	$q(1,1)$	$q(1,2)$	$q(1,3)$
2	$q(2,1)$	$q(2,2)$	
3	$q(3,1)$		

- Transforming to a log scale:

	1	2	3
1	$\ln[q(1,1)]$	$\ln[q(1,2)]$	$\ln[q(1,3)]$
2	$\ln[q(2,1)]$	$\ln[q(2,2)]$	
3	$\ln[q(3,1)]$		

- Specify a system of equations with vectors α_w and β_d:

$$\ln[q(1,1)] = 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 0\beta_2 + 0\beta_3$$
$$\ln[q(2,1)] = 0\alpha_1 + 1\alpha_2 + 0\alpha_3 + 0\beta_2 + 0\beta_3$$
$$\ln[q(3,1)] = 0\alpha_1 + 0\alpha_2 + 1\alpha_3 + 0\beta_2 + 0\beta_3$$
$$\ln[q(1,2)] = 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 1\beta_2 + 0\beta_3$$
$$\ln[q(2,2)] = 0\alpha_1 + 1\alpha_2 + 0\alpha_3 + 1\beta_2 + 0\beta_3$$
$$\ln[q(1,3)] = 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 1\beta_2 + 1\beta_3$$

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ODP Bootstrap Model

ODP Bootstrap Overview

- Converting to matrix notation we have:

$$Y = X \times A$$

- Where:

$$Y = \begin{bmatrix} \ln[q(1,1)] & 0 & 0 & 0 & 0 & 0 \\ 0 & \ln[q(2,1)] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ln[q(3,1)] & 0 & 0 & 0 \\ 0 & 0 & 0 & \ln[q(1,2)] & 0 & 0 \\ 0 & 0 & 0 & 0 & \ln[q(2,2)] & 0 \\ 0 & 0 & 0 & 0 & 0 & \ln[q(1,3)] \end{bmatrix},$$

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \text{ and } A = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

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ODP Bootstrap Overview

- Solving for the parameters in A that minimize the difference between Y and W, where:

$$W = \begin{bmatrix} \ln[m_{1,1}] & 0 & 0 & 0 & 0 & 0 \\ 0 & \ln[m_{2,1}] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ln[m_{3,1}] & 0 & 0 & 0 \\ 0 & 0 & 0 & \ln[m_{1,2}] & 0 & 0 \\ 0 & 0 & 0 & 0 & \ln[m_{2,2}] & 0 \\ 0 & 0 & 0 & 0 & 0 & \ln[m_{1,3}] \end{bmatrix}$$

- X = Design Matrix, and W = Weight Matrix
- Then we have:

$$\begin{aligned} \ln[m_{1,1}] &= \eta_{1,1} = \alpha_1 \\ \ln[m_{2,1}] &= \eta_{2,1} = \alpha_2 \\ \ln[m_{3,1}] &= \eta_{3,1} = \alpha_3 \\ \ln[m_{1,2}] &= \eta_{1,2} = \alpha_1 + \beta_2 \\ \ln[m_{2,2}] &= \eta_{2,2} = \alpha_2 + \beta_2 \\ \ln[m_{1,3}] &= \eta_{1,3} = \alpha_1 + \beta_2 + \beta_3 \end{aligned}$$

and

	1	2	3
1	$\ln[m_{1,1}]$	$\ln[m_{1,2}]$	$\ln[m_{1,3}]$
2	$\ln[m_{2,1}]$	$\ln[m_{2,2}]$	
3	$\ln[m_{3,1}]$		

Finally, exponentiating we get:

	1	2	3
1	$m_{1,1}$	$m_{1,2}$	$m_{1,3}$
2	$m_{2,1}$	$m_{2,2}$	
3	$m_{3,1}$		

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ODP Bootstrap Overview

- Using this “GLM Framework” and setting z=1 (Poisson), the solution exactly replicates the “fitted” values using volume-weighted average age-to-age ratios!
- This is generally referred to as the Over-Dispersed Poisson (ODP) Bootstrap model.
- Instead of solving the GLM, we can simplify by using the volume-weighted average ratios.
- We refer to this as the “ODP Bootstrap”
- The “ODP Bootstrap” also improves issues with negative incremental values.

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ODP Bootstrap Model

ODP Bootstrap Overview

- Using a model fit to the data, bootstrapping involves sampling the residuals with replacement, using:

$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}}$$

- From the sampled residuals and fitted incremental values, we can derive a sample triangle using:

$$q'(w,d) = r^{*} \times \sqrt{m_{w,d}^{z}} + m_{w,d}$$

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ODP Bootstrap Overview

- However, in order to correct for a bias in (and standardize) the residuals, the GLM framework requires a hat matrix adjustment factor:

$$H = X\left(X^{T}WX\right)^{-1}X^{T}W$$

$$f_{w,d}^{H} = \sqrt{\frac{1}{1-H_{i,i}}}$$

Can approximate with Degrees of Freedom Adjustment Factor:

$$f^{DoF} = \sqrt{\frac{N}{N-p}}$$

Where: N = Number of Data Cells [e.g., n x (n+1) ÷ 2]
p = Number of Parameters [e.g., 2 x n -1]

- Standardized Residuals:

$$r_{w,d}^{H} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}} \times f_{w,d}^{H}$$

Or Scaled Residuals:

$$r_{w,d}^{S} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}} \times f^{DoF}$$

- Continuing the bootstrap process…

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ODP Bootstrap Overview

- Each sample incremental triangle can be converted to cumulative values
- Sample age-to-age factors can be calculated (parameter risk)
- A point estimate can be calculated
- We can add process variance to the future incremental values (from the point estimate) using a Poisson (or Gamma) distribution assuming each incremental cell is the mean and the variance is the cell value times the scale parameter (i.e., to over-disperse the variance):

$$\phi = \frac{\sum r_{w,d}^{2}}{N-p}$$

- Repeat a significant number of iterations.

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ODP Bootstrap Model

"Fitted" Incremental Triangle

- Work Backwards from observations on diagonal to create estimated cumulative triangle
  - $\hat{c}(w,n\text{-}1) = c(w,n)/F(n\text{-}1)$
  - $\hat{c}(w,n\text{-}2) = \hat{c}(w,n\text{-}1)/F(n\text{-}2)$
  - Fill in all cumulative entries on triangle
- Compute estimated or "fitted" incremental triangle

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Exercises

- Compute fitted incremental triangle from "Exercise" data
  - Use weighted average loss development factors
  - Compute fitted cumulative triangle
  - Compute fitted incremental triangle

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Standardized Residuals

- In "Diagnostics" section, we used standardized residual:

$$z = \frac{x_i - \mu}{\sigma}$$

- More general Pearson Residual used with GLM models:

$$r = \frac{x - \mu}{\sqrt{Var(\mu)}}$$

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Exercise

- Compute Unscaled Pearson Residuals from the “Exercise” incremental data
  - Assume Var(x) = E(x) = fitted incremental value

$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}}}$$

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Pearson Residuals

- If data assumed Normally distributed, Pearson Residual = standardized residual
- If data assumed Poisson, then:

$$Var(\mu) = \mu, \text{ so } r = \frac{x_i - \mu}{\sqrt{\mu}}$$

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Over-Dispersed Poisson

- Often Poisson distributed: Var > Mean
- One of the “Over-Dispersed Poisson” models uses the constant ϕ to inflate Variance:

$$Var(\mu) = \phi\mu, \text{ and Scaled Pearson Residual is } \frac{x - \mu}{\sqrt{\phi\mu}}$$

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ODP Bootstrap Model

Scale Parameter

- Using the Chi-Squared statistic:
  - N = sample size, p = # parameters

$$\phi = X^2 = \sum_N \frac{[x_i - E(x_i)]^2}{(N-p)E(x_i)}$$

- Scaled Residual is:

$$r_i = \frac{x_i - E(x_i)}{\sqrt{\phi E(x_i)}}$$

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Poisson Triangles

- The Poisson has been a useful Parametric assumption in modeling loss development triangles
- The "semi" Parametric bootstrap does not require a distribution assumption but
  - It uses a Pearson Residual
  - The Standardized (or Scaled) Pearson Residual follows the Poisson assumption

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Triangle of Residuals

- Using actual and "fitted" incremental triangles, compute Unscaled Pearson Residuals:

$$e_U(w,d) = [q(w,d) - \hat{q}(w,d)] / \sqrt{\hat{q}(w,d)}$$

- Calculate Degrees of Freedom Adjustment:

$$f^{DoF} = \sqrt{\frac{N}{N-p}}$$

- Calculate Scaled Pearson Residuals:

$$e_s(w,d) = e_u(w,d) \times f^{DoF}$$

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ODP Bootstrap Model

Scale Parameter

- Use Unscaled Pearson Residuals and Degrees of Freedom to calculate the Scale Parameter:

$$\phi = \frac{\sum e_u(w,d)^2}{N-p}$$

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Exercise

- Using "Exercise" data and results of prior exercises
  1. Compute triangle of square of Unscaled Pearson Residuals
  2. Compute the triangle's degrees of freedom
  3. Using 1. and 2. compute the Scale Parameter for "Exercise" data
  4. Compute the triangle's Degrees of Freedom Adjustment Factor
  5. Compute triangle of Scaled Pearson Residuals

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Bootstrap Residuals

- For each cell in the triangle, randomly select a Scaled Pearson Residual (with replacement)
- Transform residual into an incremental value for the triangle

$$q_s(w,d) = \hat{q}(w,d) + [e_s(w,d) \times \sqrt{\hat{q}(w,d)}]$$

- Calculate cumulative sample triangle
- Compute age-to-age factors

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ODP Bootstrap Model

Exercise

- Create a table of Scaled Pearson Residuals, using results of previous exercise
- Simulate a bootstrap triangle of residuals
- Create a triangle of incremental values from bootstrapped residuals
- Compute a cumulative triangle
- Compute weighted average age-to-age factors

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Process Variance

- Use Age-to-Age factors to compute ultimate for sample data
- Calculate incremental values for completed triangle
- Use the Gamma distribution to simulate random incremental values with:
  - Mean = sample incremental
  - Variance = sample incremental x Scale Parameter

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Distribution of Estimates

- Add incremental values after process variance to get ultimate and unpaid estimates
- Sum the unpaid amounts to get total unpaid
- Repeat many times...

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ODP Bootstrap Model

Distribution of Estimates

- A “trick” is needed to easily compute a distribution of unpaid amounts
- Use the Data Table (menu) function of Excel
- ={Table(,input cell=a constant)}
- Or use VBA to write a macro

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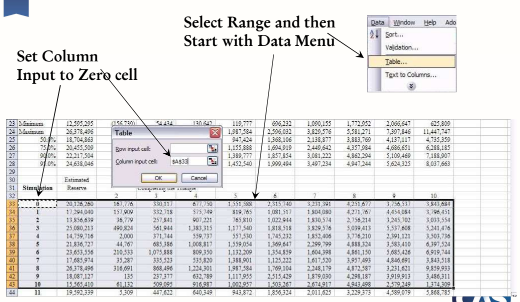
Data Table Function



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Exercise

- Review bootstrap calculation in “Bootstrap Models” spreadsheet
- Where is the calculation of
  - The fitted cumulative triangle?
  - The fitted incremental triangle?
  - The residuals?
- What formula is used to resample residuals?
- Where is estimation of bootstrapped unpaid?
- What diagnostics are used?
- Paste value a new triangle into the Inputs sheet and run a new model for 100 iterations

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ODP Bootstrap Model

Parametric Bootstrap

- Sampling of residuals is limited to past observations
- For a 10 x 10 triangle, 53 residuals are sampled so extremes could be considered 1 in 53 events, but we usually want 1 in 100 or 1 in 200 events.
- Solution: Parameterize residuals and simulate from a distribution

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Questions on ODP Bootstrap Model?

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