Introduction

Instructional Approach to Mack and ODP Bootstrap Models

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Overview

- Introductions
- Notation
- Ranges vs. Distributions
- Mack Model
- ODP Bootstrap Model

Page I.2

Logistics Overview

- Facilities
- Time / Breaks
- Language, Terminology & Currency
- Your view blocked? / Can't See?
- Feedback forms
- Don't Skip Ahead
- Have Fun / Ask Questions
- Examples NOT Commercial Software

Page I.3

Introduction

Notation

- w – denotes an Accident (or Policy) period ("when").
- d – denotes a development age ("delay").
- k – denotes a diagonal or constant $w + d$.
- $c(w,d)$ – denotes cumulative losses for accident (or policy) period w, and development age d.
- $c(w,n)$ – denotes ultimate losses for accident (or policy) period w [also denoted as $U(w)$].
- $R(w,d)$ – denotes future development for accident (or policy) period w, and development age d.

Note: $R(w,d) = U(w) – c(w,d)$

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Page I.4

Notation

- $q(w,d)$ – denotes incremental losses for accident (or policy) period w, and development age d.
- $f(d)$ – denotes the factor applied to $c(w,d)$ to estimate $q(w,d+1)$.
- $F(d)$ – denotes the factor applied to $c(w,d)$ to estimate $c(w,d+1)$. For example, the d to $d+1$ age-to-age factor.

[Note: WP Report uses $c(w,n)$]

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Page I.5

Notation

- $G(w)$ – denotes a factor relating to accident (or policy) year w.
- $h(w+d)$ – denotes a factor relating to diagonal k (or calendar year) in which $w+d$ is constant.
- $e(w,d)$ – denotes the mean of zero random fluctuation which occurs in cell w, d.

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Page I.6

Notation

- $E(x)$ – denotes the expectation of the random variable *x*. (Also μ_x)
- $Var(x)$ – denotes the variance of the random variable *x*. (Also σ_x^2 or $Sigma_x^2$)
- W – Weight
- N – Total number of Accident (Policy) periods
- n – Total number of Development periods
- $\hat{x}$ – Estimate of *x*.

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Page I.7

Notation Example

Cumulative Losses

AY	0	1	2
1981	5,012	8,269	10,907
1982	106	4,285	
1983	3,410		

w Can be Year (i.e., 1981) or Index (i.e., 1)

Notation

AY	0	1	2
1981	c(1981,0)	c(1981,1)	c(1981,2)
1982	c(1982,0)	c(1982,1)	
1983	c(1983,0)		

d

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Page I.8

Notation Example

Incremental Losses

AY	0	1	2
1981	5,012	3,257	2,638
1982	106	4,179	
1983	3,410		

Calendar Effect denoted $h(1982+1)$ or $h(1983)$ or $h(3)$

Notation

AY	0	1	2
1981	q(1981,0)	q(1981,1)	q(1981,2)
1982	q(1982,0)	q(1982,1)	
1983	q(1983,0)		

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Page I.9

Notation Example

Age-to-Age Factors			Notation		
AY	1	2	AY	1	2
1981	1.650	1.319	1981	F(1981,1)	F(1981,2)
1982	40.425		1982	F(1982,1)	
Mean	21.037	1.319	Mean	F(1)	F(2)

Page I.10

Ranges vs. Distributions

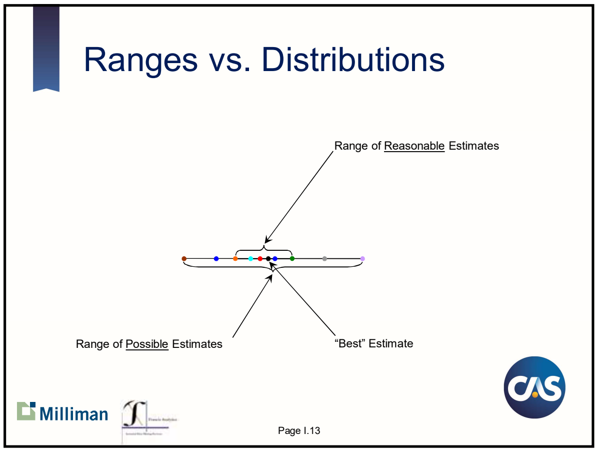
- A **Range** is not the same as a **Distribution**
- *A Range of Reasonable Estimates* is a range of estimates that could be produced by appropriate actuarial methods or alternative sets of assumptions that the actuary judges to be reasonable.
- A *Distribution* is a statistical function that attempts to quantify probabilities of all possible outcomes.

Page I.11

Ranges vs. Distributions

- A **Range** is generally considered to be either a subset of "*central estimates*" or a subset of the "*possible outcomes*".
- For a "*central estimate*" the incremental values will essentially have the random movements "*averaged*" or "*smoothed*" out.
- A "*possible outcome*" will generally include random movements in the incremental values (e.g., calendar period payments within each accident period).

Page I.12


Ranges vs. Distributions
Range of Reasonable Estimates
Range of Possible Estimates
"Best" Estimate
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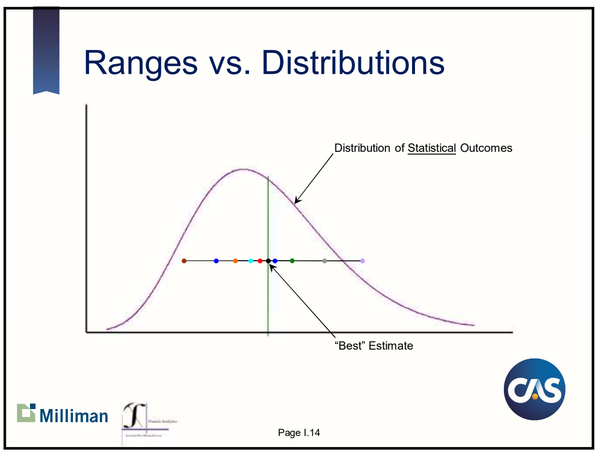
Page I.13


Ranges vs. Distributions
Distribution of Statistical Outcomes
"Best" Estimate
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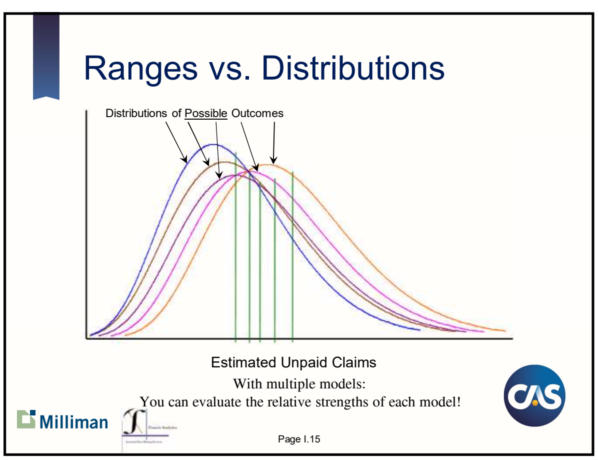
Page I.14


Ranges vs. Distributions
Distributions of Possible Outcomes
Estimated Unpaid Claims
With multiple models:
You can evaluate the relative strengths of each model!
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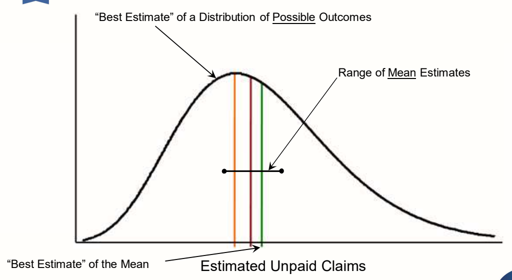
Page I.15

Ranges vs. Distributions



With multiple models:
You can use credibility weights to get your "best estimate"!

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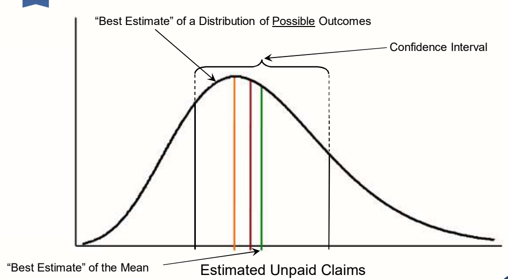
Page I.16

Ranges vs. Distributions



With multiple models:
You can calculate confidence intervals.

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Page I.17

Basic Models

- And Now for Something Completely Different…
  - Mack Model
  - ODP Bootstrap Model

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Page I.18

Final Questions?

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Page I.19

Suggested Bibliography

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Page I.20