Chasing our Tails

With our Risk Models

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Fat Tails

Many risks taken by insurers have Fat Tails

Fat Tails
So Why is that a Problem?

1. We model risks
2. We have no data to fit to tails
3. So we extrapolate
4. And we validate our models by validating our extrapolation process
5. We also explain our models with a process description
6. That leaves non-modelers in the dust
7. Which may be a problem

Today's Talk
"Chasing our Tails with Risk Models"

- How different people make decisions
- How we might bridge the gap between modelers and non-modelers regarding Fat Tails
  - Suggest using a new/old metric
    - **Coefficient of Risk (COR)**
  - **Provide a variety of examples of COR values and use**

Decision Making Models of the World

Natural Decision Making *From the Gut*	**Newtonian** *Logical*
Statistical *Future as Multiverse*	**Systems Analysis** *Complex Independencies*

Natural Decision Making (NDM)

Gut

- Pragmatic / Reactive
- Trial and Error
- Heuristics and Biases
- Behavioral Economics

Natural Decision Making

Heuristics and Gut Reactions

Advantages

- **Fast and Frugal** (Gigerenzer)
  - Our brains automatically sort through thousands of factors and identify just a few that are actually needed to make a good decision.
- **Trust your Gut**
  - The more you trust your gut the better your intuition gets
  - Natural process of developing Heuristics
- **Decision making requires emotion**

Disadvantages

- **Biases**
  - Humans tend to make systematic and predictable mistakes
- **Luck vs. Skill**
  - Hard to distinguish between luck and skill
- **Hard to know**
  - When your gut doesn't have a clue
- **Tend to like**
  - Out of the money puts

My Favorite Biases

Anchoring	Availability heuristic	Confirmation bias	Endowment effect
Framing effect	Gambler's fallacy	Hindsight bias	Illusion of control
Overconfidence effect	Status quo bias	Survivorship bias	Ostrich Effect

Actuaries' Guts

- While early actuarial work usually didn't fall under NDM
  - Actuarial assumptions almost universally incorporated what came to be called **Provisions for Adverse Deviation**
    - For the longest time, PADs were totally from the actuary's gut
    - But only very experienced actuaries had guts
    - Eventually, Australians replaced the gut with the 75%tile

Newtonian

Logical

Deterministic World

Maximum Likely Scenario

Cause effect

Risk Reward

Single Frequency/Severity view of risk

Newtonian

Scientific Cause and Effect

Advantages

- **"Scientific Method"**
  - Provides a clear path to proceed with decision making
  - Eliminates guesswork and subjectivity
  - Reduces errors
- **Can be applied to complex problems**
  - Usually by breaking a big problem up into smaller more tractable problems
- **Decision making without emotion**

Disadvantages

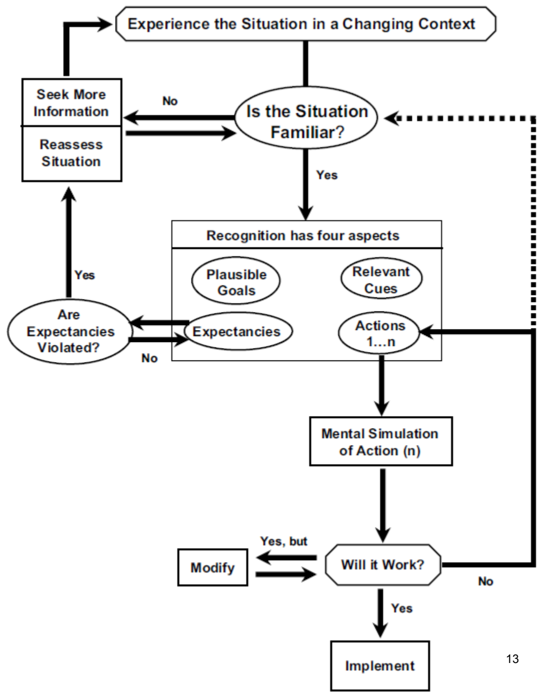
- **Requires high analytical competence**
  - To break a problem up into the right pieces that can be solved
- **Can be slow and painstaking**
  - Need to examine many parts to solve a problem
  - Only deals with one possible outcome at a time
- **The whole may be different from the sum of the parts!**
- **Decision making without emotion**

Rational Decision Making

1. Study the problem
2. Develop a list of possible solutions
3. Evaluate the effectiveness of each possible solution
4. Choose the best alternative

Expert Problem Solving

Uses Natural Decision Making



Klein, Naturalistic Decision Making, 2008

Statistical

The Future as Multiverse

- Risk and Uncertainty
- Risk as probability distribution
- Expected value
- Rational Expectations
- Value at Risk
- Statistical Inference ???

Statistical
Probability Distributions

Advantages

- **Takes many possibilities into account all at once**
  - Our computer models sort through thousands of factors and determine the full range of outcomes.
- **Fit models to experience or modify to reflect trends**
  - Experience varies – so model varies

Disadvantages

- **Complexity**
  - Biases apply to model assumptions as well as to NDM
  - May scare away some users
  - May cause over reliance by others
- **Lack of Data**
  - Hard to calibrate
  - Biases apply to how we react to areas with low data
- **Hard to know**
  - When your model doesn't have a clue

We consider every possibility

And somehow we know the likelihood of every possibility

Two broad approaches to that…

- The future is assumed to be some minor variation on the past
  - Observed frequency = Likelihood
    - *May apply expert judgment to make minor adjustments to that*
- The collective wisdom of the market is correct about the future
  - Likelihood is inferred from prices of various securities
    - *Any variation from that infers that arbitrage opportunities exist*

Expected Values were the focus

- Actuarial work focused on reviewing statistical data to determine best estimate
  - Which may or may not be close to Expected Value
- Actuarial Cost came to be the term for the present value without PAD's
- Even when actuaries worked with full loss distributions
  - Tended to focus on expected values for a part of the loss distribution

Statistical inference

- Used extensively for medical decision making
- Used by consumer product companies
- But rarely used by insurers or actuaries

Advent of Risk Management

and Enterprise Risk Modeling

- Focus on Risk – contingent future events
  - Quantifying risk – usually in terms of an amount of loss for a particular frequency (VaR) or average loss for a range of frequencies (CTE)
  - High focus on Extreme Values
  - 99.5%
  - Everyone acts as if they can know what a 99.5% loss is
  - The statistical models that were developed for other purposes (Pricing, Hedging, Reinsurance) are adapted to create 99.5% values
- **We all then try very, very hard not to think of what Statistical inference would say about our results!**

Systems Analysis

Interdependencies

- Homeostasis and adaptability
- Positive and negative feedback loops
- Systems capacity
- Complex adaptive systems
- Fragility

Systems Analysis
Interdependencies

Advantages

- **Systems Model more closely resembles real world**
  - Everything is not extrapolation
  - Many systems cannot be understood properly by taking them apart
- **Builds a story**
  - That can be shared with users
- **Systems Models can reveal things that can happen in the tails**
  - Even if they have never happened before

Disadvantages

- **Biases**
  - Humans will tend to bring their biases into systems analysis
- **Complicated**
  - While you do not “take system apart” you need to identify pieces, their interaction and how/when they “break”
  - May scare away some users
  - May cause over reliance by others
- **Hard to know**
  - When your systems model is wrong

Equity Market Risk

- In many seasons, the equity performs the expected random walk with some noticeable long term alpha
- On occasion, the markets break down
  - **Positive feedback loops** cause market prices to rise far ahead of fundamentals (Internet Boom in late 1990’s)
  - **Negative feedback loops** cause market prices to fall so far that they invalidate market valuations before the fall (2001, 2008)
- These excesses on the upside and downside suggest that Gaussian model of stock market that is associated with Random Walk paradigm is insufficient
- Stock Market has **Fat Tails** that are due to systems effects

Credit Market Risk

Minsky Financial Instability Hypothesis

- **Hedge Finance** – Borrowing levels are supportable by cash flows. Businesses can afford to repay both interest and principle from cash flows.
- **Speculative Finance** – Borrowing is not fully supportable by cash flows. Businesses can afford to repay interest from cash flows. Expect to refinance principle.
- **Ponzi Finance** – Borrowing is totally unsupportable from cash flows. Businesses cannot afford to repay interest or principle from cash flows. Expect to increase borrowing to fund future interest payments.

1998 Asian Credit Crunch – 12 economies impacted, sharp contraction of credit availability

2001 US Credit event – default losses were **twice** the level of other post WWII credit events

2008 Global Financial Crisis – Minsky cycle hits US/UK housing markets

Natural Catastrophes

- Earthquakes, Hurricanes, Typhoons, Tsunamis, Floods are all the end stage of a system that has exceeded its capacity
- When capacity is exceeded, things are thrown into a different system where great deals of energy are released, rather than being dampened within the system.

Why do big complex systems fail

A **Bias** of many systems analysts

- Some believe that complex systems are inherently fragile
  - The bigger systems get the more complex they get
    - And the more fragile they get
- Natural systems usually develop natural control systems
  - Dynamic balance of predators and prey for example
  - Very complex natural systems can become fragile when humans eliminate major parts of the natural control systems
- Big complicated human systems are sometimes fragile
  - Humans mash together smaller systems that are minimally controlled and fail to realize that the new larger, more complex systems needs more controls
- Ashby's Law – the Law of Requisite Variety

Fat Tails

What do they mean to each type of thinker?

Natural Decision Making *From the Gut*	**Newtonian** *Logical*
Statistical *Future as Multiverse*	**Systems Analysis** *Complex Independencies*

Fat Tails
In Risk Models

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Fat Tails

- **Definition:**
  - A Fat Tail means that high severity/low probability events are more severe/more likely than would be predicted by a Gaussian distribution
- **Why is this an issue?**
  - Many risk models <u>had</u> assumed Gaussian distribution of one or all risk drivers
  - Many risks actually have Fat Tails
- **Solution:**
  - Use Fat Tailed Model

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Fat Tails

- So are we done with this talk already?
  - Perhaps not.

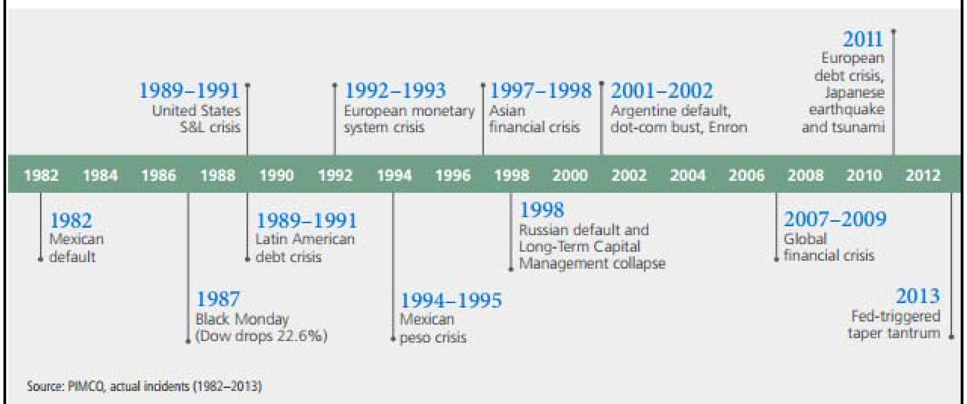
- Questions:
  - How Fat are the Tails of your Model?
  - Why should anyone believe what your model says about the tail values?
  - Are they Fat enough? Or Too Fat?
  - How do they compare with the Tails of other Models?
  - How Fat should the Tails be?
  - Who should be involved in deciding?
  - Can you explain your answer to any of the above questions to anyone who is not a modeler?

Four Models

How do they each see the world?

Natural Decision Making *From the Gut*	**Newtonian** *Logical*
Statistical *Future as Multiverse*	**Systems Analysis** *Complex Independencies*

Fat Tail Incidents



Coefficient of Riskiness

- Use 1 in 1000 loss as a proxy for the tail of the distribution of gains and losses
- With CLT assumed Extreme Loss is quick and easy to determine
- Tail is 3.09 standard deviations worse than the mean
  - For simplicity, round to 3
- Call that the **Coefficient of Riskiness (CoR)**

$$CoR = \frac{V_{.999} - \mu}{\sigma}$$

Chebyshev's Inequality

- CoR is the **k factor** in *Chebyshev's Inequality*

$$\Pr(\,|X\ -\mu| \geq k\sigma) \leq \frac{1}{k^2}$$

k	Percentile
10.00	99.00%
14.14	99.50%
15.81	99.60%
22.36	99.80%
31.62	99.90%

Preliminary Tests of COR

- The following slides show some preliminary tests of the COR calculation applied to hundreds and thousands of insurance risk models that were developed by Willis Re actuaries for our clients
- These tests show that in many cases the insurance blocks have much higher COR's than 3.09

CoR for Actuarial Model

Umbrella Products Model		Base model
Attritional	Mean	2,411,927
	Stdev	364,422
	VaR.999	3,873,352
	COV	4.01

CoR for Actuarial Model

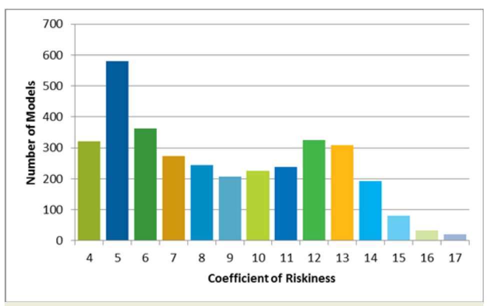
Umbrella Products Model		Base model
Attritional	Mean	2,411,927
	Stdev	364,422
	VaR.999	3,873,352
	COV	4.01
Large Losses	Mean	21,286,374
	Stdev	14,748,095
	VaR.999	92,827,916
	COV	4.85
Combined	Mean	23,698,301
	Stdev	14,838,839
	VaR.999	95,715,669
	COV	4.85

CoR for Actuarial Model

Umbrella Products Model		Base model	w/ 5x attr losses	w/ 10x attr losses	w/ 20x attr losses
Attritional	Mean	2,411,927	12,059,363	24,090,786	48,193,846
	Stdev	364,422	1,820,694	3,646,379	7,293,248
	VaR.999	3,873,352	18,919,276	38,112,250	76,268,019
	COV	4.01	3.77	3.85	3.85
Large Losses	Mean	21,286,374	21,307,221	21,265,393	21,159,575
	Stdev	14,748,095	14,738,229	14,650,350	14,566,846
	VaR.999	92,827,916	94,930,062	93,526,300	92,115,737
	COV	4.85	5.00	4.93	4.87
Combined	Mean	23,698,301	33,366,585	45,356,180	69,353,421
	Stdev	14,838,839	15,273,716	15,943,169	17,733,501
	VaR.999	95,715,669	107,109,220	121,681,495	149,855,190
	COV	4.85	4.83	4.79	4.54

Test of Coefficient of Riskiness

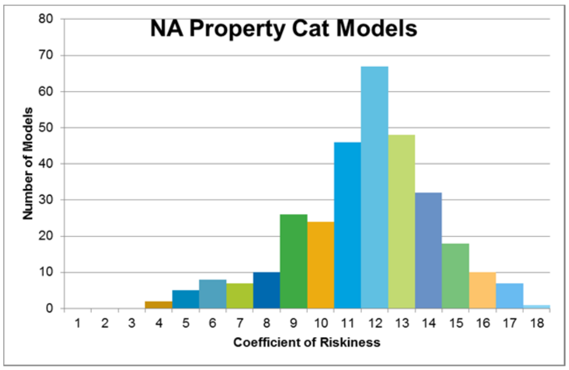
- COR was calculated for 3400 insurance models that were created by Willis Re actuaries over 2011-2014
- This is a plot of all of those 3400 mixed insurance risk models.
- Next step will be to stratify those 3400 models by type.
- For instance, we note that the model with the highest COR is a Homeowner only model for a single state company in a Nat Cat zone.



Note: COR 4 indicates value is 3 – 4, etc

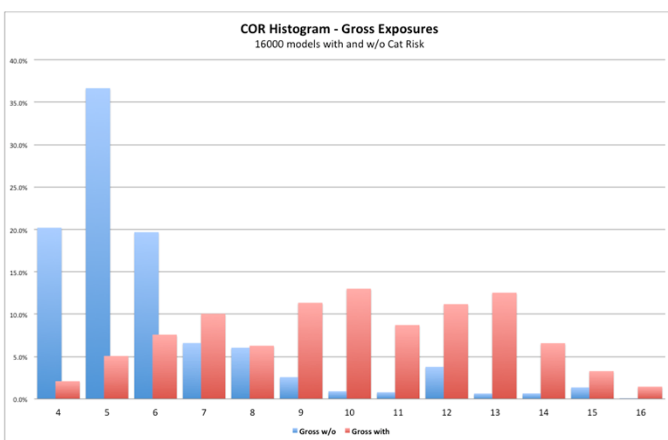
Stratification of Models

- This plot looks at 400 models of Property Risk Natural Catastrophe (Windstorm &/or Earthquake) losses



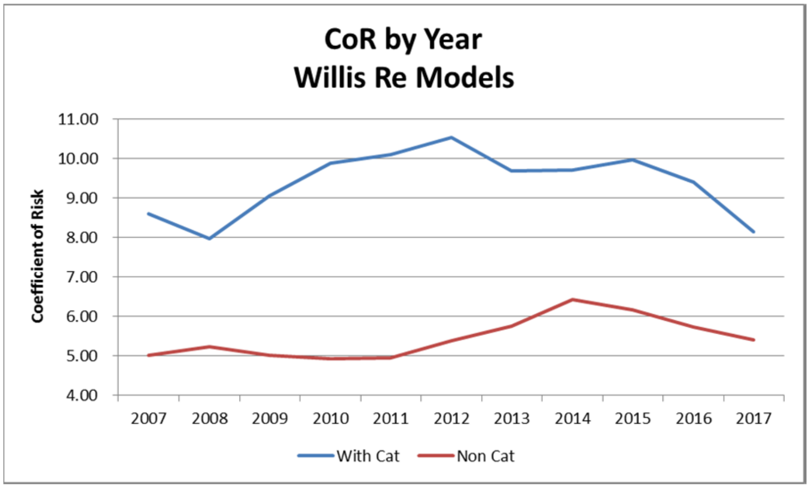
Insurance Models

with and without cat risk



COR over time

Willis Re Insurance Models



COR – Values for ESG output

Fat Tails

12/31/2016	Mean	Sigma	CoV	0.001	COR.001
Rate of Price Inflation	1.25%	0.76%	0.609	0.07	7.59
US Commodities	2.46%	9.47%	3.845	-0.604	6.64
US Mortgages_ABS_CMBS	2.71%	5.40%	1.994	-0.24	4.95
US Hedge_Fund	3.44%	6.53%	1.899	-0.257	4.46
US Property_Equity	4.91%	14.18%	2.89	-0.567	4.34
US Rate of Medical Inflation	3.57%	1.61%	0.451	0.10	4.07
HY_Global	4.18%	10.20%	2.438	-0.364	3.98
US Unemployment Rate	5.15%	0.89%	0.172	0.09	3.91
JPM_EM_Global	6.77%	10.79%	1.594	-0.326	3.65
Global_Equity	6.37%	17.72%	2.78	-0.559	3.51
US Infrastructure	5.88%	16.49%	2.803	-0.507	3.43

COR – Values

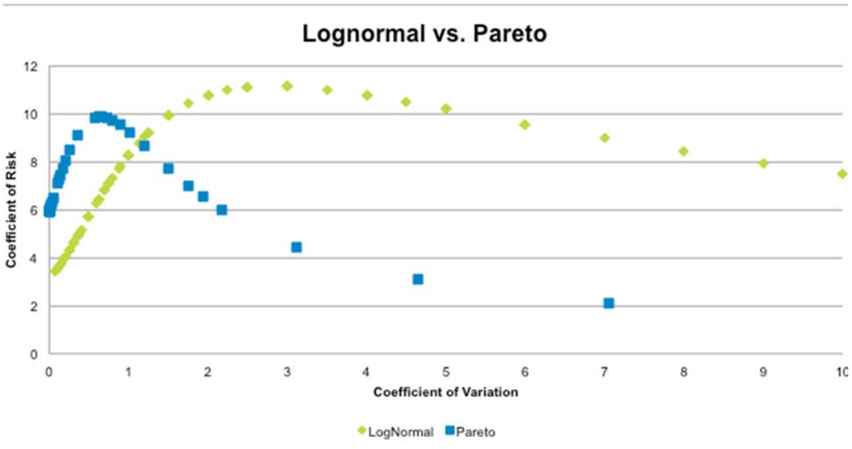
Not Fat Tails

12/31/2016	Mean	Sigma	CoV	0.001	COR.001
US_HY	5.79%	9.96%	1.721	-0.279	3.38
Private_Equity, European	6.21%	22.15%	3.567	-0.683	3.36
Commodities_Gold	2.11%	13.06%	6.184	-0.415	3.34
Rate of Wage Inflation	1.82%	1.14%	0.626	0.05	3.21
GDP	2.98%	2.38%	0.799	-0.05	3.20
US Equity_Total_Return	5.80%	18.00%	3.10	-0.508	3.14
Equities_GlobalSmallCap	6.49%	20.60%	3.176	-0.580	3.13
US HighYield_BB	6.95%	20.72%	2.98	-0.555	3.01
Change in Property Value Total Return	4.21%	9.58%	2.272	-0.23	2.85
UK Structured Credit	2.89%	6.71%	2.322	-0.158	2.79
Emerging Market Equity	7.86%	25.25%	3.213	-0.619	2.76
Emerging Equities_Small Cap	9.12%	26.22%	2.876	-0.633	2.76
US Real Assets Timberland	10.60%	11.66%	1.1	-0.065	1.47
US Real Assets Agricultural Land	10.53%	8.21%	0.78	-0.003	1.32

US Equities

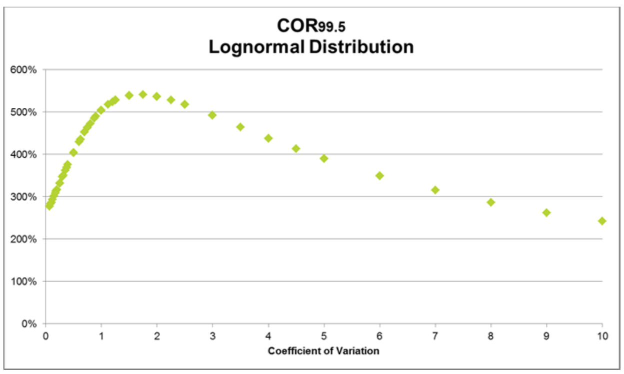
	Mean	Sigma	CV	1 in 1000	CoR
Equity Total Return – Jump Diffusion	5.80%	18.00%	310%	50.81%	3.14
DJIA	7.53%	15.71%	209%	48.03%	3.54
S&P 500	7.96%	16.02%	201%	47.96%	3.49
Equity Returns – Regime Switching	10.68%	19.92%	187%	59.25%	3.51

Distributions



What about 99.5%tile?

- All of this discussion applies equally to 99.5%tile



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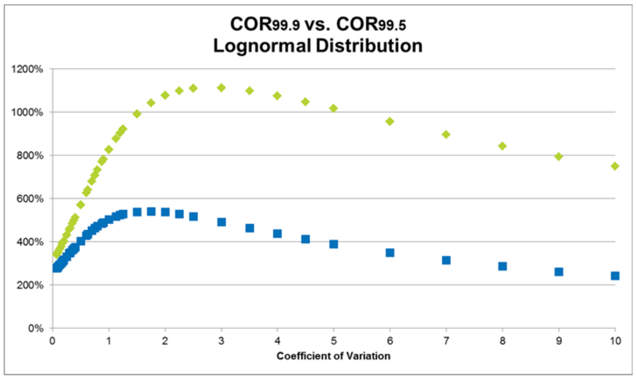
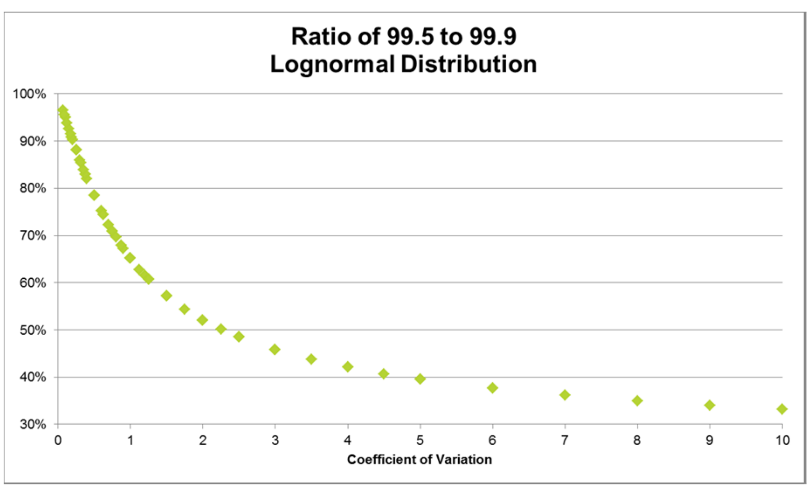


Relationship between 99.9 and 99.5%tile



Risk Tic Tac Toe

(From insurer's point of view)

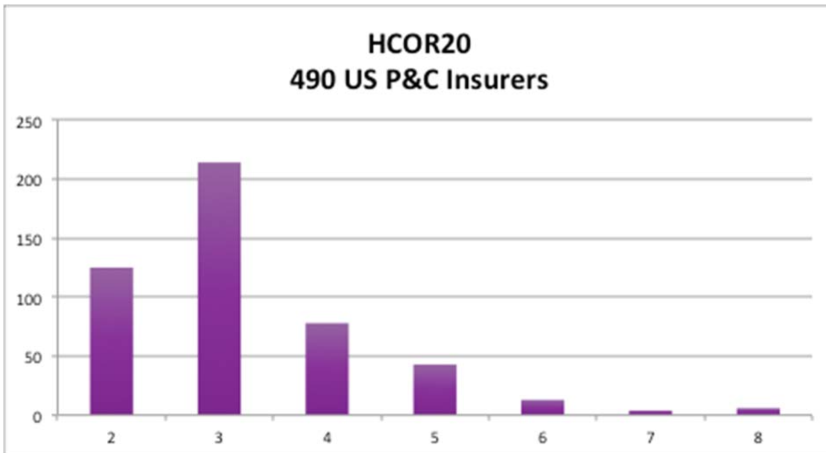
Volatility (CoV)		Low	Medium	High
	High	Reinsured (Type A)	Trouble	X
	Medium	Not reinsured	Reinsured (Type B)	Trouble
	Low	Not insured	Not reinsured	Reinsured (Type C)

Fat Tail (CoR)

Historical Coefficient of Riskiness (HCOR)

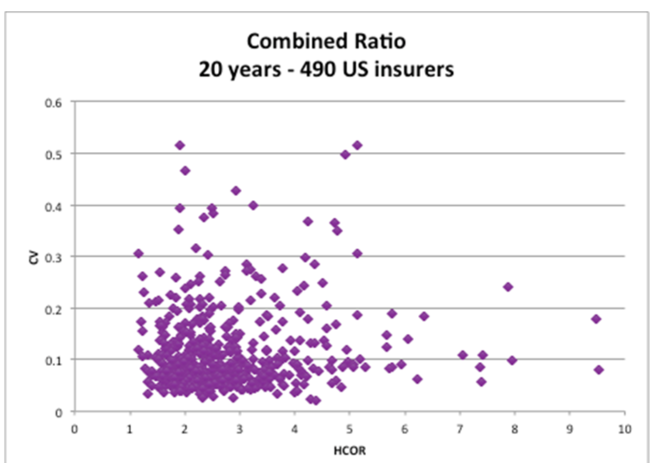
- COR is, of course, always an extrapolation
- HCOR however can be calculated in any cases where there is a good sized set of observations
  - Define HCOR as the historical worst observation less the sample mean divided by the standard deviation
    - Where the historical worst observation is excluded from the calculation of the sample mean and standard deviation

Actual Insurance Company HCOR20

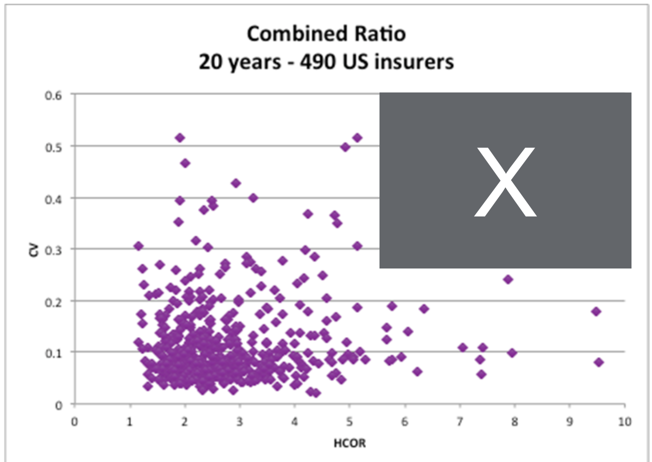


Insurance Models

CV vs. COR plot



Empty Region



Pareto Distribution

- Some risks are modeled with Pareto Distributions
  - Really fat tails
- Pareto Distributions can have infinite variances
  - Alpha 1 – 2
- And can have infinite Mean
  - Alpha <1
- Which makes calculating CoR impossible for those models

Wild and Extreme Randomness

- Mandelbrot describes seven states of randomness
- Proper mild randomness (the normal distribution)
- Borderline mild randomness: (the exponential distribution with $\lambda=1$)
- Slow randomness with finite and delocalized moments
- Slow randomness with finite and localized moments (such as the lognormal distribution)
- Pre-wild randomness (Pareto distribution with $\alpha=2$ - 3)
- Wild randomness: infinite second moment (Variance is infinite. Pareto distribution with $\alpha=1$ - 2)
- Extreme randomness: (Mean is infinite. Pareto distribution with $\alpha<=1$)

- B. Mandelbrot, *Fractals and Scaling in Finance*, Springer,1997.

Coefficient of Risk

How will our Four Thinkers use COR?

Natural Decision Making *From the Gut*	**Newtonian** *Logical*
Statistical *Future as Multiverse*	**Systems Analysis** *Complex Independencies*

Next Steps

- Starting Asking about the COR of Risk Models
- Start looking at HCOR
- Then we can start to develop:
  - Language for discussing model tail risk
  - Processes for using it to validate models
  - Procedure for estimating risk capital using company's own risk volatilities

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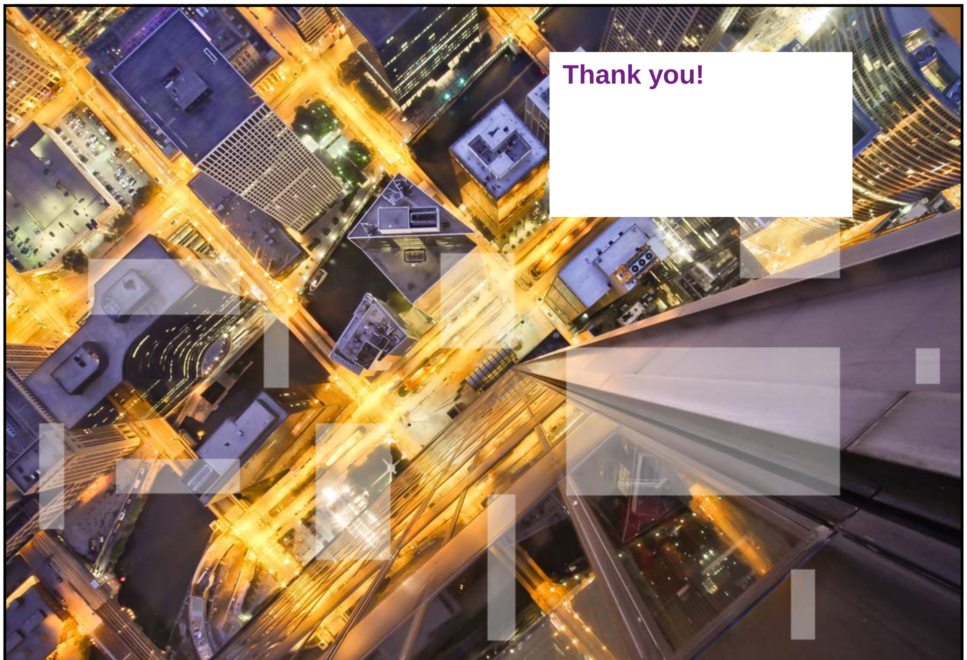

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