Casualty Actuarial Society Loss Reserve Seminar

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Testing Age to Age Assumptions

- Regression analysis produces estimates for the standard deviation of each parameter estimated.
- Usually the absolute value of a factor is required to be at least twice its standard deviation for the factor to be regarded as significantly different from zero.
- This test is failed by many development triangles, which means that the chain ladder method is not optimal for these triangles.

Testing Age to Age Assumptions

- Gary Venter's PCAS paper, "Testing the Assumptions of Age-to-Age Factors" provides the actuary with a well written documentation of the issues to be considered in deciding on a loss reserving methodology for a data set under study.

Testing Age to Age Assumptions

All the materials presented in this presentation are based on the PCAS paper, “Testing the Assumptions of Age-to-Age Factors,” by Gary Venter, FCAS, MAAA.

Testing Age to Age Assumptions Objective

- The actuary has many data, resource and time constraints in performing loss reserving analyses.
- The objective of this presentation is to develop an approach to efficiently implement the theories in his paper to allow the actuary more time to evaluate the results.

Testing Age to Age Assumptions

- The ***Stanard*** loss generation scheme illustrates how far off reserves can be when one reserving technique is applied to losses that have an emergence process different from the one underlying the technique.

- "A simulation Test of Prediction Errors of Loss Reserve Estimation Techniques," PCAS LXXII

Testing Age to Age Assumptions

- In this exercise, we will test the assumption that the expected emergence in the next period is proportional to the losses emerged to date. To test this assumption against its alternatives, the development method that leads from each alternative needs to be fit, and then a goodness-of-fit measure applied.

Testing Age to Age Assumptions

- Thomas Mack showed that some specific assumptions on the process of loss generation are needed for the chain ladder method to be optimal. Thus if actuaries find themselves in disagreement with one or another of these assumptions, they should look for some other method of development that is more in harmony with their intuition about the loss generation process.

Chain Ladder Assumptions for Optimality

1. The expected value of the incremental losses to emerge in the next period is proportional to the total losses merged to date by accident year.
2. Except for the same accident year, the increments are independent.
3. The variance of the next increment observation is a function of the age and the cumulative losses to date

Testing Age to Age Assumptions

- If the chain ladder fails the assumption of least squares optimality, test the underlying assumptions for the Bornhuetter-Ferguson, Cape Cod, combination of Bornhuetter-Ferguson/Cape Cod or other creative approaches that the data might suggest.

Test the following Assumptions

1. Expected losses emerge proportional to losses to date
2. Expected losses emerge proportional to losses to date plus a constant
3. Expected losses emerge proportional to ultimate losses
4. Expected losses emerge as a constant
5. Expected losses emerge as a constant with an adjustment

Testing Age to Age Assumptions

- The results of the tests should increase the actuary's confidence in the hypothesis, still recognizing that no hypothesis can ever be fully verified.

Testing Age to Age Assumptions

- This is a work in progress. It is hoped that further research of Gary Venter's paper will foster the development of efficient testing of loss reserving approaches for the practicing actuary.

- COTOR is currently developing materials for class room instructions.

Tests for five data sets

1. Auto liability, small volume
2. Auto liability, stable development
3. Part 1A, small volume
4. Part 1 H, small volume
5. Part 1 H, small volume

Example 1
Stable development, auto liability

Test of factor assumption to alternative emergence patterns

Summary

Analysis	Sum Squared Error	IBNR
Regression w/o constant	24,320	46,615
Regression w/ constant	28,120	39,091
Constant	101,140	39,414
Bornheutter-Ferguson	41,328	46,667
Cape Cod	101,140	39,414
BF/Cape Cod	23,780	48,443
Loss Development Factor		
Selected	29,058	44,054
Three year average	28,739	45,088
Five year average	29,334	42,817
Five year excl high/low	28,901	43,625
Four year average	30,698	43,447
All years excl high/low	24,700	46,818

Correlation Statistics

Age	Correlation	Sample T	n	Probability	Significant from Zero
2 vs 1	-0.373886	-0.987444	8	0.3616	**No**
3 vs 2	-0.230437	-0.529523	7	0.6191	**No**
4 vs 3	0.451661	1.012477	6	0.3686	**No**
5 vs 4	-0.099289	-0.172828	5	0.8738	**No**
6 vs 5	-0.363087	-0.551092	4	0.6369	**No**

Number of significant correlations 0
Number of required significant correlations 2.736068

Significance of Factors

Regression with constant

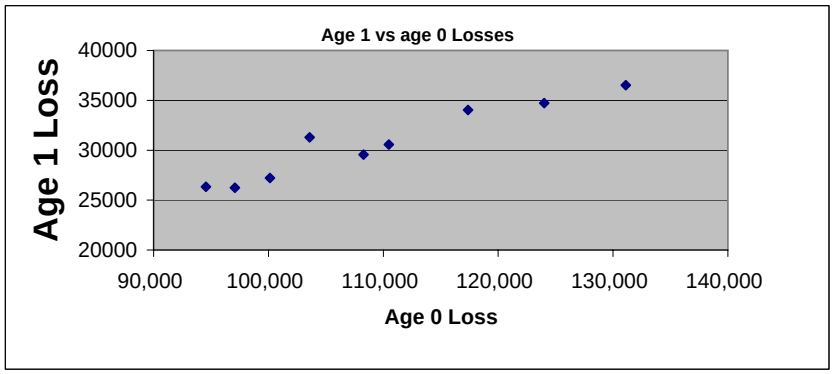
		Intercept				Significant	X variable				
Age	R^2	Coefficient	SE	t statistic	P-Value	from Zero	Coefficient	SE	t statistic	P-Value	P-Value
1 vs 0	92.3%	-1151.5	3494.5	-0.330	0.751	No	0.291	0.032	9.173	0.000	Yes
2 vs 1	35.2%	12559.7	4883.4	2.572	0.042	Yes	-0.064	0.036	-1.806	0.121	No
3 vs 2	5.9%	2603.1	3379.4	0.770	0.476	No	-0.014	0.024	-0.560	0.599	No
4 vs 3	32.5%	4106.8	2853.7	1.439	0.224	No	-0.029	0.021	-1.389	0.237	No
5 vs 4	28.5%	-2556.8	2227.3	-1.148	0.334	No	0.018	0.017	1.093	0.354	No
6 vs 5	93.4%	1854.3	352.8	5.256	0.034	Yes	-0.014	0.003	-5.337	0.033	Yes
7 vs 6	53.8%	-1768.2	1559.0	-1.134	0.460	No	0.013	0.012	1.080	0.476	No

Regression without constant

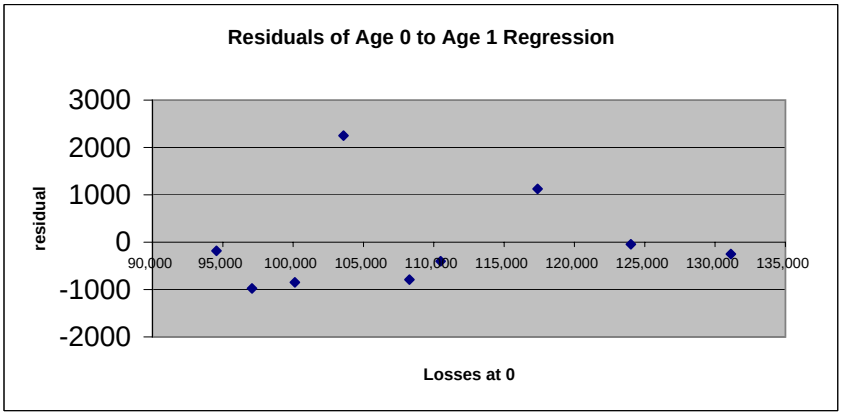
		Slope				Significant
Age	R^2	Coefficient	SE	t statistic	P-Value	from Zero
1 vs 0	99.9%	0.280	0.003	88.036	0.000	**Yes**
2 vs 1	84.6%	0.027	0.004	6.211	0.000	**Yes**
3 vs 2	63.6%	0.005	0.002	3.239	0.018	**Yes**
4 vs 3	14.9%	0.001	0.001	0.935	**0.393**	**No**
5 vs 4	24.5%	-0.001	0.001	-1.141	**0.318**	**No**
6 vs 5	16.0%	0.000	0.000	-0.757	**0.504**	**No**
7 vs 6	72.1%	-0.001	0.000	-2.276	**0.151**	**No**

Example 1
Stable development, auto liability

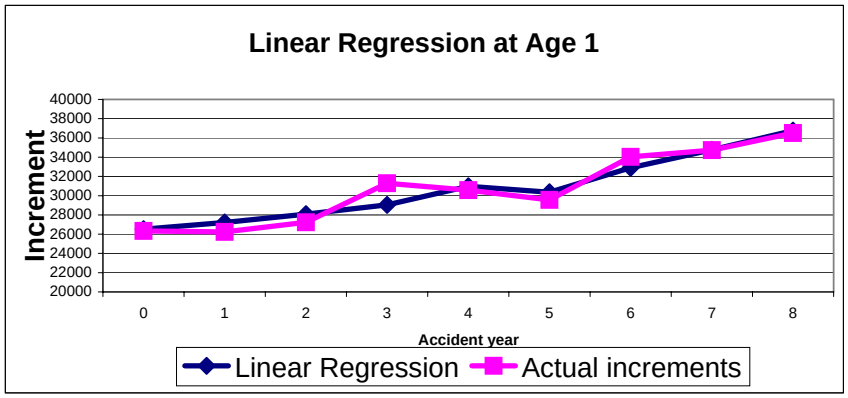
Significance of Factors



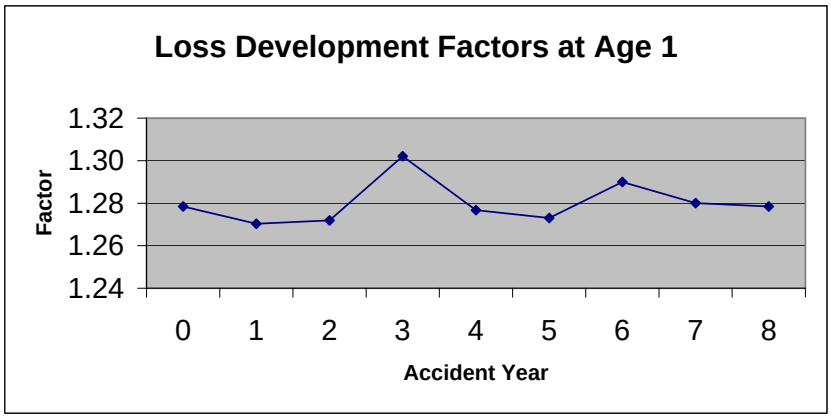
Linearity of Model



Linear Regression



Stability of Factors



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Stable development, auto liability

Significance of Factors

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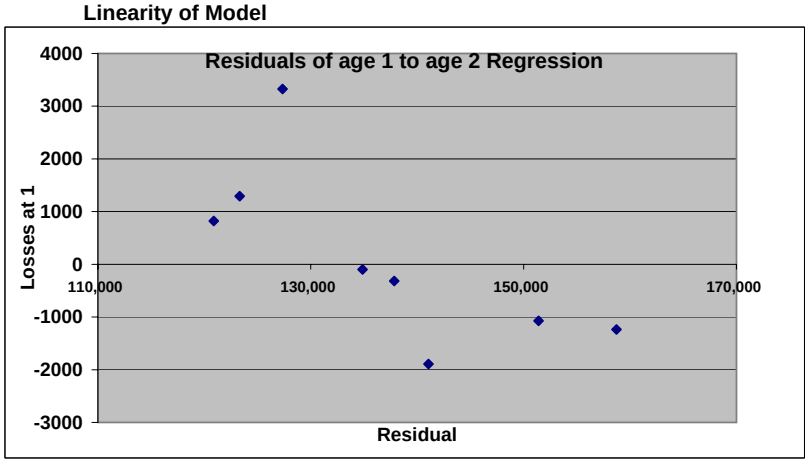

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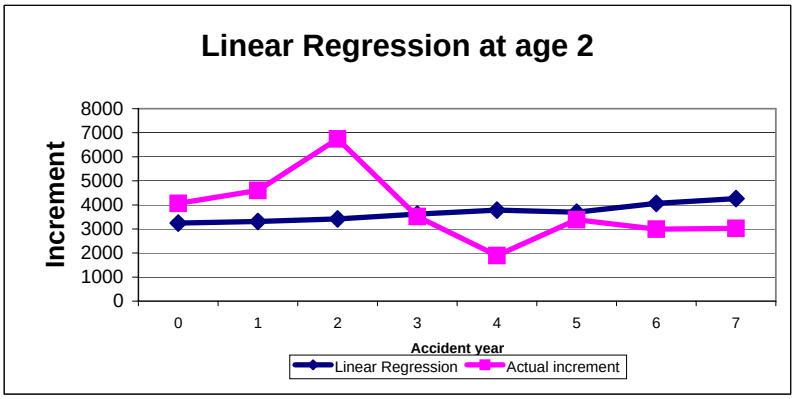

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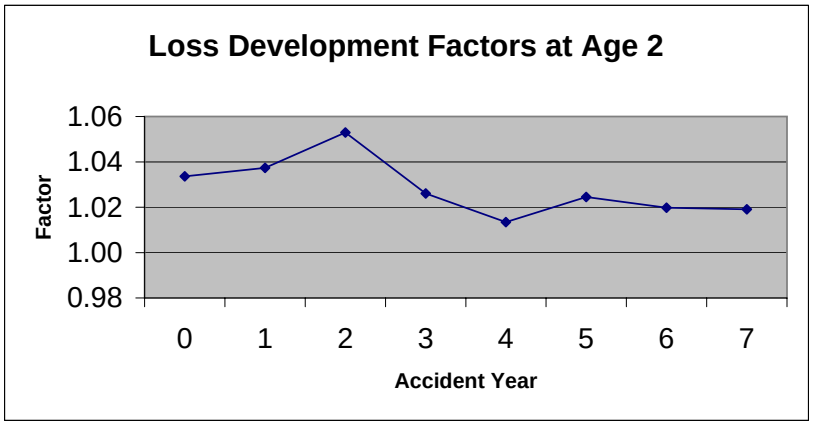

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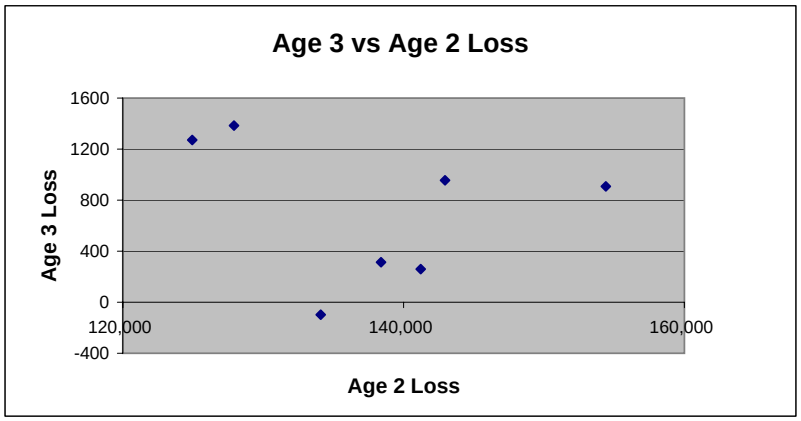
Stable development, auto liability

Significance of Factors



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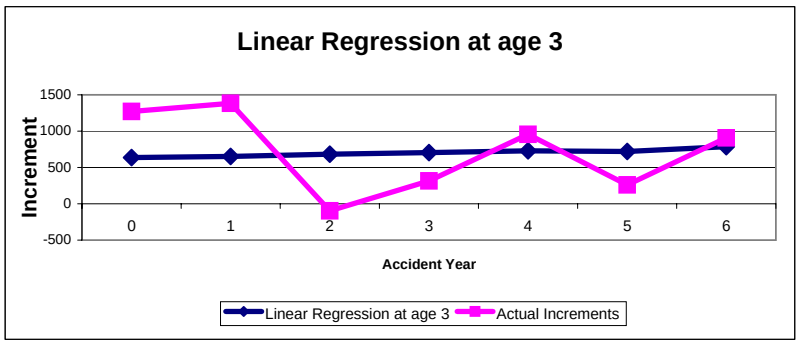

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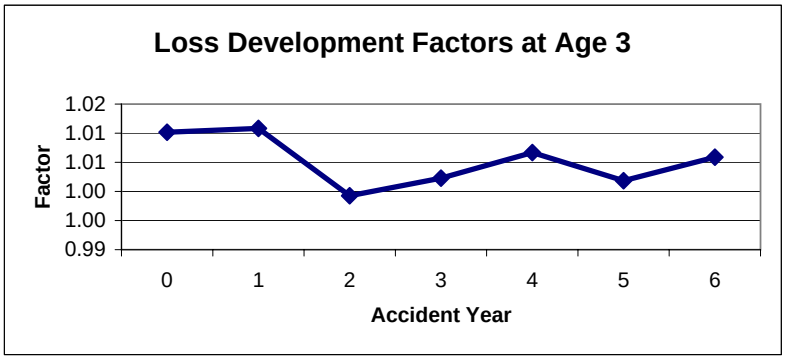

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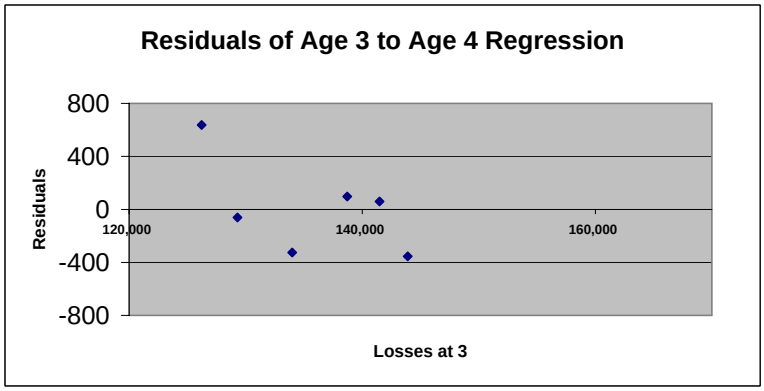
Stable development, auto liability

Significance of Factors

Linearity of Model



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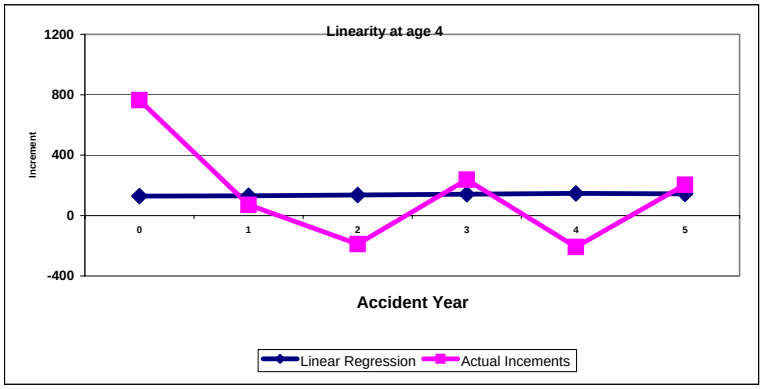

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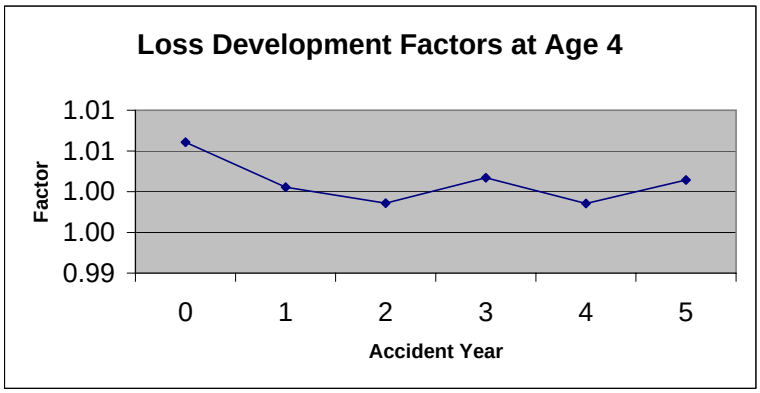

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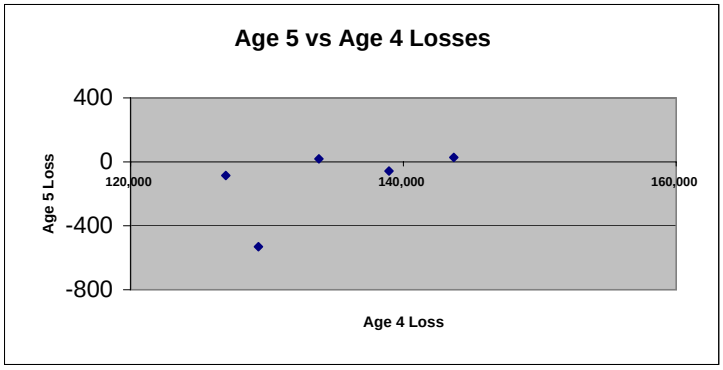

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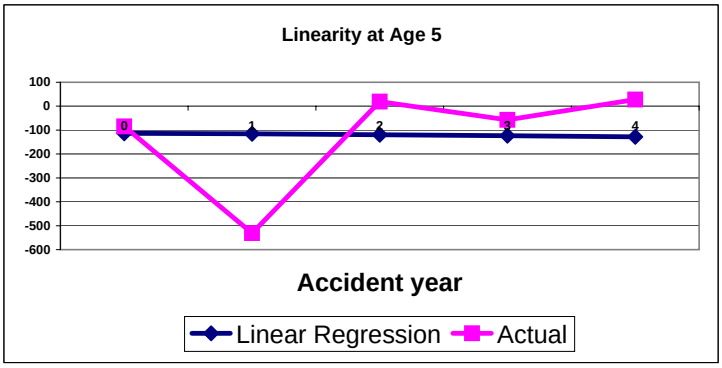

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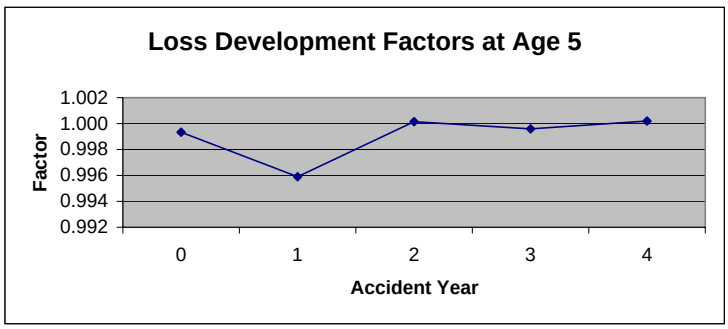


Example 2
Small volume, auto liability

Test of factor assumption to alternative emergence patterns

Summary

Analysis	Sum Squared Error	IBNR
Regression w/o constant	3,383	7,968
Regression w/ constant	5,226	8,930
Constant	4,666	6,924
Bornheutter-Ferguson	5,712	7,823
Cape Cod	4,666	6,924
BF/Cape Cod	3,557	10,021
Loss Development Factor		
Selected	3,517	8,731
Three year average	3,766	9,258
Five year average	3,627	8,274
Five year excl high/low	3,440	7,649
Four year average	3,657	9,290
All years excl high/low	3,590	7,101

Correlation Statistics

Age	Correlation	Sample T	n	Probability	Significant from Zero
2 vs 1	0.232208	0.584776	8	0.5800	**No**
3 vs 2	0.488312	1.251217	7	0.2662	**No**
4 vs 3	0.633788	1.638738	6	0.1766	**slight**
5 vs 4	0.410577	0.779908	5	0.4923	**No**
6 vs 5	0.098663	0.140214	4	0.9013	**No**

Number of significant correlations 0
Number of required significant correlations 2.736068

Significance of Factors

Regression with constant

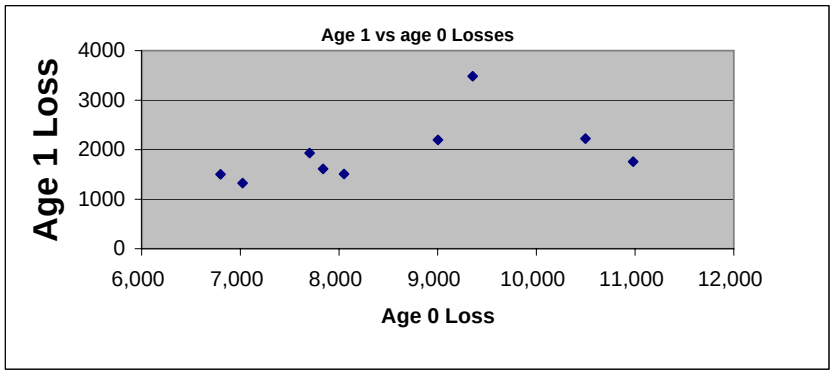
Age	R^2	Intercept Coefficient	SE	t statistic	P-Value	Significant from Zero	X variable Coefficient	SE	t statistic	P-Value	P-Value
1 vs 0	22.1%	161.0	1286.2	0.125	0.904	No	0.208	0.148	1.408	0.202	No
2 vs 1	68.8%	-1389.8	650.9	-2.135	0.077	slight	0.228	0.063	3.641	0.011	Yes
3 vs 2	44.4%	-836.1	629.8	-1.328	0.242	No	0.116	0.058	1.996	0.102	No
4 vs 3	20.6%	789.7	555.9	1.421	0.228	No	-0.054	0.053	-1.019	0.366	No
5 vs 4	5.4%	-471.8	1592.4	-0.296	0.786	No	0.064	0.154	0.414	0.707	No
6 vs 5	43.3%	-278.6	247.5	-1.125	0.377	No	0.030	0.024	1.236	0.342	No
7 vs 6	11.2%	-5.9	18.5	-0.318	0.804	No	0.001	0.002	0.354	0.783	No

Regression without constant

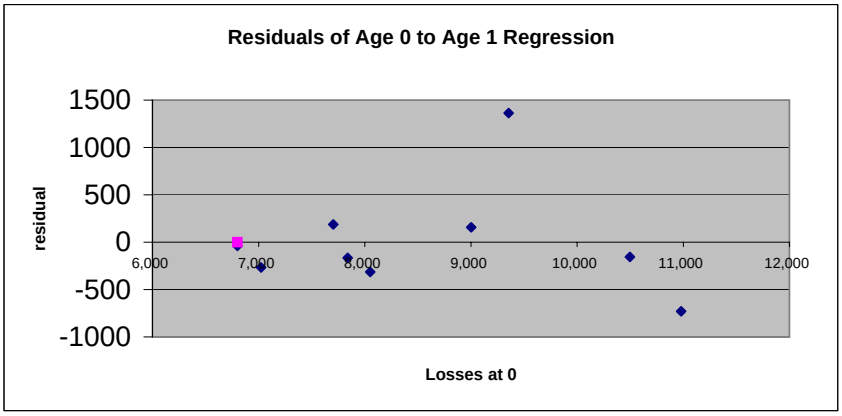
Age	R^2	Slope Coefficient	SE	t statistic	P-Value	Significant from Zero
1 vs 0	92.9%	0.227	0.022	10.226	0.000	**Yes**
2 vs 1	89.5%	0.096	0.012	7.722	0.000	**Yes**
3 vs 2	73.4%	0.040	0.010	4.073	0.007	**Yes**
4 vs 3	73.2%	0.021	0.006	3.692	0.014	**Yes**
5 vs 4	47.7%	0.018	0.010	1.910	**0.129**	**No**
6 vs 5	55.3%	0.003	0.001	1.928	**0.149**	**No**
7 vs 6	23.9%	0.000	0.000	0.793	**0.511**	**No**

Example 2
Small volume, auto liability

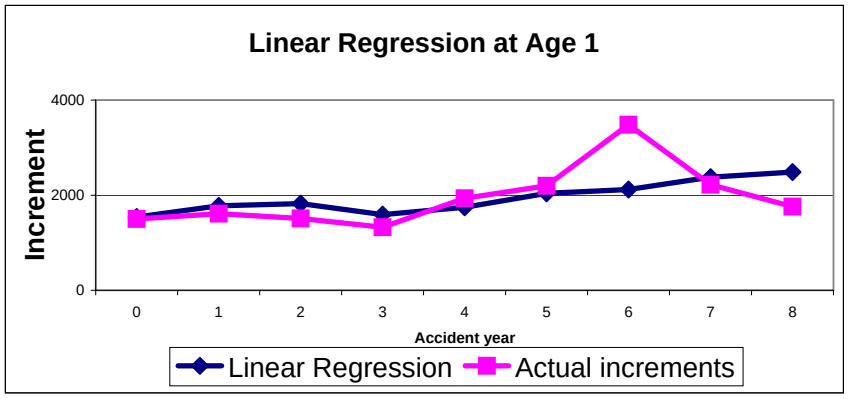
Significance of Factors



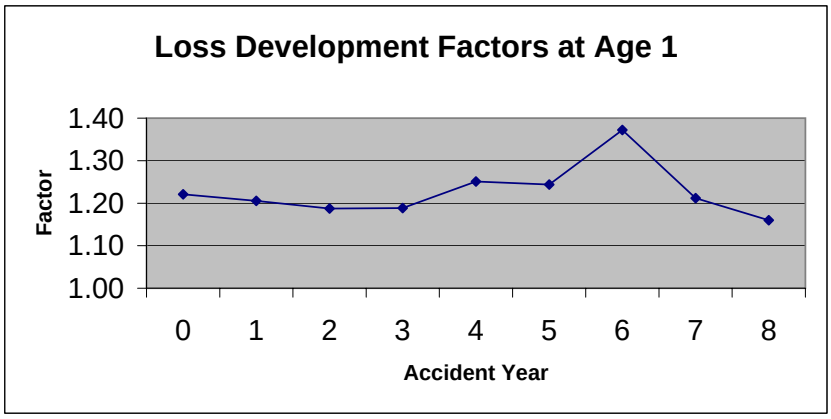
Linearity of Model



Linear Regression



Stability of Factors



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Small volume, auto liability

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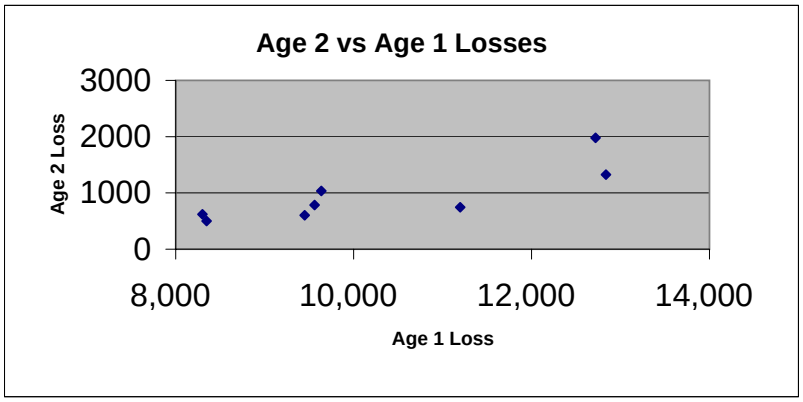

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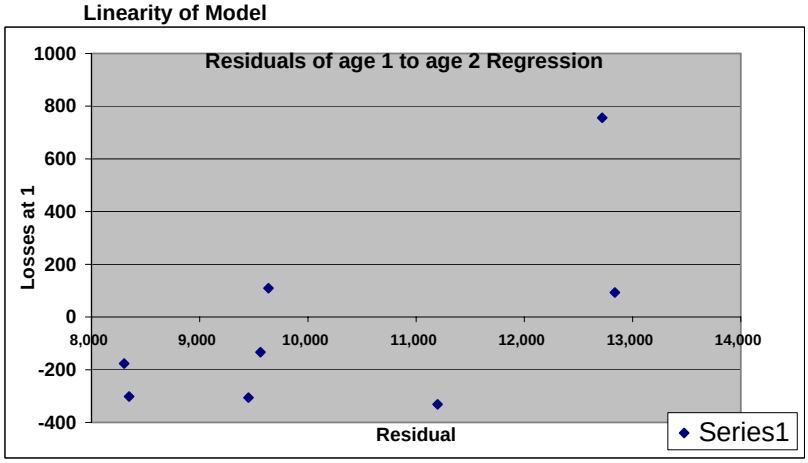

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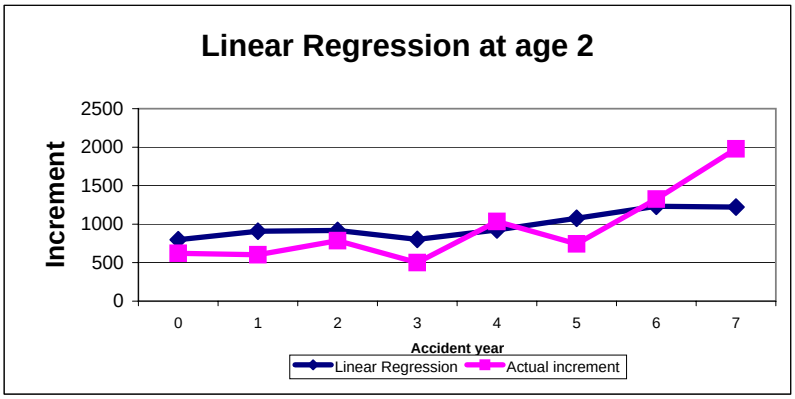

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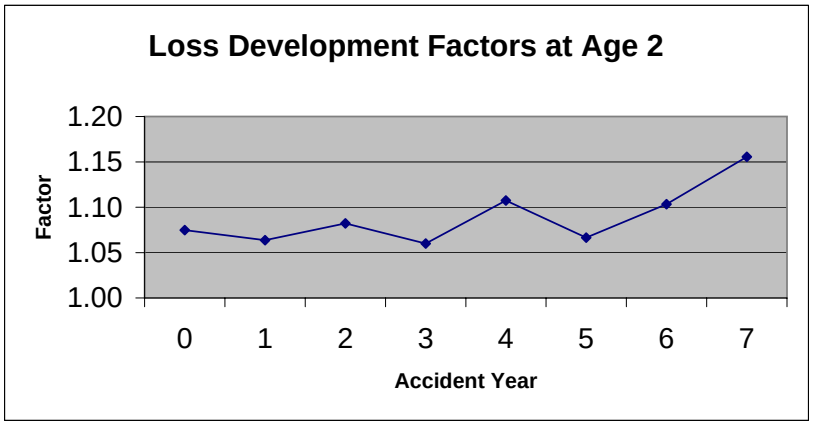
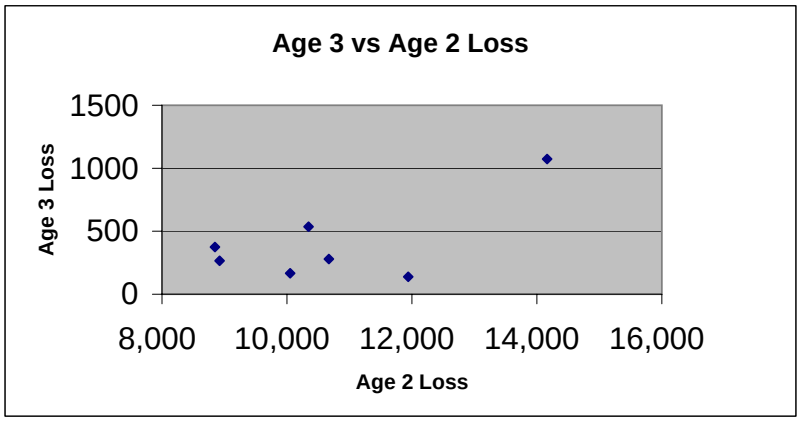
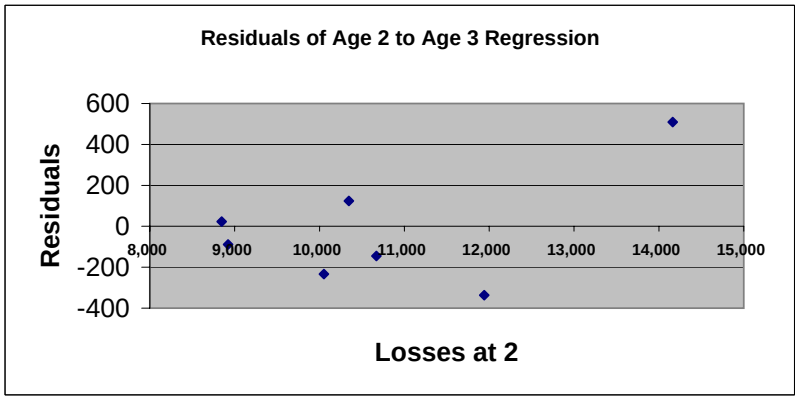


Example 2
Small volume, auto liability

Significance of Factors



Linearity of Model



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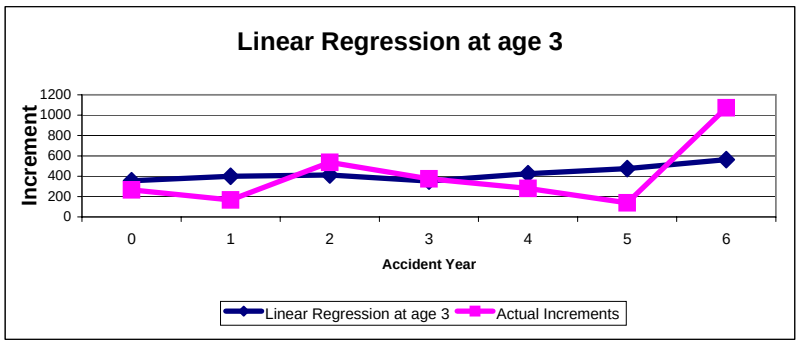

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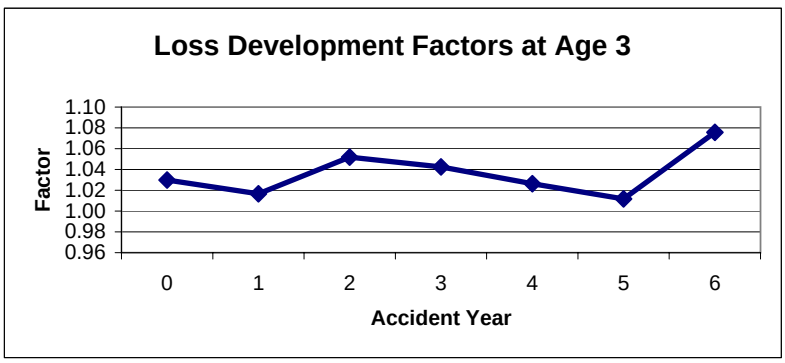

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Small volume, auto liability

Significance of Factors

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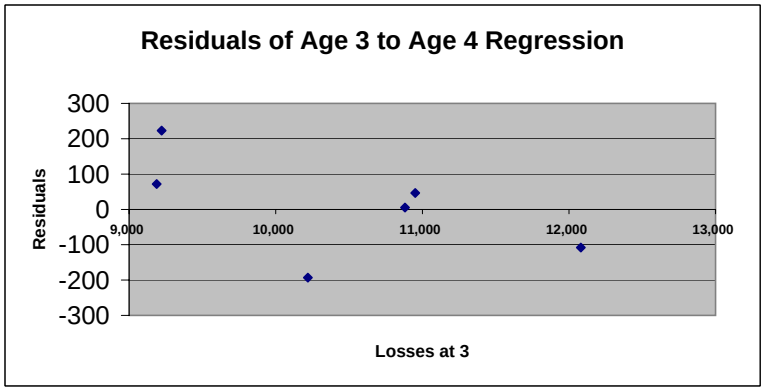

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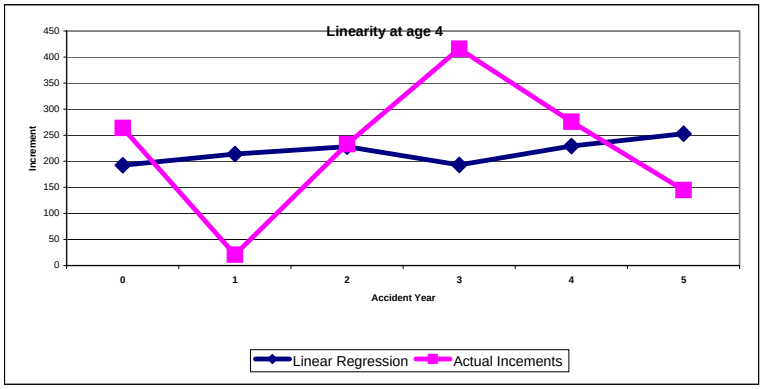

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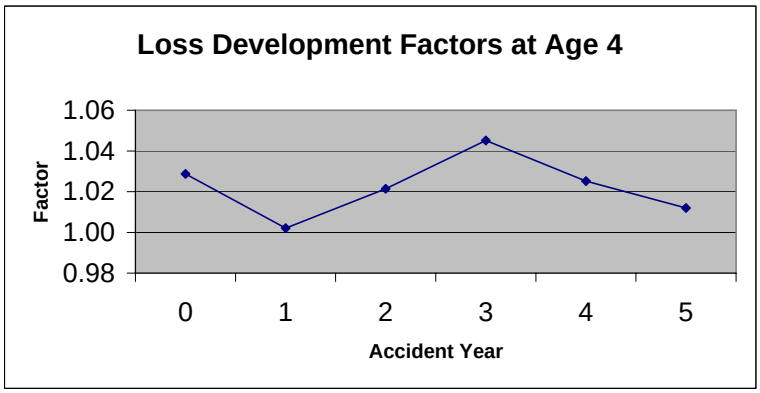

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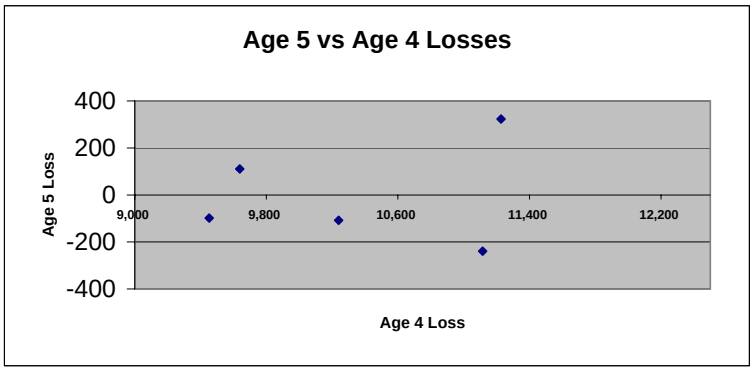

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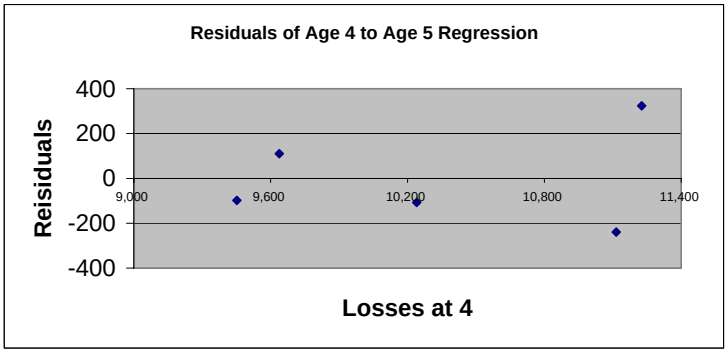

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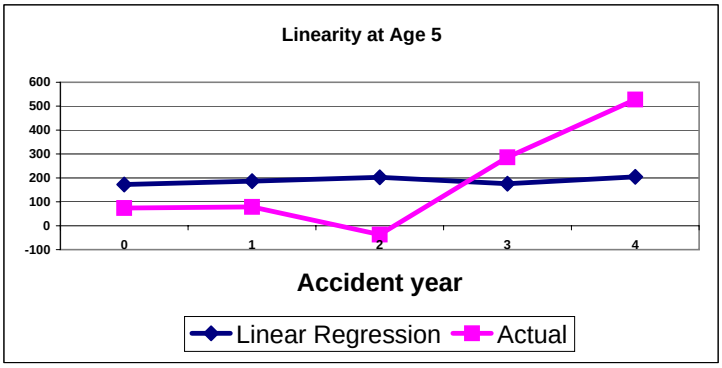
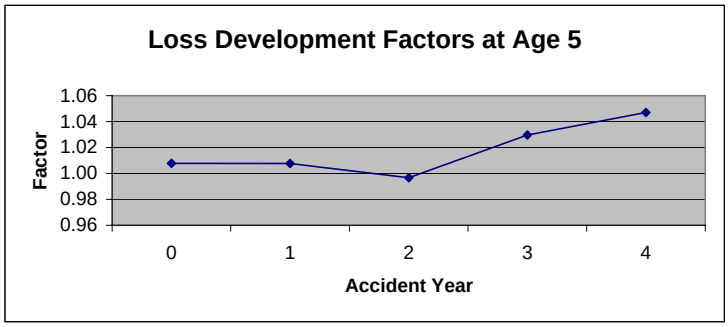


Stability of Factors



Example 3
Small volume, property package

Test of factor assumption to alternative emergence patterns

Summary

Analysis	Sum Squared Error	IBNR
Regression w/o constant	10,169	1,332
Regression w/ constant	10,023	-1,210
Constant	7,370	1,003
Bornheutter-Ferguson	18,024	1,343
Cape Cod	7,370	1,003
BF/Cape Cod	8,313	1,442
Loss Development Factor		
Selected	10,279	462
Three year average	10,701	19
Five year average	13,794	2,262
Five year excl high/low	11,099	1,812
Four year average	10,778	-434
All years excl high/low	10,316	1,707

Correlation Statistics

Age	Correlation	Sample T	n	Probability	Significant from Zero
2 vs 1	-0.104693	-0.25786	8	0.8051	**No**
3 vs 2	0.038944	0.087148	7	0.9339	**No**
4 vs 3	-0.497408	-1.146741	6	0.3154	**No**
5 vs 4	0.921418	4.107203	5	0.0261	**Yes**
6 vs 5	-0.728335	-1.503194	4	0.2717	**No**

Number of significant correlations	1
Number of required significant correlations	2.736068

Significance of Factors

Regression with constant

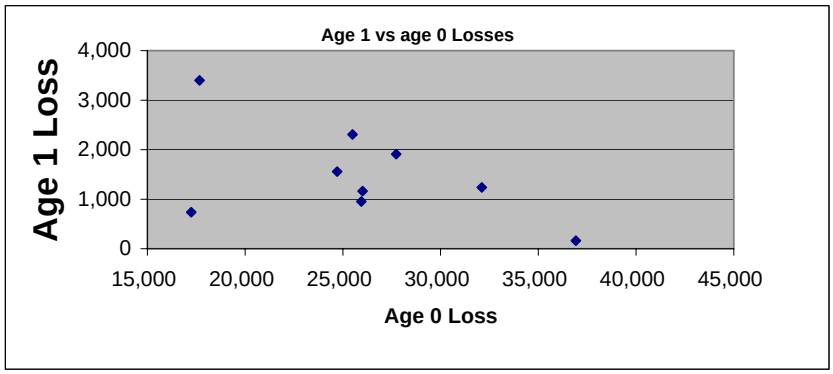
Age	R^2	Intercept Coefficient	SE	t statistic	P-Value	Significant from Zero	X variable Coefficient	SE	t statistic	P-Value	P-Value
1 vs 0	28.2%	3612.7	1312.3	2.753	0.028	Yes	-0.082	0.049	-1.657	0.142	No
2 vs 1	4.1%	735.7	897.6	0.820	0.444	No	-0.017	0.034	-0.505	0.631	No
3 vs 2	18.4%	680.8	865.0	0.787	0.467	No	-0.035	0.033	-1.061	0.337	No
4 vs 3	0.2%	-7.2	733.2	-0.010	0.993	No	-0.002	0.029	-0.078	0.942	No
5 vs 4	19.5%	-260.7	271.1	-0.962	0.407	No	0.009	0.011	0.852	0.457	No
6 vs 5	13.9%	-151.4	261.7	-0.579	0.621	No	0.006	0.010	0.569	0.627	No
7 vs 6	61.1%	-21.5	23.6	-0.910	0.530	No	0.001	0.001	1.254	0.428	No

Regression without constant

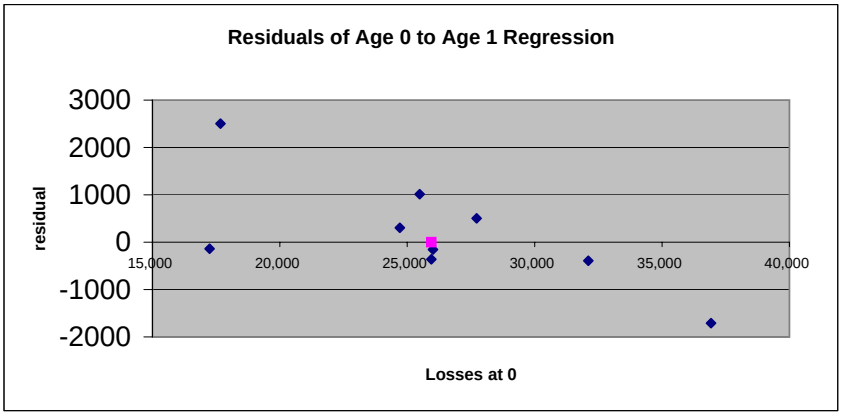
Age	R^2	Slope Coefficient	SE	t statistic	P-Value	Significant from Zero
1 vs 0	60.1%	0.051	0.015	3.473	0.008	**Yes**
2 vs 1	32.7%	0.010	0.006	1.843	**0.108**	**No**
3 vs 2	38.0%	-0.009	0.005	-1.917	**0.104**	**No**
4 vs 3	7.0%	-0.003	0.004	-0.616	**0.565**	**No**
5 vs 4	7.2%	-0.001	0.002	-0.557	**0.607**	**No**
6 vs 5	0.0%	0.000	0.001	-0.008	**0.994**	**No**
7 vs 6	70.0%	0.000	0.000	2.161	**0.163**	**No**

Example 3
Small volume, property package

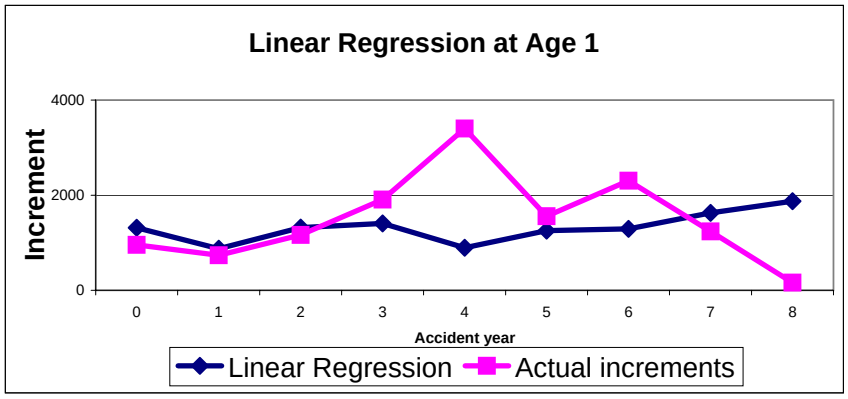
Significance of Factors



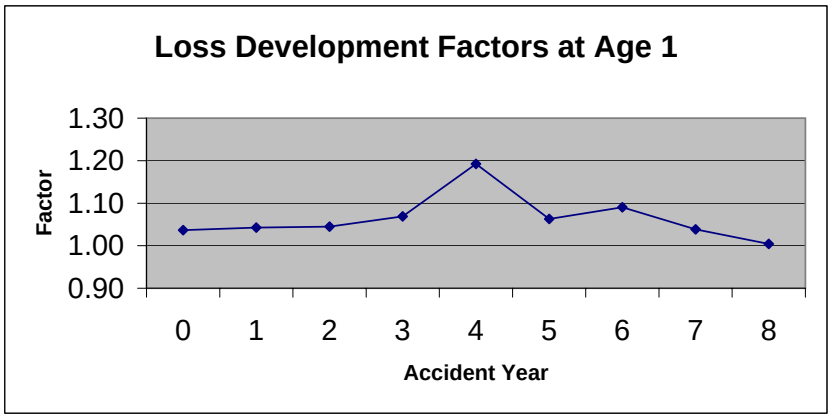
Linearity of Model



Linear Regression

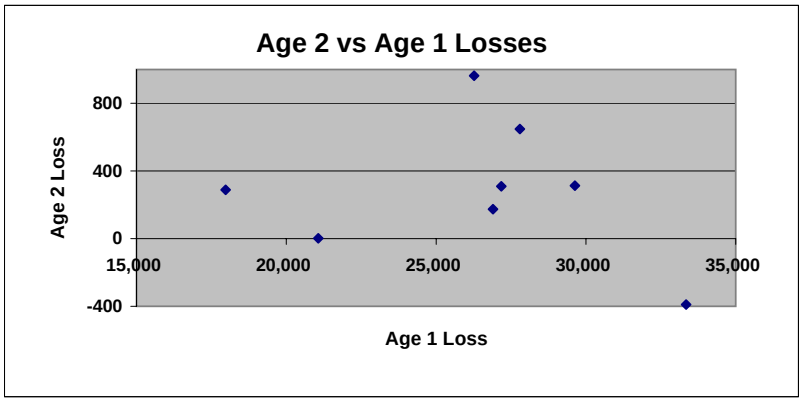


Stability of Factors



Example 3
Small volume, property package

Significance of Factors



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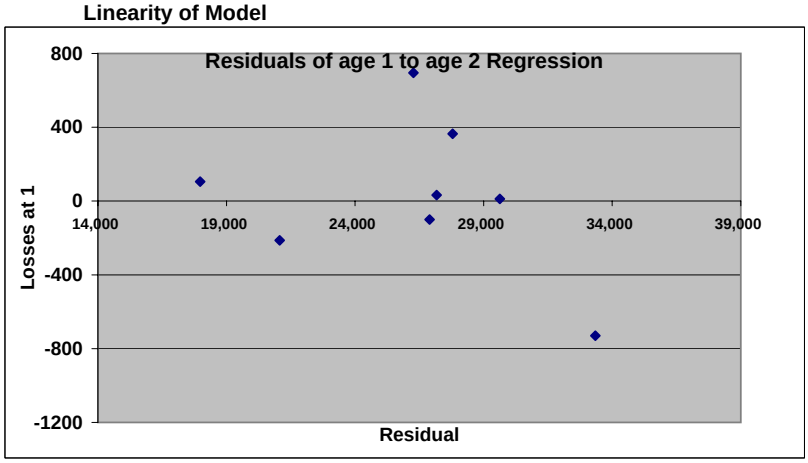
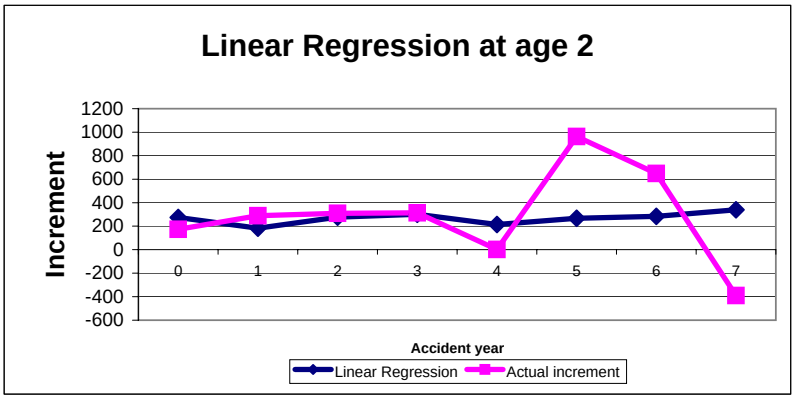
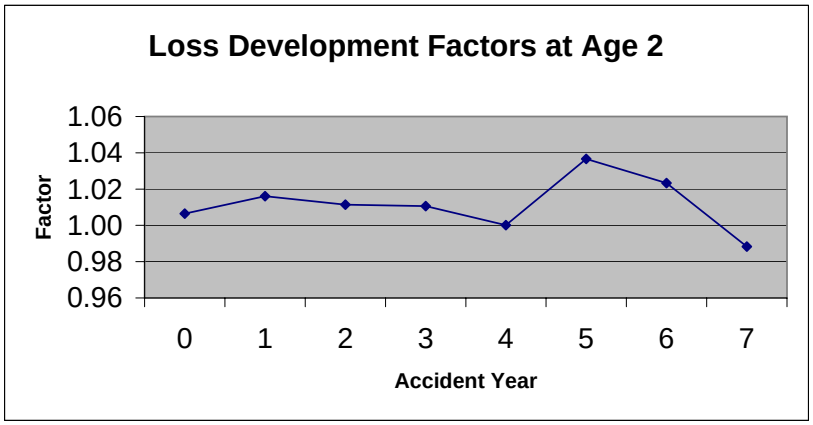


Linear Regression

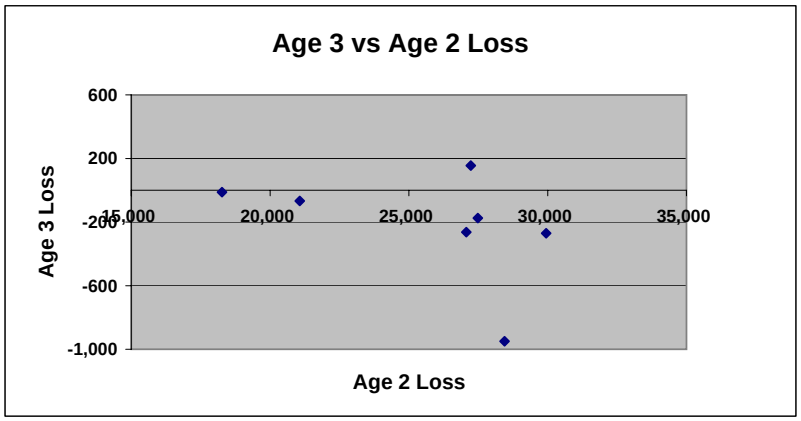


Stability of Factors



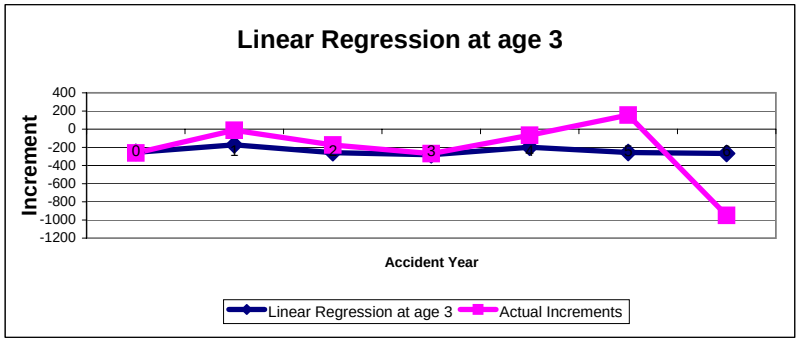
Example 3
Small volume, property package

Significance of Factors

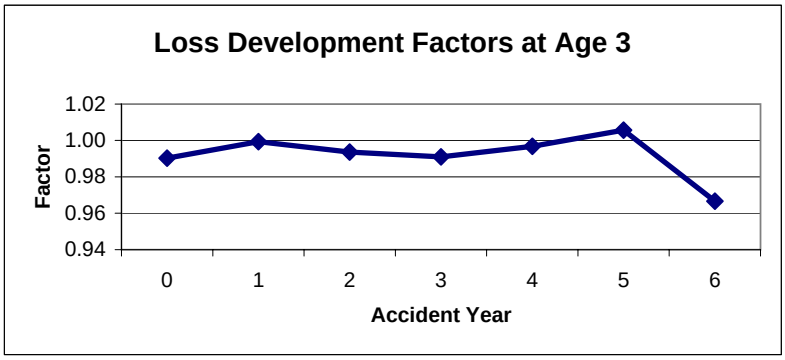


Linearity of Model

Linear Regression

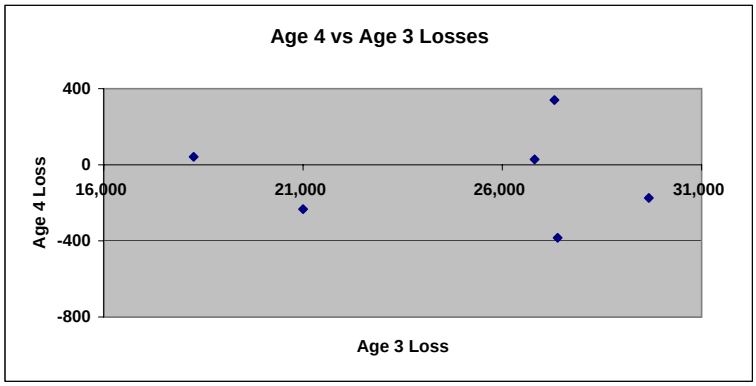


Linearity of Model

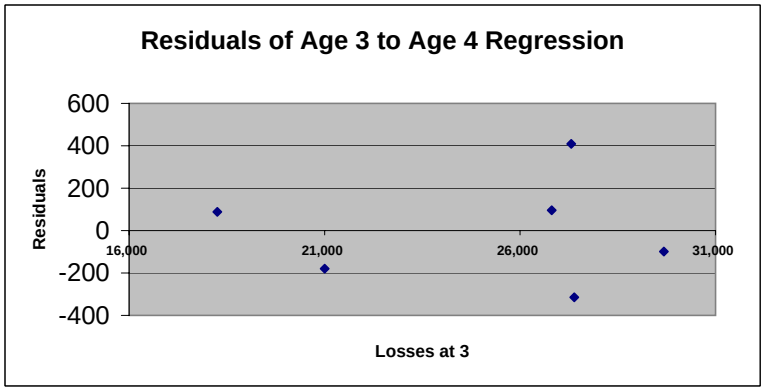


Example 3
Small volume, property package

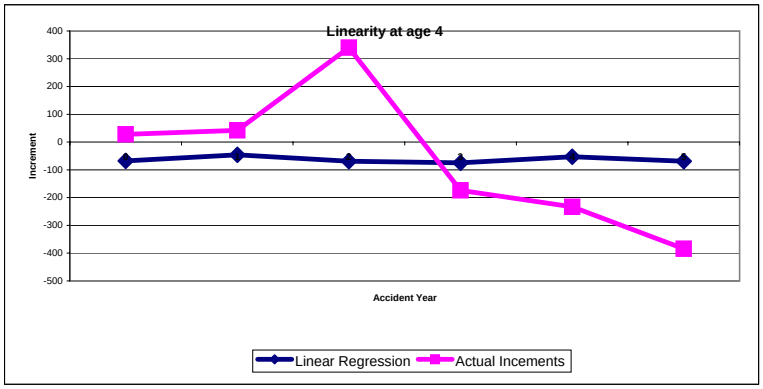
Significance of Factors



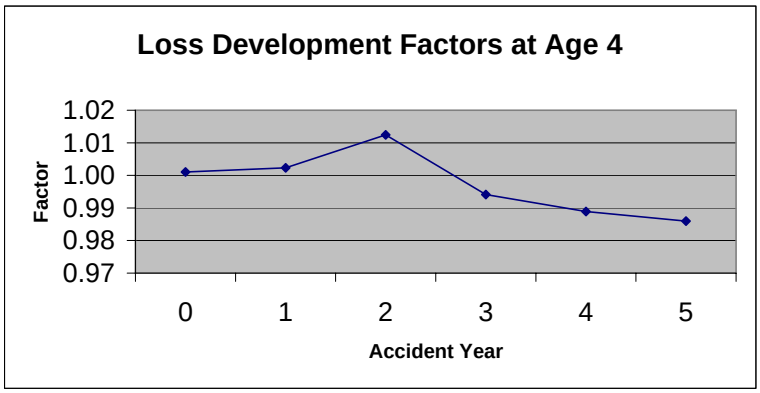
Linearity of Model



Linear Regression

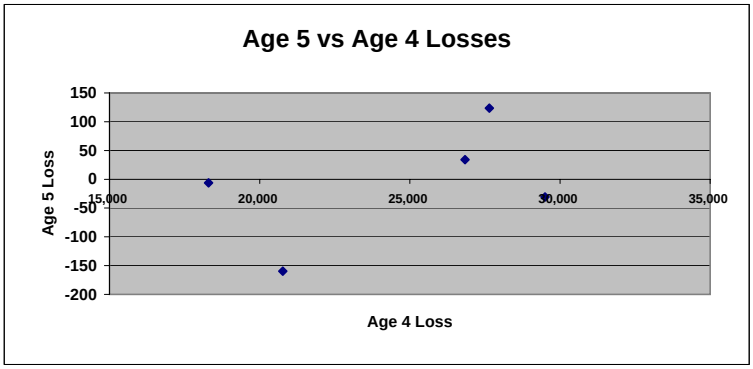


Stability of Factors

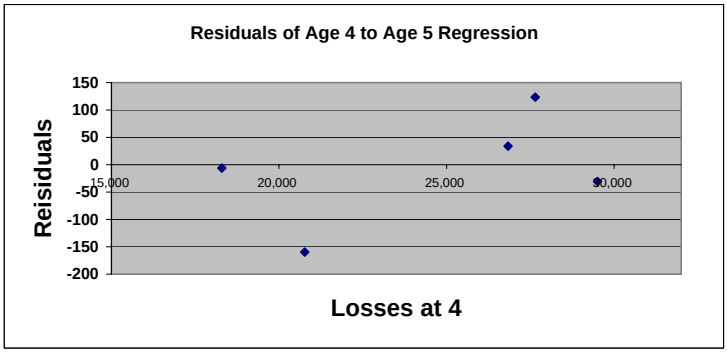


Example 3
Small volume, property package

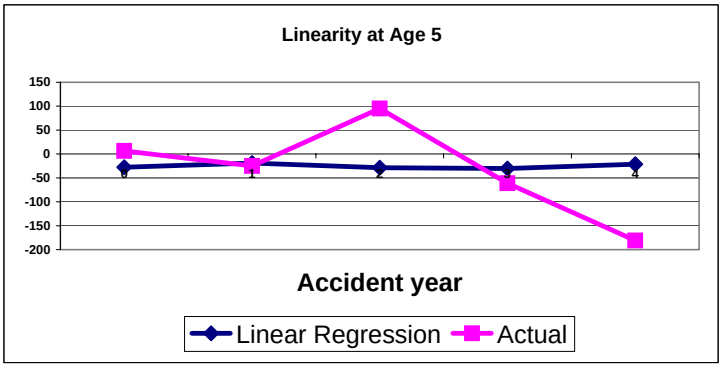
Significance of Factors



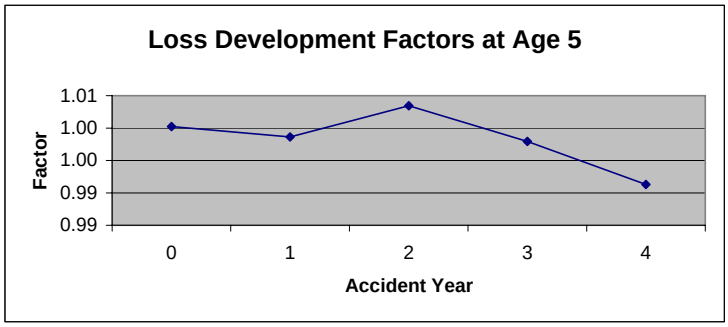
Linearity of Model



Linear Regression



Stability of Factors



Example 4
Small volume, Other liability

Test of factor assumption to alternative emergence patterns

Summary

Analysis	Sum Squared Error	IBNR
Regression w/o constant	138	532
Regression w/ constant	222	701
Constant	137	252
Bornheutter-Ferguson	185	199
Cape Cod	140	198
BF/Cape Cod	88	304
Loss Development Factor		
Selected	196	648
Three year average	326	813
Five year average	404	1,130
Five year excl high/low	153	428
Four year average	209	684
All years excl high/low	143	396

Correlation Statistics

Age	Correlation	Sample T	n	Probability	Significant from Zero
2 vs 1	0.433257	1.177515	8	0.2836	**No**
3 vs 2	-0.412386	-1.012199	7	0.3579	**No**
4 vs 3	-0.375146	-0.809406	6	0.4637	**No**
5 vs 4	-0.565376	-1.187222	5	0.3206	**No**
6 vs 5	0.373175	0.568841	4	0.6268	**No**

Number of significant correlations 0
Number of required significant correlations 2.736068

Significance of Factors

Regression with constant

Age	R^2	Intercept Coefficient	SE	t statistic	P-Value	Significant from Zero	X variable Coefficient	SE	t statistic	P-Value	P-Value
1 vs 0	13.3%	48.4	53.5	0.905	0.396	No	0.261	0.252	1.038	0.334	No
2 vs 1	74.6%	-24.2	17.9	-1.352	0.225	No	0.236	0.056	4.194	0.006	Yes
3 vs 2	9.2%	29.3	64.7	0.453	0.670	No	-0.119	0.167	-0.710	0.510	No
4 vs 3	9.5%	53.8	46.8	1.149	0.315	No	-0.075	0.116	-0.650	0.551	No
5 vs 4	4.2%	6.4	26.4	0.243	0.824	No	-0.025	0.067	-0.365	0.739	No
6 vs 5	59.3%	-8.5	9.8	-0.867	0.477	No	0.041	0.024	1.708	0.230	No
7 vs 6	1.9%	-2.6	4.3	-0.605	0.654	No	0.001	0.009	0.138	0.913	No

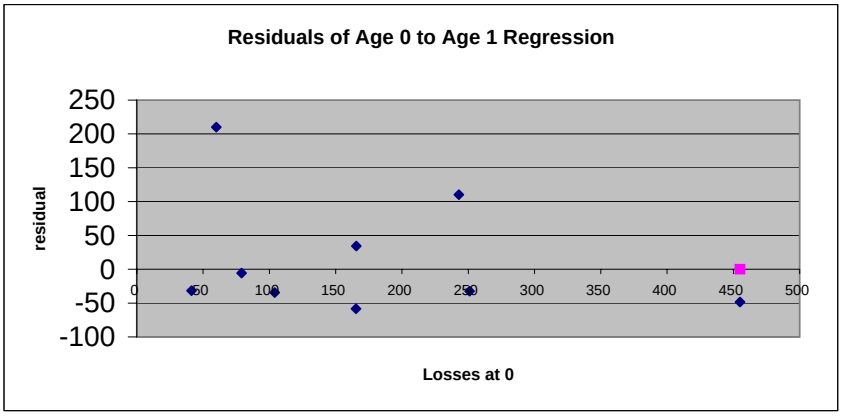
Regression without constant

Age	R^2	Slope Coefficient	SE	t statistic	P-Value	Significant from Zero
1 vs 0	55.0%	0.448	0.143	3.128	0.014	**Yes**
2 vs 1	78.6%	0.174	0.034	5.065	0.001	**Yes**
3 vs 2	5.9%	-0.060	0.098	-0.611	**0.564**	**No**
4 vs 3	3.9%	0.032	0.071	0.452	**0.670**	**No**
5 vs 4	2.6%	-0.011	0.035	-0.327	**0.760**	**No**
6 vs 5	50.2%	0.024	0.014	1.739	**0.180**	**No**
7 vs 6	16.7%	-0.003	0.005	-0.632	**0.592**	**No**

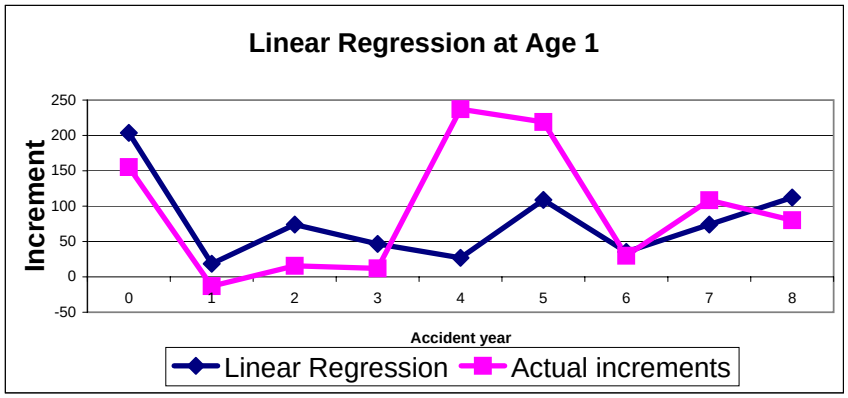
Example 4
Small volume, Other liability

Significance of Factors

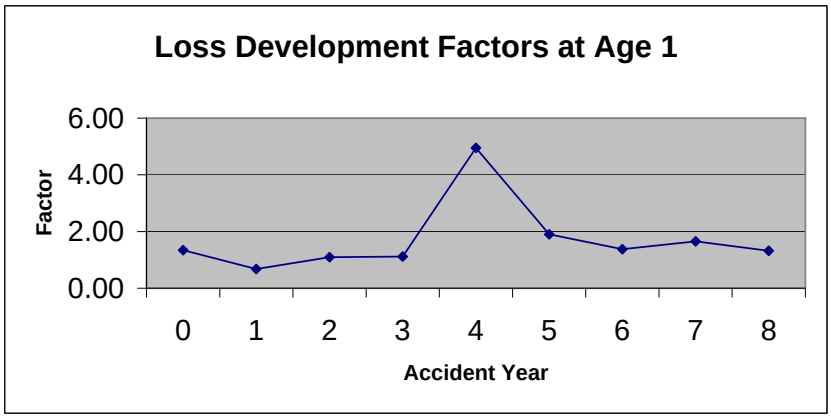
Linearity of Model



Linear Regression

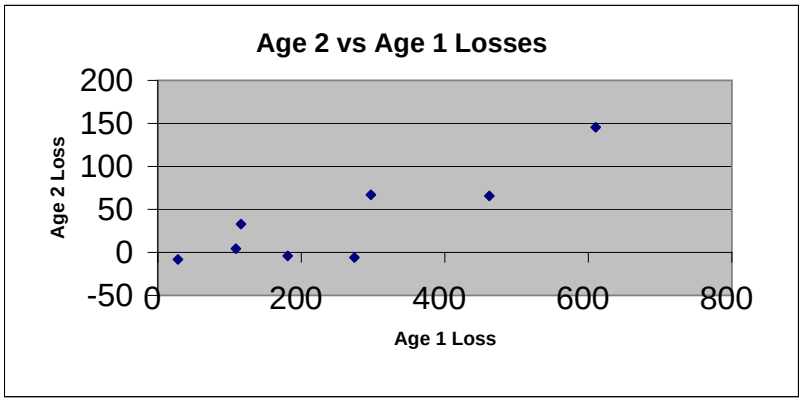


Stability of Factors

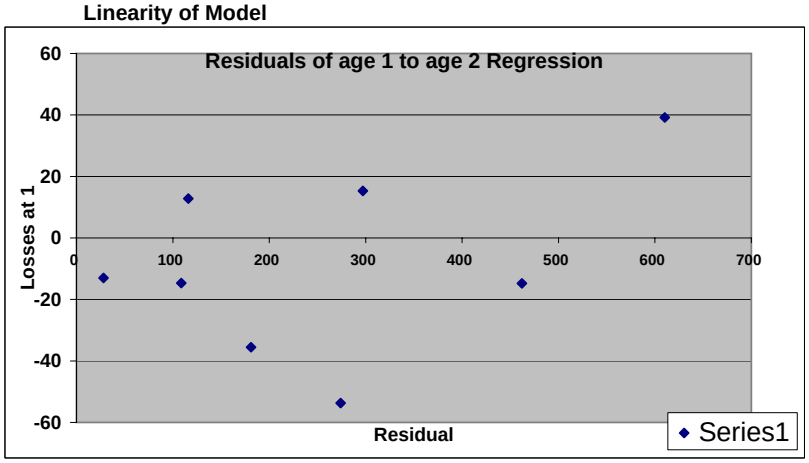


Example 4
Small volume, Other liability

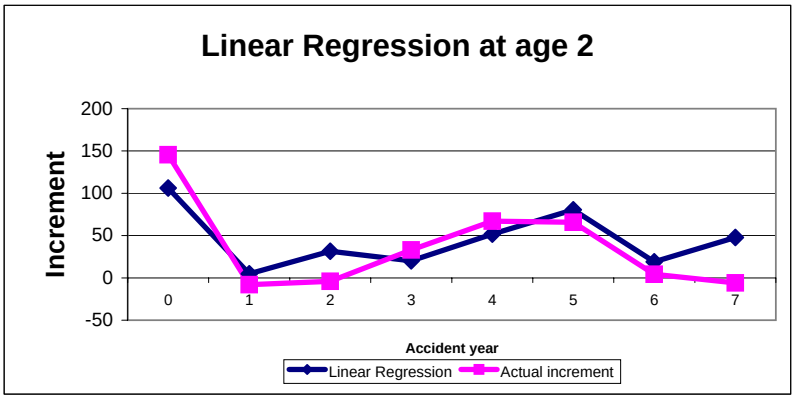
Significance of Factors



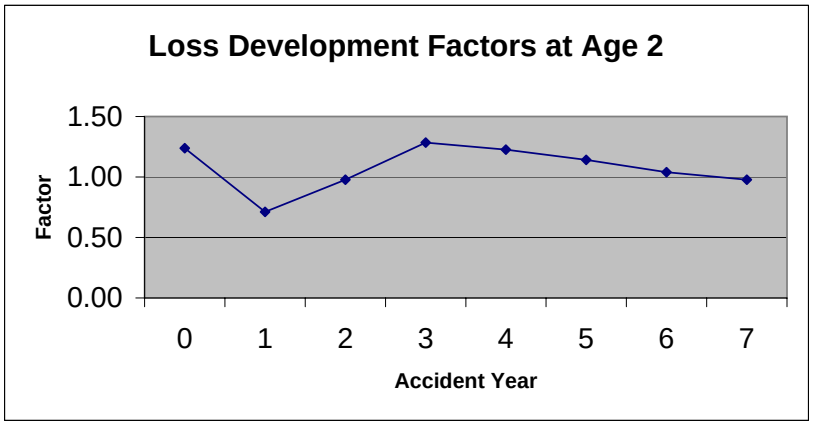
Linearity of Model



Linear Regression

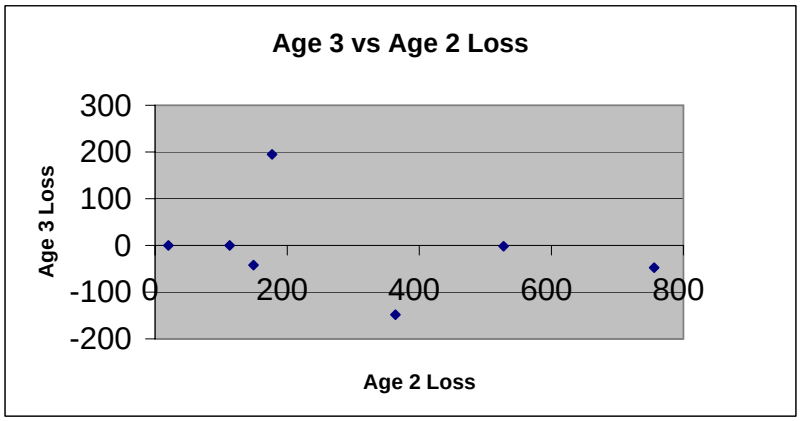


Stability of Factors

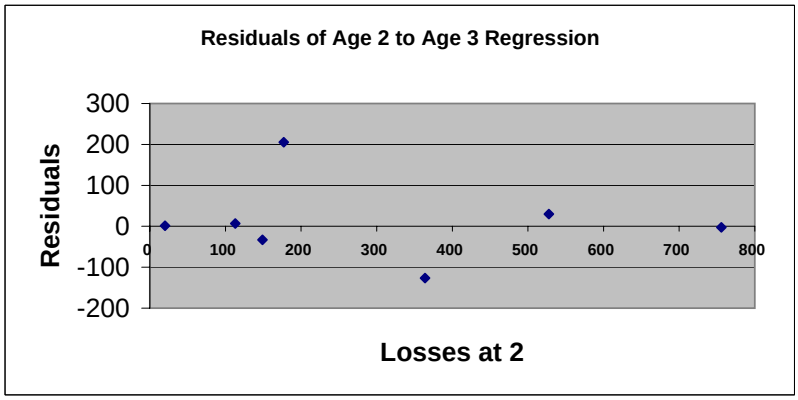


Example 4
Small volume, Other liability

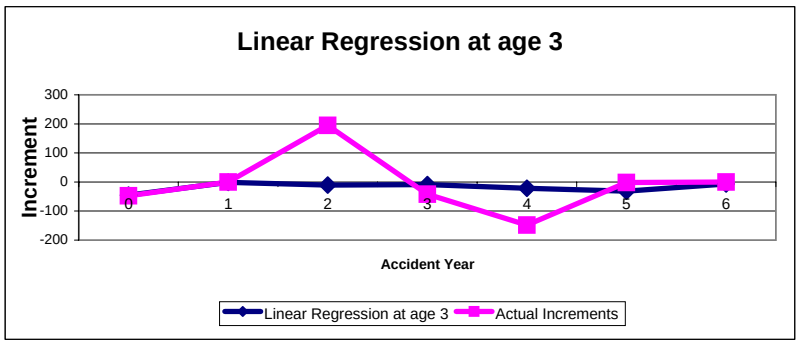
Significance of Factors



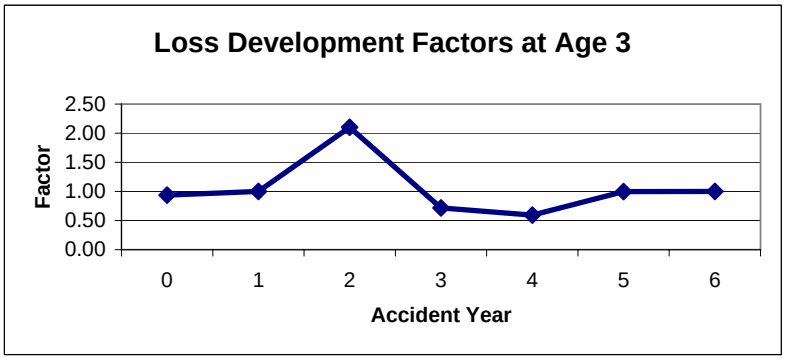
Linearity of Model



Linear Regression



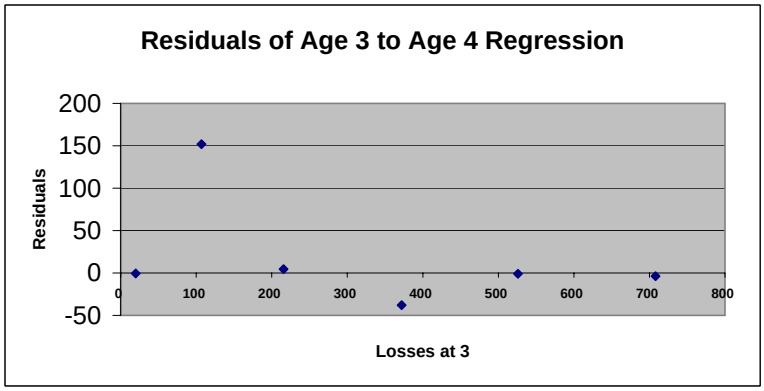
Linearity of Model



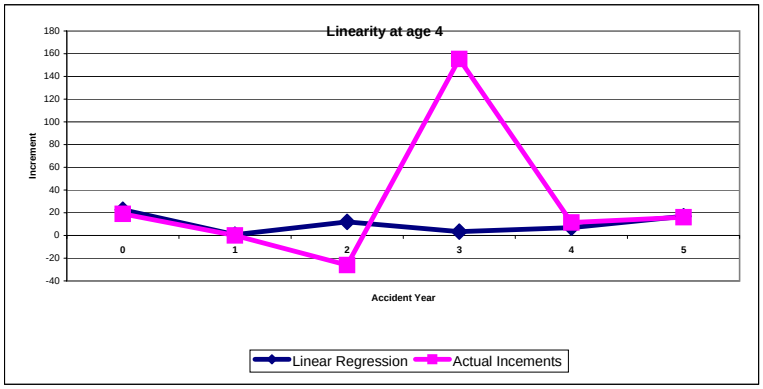
Example 4
Small volume, Other liability

Significance of Factors

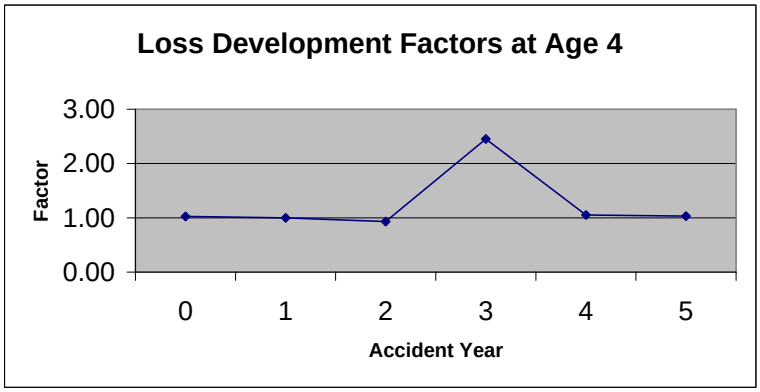
Linearity of Model



Linear Regression

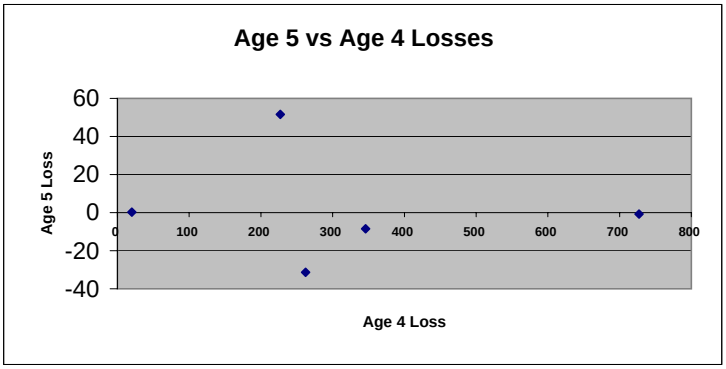


Stability of Factors

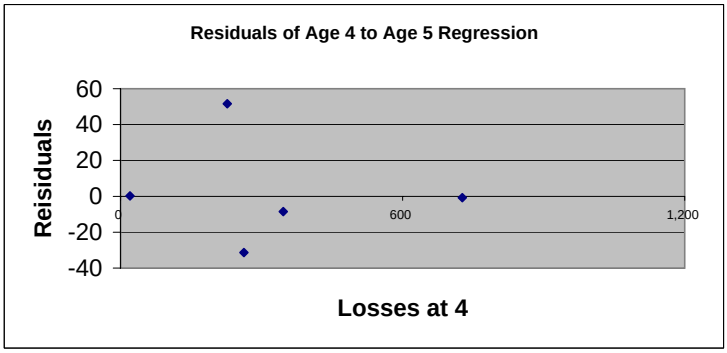


Example 4
Small volume, Other liability

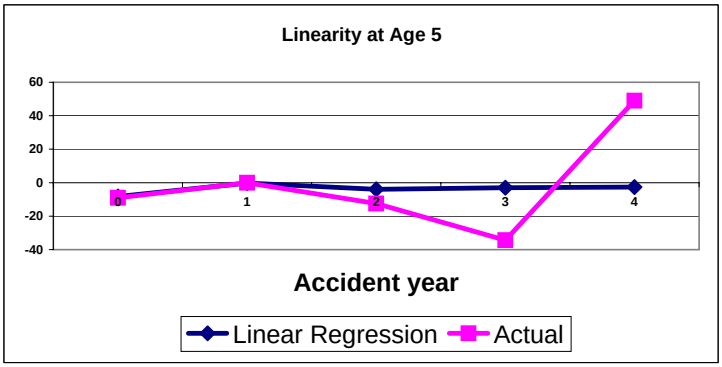
Significance of Factors



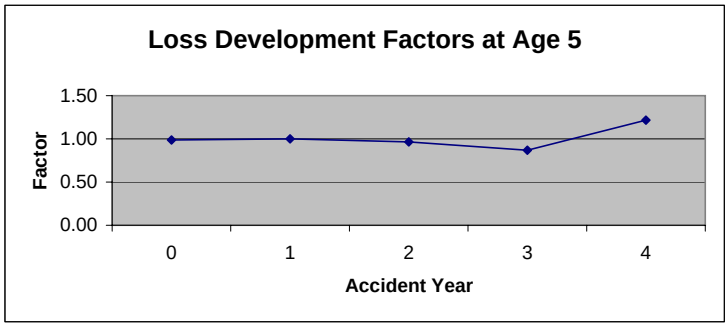
Linearity of Model



Linear Regression



Stability of Factors



Example 5
Small volume, Other liability

Test of factor assumption to alternative emergence patterns

Summary

Analysis	Sum Squared Error	IBNR
Regression w/o constant	369	626
Regression w/ constant	511	840
Constant	316	789
Bornheutter-Ferguson	476	830
Cape Cod	316	789
BF/Cape Cod	329	299
Loss Development Factor		
Selected	458	537
Three year average	478	819
Five year average	472	864
Five year excl high/low	444	123
Four year average	467	685
All years excl high/low	412	242

Correlation Statistics

Age	Correlation	Sample T	n	Probability	Significant from Zero
2 vs 1	-0.251454	-0.636382	8	0.5480	**No**
3 vs 2	-0.082979	-0.186189	7	0.8596	**No**
4 vs 3	-0.93184	-5.135936	6	0.0068	**Yes**
5 vs 4	-0.035499	-0.061524	5	0.9548	**No**
6 vs 5	-0.056533	-0.080077	4	0.9435	**No**

Number of significant correlations	1
Number of required significant correlations	2.736068

Significance of Factors

Regression with constant

Age	R^2	Intercept Coefficient	SE	t statistic	P-Value	Significant from Zero	X variable Coefficient	SE	t statistic	P-Value	P-Value
1 vs 0	15.3%	274.6	160.6	1.709	0.131	No	-0.712	0.632	-1.126	0.297	No
2 vs 1	2.4%	175.5	149.2	1.176	0.284	No	-0.157	0.410	-0.384	0.714	No
3 vs 2	31.8%	12.2	33.8	0.361	0.733	No	-0.099	0.065	-1.527	0.187	No
4 vs 3	15.2%	11.9	53.4	0.223	0.834	No	-0.103	0.121	-0.846	0.445	No
5 vs 4	13.3%	56.8	57.4	0.989	0.395	No	-0.097	0.142	-0.680	0.545	No
6 vs 5	33.9%	-13.1	18.3	-0.719	0.547	No	0.043	0.043	1.012	0.418	No
7 vs 6	1.3%	47.9	283.5	0.169	0.893	No	0.075	0.639	0.117	0.926	No

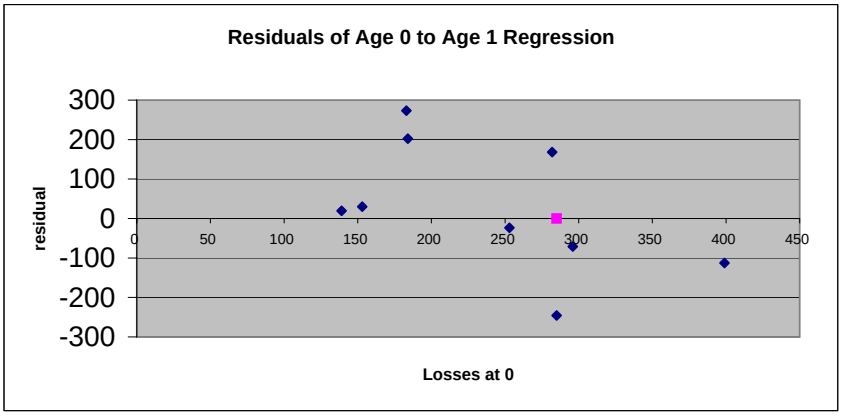
Regression without constant

Age	R^2	Slope Coefficient	SE	t statistic	P-Value	Significant from Zero
1 vs 0	20.5%	0.315	0.219	1.434	**0.189**	**No**
2 vs 1	30.9%	0.288	0.162	1.771	**0.120**	**No**
3 vs 2	71.3%	-0.077	0.020	-3.861	0.008	**Yes**
4 vs 3	50.0%	-0.077	0.035	-2.234	0.076	**slight**
5 vs 4	11.4%	0.035	0.049	0.716	**0.514**	**No**
6 vs 5	27.8%	0.014	0.013	1.074	**0.362**	**No**
7 vs 6	31.2%	0.174	0.182	0.953	**0.441**	**No**

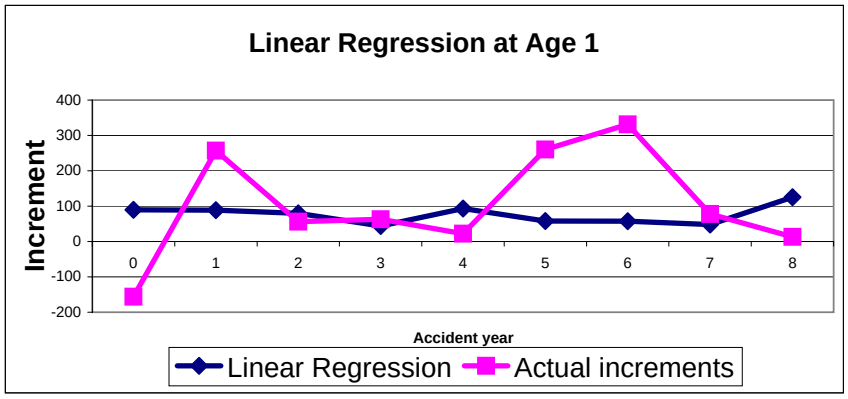
Example 5
Small volume, Other liability

Significance of Factors

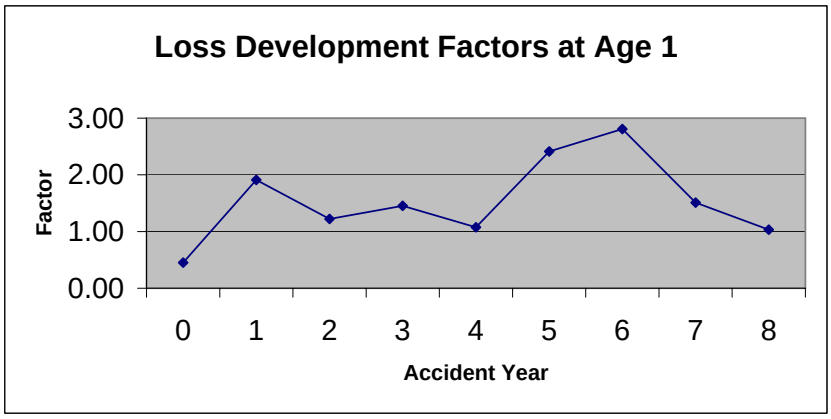
Linearity of Model



Linear Regression



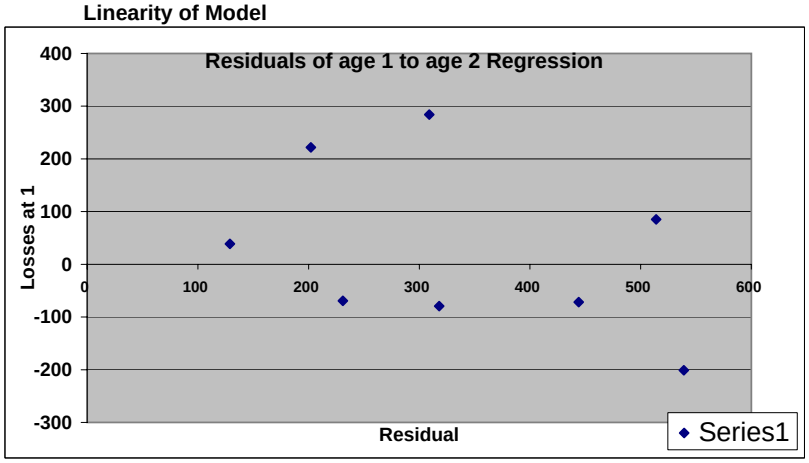
Stability of Factors



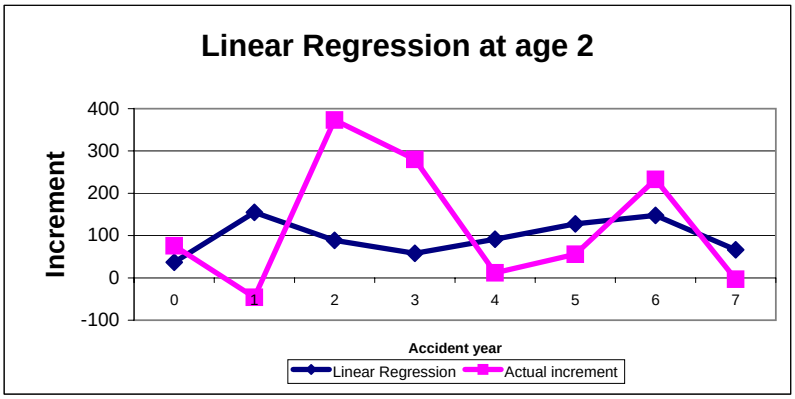
Example 5
Small volume, Other liability

Significance of Factors

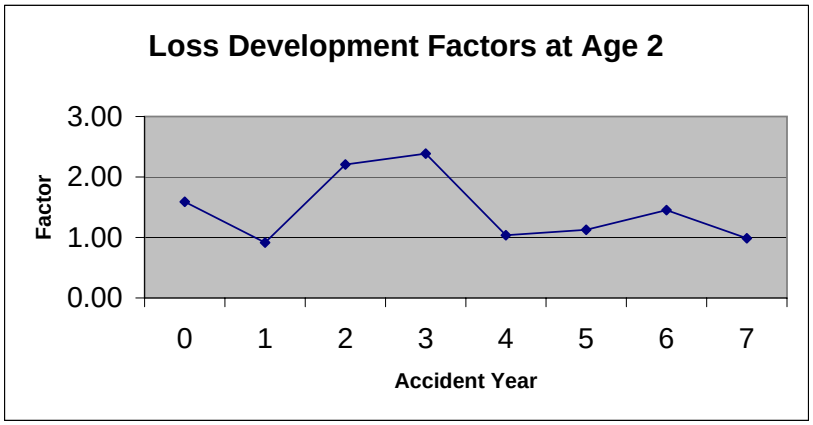
Linearity of Model



Linear Regression



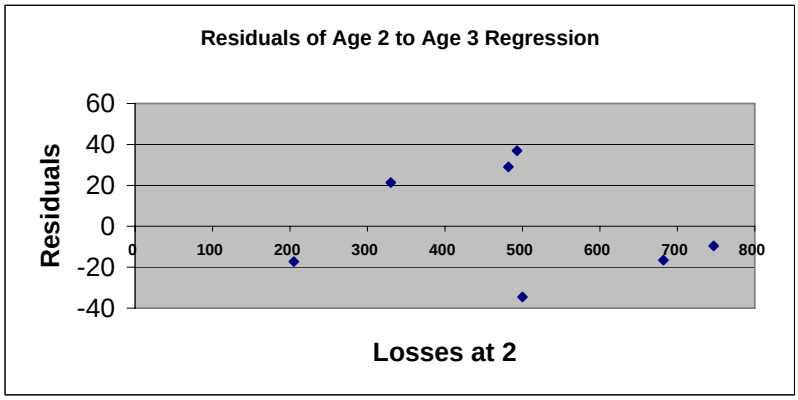
Stability of Factors



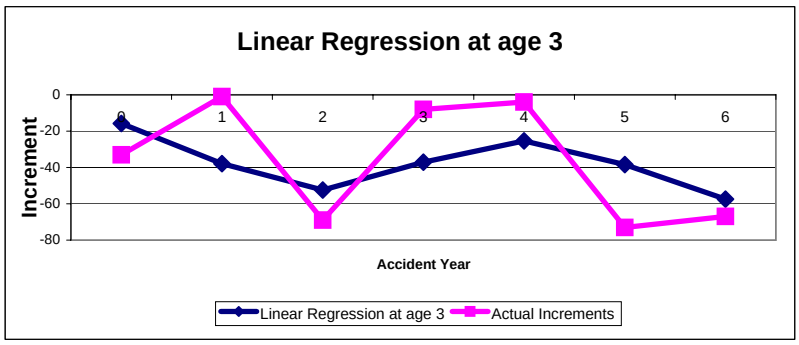
Example 5
Small volume, Other liability

Significance of Factors

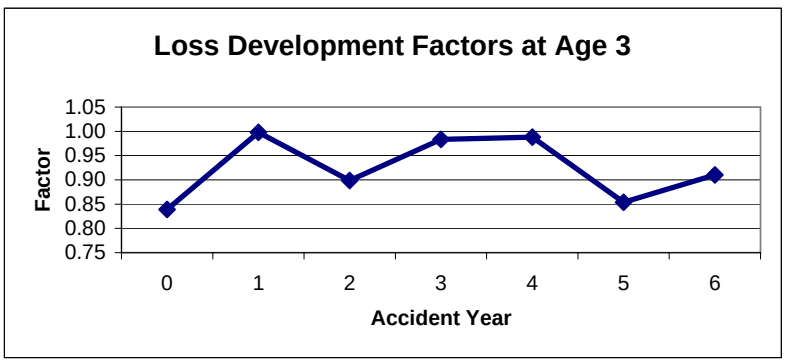
Linearity of Model



Linear Regression

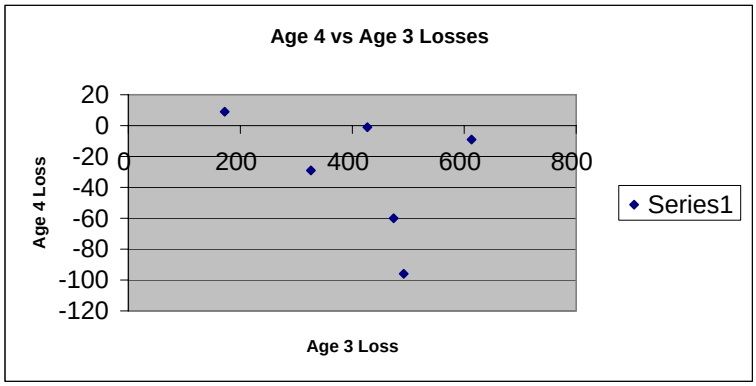


Linearity of Model

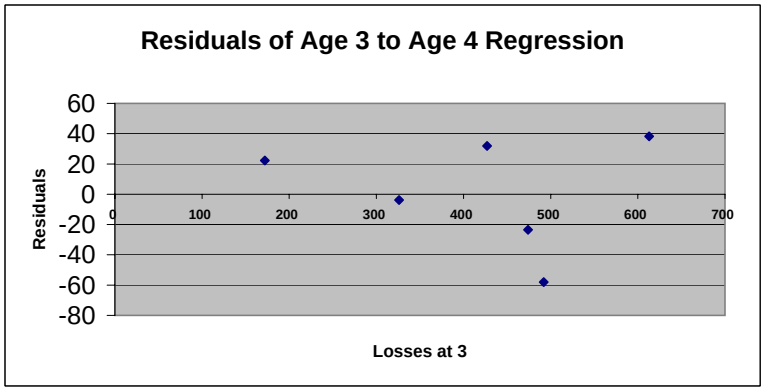


Example 5
Small volume, Other liability

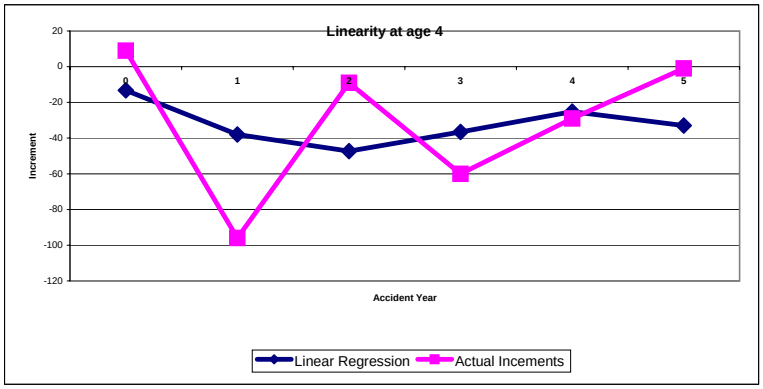
Significance of Factors



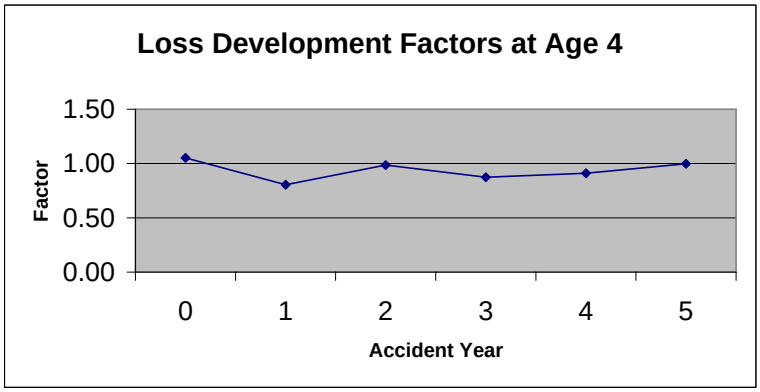
Linearity of Model



Linear Regression

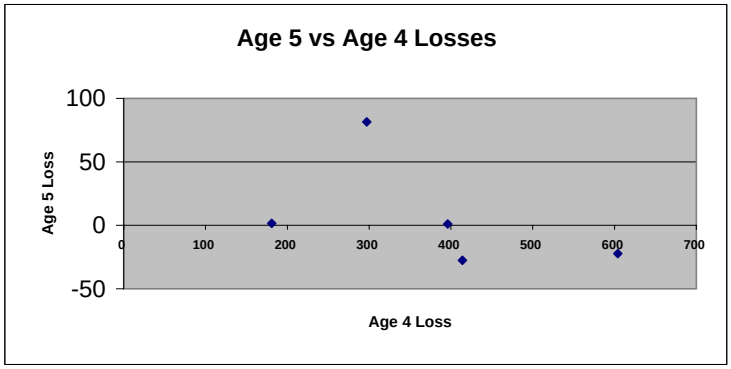


Stability of Factors

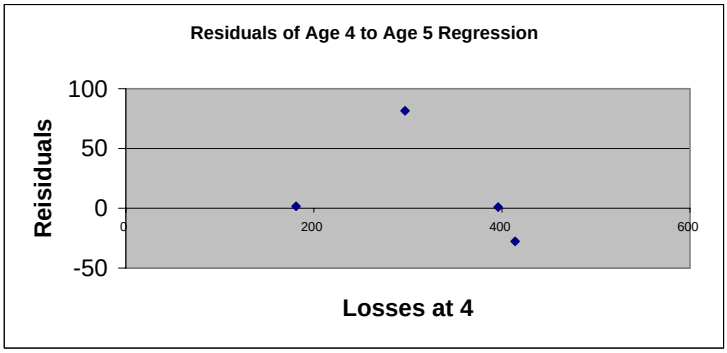


Example 5
Small volume, Other liability

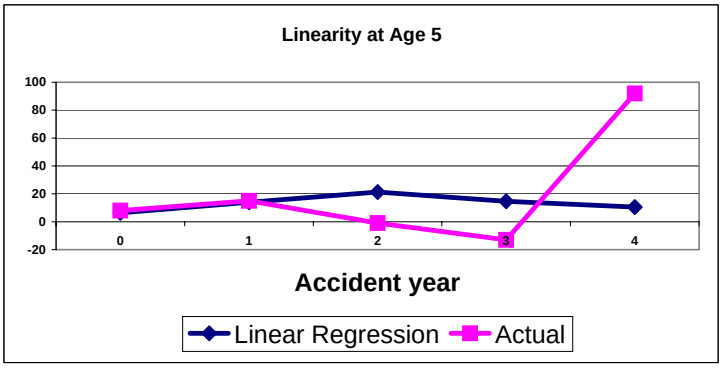
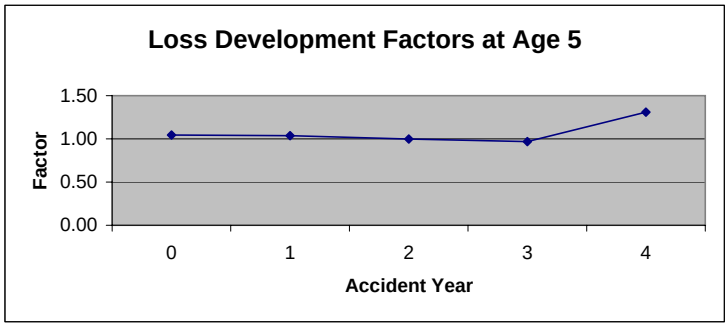
Significance of Factors



Linearity of Model



Linear Regression



Stability of Factors