Methods and Models of Loss Reserving Based on Run–Off Triangles: A Unifying Survey

Klaus D. Schmidt

Lehrstuhl für Versicherungsmathematik
Technische Universität Dresden

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The Run–Off Triangle for Cumulative Losses (1)

An example from the *Claims Reserving Manual*:

Accident Year	Development Year 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					

The enumeration of the development years represents delays with respect to the accident years.

The Run–Off Triangle for Cumulative Losses (2)

Accident Year	**Development Year** 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

The Run–Off Triangle for Cumulative Losses (2)

Accident Year	**Development Year** 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

The Run–Off Triangle for Cumulative Losses (2)

Accident Year	Development Year 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

- observable if $i + k \leq n$.

The Run–Off Triangle for Cumulative Losses (2)

Accident Year	**Development Year** 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

- observable if $i + k \leq n$.
- non–observable or future if $i + k > n$.

The Run–Off Triangle for Cumulative Losses (2)

Accident Year	Development Year 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
n	$S_{n,0}$	$S_{n,1}$	...	$S_{n,k}$	...	$S_{n,n-i}$	...	$S_{n,n-1}$	$S_{n,n}$

A cumulative loss $S_{i,k}$ is said to be

- observable if $i+k \leq n$.
- non–observable or future if $i+k > n$.
- present if $i+k = n$.

The Run–Off Triangle for Cumulative Losses (2)

Accident Year	Development Year 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
$\vdots$	$\vdots$	$\vdots$		$\vdots$		$\vdots$		$\vdots$	$\vdots$
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
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A cumulative loss $S_{i,k}$ is said to be

- observable if $i+k \leq n$.
- non–observable or future if $i+k > n$.
- present if $i+k = n$.
- ultimate if $k = n$.

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Accident Year	**Development Year** 0	1	...	k	...	$n-i$	...	$n-1$	n
0	$S_{0,0}$	$S_{0,1}$	...	$S_{0,k}$	...	$S_{0,n-i}$	...	$S_{0,n-1}$	$S_{0,n}$
1	$S_{1,0}$	$S_{1,1}$	...	$S_{1,k}$	...	$S_{1,n-i}$	...	$S_{1,n-1}$	$S_{1,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
i	$S_{i,0}$	$S_{i,1}$	...	$S_{i,k}$	...	$S_{i,n-i}$	...	$S_{i,n-1}$	$S_{i,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-k$	$S_{n-k,0}$	$S_{n-k,1}$	...	$S_{n-k,k}$	...	$S_{n-k,n-i}$	...	$S_{n-k,n-1}$	$S_{n-k,n}$
⋮	⋮	⋮		⋮		⋮		⋮	⋮
$n-1$	$S_{n-1,0}$	$S_{n-1,1}$	...	$S_{n-1,k}$	...	$S_{n-1,n-i}$	...	$S_{n-1,n-1}$	$S_{n-1,n}$
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- the future cumulative losses $S_{i,k}$
- the future incremental losses $Z_{i,k} := S_{i,k} - S_{i,k-1}$

with $i + k \geq n + 1$

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- the future cumulative losses $S_{i,k}$
- the future incremental losses $Z_{i,k} := S_{i,k} - S_{i,k-1}$
- the calendar year reserves $\sum_{j=p-n}^{n} Z_{j,p-j}$

with $i + k \geq n + 1$ and $p = n + 1, \dots, 2n$.

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More generally: The aim is to predict

- the future cumulative losses $S_{i,k}$
- the future incremental losses $Z_{i,k} := S_{i,k} - S_{i,k-1}$
- the calendar year reserves $\sum_{j=p-n}^{n} Z_{j,p-j}$
- the total reserve $\sum_{j=1}^{n} \sum_{l=n-j+1}^{n} Z_{j,l}$

with $i + k \geq n + 1$ and $p = n + 1, \ldots, 2n$.

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- the calendar year reserves $\sum_{j=p-n}^{n} Z_{j,p-j}$
- the total reserve $\sum_{j=1}^{n} \sum_{l=n-j+1}^{n} Z_{j,l}$

with $i + k \geq n + 1$ and $p = n + 1, \ldots, 2n$.

Thus: The principal task of loss reserving is to predict the future cumulative losses.

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Development Patterns

- A development pattern for quotas consists of parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ with

$$\gamma_k = \mathrm{E}[S_{i,k}]/\mathrm{E}[S_{i,n}]$$

for all $k = 0, 1, \ldots, n$ and for all $i = 0, 1, \ldots, n$.
These parameters are called development quotas (percentages reported).

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  These parameters are called development quotas (percentages reported).

- A development pattern for factors consists of parameters $\varphi_1, \ldots, \varphi_n$ with

  $$\varphi_k = E[S_{i,k}]/E[S_{i,k-1}]$$

  for all $k = 1, \ldots, n$ and for all $i = 0, 1, \ldots, n$.
  These parameters are called development factors (age–to–age factors).

Development Patterns: Cumulative Losses and Quotas

Accident Year	Development Year 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4044
2	1265	2433	3233	3977	4477	4677
3	1490	2873	3880	4880	5380	5680
4	1725	3261	4361	5461	5961	6361
5	1889	3489	4889	5889	6489	6889
0	0.287	0.533	0.696	0.858	0.958	1.000
1	0.275	0.520	0.686	0.846	0.951	1.000
2	0.270	0.520	0.691	0.850	0.957	1.000
3	0.262	0.506	0.683	0.859	0.947	1.000
4	0.271	0.513	0.686	0.859	0.937	1.000
5	0.274	0.506	0.710	0.855	0.942	1.000

Development Patterns: Cumulative Losses and Factors

Accident Year	Development Year 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4044
2	1265	2433	3233	3977	4477	4677
3	1490	2873	3880	4880	5380	5680
4	1725	3261	4361	5461	5961	6361
5	1889	3489	4889	5889	6489	6889
0		1.853	1.306	1.233	1.116	1.044
1		1.889	1.319	1.234	1.123	1.052
2		1.923	1.329	1.230	1.126	1.045
3		1.928	1.351	1.258	1.102	1.056
4		1.890	1.337	1.252	1.092	1.067
5		1.847	1.401	1.205	1.102	1.062

Development Patterns: Quotas and Factors

- If the parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ form a development pattern for quotas, then the parameters $\varphi_1, \ldots, \varphi_n$ with

$$\varphi_k := \frac{\gamma_k}{\gamma_{k-1}}$$

form a development pattern for factors.

Development Patterns: Quotas and Factors

- If the parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ form a development pattern for quotas, then the parameters $\varphi_1, \ldots, \varphi_n$ with

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- If the parameters $\varphi_1, \ldots, \varphi_n$ form a development pattern for factors, then the parameters $\gamma_0, \gamma_1, \ldots, \gamma_n$ with

$$\gamma_k := \prod_{l=k+1}^{n} \frac{1}{\varphi_l}$$

form a development pattern for quotas.

Development Patterns: Estimation of Quotas

For estimation of the parameter γ_k of a development pattern for quotas, the only obvious estimator provided by the run–off triangle is the estimator

$$G_{i,k} := S_{i,k}/S_{i,n}$$

Accident Year	Development Year 0	1	2	3	4	5
0	0.287	0.533	0.696	0.858	0.958	1.000
1	0.275	0.520	0.686	0.846	0.951	1.000
2	0.270	0.520	0.691	0.850	0.957	1.000
3	0.262	0.506	0.683	0.859	0.947	1.000
4	0.271	0.513	0.686	0.859	0.937	1.000
5	0.274	0.506	0.710	0.855	0.942	1.000

Development Patterns: Estimation of Factors

For estimation of the parameter φ_k of a development pattern for factors, the run–off triangle provides the estimators

$$F_{i,k} := S_{i,k} / S_{i,k-1}$$

with $i = 0, 1, \ldots, n-k$. Moreover, any weighted mean of these estimators is an estimator as well.

Accident Year	**Development Year** 0	1	2	3	4	5
0		1.853	1.306	1.233	1.116	1.044
1		1.889	1.319	1.234	1.123	1.052
2		1.923	1.329	1.230	1.126	1.045
3		1.928	1.351	1.258	1.102	1.056
4		1.890	1.337	1.252	1.092	1.067
5		1.847	1.401	1.205	1.102	1.062

Development Patterns: Chain–Ladder Factors

The chain–ladder factors

$$\widehat{\varphi}_k^{\text{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} F_{j,k}$$

are weighted means and may be used to estimate the development factors φ_k.

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\widehat{\varphi}_k^{\text{CL}}$		1.899	1.329	1.232	1.120	1.044

Development Patterns: Chain–Ladder Quotas

The chain–ladder quotas

$$\widehat{\gamma}_k^{\text{CL}} := \prod_{l=k+1}^{n} \frac{1}{\widehat{\varphi}_l^{\text{CL}}}$$

may be used to estimate the development quotas γ_k.

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\widehat{\varphi}_k^{\text{CL}}$		1.899	1.329	1.232	1.120	1.044
$\widehat{\gamma}_k^{\text{CL}}$	0.278	0.527	0.701	0.864	0.968	1

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The Bornhuetter–Ferguson Method (1)

The Bornhuetter–Ferguson method is based on the assumption, that there exists a development pattern for quotas

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The Bornhuetter–Ferguson method is based on the assumption, that there exists a development pattern for quotas and that

- prior estimators

$$\widehat{\gamma}_0, \widehat{\gamma}_1, \ldots, \widehat{\gamma}_n$$

(with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \ldots, \gamma_n$ and

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The Bornhuetter–Ferguson method is based on the assumption, that there exists a development pattern for quotas and that

- prior estimators

  $$\widehat{\gamma}_0, \widehat{\gamma}_1, \dots, \widehat{\gamma}_n$$

  (with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \dots, \gamma_n$ and
- prior estimators

  $$\widehat{\alpha}_0, \widehat{\alpha}_1, \dots, \widehat{\alpha}_n$$

  of the expected ultimate losses

  $$\alpha_i := E[S_{i,n}]$$

  with $i = 0, 1, \dots, n$

are available.

The Bornhuetter–Ferguson Method (2)

These prior estimators can be obtained from

- internal information (provided by the run–off triangle, like chain–ladder factors),

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- internal information (provided by the run–off triangle, like chain–ladder factors),
- volume measures (like premiums) for the portfolio under consideration,
- external information (market statistics or data from similar portfolios) or

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- internal information (provided by the run–off triangle, like chain–ladder factors),
- volume measures (like premiums) for the portfolio under consideration,
- external information (market statistics or data from similar portfolios) or
- a combination of these data.

The Bornhuetter–Ferguson Method (3)

The Bornhuetter–Ferguson predictors of the future cumulative losses $S_{i,k}$ are defined as

$$\widehat{S}_{i,k}^{\text{BF}} := S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right)\widehat{\alpha}_i$$

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Thus:

- The run–off triangle provides information only via the present cumulative losses.
- The predictors of the ultimate cumulative losses are obtained by linear extrapolation from the present cumulative losses.

The Bornhuetter–Ferguson Method (4)

Accident Year i	**Development Year** k 0	1	2	3	4	5	$\widehat{\alpha}_i$
0						3483	3517
1					3844		3981
2				3977			4598
3			3880				5658
4		3261					6214
5	1889						6325
$\widehat{\gamma}_k$	0.280	0.510	0.700	0.860	0.950	1.000	
$1 - \widehat{\gamma}_k$	0.720	0.490	0.300	0.860	0.050	0.000	

The Bornhuetter–Ferguson Method (4)

Accident Year i	Development Year k 0	1	2	3	4	5	$\widehat{\alpha}_i$
0						3483	3517
1					3844	4043	3981
2				3977	4391	4621	4598
3			3880	4785	4389	5577	5658
4		3261	4442	5436	5995	6306	6214
5	1889	3344	4546	5558	6127	6443	6325
$\widehat{\gamma}_k$	0.280	0.510	0.700	0.860	0.950	1.000	
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- Repeat the previous step as often as you want.

During the iteration,

- the prior estimators of the expected ultimate losses are modified in every step, but
- the prior estimators of the development quotas remain unchanged.

The Iterated Bornhuetter–Ferguson Method (2)

Formalizing this idea, the iterated Bornhuetter–Ferguson predictors of order $m = 0, 1, 2, \ldots$ are defined as

$$\widehat{S}_{i,k}^{(m)} := \begin{cases} S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right) \widehat{\alpha}_i & \text{if } m = 0 \\ S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right) \widehat{S}_{i,n}^{(m-1)} & \text{else} \end{cases}$$

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- For $m = 0$ this yields the Bornhuetter–Ferguson predictors.

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- For $m = 0$ this yields the Bornhuetter–Ferguson predictors.
- For $m = 1$ this yields the Benktander–Hovinen predictors.

The Iterated Bornhuetter–Ferguson Method (3)

The iterated Bornhuetter–Ferguson predictors can be written as

$$\widehat{S}_{i,k}^{(m)} = \widehat{\gamma}_k \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}} + \left(1-\widehat{\gamma}_{n-i}\right)^m \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right) \left(\widehat{\alpha}_i - \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}\right)$$

and this yields (in the case $0 \le \widehat{\gamma}_{n-i} \le 1$)

$$\lim_{m\to\infty} \widehat{S}_{i,k}^{(m)} = \widehat{\gamma}_k \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

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Thus:

- The iterated Bornhuetter–Ferguson method provides in every step a reduction of the influence of the prior estimators $\widehat{\alpha}_i$ of the ultimate aggregate losses.

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and this yields (in the case $0 \leq \widehat{\gamma}_{n-i} \leq 1$)

$$\lim_{m\to\infty} \widehat{S}_{i,k}^{(m)} = \widehat{\gamma}_k \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

Thus:

- The iterated Bornhuetter–Ferguson method provides in every step a reduction of the influence of the prior estimators $\widehat{\alpha}_i$ of the ultimate aggregate losses.
- For every accident year and every development year, the sequence of the iterated Bornhuetter–Ferguson predictors is monotone (decreasing or increasing) and convergent.

The Iterated Bornhuetter–Ferguson Method (4)

The following table contains the prior estimators $\widehat{\alpha}_i$ of the expected ultimate losses $E[S_{i,n}]$, the iterated Bornhuetter–Ferguson predictors

$$\widehat{S}_{i,n}^{(m)} = \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}} + (1-\widehat{\gamma}_{n-i})^{m+1}\left(\widehat{\alpha}_i - \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}\right)$$

of the ultimate losses $S_{i,n}$, and their limits:

Accident Year i	$\widehat{\alpha}_i$	Iterated Bornhuetter–Ferguson Predictors $\widehat{S}_{i,5}^{(0)}$	$\widehat{S}_{i,5}^{(1)}$	$\widehat{S}_{i,5}^{(2)}$	$\widehat{S}_{i,5}^{(3)}$	$\widehat{S}_{i,5}^{(4)}$	$\widehat{S}_{i,5}^{(5)}$	...	$\widehat{S}_{i,5}^{(\infty)}$
0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981							...	
2	4598							...	
3	5658							...	
4	6214							...	
5	6325							...	

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0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043						...	
2	4598	4621						...	
3	5658	5577						...	
4	6214	6306						...	
5	6325	6443						...	

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0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043	4046					...	
2	4598	4621	4623					...	
3	5658	5577	5553					...	
4	6214	6306	6351					...	
5	6325	6443	6528					...	

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0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043	4046	4046				...	
2	4598	4621	4623	4624				...	
3	5658	5577	5553	5546				...	
4	6214	6306	6351	6373				...	
5	6325	6443	6528	6589				...	

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0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043	4046	4046	4046			...	
2	4598	4621	4623	4624	4624			...	
3	5658	5577	5553	5546	5544			...	
4	6214	6306	6351	6373	6384			...	
5	6325	6443	6528	6589	6633			...	

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0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043	4046	4046	4046	4046		...	
2	4598	4621	4623	4624	4624	4624		...	
3	5658	5577	5553	5546	5544	5543		...	
4	6214	6306	6351	6373	6384	6389		...	
5	6325	6443	6528	6589	6633	6664		...	

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Accident Year i	$\widehat{\alpha}_i$	Iterated Bornhuetter–Ferguson Predictors $\widehat{S}_{i,5}^{(0)}$	$\widehat{S}_{i,5}^{(1)}$	$\widehat{S}_{i,5}^{(2)}$	$\widehat{S}_{i,5}^{(3)}$	$\widehat{S}_{i,5}^{(4)}$	$\widehat{S}_{i,5}^{(5)}$	...	$\widehat{S}_{i,5}^{(\infty)}$
0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043	4046	4046	4046	4046	4046	...	
2	4598	4621	4623	4624	4624	4624	4624	...	
3	5658	5577	5553	5546	5544	5543	5543	...	
4	6214	6306	6351	6373	6384	6389	6392	...	
5	6325	6443	6528	6589	6633	6664	6687	...	

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The following table contains the prior estimators $\widehat{\alpha}_i$ of the expected ultimate losses $E[S_{i,n}]$, the iterated Bornhuetter–Ferguson predictors

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of the ultimate losses $S_{i,n}$, and their limits:

Accident Year i	$\widehat{\alpha}_i$	**Iterated Bornhuetter–Ferguson Predictors** $\widehat{S}_{i,5}^{(0)}$	$\widehat{S}_{i,5}^{(1)}$	$\widehat{S}_{i,5}^{(2)}$	$\widehat{S}_{i,5}^{(3)}$	$\widehat{S}_{i,5}^{(4)}$	$\widehat{S}_{i,5}^{(5)}$	...	$\widehat{S}_{i,5}^{(\infty)}$
0	3517	3483	3483	3483	3483	3483	3483	...	3483
1	3981	4043	4046	4046	4046	4046	4046	...	4046
2	4598	4621	4623	4624	4624	4624	4624	...	4624
3	5658	5577	5553	5546	5544	5543	5543	...	5543
4	6214	6306	6351	6373	6384	6389	6392	...	6394
5	6325	6443	6528	6589	6633	6664	6687	...	6746

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The Loss–Development Method (1)

The loss–development method is based on the assumption, that there exists a development pattern for quotas

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(with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \ldots, \gamma_n$ are available.

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$$\widehat{\gamma}_0, \widehat{\gamma}_1, \ldots, \widehat{\gamma}_n$$

(with $\widehat{\gamma}_n = 1$) of the development quotas $\gamma_0, \gamma_1, \ldots, \gamma_n$ are available.

The loss–development method does not involve any prior estimators for the expected ultimate losses.

The Loss–Development Method (2)

The loss–development predictors of the future cumulative losses $S_{i,k}$ are defined as

$$\widehat{S}_{i,k}^{\mathrm{LD}} := \widehat{\gamma}_k \, \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

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The loss–development predictors of the future cumulative losses $S_{i,k}$ are defined as

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Thus:

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The loss–development predictors of the future cumulative losses $S_{i,k}$ are defined as

$$\widehat{S}_{i,k}^{\mathrm{LD}} := \widehat{\gamma}_k \, \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

Thus:

- The run–off triangle provides information only via the present cumulative losses.
- The predictors of the ultimate cumulative losses are obtained by using first the prior estimator $\widehat{\gamma}_{n-i}$ to scale the present cumulative losses to the level of the ultimate losses and using then the prior estimator $\widehat{\gamma}_k$ to scale the result to the level of the cumulative losses of development year k.

The Loss–Development Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0						3483
1					3844	
2				3977		
3			3880			
4		3261				
5	1889					
$\widehat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

The Loss–Development Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0						3483
1					3844	4046
2				3977		4624
3			3880			5543
4		3261				6394
5	1889					6746
$\widehat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

The Loss–Development Method (3)

Accident Year i	Development Year k 0	1	2	3	4	5
0						3483
1					3844	4046
2				3977	4393	4624
3			3880	4767	5266	5543
4		3261	4476	5499	6074	6394
5	1889	3440	4722	5802	6409	6746
$\widehat{\gamma}_k$	0,280	0,510	0,700	0,860	0,950	1,000

The Loss–Development Method (4)

- Because of the definition

$$\widehat{S}_{i,k}^{\text{LD}} := \widehat{\gamma}_k \, \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

the loss–development predictors coincide with the limits of the iterated Bornhuetter–Ferguson method.

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  the loss–development predictors coincide with the limits of the iterated Bornhuetter–Ferguson method.

- Since

$$\widehat{S}_{i,k}^{\text{LD}} = S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i} \right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

  the loss–development predictors can be interpreted as Bornhuetter–Ferguson predictors with respect to the internal prior estimators

$$\widehat{\alpha}_i^{\text{LD}} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}}$$

  of the expected ultimate losses.

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The Chain–Ladder Method (1)

The chain–ladder method is based on the assumption that there exists a development pattern for factors.

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The chain–ladder method relies completely on the observable cumulative losses of the run–off triangle and involves no prior estimators at all.

As estimators of the development factors, the chain–ladder method uses the chain–ladder factors

$$\widehat{\varphi}_k^{\text{CL}} := \frac{\sum_{j=0}^{n-k} S_{j,k}}{\sum_{j=0}^{n-k} S_{j,k-1}} = \sum_{j=0}^{n-k} \frac{S_{j,k-1}}{\sum_{h=0}^{n-k} S_{h,k-1}} \, \frac{S_{j,k}}{S_{j,k-1}}$$

The Chain–Ladder Method (2)

The chain–ladder predictors of the future cumulative losses $S_{i,k}$ are defined as

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^{k} \widehat{\varphi}_l^{\text{CL}}$$

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Thus:

The Chain–Ladder Method (2)

The chain–ladder predictors of the future cumulative losses $S_{i,k}$ are defined as

$$\widehat{S}_{i,k}^{\text{CL}} := S_{i,n-i} \prod_{l=n-i+1}^{k} \widehat{\varphi}_l^{\text{CL}}$$

Thus:

- The chain–ladder method consists in successive scaling of the present cumulative loss $S_{i,n-i}$ to the level of the future cumulative loss $S_{i,k}$.

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889					
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261				
5	1889	3587				
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0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	3261	4334			
5	1889	3587	4767			
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880	4780		
4	1725	3261	4334	5339		
5	1889	3587	4767	5873		
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977	4454	
3	1490	2873	3880	4780	5354	
4	1725	3261	4334	5339	5980	
5	1889	3587	4767	5873	6578	
$\widehat{\varphi}_k^{\mathrm{CL}}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (3)

Accident Year i	**Development Year** k 0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	4013
2	1265	2433	3233	3977	4454	4650
3	1490	2873	3880	4780	5354	5590
4	1725	3261	4334	5339	5980	6243
5	1889	3587	4767	5873	6578	6867
$\widehat{\varphi}_k^{CL}$		1,899	1,329	1,232	1,120	1,044

The Chain–Ladder Method (4)

Since

$$\widehat{S}_{i,k}^{\text{CL}} = S_{i,n-i} \prod_{l=n-i+1}^{k} \widehat{\varphi}_l^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \, \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

the chain–ladder predictors can be interpreted as loss–development predictors with respect to the chain–ladder quotas.

The Chain–Ladder Method (5)

Since

$$\widehat{S}_{i,k}^{\text{CL}} = \widehat{\gamma}_k^{\text{CL}} \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}} = S_{i,n-i} + \left(\widehat{\gamma}_k^{\text{CL}} - \widehat{\gamma}_{n-i}^{\text{CL}} \right) \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

the chain–ladder predictors can also be interpreted as Bornhuetter–Ferguson predictors with respect to the chain–ladder quotas and the prior estimators

$$\widehat{\alpha}_i^{\text{CL}} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}^{\text{CL}}}$$

of the expected ultimate losses.

Table of Contents

Conclusion and Extensions (1)

Thesis:

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- The Bornhuetter–Ferguson method is not just a method.
- The Bornhuetter–Ferguson method is a principle under which various methods of loss reserving can be summarized.

Conclusion and Extensions (2)

Bornhuetter–Ferguson predictors:

$$\widehat{S}_{i,k} := S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right)\widehat{\alpha}_i$$

prior estimators	$\widehat{\alpha}_i$	$\widehat{\gamma}_k$
Bornhuetter–Ferguson	arbitrary	arbitrary
Loss–Development	$\widehat{\alpha}_i^{LD}$	arbitrary
Chain–Ladder	$\widehat{\alpha}_i^{LD}$	$\widehat{\gamma}_k^{CL}$
Cape–Cod	$\widehat{\alpha}_i^{CC}$	arbitrary
Additive	$\widehat{\alpha}_i^{CC}$	$\widehat{\gamma}_k^{AD}$

with

$$\widehat{\alpha}_i^{LD} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}} \qquad \text{und} \qquad \widehat{\alpha}_i^{CC} := v_i \frac{\sum_{j=0}^{n} S_{j,n-j}}{\sum_{j=0}^{n} \widehat{\gamma}_{n-j}\, v_j}$$

Conclusion and Extensions (2)

Bornhuetter–Ferguson predictors:

$$\widehat{S}_{i,k} := S_{i,n-i} + \left(\widehat{\gamma}_k - \widehat{\gamma}_{n-i}\right)\widehat{\alpha}_i$$

prior estimators	$\widehat{\alpha}_i$	$\widehat{\gamma}_k$
Bornhuetter–Ferguson	arbitrary	arbitrary
Loss–Development	$\widehat{\alpha}_i^{\text{LD}}$	arbitrary
Chain–Ladder	$\widehat{\alpha}_i^{\text{LD}}$	$\widehat{\gamma}_k^{\text{CL}}$
Cape–Cod	$\widehat{\alpha}_i^{\text{CC}}$	arbitrary
Additive	$\widehat{\alpha}_i^{\text{CC}}$	$\widehat{\gamma}_k^{\text{AD}}$

with

$$\widehat{\alpha}_i^{\text{LD}} := \frac{S_{i,n-i}}{\widehat{\gamma}_{n-i}} \qquad \text{und} \qquad \widehat{\alpha}_i^{\text{CC}} := v_i \, \frac{\sum_{j=0}^{n} S_{j,n-j}}{\sum_{j=0}^{n} \widehat{\gamma}_{n-j}\, v_j}$$

Input Prior Development Pattern

Development Year	0	1	2	3	4	5	Check
Incremental Quotas							
Quotas	0.2800	0.5100	0.7000	0.8600	0.9500	1.0000	**O.K.**
Factors							

Input Prior Estimates of Expected Ultimates / Volume Measures

Accident Year	0	1	2	3	4	5	
Ultimates	3517	3981	4598	5658	6214	6325	**O.K.**
Volumes	4025	4456	5315	5986	6939	8158	**O.K.**

Input Run-Off Triangle

Incremental Losses	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						
Check						

Cumulative Losses	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	4261				
5	1889					
Check	**O.K.**					

Methods with an O.K. work with the given data.

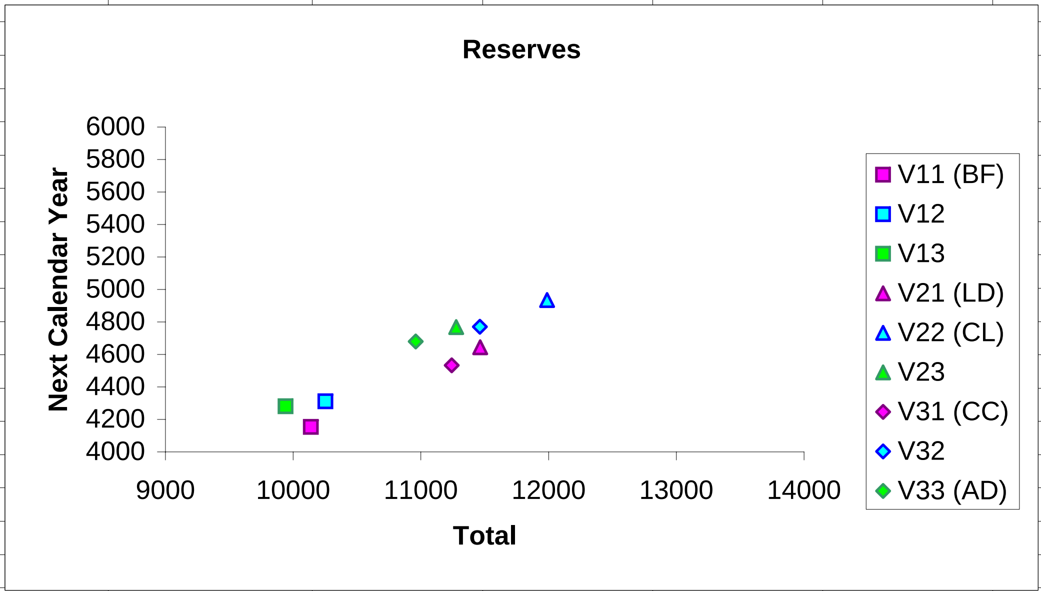
	Prior Quotas	Chain-Ladder Quotas	Additive Quotas
Priori Ultimates	**O.K.**	**O.K.**	**O.K.**
Loss-Development Ultimates	**O.K.**	**O.K.**	**O.K.**
Cap-Cod Ultimates	**O.K.**	**O.K.**	**O.K.**

Names of known methods are inserted.

	Prior Quotas	Chain-Ladder Quotas	Additive Quotas
Prior Ultimates	Bornhuetter-Ferguson		
Loss-Development Ultimates	Loss-Development	Chain-Ladder	
Cap-Cod Ultimates	Cape-Cod		Additive Method

	Development Pattern					
	0	1	2	3	4	5
Prior	0.2800	0.5100	0.7000	0.8600	0.9500	1.0000
Chain-Ladder	0.2546	0.5222	0.6939	0.8549	0.9575	1.0000
Additive	0.2627	0.5428	0.7091	0.8624	0.9603	1.0000

			"Prior" Ultimates						Reserves	
	Ultimates	Quotas	0	1	2	3	4	5	Total	Next Calendar Year
V11	Prior	Prior	3517	3981	4598	5658	6214	6325	10139	4154
V12	Prior	Chain-Ladder	3517	3981	4598	5658	6214	6325	10252	4312
V13	Prior	Additive	3517	3981	4598	5658	6214	6325	9941	4281
V21	Loss-Development	Prior	3483	4046	4624	5543	8355	6746	11464	4644
V22	Loss-Development	Chain-Ladder	3483	4015	4652	5592	8160	7420	11987	4935
V23	Loss-Development	Additive	3483	4003	4612	5471	7850	7191	11276	4769
V31	Cap-Cod	Prior	3759	4162	4964	5591	6481	7619	11242	4533
V32	Cap-Cod	Chain-Ladder	3785	4190	4998	5628	6524	7671	11461	4770
V33	Cap-Cod	Additive	3727	4126	4921	5542	6425	7553	10959	4679
								Maximum	11987	4935
								Minimum	9941	4154



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Input Prior Development Pattern

Development Year	0	1	2	3	4	5	Check
Incremental Quotas							
Quotas	0.2800	0.5100	0.7000	0.8600	0.9500	1.0000	**O.K.**
Factors							

Input Prior Estimates of Expected Ultimates / Volume Measures

Accident Year	0	1	2	3	4	5	
Ultimates	3517	3981	4598	5658	6214	6325	**O.K.**
Volumes	4025	4456	5315	5986	6939	8158	**O.K.**

Input Run-Off Triangle

Incremental Losses	0	1	2	3	4	5
0						
1						
2						
3						
4						
5						

Check

Cumulative Losses	0	1	2	3	4	5
0	1001	1855	2423	2988	3335	3483
1	1113	2103	2774	3422	3844	
2	1265	2433	3233	3977		
3	1490	2873	3880			
4	1725	5261				
5	1889					

Check **O.K.**

Methods with an O.K. work with the given data.

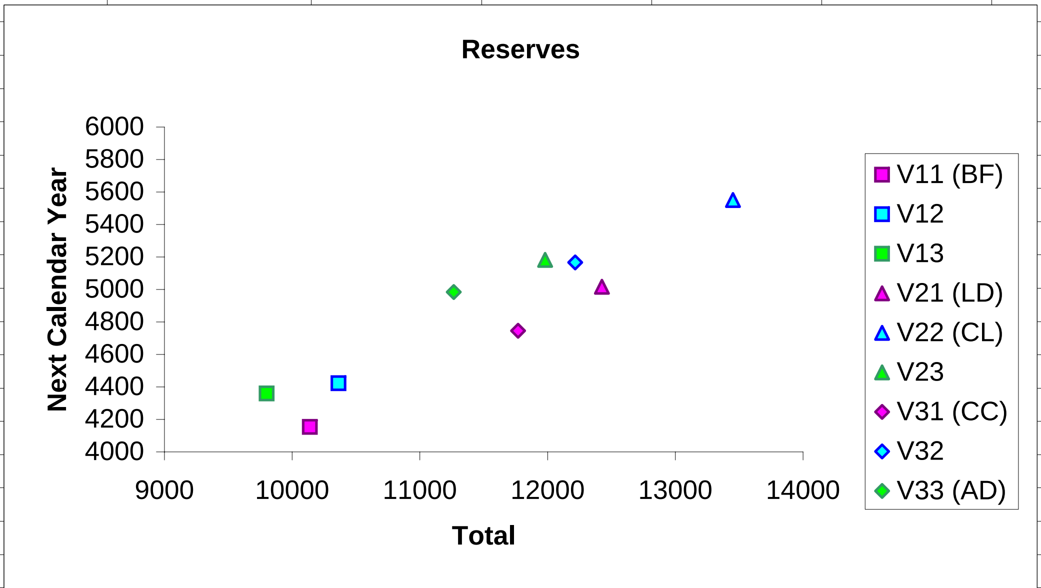
	Prior Quotas	Chain-Ladder Quotas	Additive Quotas
Priori Ultimates	**O.K.**	**O.K.**	**O.K.**
Loss-Development Ultimates	**O.K.**	**O.K.**	**O.K.**
Cap-Cod Ultimates	**O.K.**	**O.K.**	**O.K.**

Names of known methods are inserted.

	Prior Quotas	Chain-Ladder Quotas	Additive Quotas
Prior Ultimates	Bornhuetter-Ferguson		
Loss-Development Ultimates	Loss-Development	Chain-Ladder	
Cap-Cod Ultimates	Cape-Cod		Additive Method

		Development Pattern					
	0	1	2	3	4	5	
Prior	0.2800	0.5100	0.7000	0.8600	0.9500	1.0000	
Chain-Ladder	0.2371	0.5222	0.6939	0.8549	0.9575	1.0000	
Additive	0.2525	0.5606	0.7204	0.8677	0.9618	1.0000	

			"Prior" Ultimates						Reserves	
	Ultimates	Quotas	0	1	2	3	4	5	Total	Next Calendar Year
V11	Prior	Prior	3517	3981	4598	5658	6214	6325	10139	4154
V12	Prior	Chain-Ladder	3517	3981	4598	5658	6214	6325	10363	4422
V13	Prior	Additive	3517	3981	4598	5658	6214	6325	9801	4360
V21	Loss-Development	Prior	3483	4046	4624	5543	10316	6746	12425	5017
V22	Loss-Development	Chain-Ladder	3483	4015	4652	5592	10075	7969	13451	5550
V23	Loss-Development	Additive	3483	3997	4583	5386	9385	7482	11981	5182
V31	Cap-Cod	Prior	3935	4357	5197	5853	6785	7976	11769	4746
V32	Cap-Cod	Chain-Ladder	3987	4414	5265	5930	6874	8081	12216	5167
V33	Cap-Cod	Additive	3877	4292	5120	5766	6684	7859	11265	4985
								Maximum	13451	5550
								Minimum	9801	4154



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Further Topics

Loss reserving based on run–off triangles can also be studied in more sophisticated models and with regard to classical principles of statistical inference:

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Loss reserving based on run–off triangles can also be studied in more sophisticated models and with regard to classical principles of statistical inference:

- Linear Models and Gauss–Markov Prediction
- Credibility Models and Best Affine–Linear Prediction
- Parametric Models and Maximum–Likelihood Estimation

These topics are also discussed in the paper.