Casualty Loss Reserve Seminar

The Language of Uncertainty: Reserve Ranges – Real and Imagined

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PRICEWATERHOUSECOOPERS

Overview

- Present reserve ranges based on applying 4 methods to 2 bodies of data (“imagined”)
- Present implicit reserve ranges in carried reserves based on subsequent development (“real”)

Estimating Reserve Ranges

Overview

- Attempt to Quantify the Amount of Variability Suggested by Methods Currently Available
- Focused on Four Methods that were Relatively Simple and Only Required a Triangle of Loss Data to Apply
  - Simulation
  - CAS Working Party “Estimated Range” Method (CAS Forum, Fall 2005)
  - Thomas Mack
  - Bootstrapping
- Two Sets of Data, ABC & XYZ Company
- Data valued as of 12/31/2005
- Both Paid and Incurred Triangles

Estimating Reserve Ranges

Data Samples

- ABC Company @ December 31, 2005

Incurred Loss Development As of December 31, 2005 Net Layer										
Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-Ult
1996	1.2283	1.1219	1.0219	1.0127	1.0058	1.0033	1.0019	1.0003	1.0001	
1997	1.1608	1.0727	1.0450	1.0199	1.0071	1.0020	1.0011	1.0003		
1998	1.0374	1.0992	1.0526	1.0224	1.0066	1.0041	1.0018			
1999	1.1796	1.1477	1.0634	1.0253	1.0109	1.0046				
2000	1.2240	1.1445	1.0614	1.0294	1.0114					
2001	1.2122	1.1427	1.0692	1.0284						
2002	1.2077	1.1544	1.0689							
2003	1.2150	1.1583								
2004	1.2109									
Volume Weighted	1.1837	1.1296	1.0544	1.0230	1.0083	1.0035	1.0016	1.0003	1.0001	
5 Year Volume W	1.2139	1.1496	1.0633	1.0251	1.0083	1.0035	1.0016	1.0003	1.0001	
4 Year Volume W	1.2114	1.1500	1.0658	1.0264	1.0090	1.0035	1.0016	1.0003	1.0001	
3 Year Volume W	1.2111	1.1518	1.0666	1.0277	1.0097	1.0035	1.0016	1.0003	1.0001	
2 Year Volume W	1.2130	1.1563	1.0690	1.0289	1.0111	1.0043	1.0014	1.0003	1.0001	
Selected	1.2130	1.1563	1.0690	1.0289	1.0111	1.0043	1.0016	1.0003	1.0002	1.0002
Cumulative	1.5701	1.2945	1.1194	1.0471	1.0177	1.0065	1.0022	1.0006	1.0003	1.0002
Ratio to Ultimate	0.6369	0.7725	0.8933	0.9550	0.9826	0.9935	0.9978	0.9994	0.9997	0.9998

Estimating Reserve Ranges

Data Samples (Continued)

- XYZ Company @ December 31, 2005

Incurred Loss Development
As of 12/31/2005
Direct Layer

Accident Year	12-24	24-36	36-48	48-60	60-72	72-84	84-96	96-108	108-120	120-132	132-144	144-156	156-168	168-180
1988	1.445	1.462	2.661	1.074	0.890	1.140	1.090	0.827	1.179	0.998	1.000	1.000	0.999	1.000
1989	1.582	3.664	0.820	1.099	0.879	1.154	0.968	1.192	0.965	1.028	0.972	1.001	0.999	1.000
1990	1.698	1.005	1.026	1.480	1.216	1.257	1.023	0.948	1.003	1.058	0.993	0.983	1.015	1.003
1991	1.182	4.982	1.436	1.392	1.084	1.024	1.027	1.044	0.949	1.023	1.000	1.000	0.986	1.000
1992	6.935	2.136	2.200	1.651	1.057	0.871	0.861	1.105	1.038	1.092	0.983	1.000	1.001	
1993	28.465	1.416	1.673	5.309	1.005	1.000	1.086	0.993	0.994	1.000	1.000	1.000		
1994	10.747	5.461	1.283	1.016	1.008	1.095	1.000	1.019	0.928	1.000	1.000			
1995	5.457	0.963	1.624	0.935	1.289	1.052	1.121	0.998	1.004	1.002				
1996	4.872	1.494	0.756	0.950	0.917	0.913	1.018	1.091	1.000					
1997	1.571	1.786	1.212	1.035	1.064	0.988	1.215	1.000						
1998	51.688	1.219	1.967	1.218	0.993	0.997	1.009							
1999	6.591	1.078	1.297	1.894	1.021	1.005								
2000	2.442	1.626	1.052	1.151	1.082									
2001	4.016	3.123	1.306	1.267										
2002	11.046	1.142	1.098											
2003	2.405	1.235												
2004	3.160													
Volume Weigh	3.748	1.493	1.263	1.269	1.051	1.023	1.054	1.013	0.997	1.019	0.993	0.997	1.001	1.001
Volume Weigh	3.487	1.502	1.287	1.255	1.032	1.019	1.043	1.012	0.996	1.013	0.995	1.000	1.000	1.000
10 Year Avera	9.325	1.913	1.327	1.643	1.052	1.020	1.033	1.022	1.007	1.025	0.993	0.997	1.000	1.001
5 Year Volume	3.607	1.459	1.271	1.280	1.028	1.003	1.074	1.010	0.993	1.014	0.996	0.996	1.001	1.001
3 Year Volume	4.224	1.488	1.145	1.350	1.032	0.998	1.068	1.012	0.985	1.001	0.996	1.000	1.001	1.001

Method Descriptions

Simulation Method

- Calculate Average LDF and Standard Deviation of LDF for each Development Period from Basic Chain-Ladder Approach; for Periods Where the Number of Observed Development Points is Minimal, Use the Standard Deviation of Earlier Periods as an Estimate
- Fit Lognormal Distribution for each Development Period
- Randomly Simulate an LDF for each Development Period; Estimate Reserves Based on Simple Development Method
- Run N Simulations (We Ran 5,000); Results Given as Percentiles

Method Descriptions

Thomas Mack Method

- Select Age-to-Age LDFs from Basic Chain Ladder Approach
- Estimate Reserves Based on Simple Development Method – Set Reserves Equal to X
- Estimate Variance of Reserve Estimate for each Accident Year per Mack's Approach; This is Estimated as the Loss-Weighted Average Square Error between Observed and Selected Age-to-Age LDFs
- Estimate Variance of Overall Reserve Estimate per Mack's Approach; an Implicit Correlation is Calculated from the Estimates by Accident Year – Set Overall Variance Equal to Y
- Derive Confidence Level of Overall Reserve Estimate Using a Lognormal Distribution; $u = \ln(X) - o^2 / 2$ and $o^2 = \ln(1 + (Y/X)^2)$
- Using Distribution Above, Can Calculate Any Desired Confidence Interval

Method Descriptions

CAS Working Party “Estimated Range” Method

- For each Age d, Calculate Average LDF = f(d); Cumulative LDF = F(d)
- For each d, Calculate Average Squared Deviation $s^2(d)$; for n Observed Factors, Divide by (n-1) when Calculating $s^2(d)$ to Adjust for Uncertainty about f(d)
- Calculate $S^2(d)$ Working Backwards through Triangle Such That $S^2(d) = f(d)^2 * S^2(d+1) + F(d+1)^2 * s^2(d) + s^2(d) * S^2(d+1)$
- Last Diagonal of Observed Loss = c(w,d) for each Accident Year w
- Estimate Ultimate Loss by Accident Year as c(w,d) * F(d) and Variance of Ultimate Loss by Accident Year as $c(w,d)^2 * S^2(d)$
- Total Ultimate Loss Mean and Variance for All Accident Years Combined Is Equal to Sum of Individual Accident Year Estimates
- Assume Lognormal Distribution of Total Ultimate Loss with Parameters as Described; Use That Distribution to Estimate Percentiles of Outcomes

Method Descriptions

CAS Working Party "Estimated Range" Method

- XYZ Company @ December 31, 2005

AY Starting 1/1/xx	12-24	24-36	36-48	48-60	60-72	72-84
1988	1.445	1.462	2.661	1.074	0.890	1.140
1989	1.582	3.664	0.820	1.099	0.879	1.154
1990	1.698	1.005	1.026	1.480	1.216	1.257
1991	1.182	4.982	1.436	1.392	1.084	1.024
1992	6.935	2.136	2.200	1.651	1.057	0.871
1993	28.465	1.416	1.673	5.309	1.005	1.000
1994	10.747	5.461	1.283	1.016	1.008	1.095
1995	5.457	0.963	1.624	0.935	1.289	1.052
1996	4.872	1.494	0.756	0.950	0.917	0.913
1997	1.571	1.786	1.212	1.035	1.064	0.988
1998	51.688	1.219	1.967	1.218	0.993	0.997
1999	6.591	1.078	1.297	1.894	1.021	1.005
2000	2.442	1.626	1.052	1.151	1.082	
2001	4.016	3.123	1.306	1.267		
2002	11.046	1.142	1.098			
2003	2.405	1.235				
2004	3.160					

Actual Parameters

d =	1	2	3	4	5	6
f(d) =	8.547	2.112	1.427	1.534	1.039	1.041
F(d) =	46.167	5.401	2.558	1.792	1.168	1.124
$s^2(d)$=	177.423	2.179	0.294	1.355	0.015	0.012
$S^2(d)$=	18510.678	53.235	5.872	2.115	0.072	0.048

Method Descriptions

Bootstrap Method

- Calculate Average LDFs from Basic Chain Ladder Approach
- Calculate Incremental Data Triangle
- Calculate Proxy Cumulative Data Triangle by Fixing Most Recent Calendar Year Diagonal and Working Backwards with Average LDFs; Resulting Triangle Will Have Same Diagonal, LDFs, and IBNR Estimates as Original
- Calculate Proxy Incremental Triangle
- Calculate Pearson Residuals as (Actual Incremental Loss – Proxy Incremental Loss) / (ABS (Proxy Incremental))$^{1/2}$
- Adjust Residuals to Reflect Degrees of Freedom in Triangle (N /(N-p)) $^{1/2}$; Create Triangle of Scaled (Adjusted) Pearson Residuals
- Randomly Reorder Scaled Residuals – Major Bootstrap Assumption is That These Residuals Appear Randomly in Claim Development and Are Independently Distributed; Hence, Sampling with Replacement is Possible

Method Descriptions

Bootstrap Method (Continued)

- Calculate “False History” Triangle Based on Formula: False Incremental = Reordered Residual * (Proxy Incremental)$^{1/2}$ + Proxy Incremental
- Calculate Resulting Cumulative “False History” Triangle and New Average LDFs
- Simulate Future Incremental Losses Based on Normal Assumption; Mean = Expected Incremental per New Average LDFs and Variance = Mean * Scale Parameter Based on the Squared Residuals
- Sum of Future Incremental Losses is Reserve Estimate; Repeat Process Starting with Reordering of Residuals Step (Can Also Vary Normal Simulation Step)
- The Reordering of Residuals Models the Estimation Error, the Normal Simulation Models the Process Error
- Key Bootstrap Assumption is That the Variance Observed within the Historical Triangle is All the Variance Needed to Run a Simulation

Results

ABC Company @ 12/2005, Incurred & Paid

Incurred	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	94.3%	93.8%	93.7%	91.5%	93.4%
25th Percentile	97.0%	96.6%	96.7%	95.2%	96.4%
50th Percentile	100.0%	99.9%	100.0%	99.6%	99.9%
75th Percentile	103.0%	103.2%	103.3%	104.2%	103.4%
90th Percentile	105.7%	106.3%	106.3%	109.2%	106.9%
95th Percentile	107.3%	108.2%	108.2%	112.3%	109.0%
99th Percentile	110.4%	111.9%	111.7%	119.0%	113.2%

Paid	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	95.8%	95.4%	96.3%	93.4%	95.2%
25th Percentile	97.8%	97.5%	98.0%	96.2%	97.4%
50th Percentile	100.0%	99.9%	99.9%	99.5%	99.8%
75th Percentile	102.2%	102.4%	101.8%	103.5%	102.5%
90th Percentile	104.3%	104.7%	103.7%	107.1%	104.9%
95th Percentile	105.5%	106.1%	105.0%	109.5%	106.5%
99th Percentile	107.7%	108.7%	108.5%	113.7%	109.7%

Results

ABC Company @ 12/2004, Incurred & Paid

Incurred	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	94.0%	93.5%	93.5%	90.3%	92.8%
25th Percentile	96.8%	96.5%	96.5%	94.5%	96.1%
50th Percentile	100.0%	99.9%	100.0%	99.5%	99.8%
75th Percentile	103.2%	103.4%	103.4%	105.0%	103.8%
90th Percentile	106.1%	106.7%	106.5%	110.6%	107.5%
95th Percentile	107.8%	108.7%	108.4%	114.0%	109.7%
99th Percentile	111.0%	112.6%	112.0%	119.7%	113.8%

Paid	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	94.7%	94.3%	95.3%	92.3%	94.1%
25th Percentile	97.2%	96.9%	97.5%	95.8%	96.8%
50th Percentile	100.0%	99.9%	99.9%	99.7%	99.9%
75th Percentile	102.8%	103.0%	102.3%	103.9%	103.0%
90th Percentile	105.3%	105.9%	104.7%	108.2%	106.0%
95th Percentile	106.8%	107.6%	106.4%	111.2%	108.0%
99th Percentile	109.7%	111.0%	110.4%	116.0%	111.8%

Results

XYZ Company @ 12/2005, Incurred & Paid

Incurred	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	NM	52.7%	55.4%	12.5%	40.2%
25th Percentile	NM	68.5%	73.5%	26.4%	56.1%
50th Percentile	39.6%	91.1%	95.1%	52.9%	69.7%
75th Percentile	133.5%	121.2%	121.8%	106.0%	120.6%
90th Percentile	289.3%	157.5%	153.6%	211.5%	203.0%
95th Percentile	436.6%	183.6%	178.3%	322.6%	280.2%
99th Percentile	898.6%	245.2%	249.7%	786.4%	545.0%

Paid	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	NM	2.7%	48.9%	1.4%	17.7%
25th Percentile	1.5%	7.6%	65.0%	4.0%	19.5%
50th Percentile	10.3%	23.9%	86.2%	14.0%	33.6%
75th Percentile	46.0%	74.8%	115.7%	53.3%	72.4%
90th Percentile	166.0%	209.0%	160.5%	187.5%	180.7%
95th Percentile	355.1%	386.4%	204.6%	394.4%	335.1%
99th Percentile	1470.4%	1224.1%	374.4%	1569.6%	1159.6%

Results

XYZ Company @ 12/2004, Incurred & Paid

Incurred	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	NM	56.7%	56.0%	14.6%	42.5%
25th Percentile	15.1%	71.7%	74.3%	31.7%	48.2%
50th Percentile	65.1%	92.9%	96.3%	61.3%	78.9%
75th Percentile	143.9%	120.4%	122.4%	119.3%	126.5%
90th Percentile	252.8%	152.0%	151.7%	218.7%	193.8%
95th Percentile	342.8%	174.8%	174.5%	311.3%	250.9%
99th Percentile	584.2%	227.2%	245.3%	612.6%	417.3%

Paid	CAS WP "ER"	Mack	Bootstrap	Simulation	Average
10th Percentile	NM	8.6%	61.5%	3.5%	24.5%
25th Percentile	NM	18.7%	77.0%	7.8%	34.5%
50th Percentile	7.5%	44.3%	96.3%	20.9%	42.2%
75th Percentile	46.8%	104.7%	119.3%	61.9%	83.2%
90th Percentile	175.3%	227.3%	146.3%	180.0%	182.2%
95th Percentile	373.6%	361.4%	167.2%	389.2%	322.9%
99th Percentile	1513.2%	862.7%	233.2%	1413.6%	1005.7%

Results

General Observations

- ABC Company
  - All 4 Methods Reasonably Similar, Simulation Slightly Higher and Slightly Larger Right-Hand Tail
  - Paid Results Similar to Incurred at 2004 Evaluation, Slightly Lower at 2005 Evaluation
  - A Range of -5% to +5% Would Be Approximately an 70-80% Confidence Interval, Representing Endpoints Between the 10^{th}-15^{th} Percentile and the 85^{th}-90^{th} Percentile

Results

General Observations (Continued)

- XYZ Company
  - Thomas Mack and Bootstrap Methods Yield Similar Results, CAS Working Party “Estimated Range” Method and Simulation Method Yield Separate, Similar Results
  - “Estimated Range” Method and Simulation Method Actually Yield Negative Results in Certain Cases
  - Paid Methodology for “Estimated Range” Method and Simulation Method Not Usable
  - A Range of -10% to +10% Would Be Approximately a 10-20% Confidence Interval (Based on Thomas Mack & Bootstrap Results), Representing Endpoints Between the 45th-50th Percentile and the 60th-65th Percentile

Limitations

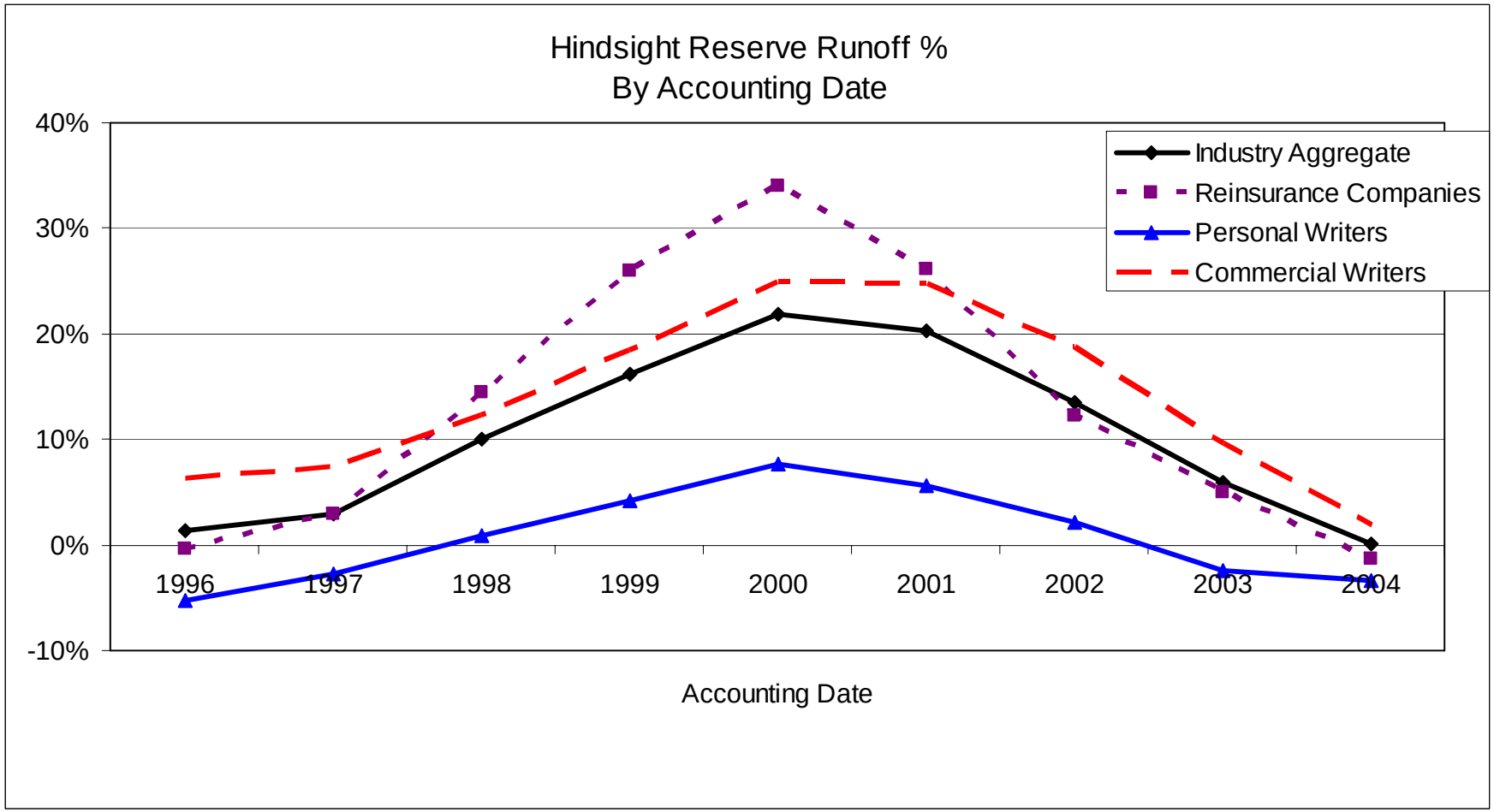
- Assumes triangles are available
- Not readily applicable to BF, frequency/severity or other projection methods
- Impact of varying the tail
- Correlation between lines of business

“Actual” Reserve Ranges/Variability

- Industry Reserve Development based on Schedule P data
- Runoff through December 31, 2005

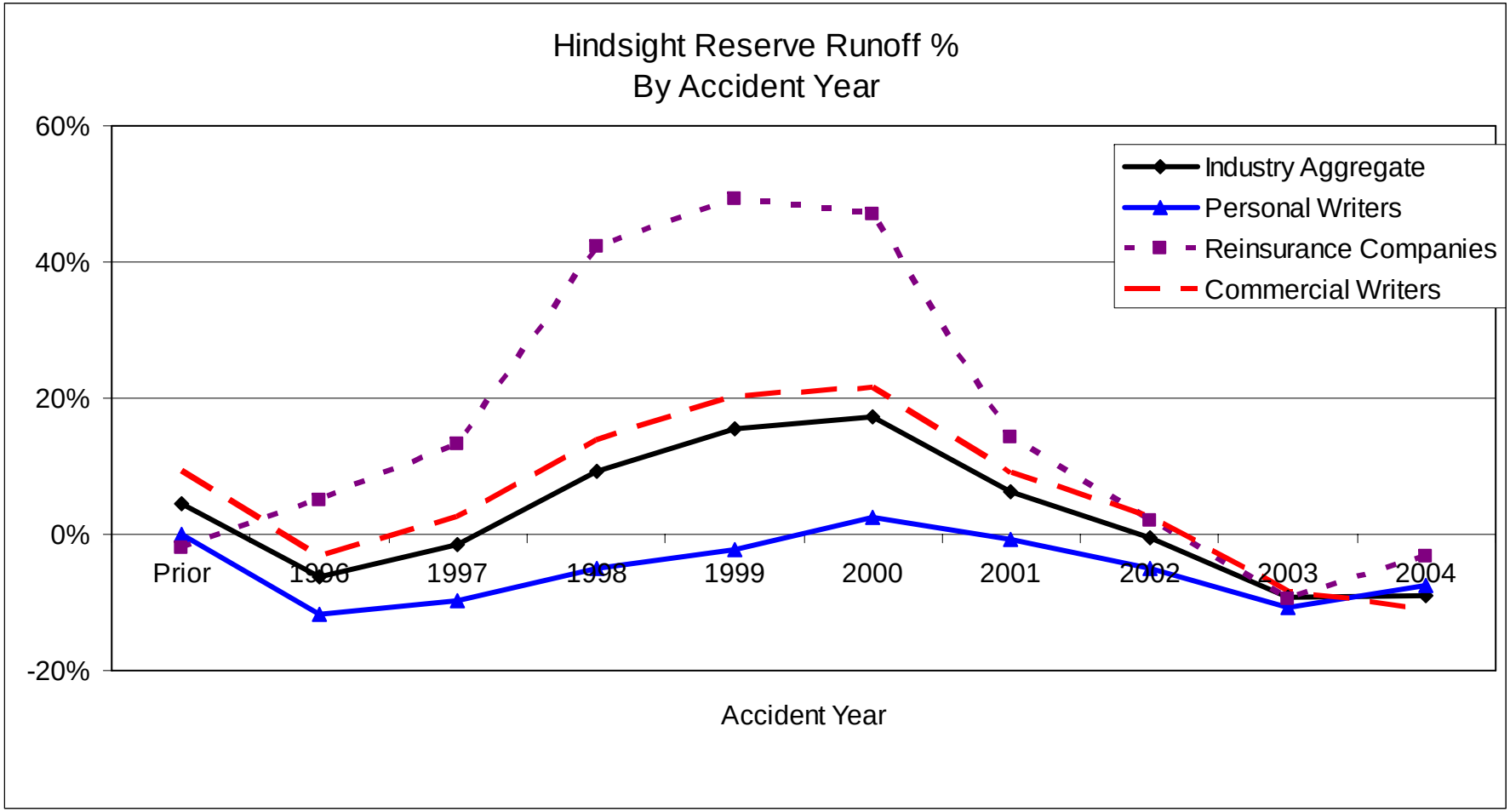
Runoff of 12/31/XXXX Reserves through 12/31/2005

Industry segregated Personal/Commercial/Reinsurer



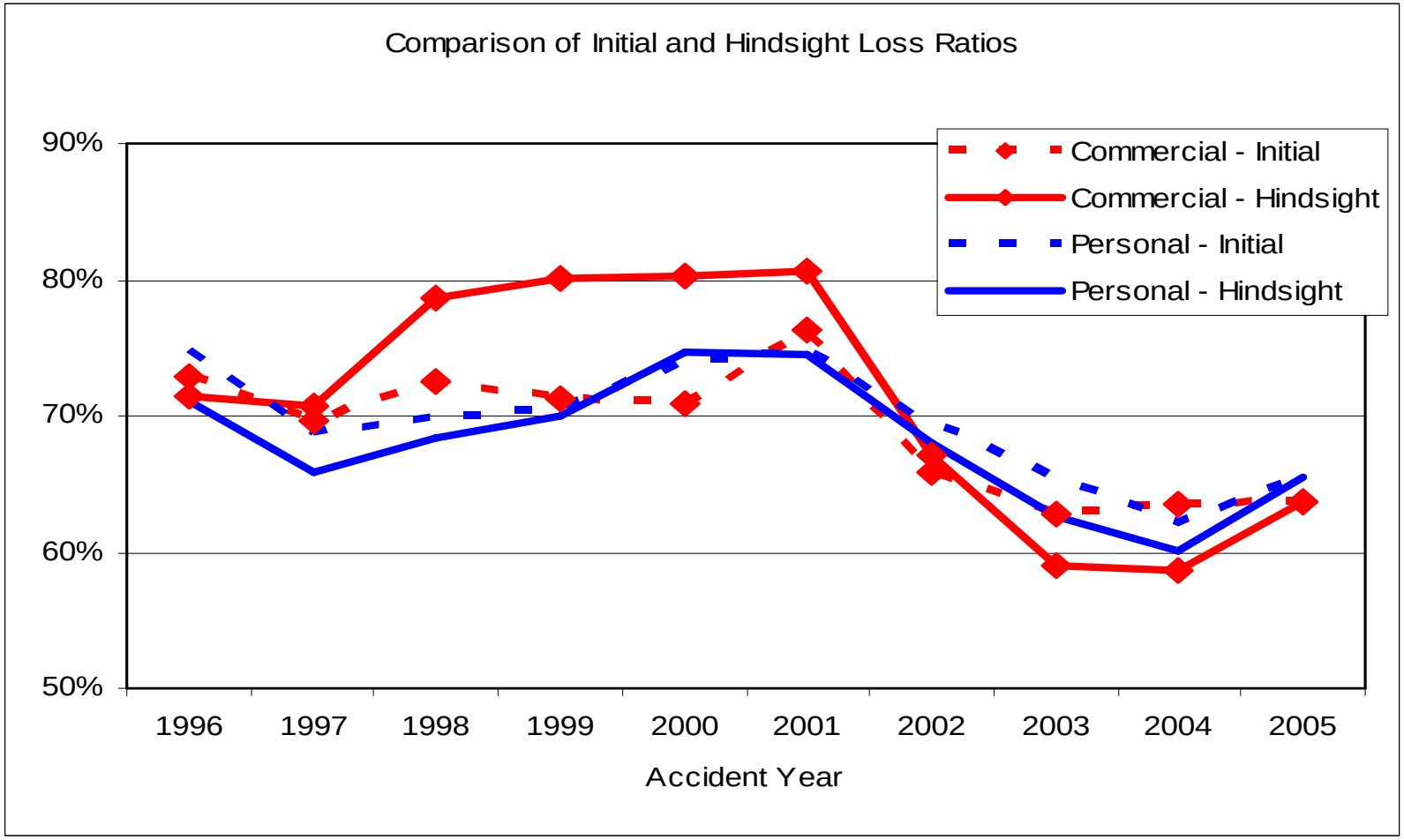
Runoff of Initial AY XXXX Reserves through 12/31/2005

Industry segregated Personal/Commercial/Reinsurer



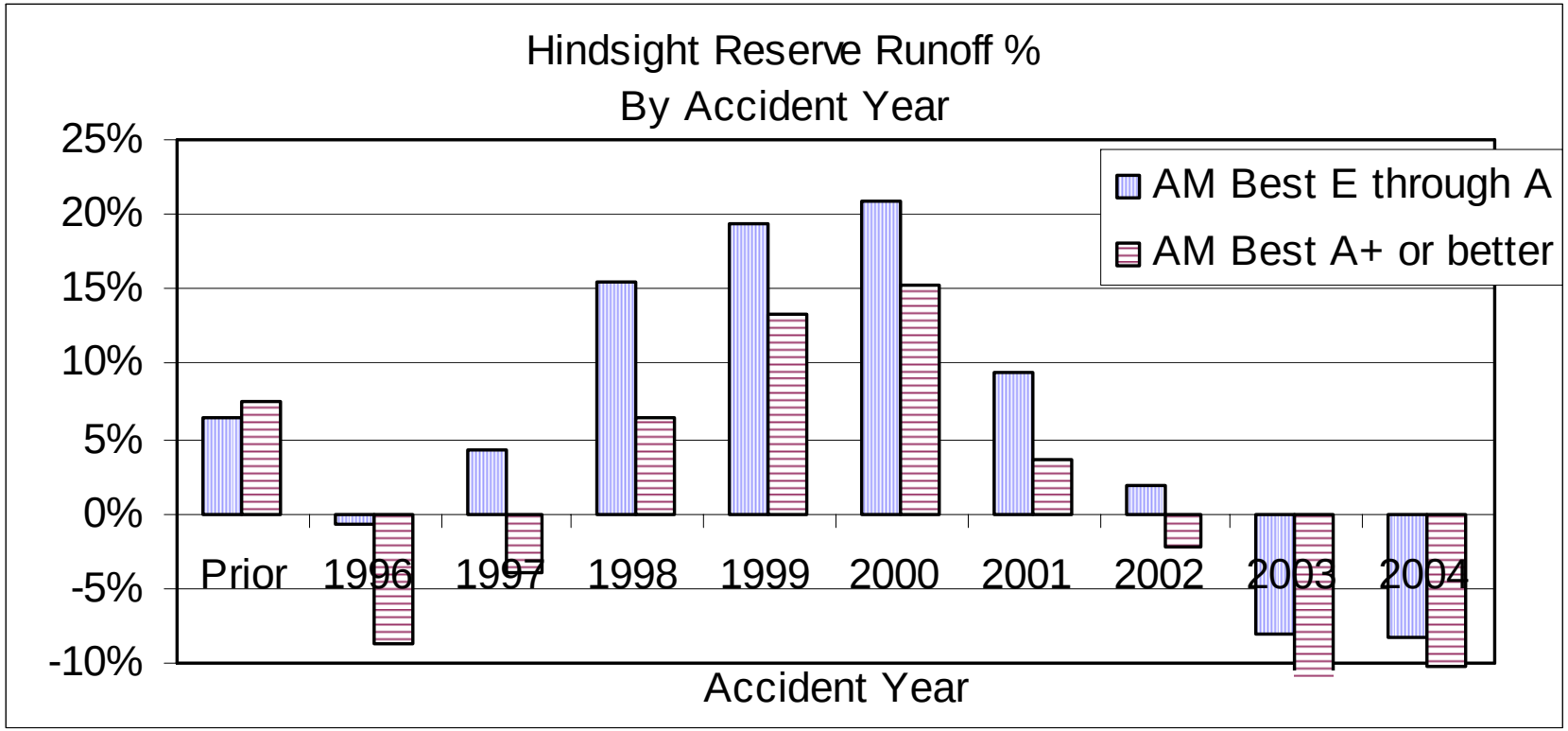
Runoff of Initial AY XXXX Loss Ratios through 12/31/2005

Industry segregated Personal/Commercial



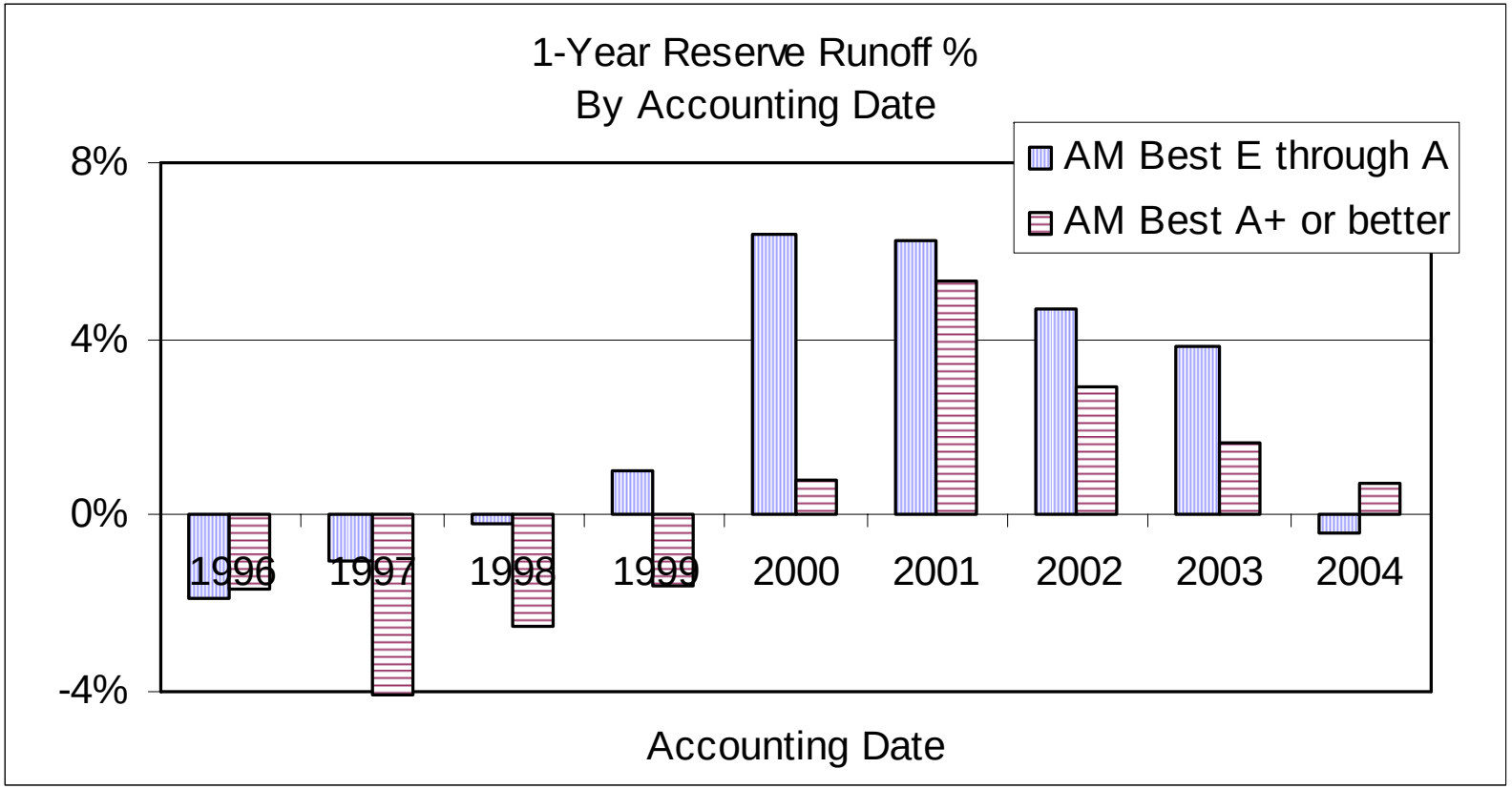
Runoff of Initial AY XXXX Reserves through 12/31/2005

Industry segregated 2005 A. M. Best Rating



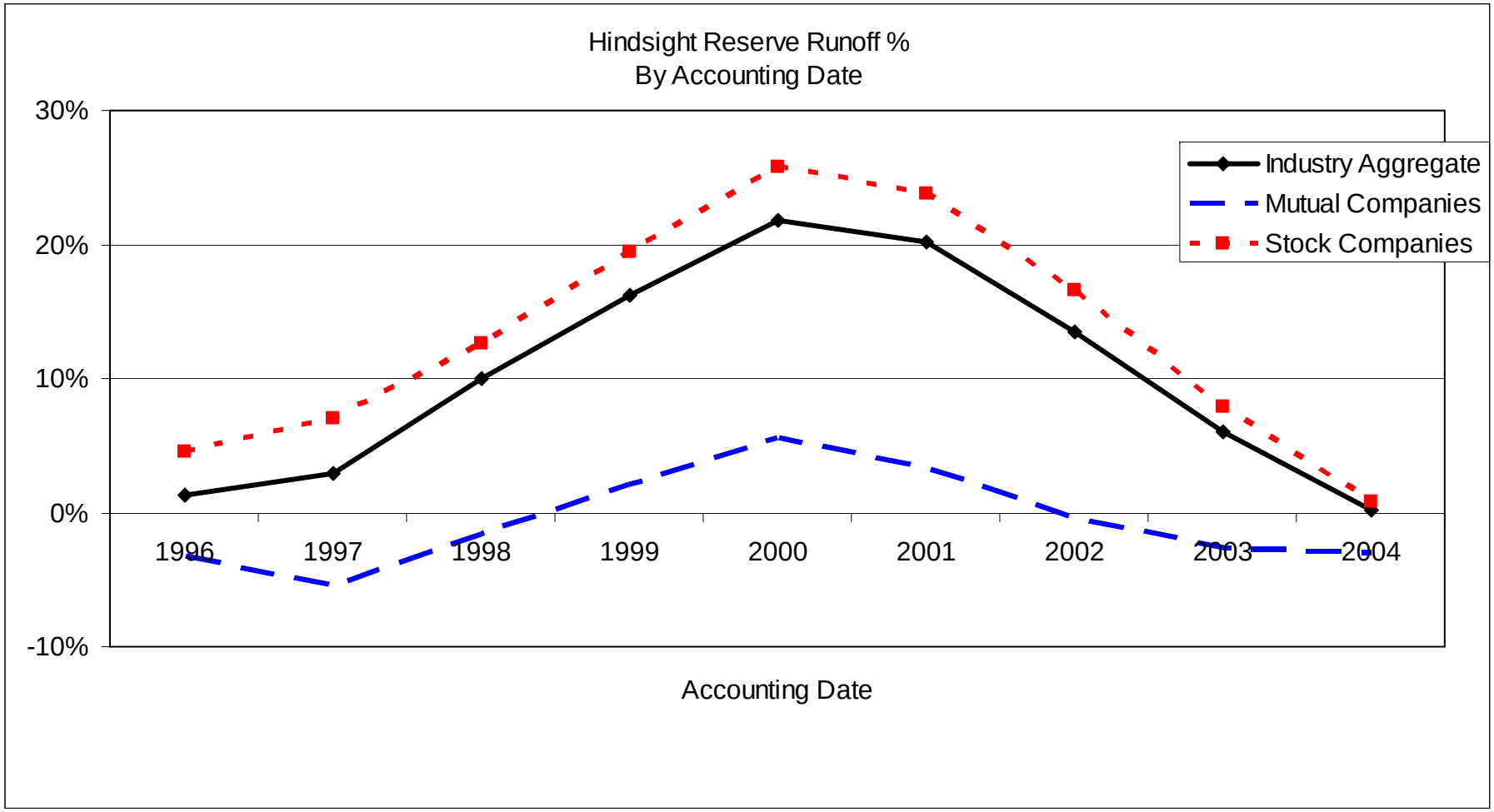
Runoff of 12/31/XXXX Reserves

Industry segregated 2005 A. M. Best Rating



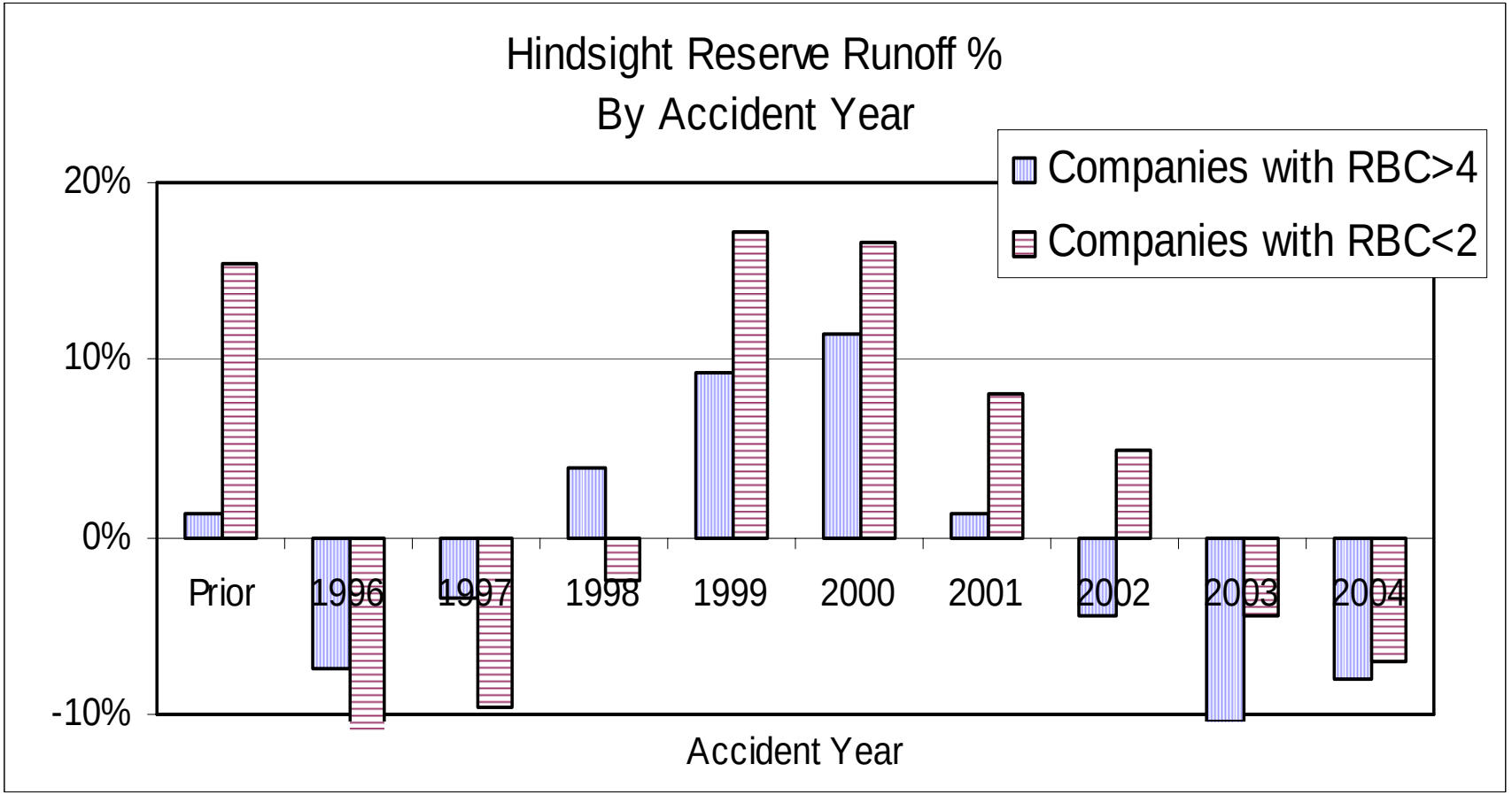
Runoff of 12/31/XXXX Reserves through 12/31/2005

Industry segregated Mutual/Stock



Runoff of Initial AY XXXX Reserves through 12/31/2005

Industry segregated by 12/31/2005 RBC Level



Q & A