CLRS September 2008

Manually Adjustable Link Ratio Model for Reserving

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MALRM for Reserving

Agenda

- **Theory**
- **Examples**
- **Questions**

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- ➲ **Theory**
- ➲ Examples
- ➲ Questions

Chain Ladder: Method vs. Model

Method	Model
▪ Mathematical algorithm	▪ Mathematical description of the world
▪ Parameters are selected	▪ "Best-Fitted" Parameters
▪ Selections assumed appropriate	▪ Selections can be tested
▪ Chain Ladder algorithm	▪ Mack/Murphy, ODP models

Chain Ladder “Method”

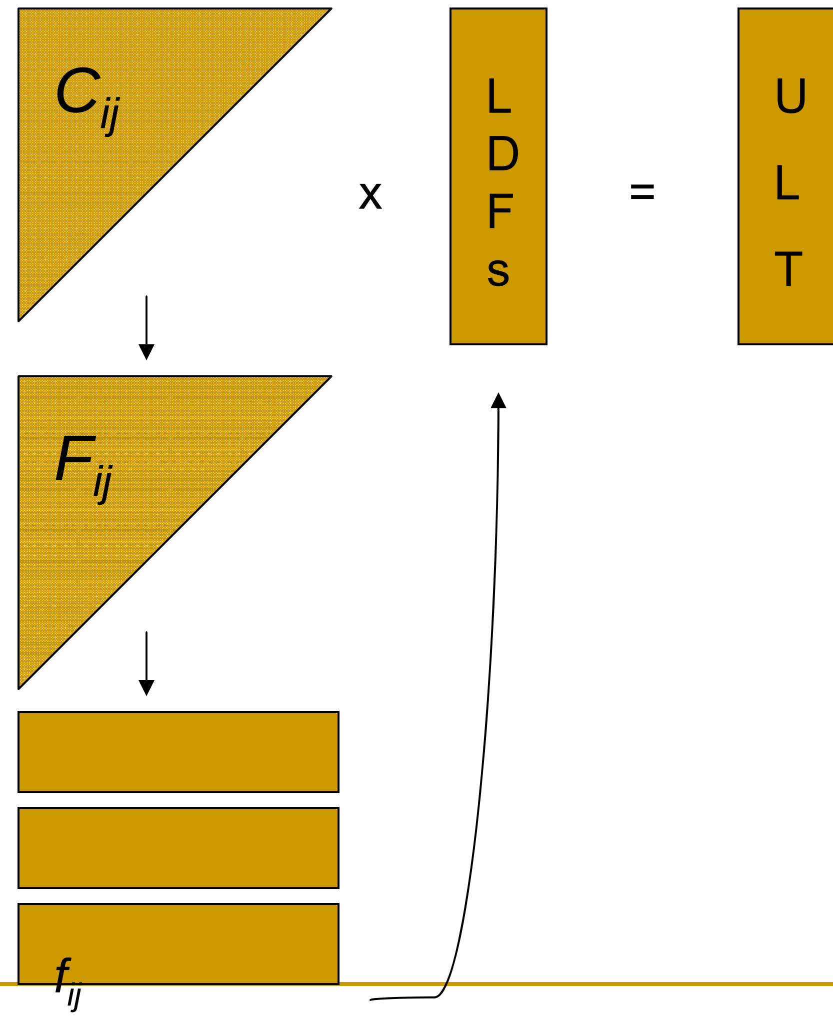
Loss Triangle

Triangle of Report-to-Report (RTR) Factors

Various Averages

Benchmark RTRs

***Selected* RTRs**



Chain Ladder "Model"

Requirements of a chain ladder model:

1. Consistent with the standard chain ladder method
   - Must produce identical reserve estimates

2. Capable of testing the CL parameter assumptions
   - Statistical model is the framework that allows us to validate our actuarial assumptions (our link ratio "picks")

The Manually Adjustable Link Ratio Model (MALRM)

(1) $$C_{ik+1} = f_k C_{i,k} + \sigma_k \varepsilon_{i,k} C_{i,k}^{\alpha_k/2}$$

(2) $$\varepsilon_{i,k} \overset{iid}{\sim} \aleph(0,1), 1 \le i \le I, 1 \le k \le I+1-i$$

The random component of the error term is assumed to be independent and identically distributed (i.i.d.) with mean zero and unity variance

- **The variance of the error term is $\sigma_k^2 C_{i,k}^{\alpha_k}$ which is a function of the parameter α_k**
- **Standard values for alpha are**

  1 – consistent with the volume weighted average link ratio in the chain ladder method

  2 – consistent with the simple average link ratio

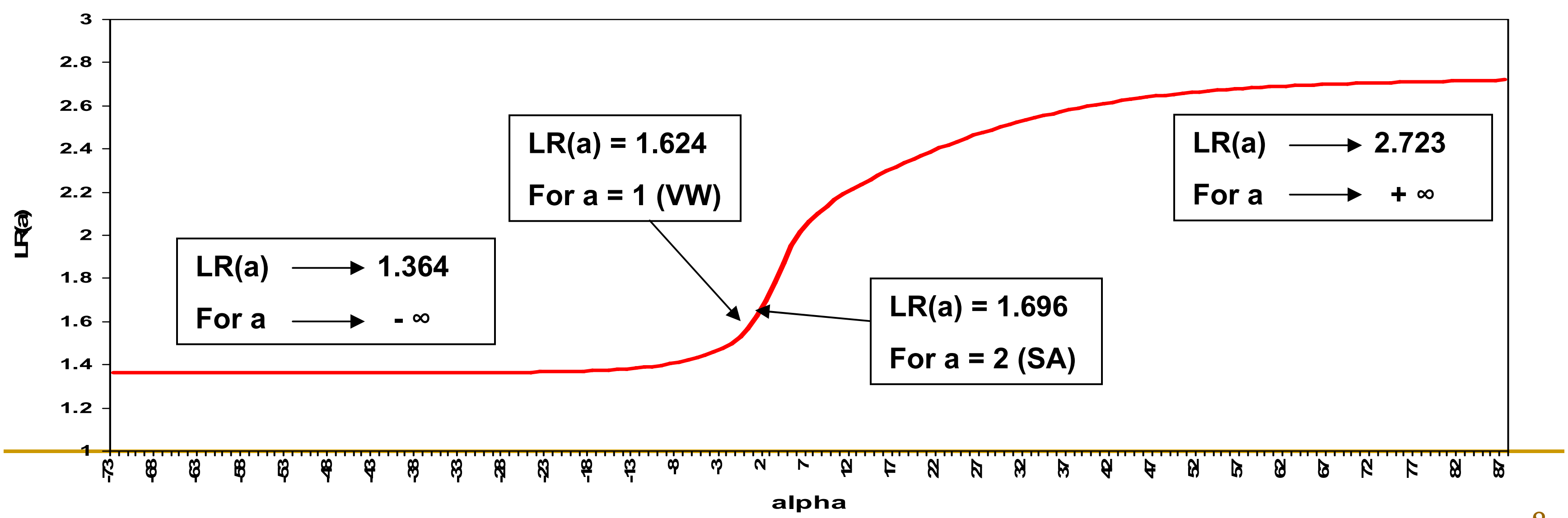
Maximum Likelihood Solution

$$(3)\quad \begin{cases} \hat{f}_k^{\alpha_k} := LR_k(\alpha_k) = \sum_{i=1}^{n-k} \frac{C_{i,k}^{1-\alpha_k}}{\sum_{j=1}^{n-k} C_{j,k}^{2-\alpha_k}} C_{i+1,k} = \sum_{i=1}^{n-k} w_{i,k}^{\alpha_k} \cdot F_{i,k}, \\ w_{i,k}^{\alpha_k} := \frac{C_{i,k}^{2-\alpha_k}}{\sum_{j=1}^{n-k} C_{j,k}^{2-\alpha_k}}, \quad F_{i,k} := \frac{C_{i,k+1}}{C_{i,k}} \end{cases}$$

- **The α_k superscript for f_k is not an exponent, but designates that the link ratio depends on the value of alpha parameter in the model**
- **By altering the variance parameter, the resulting model will have an indicated solution other than the volume weighted or simple average link ratio**

Maximum Likelihood Solution (cont)

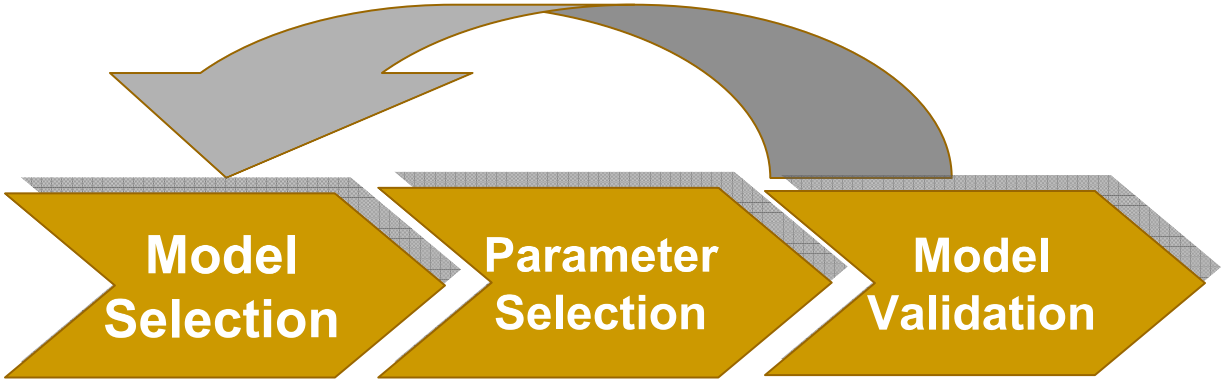
- **Put another way, under certain "reasonable" (see paper) conditions, when the actuary's selected link ratio is something other than the simple or volume weighted average, it still may be the indicated solution of model (1),(2) for some parameter *α* (numerically determinable)**
- **The graph below is an illustration of the Link Ratio function (first equation in (3) above), i.e., all potential average link ratios for** $\alpha \in \Re$



Maximum Likelihood Solution (cont)

- **When the selected link ratio is not "reasonable" relative to the data – e.g., when a benchmark is selected – a different formulation of the model should be considered, e.g.:**
  - **Bornhuetter-Ferguson**
  - **Inclusion of an intercept**
- **Having an error term in our model enables the estimation of variances and ranges of the chain ladder projection based on the actuary's own picks**
  - **This is a generalization of Mack, Murphy for the cases $\alpha = 1, 2$**
  - **Supplants custom of "scaling" Mack CVs to selected ultimates**

Validation of Selected Model



- With a model it is possible to validate – or invalidate – the appropriateness of the selected link ratios vis-a-vis the input data
  - With a method, the justification is "actuarial judgment"
  - Model validation tools include
    - visual aids (e.g., Q-Q plots, normality plots)
    - statistical tests (e.g., Shapiro-Francia, "goodness-of-fit", AIC/BIC criteria)
  - Residual analysis is essential to good statistical analysis
- A "bad fit" diagnosis leads to a different model
  - Reconsider one's picks
  - Different formulation from (1), (2) (see above)

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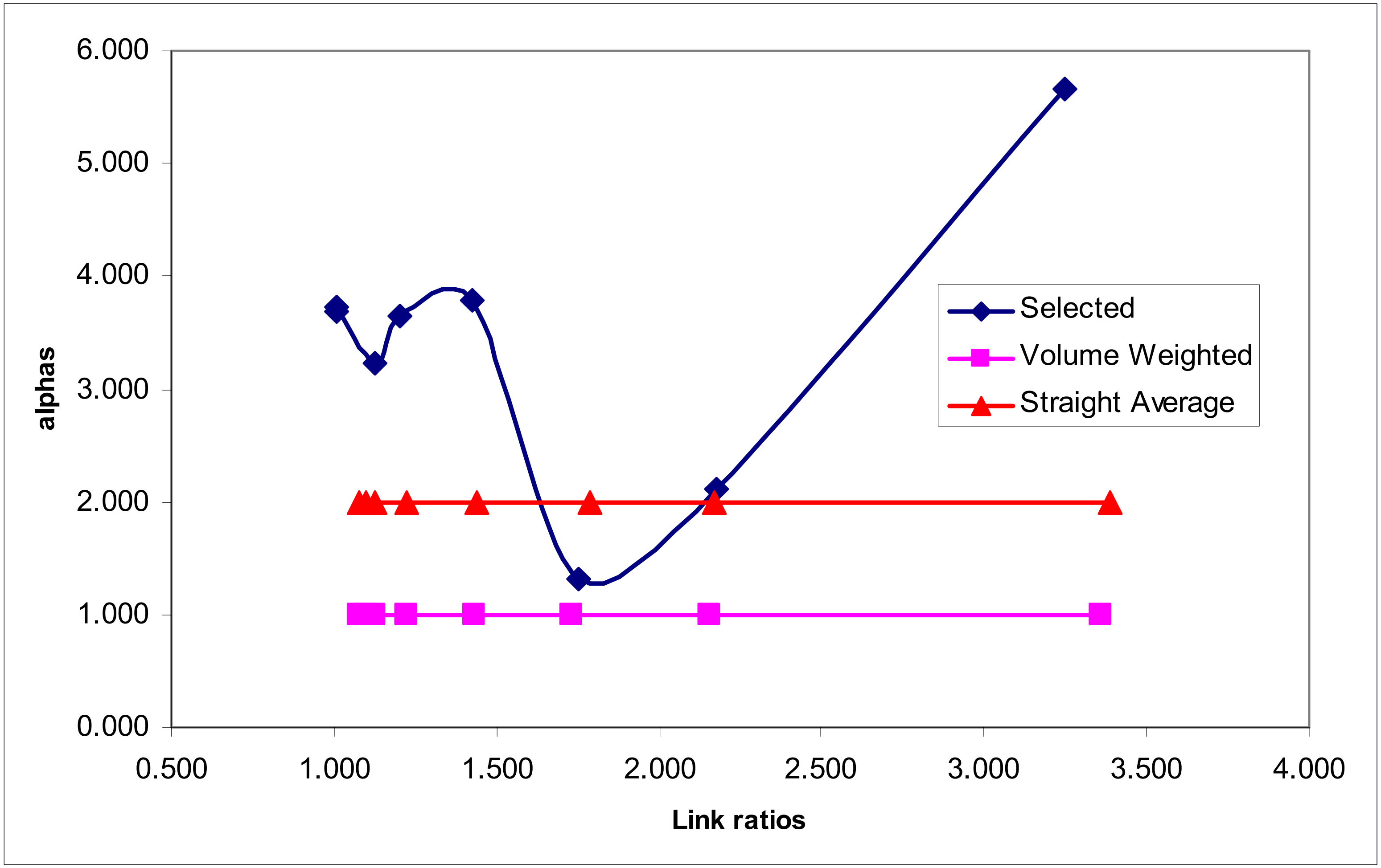
1st Example

AY	1	2	3	4	5	6	7	8	9	10
1981	42	**152**	**380**	745	1,216	1,555	1,786	1,984	2,145	2,265
1982	49	**185**	**449**	898	1,322	1,630	1,839	2,019	2,163	
1983	58	**217**	**537**	939	1,319	1,594	1,779	1,938		
1984	70	**260**	**390**	917	1,262	1,510	1,679			
1985	88	**281**	**703**	846	1,154	1,379				
1986	76	**235**	**466**	755	1,033					
1987	68	**207**	**411**	673						
1988	58	**185**	**372**							
1989	53	**167**								
1990	50									

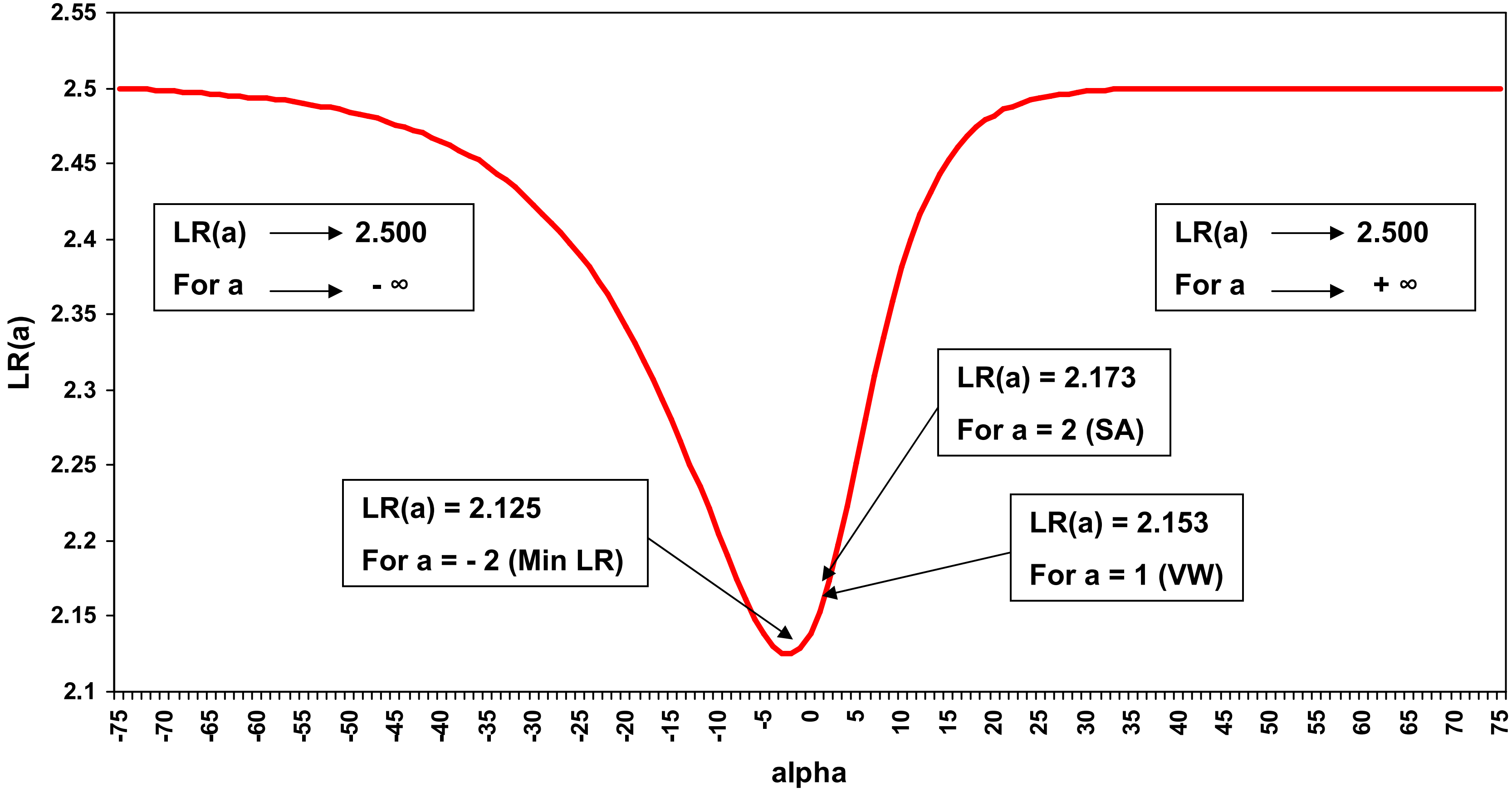
	1 to 2	2 to 3	3 to 4	4 to 5	5 to 6	6 to 7	7 to 8	8 to 9	9 to 10
1981	3.619	**2.500**	1.961	1.632	1.279	1.149	1.111	1.081	1.056
1982	3.776	**2.427**	2.000	1.472	1.233	1.128	1.098	1.071	
1983	3.741	**2.475**	1.749	1.405	1.208	1.116	1.089		
1984	3.714	**1.500**	2.351	1.376	1.197	1.112			
1985	3.193	**2.500**	1.204	1.364	1.195				
1986	3.092	**1.983**	1.620	1.368					
1987	3.044	**1.986**	1.637						
1988	3.190	**2.011**							
1989	3.151								
Straight average	3.391	2.173	1.789	1.436	1.222	1.126	1.099	1.076	1.056
VW average	3.361	2.153	1.731	1.433	1.222	1.126	1.099	1.076	1.056
Selected	3.250	2.175	1.750	1.425	1.200	1.125	1.010	1.005	1.001

“indicated” alphas



Link ratio function (2 to 3) –

1) a “reasonable” link ratio exists anywhere between 2.125 and 2.500
2) for each link ratio two alphas provide an indicated solution



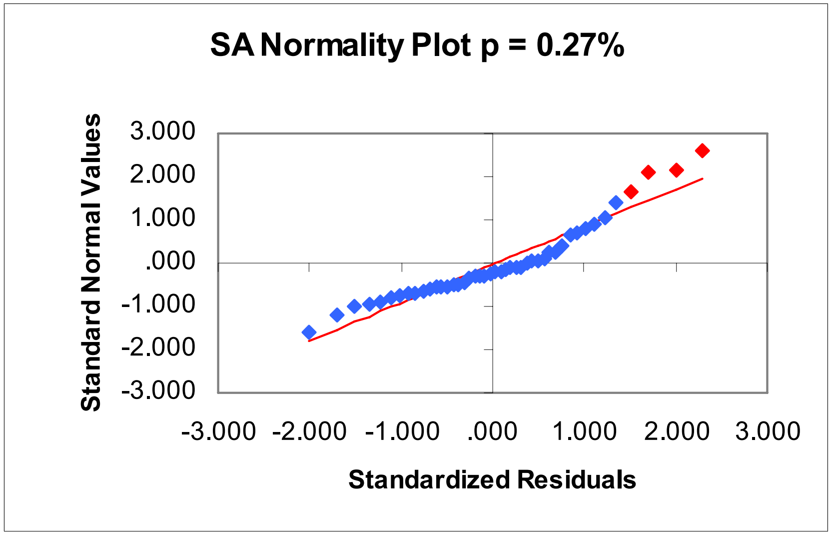
2nd Example: Taylor-Ashe triangle

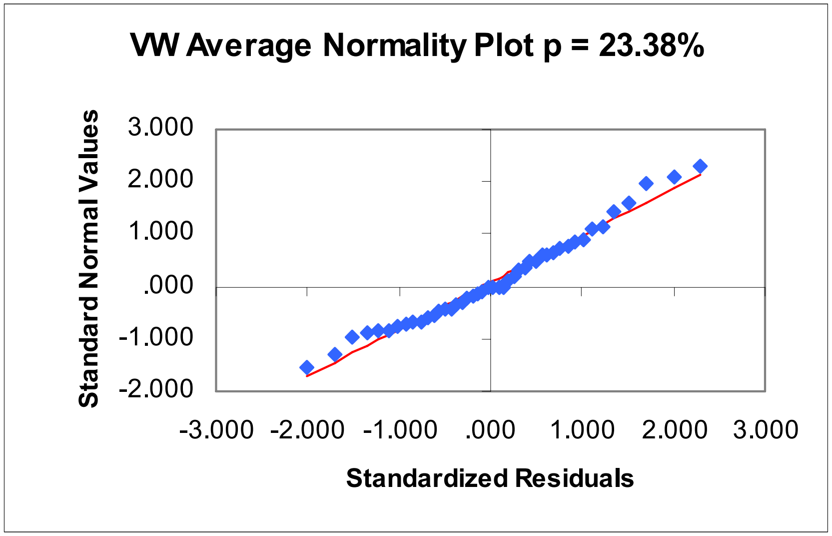
Accident Period	Evaluation Age in Months									
	12	24	36	48	60	72	84	96	108	120
1996	5,012	8,269	10,907	11,805	13,539	16,181	18,009	18,608	18,662	18,834
1997	106	4,285	5,396	10,666	13,782	15,599	15,496	16,169	16,704	
1998	3,410	8,992	13,873	16,141	18,735	22,214	22,863	23,466		
1999	5,655	11,555	15,766	21,266	23,425	26,083	27,067			
2000	1,092	9,565	15,836	22,169	25,955	26,180				
2001	1,513	6,445	11,702	12,935	15,852					
2002	557	4,020	10,946	12,314						
2003	1,351	6,947	13,112							
2004	3,133	5,395								
2005	2,063									

Accident Period	Age Interval in Months									
	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - Ult
1996	1.650	1.319	1.082	1.147	1.195	1.113	1.033	1.003	1.009	
1997	40.425	1.259	1.977	1.292	1.132	.993	1.043	1.033		
1998	2.637	1.543	1.163	1.161	1.186	1.029	1.026			
1999	2.043	1.364	1.349	1.102	1.113	1.038				
2000	8.759	1.656	1.400	1.171	1.009					
2001	4.260	1.816	1.105	1.226						
2002	7.217	2.723	1.125							
2003	5.142	1.887								
2004	1.722									
2005										

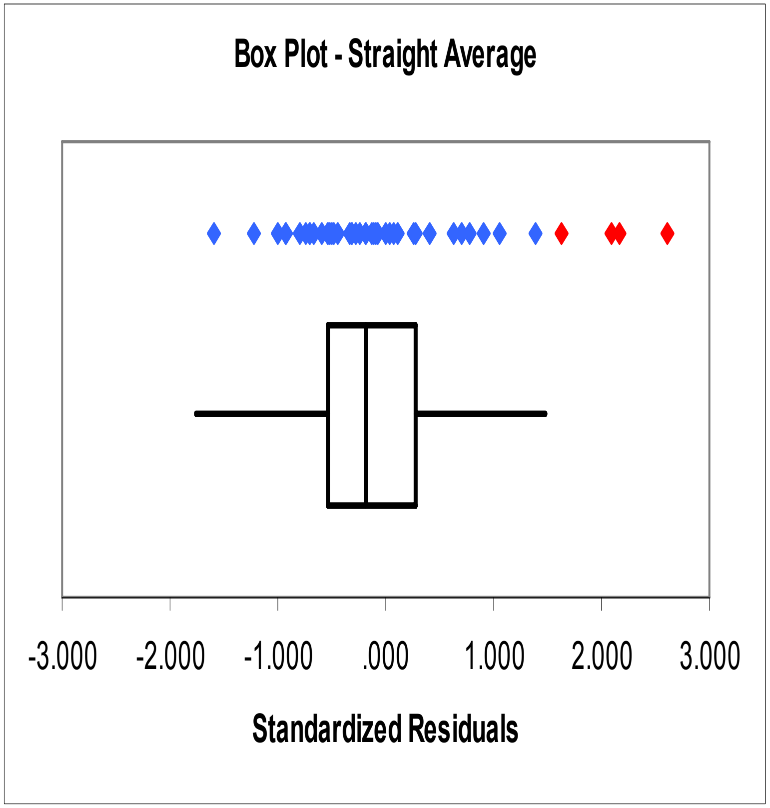
Methods	12 - 24	24 - 36	36 - 48	48 - 60	60 - 72	72 - 84	84 - 96	96 - 108	108 - 120	120 - Ult
SA All Values	8.206	1.696	1.315	1.183	1.127	1.043	1.034	1.018	1.009	1.000
VW All Values	2.999	1.624	1.271	1.172	1.113	1.042	1.033	1.017	1.009	1.000

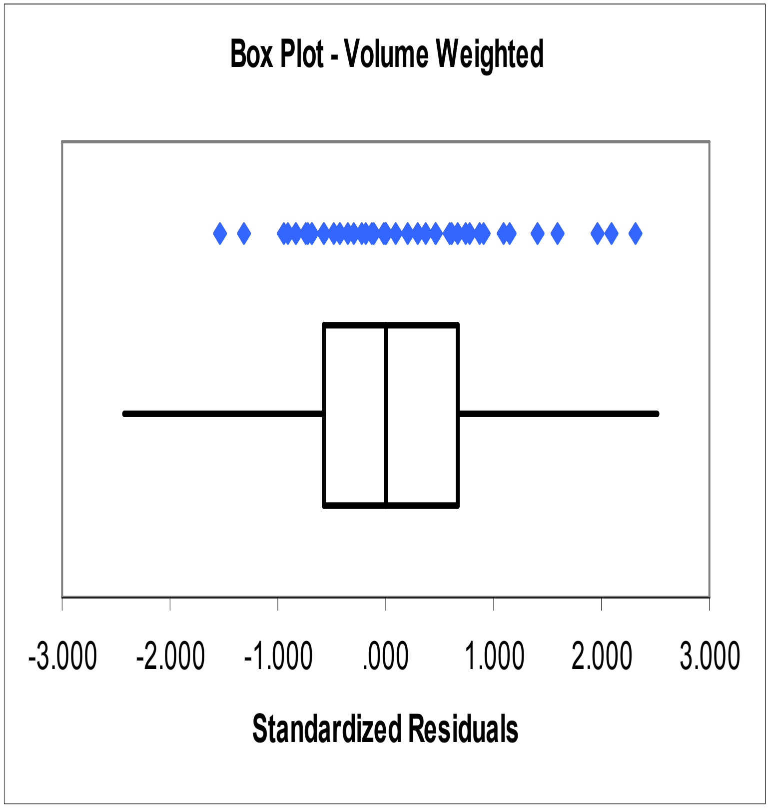
Comparison of models





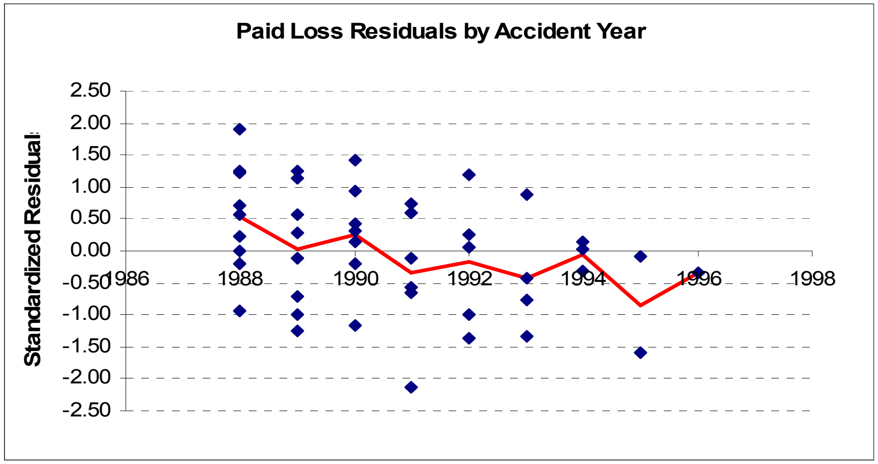
Comparison of models

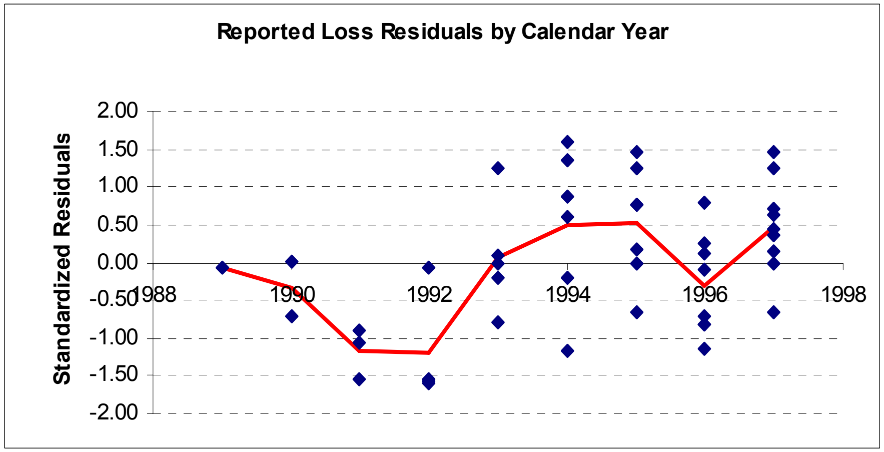




Model (1),(2) assists in the testing for Stationarity (Diagnostics)

- A key component of any regression exercise should be a diagnostic analysis of residuals
- Residuals compare actual losses vs. projected losses
- In the absence of any trends, the residuals should be randomly scattered
- Trends suggests that data is non-stationary
- Patterns in standardized residuals can reveal nonstationarity in the underlying data, including
  - Accident year trends
  - Calendar year trends
  - Case reserve strengthening





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