Meaningful Intervals

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More information will be available at

CABINET ROOM

6:30pm-11:30pm

Thursday September 18th

Summary

- Uncertainty and variability are distinct concepts
- fundamental difference between intervals for the mean vs process (CI/PI)
- Intervals only meaningful in a probabilistic framework
- Model assumptions must be explicit, interpretable, testable, and related to past volatility
- sophisticated methods (e.g. bootstrap) don't avoid need to check suitability of model. Bootstrap does not make Mack right.
- Regression formulation of standard link ratio methods and extensions- **ELRF modeling framework includes Mack , Murphy & much more**
- Link ratio methods - Mack & quasi-Poisson GLM are structure-less, give uninformative indications, no descriptors of the features in the data. Give incorrect calendar period liability stream
- Even if diagnostics are perfect, mean reserve may still be wide of the mark
- CVs by accident year fail on basic principle of insurance (statistics)
- On updating, estimates of mean ultimates may be grossly inconsistent
- Comments also apply to Munich Chain Ladder

Summary c'td

- **PTF (and MPTF) modeling framework** for building single-/multi-triangle models that can capture trend structure and volatility in real data
- identified model in PTF framework describes the trend structure and volatility succinctly (*four pictures*). All assumptions tested and validated.
- Model satisfies axiomatic trend properties
- Real loss triangle can be regarded as sample path from fitted probabilistic model. Can't tell the difference between real and simulated
- Distributions, prediction intervals are conditional on an explicit set of assumptions that are tested and validated by the data
- Readily obtain percentile and V@R tables for total reserve and aggregates, by calendar year and accident year.
- Obtain consistent estimates of prior year ultimates on updating
- Calendar year liability stream is critical for capital allocation and cost of capital calculations. (What does it depend on?)
- Pricing future underwriting years
- **No two companies are the same in respect of volatility and correlations**
- **All the above illustrated with many real data sets including <u>data from Murphy et al "*Manually Adjustable Link Ratio Model for Reserving*"</u>**. A surprising finding!

Variability and Uncertainty

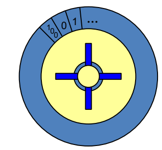
- different concepts; not interchangeable

"***Variability is a phenomenon in the physical world to be measured, analyzed and where appropriate explained. By contrast uncertainty is an aspect of knowledge.***"

Sir David Cox

Example: Coin vs Roulette Wheel

<u>Coin</u>	<u>"Roulette Wheel"</u>
100 tosses *fair* coin (#*H*?)	No. 0,1, …, 100
Mean = 50	Mean = 50
Std Dev = 5	Std Dev = 29
CI [50,50]	CI [50,50]
In 95% of experiments with the coin the number of heads will be in interval [40,60].	In 95% of experiments with the wheel, observed number will be in interval [2, 97].



Where do you need more risk capital?

Introduce uncertainty into our knowledge - if coin or roulette wheel are mutilated then conclusions could be made only on the basis of observed data.

A basic forecasting problem

Consider the following simple example –

n observations $Y_1 \ldots Y_n \overset{iid}{\approx} N(\mu,\sigma^2)$

$Y_i = \mu + \varepsilon_i \quad \varepsilon_i \overset{iid}{\approx} N(0, \sigma^2)$

Now want to forecast another observation...

(Actually, don't really need normality for most of the exposition, but it's a handy starting point.)

$$Y_{n+1} = \mu + \varepsilon_{n+1}$$

forecast of the error term

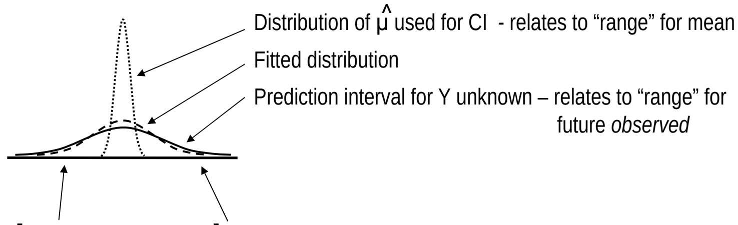
$$\hat{Y}_{n+1} = \hat{\mu} + \hat{\varepsilon}_{n+1}$$

Variance of the forecast is

$= \sigma^2 /n + \sigma^2$ = "parameter uncertainty+ process variability"

So again, imagine that the distributions are normal.
Prediction distribution wider than fitted

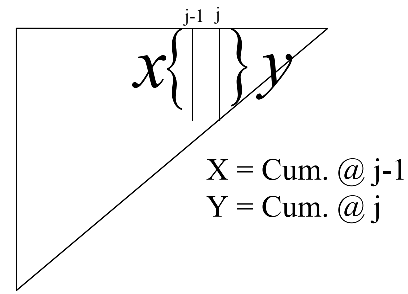
The next observation might lie –

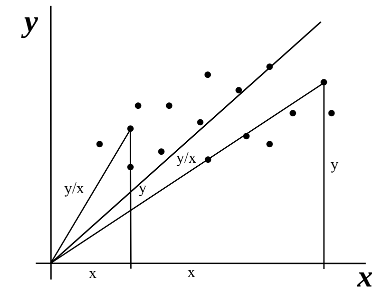


down here, or up here. **(implications for risk capital)**

ELRF
(Extended Link Ratio Family)
x is cumu. at dev. j-1 and y is cum. at dev. j

- Link Ratios are a comparison of columns
- We can graph the ratios of Y:X - *line through O?*





Using ratios => $E(Y|x) = \beta x$

Mack (1993)

$$y = bx + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

Minimize $\Sigma\, w\,(y - bx)^2$

where $w = 1 / x^\delta$

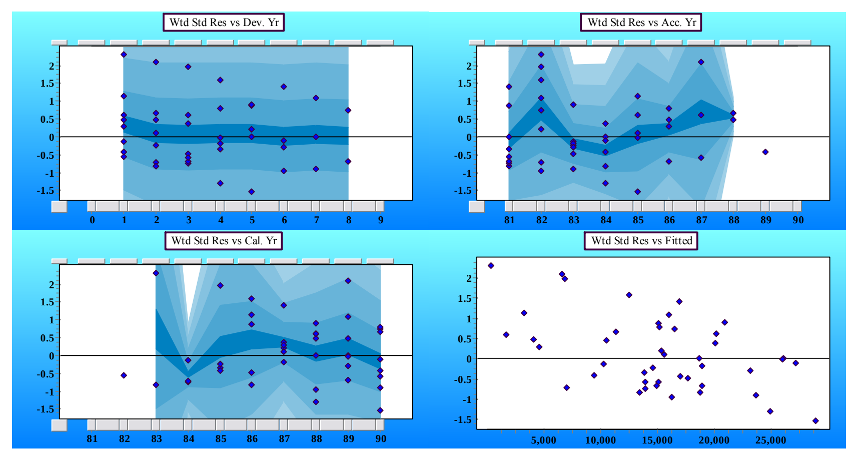
$$1.\quad \delta = 1, \qquad \hat{b} = \frac{\sum x \frac{y}{x}}{\sum x} = \frac{\sum y}{\sum x}$$

Chain Ladder Ratio (***Volume Weighted Average***)

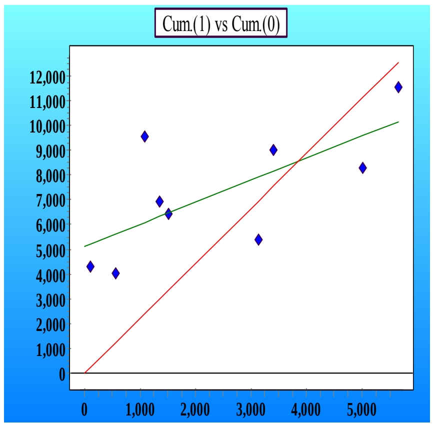
$$2.\quad \delta = 2, \qquad \hat{b} = \frac{1}{n}\sum \frac{y}{x}$$

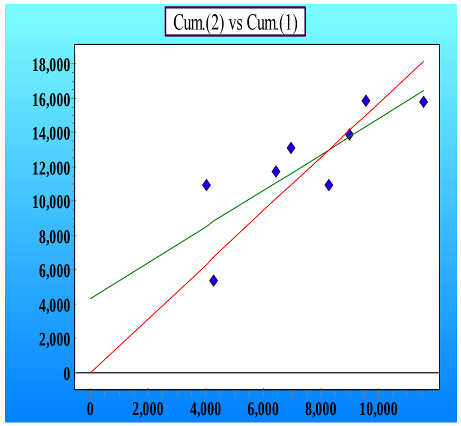
Arithmetic Average

Mack and Murphy data IL(C)
Mack (=volume weighted average) weighted standardized residuals.
Note trend versus fitted values



Mack and Murphy data IL(C)
Need intercepts





Intercept (Murphy (1994))

$$y = a + bx + \varepsilon \quad : \quad V(\varepsilon) = \sigma^2 x^\delta$$

Since ***y*** already includes ***x***: ***y = x + p, ie p = y - x***

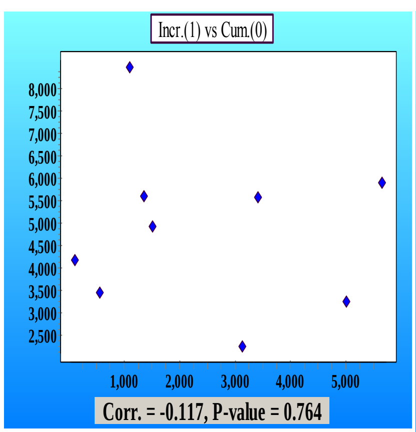
$$p = a + (b-1)x + \varepsilon \; : \; V(\varepsilon) = \sigma^2 x^\delta$$

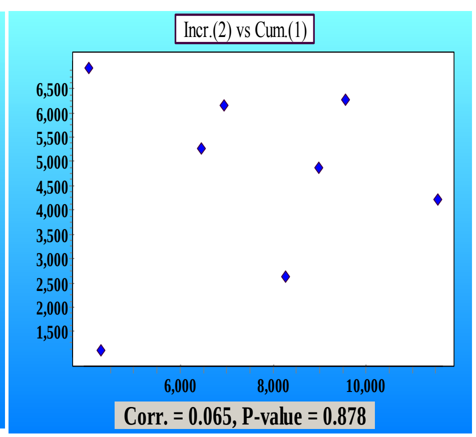
↑ ↑

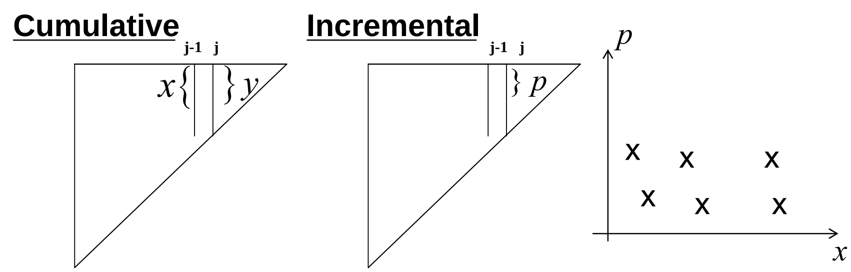
Incremental at j　　Cumulative at j -1

Is b -1 significant ?　　Venter (1996)

Mack and Murphy data IL(C)
Link Ratios=1. Zilch Predictive power







$$p = a + (b-1)x + \varepsilon \; : \; V(\varepsilon) = \sigma^2 x^\delta$$

Case (i) $b > 1 \quad a = 0$

Use link-ratios for projection

Case (ii) $b = 1 \quad a \neq 0$

$\hat{a}$ = Ave(Incrementals)

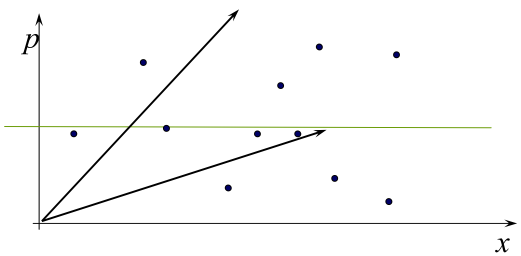
Abandon Ratios - No predictive power

Is assumption $E(p|x) = a + (b\text{-}1)\,x$ *tenable?*

Note: If corr(x, p) = 0, then corr($(b\text{-}1)x$, p) = 0

If x, p uncorrelated, *no* ratio has predictive power

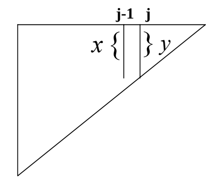
Ratio selection by actuarial judgement can't overcome zero correlation.

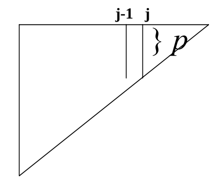


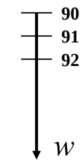
Corr. often close to 0

-Sometimes not.
Does this imply ratios are a good model?

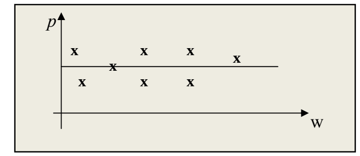
- ranges?





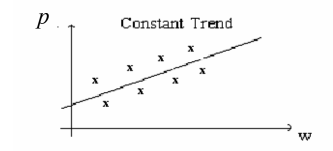


Condition 1:

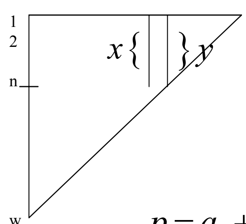


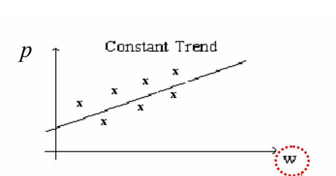
$$p = a + (b-1)x + \varepsilon \ : \ V(\varepsilon) = \sigma^2 x^\delta$$

Condition 2:



Now Introduce Trend Parameter For Incrementals





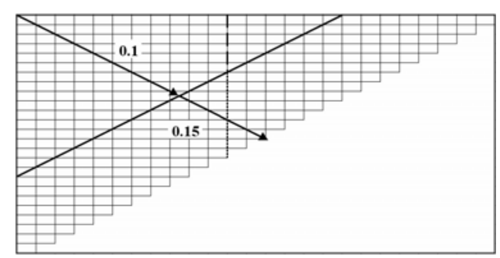
$$p = a_0 + a_1 w + (b-1)x + \varepsilon$$

a_0 = Intercept
a_1 = Trend
b = Ratio

p vs acci. yr, and previous cumulative

Condition 3:

Incremental



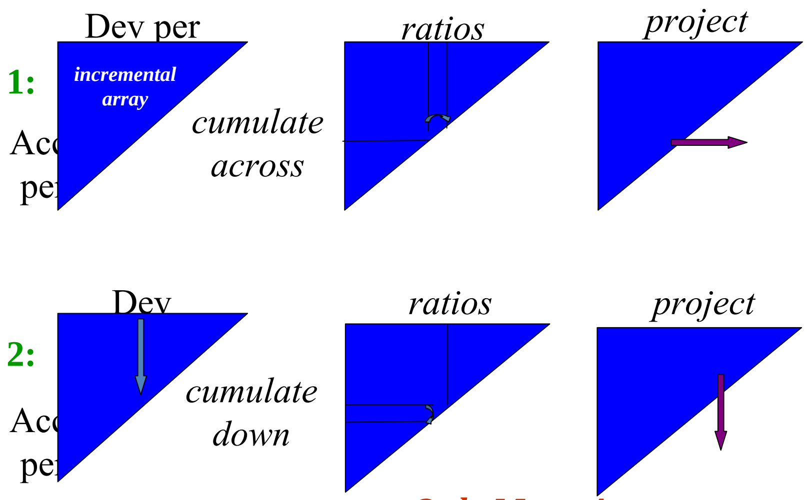
Review 3 conditions:

Condition 1: Zero trend

Condition 2: Constant trend, positive or negative

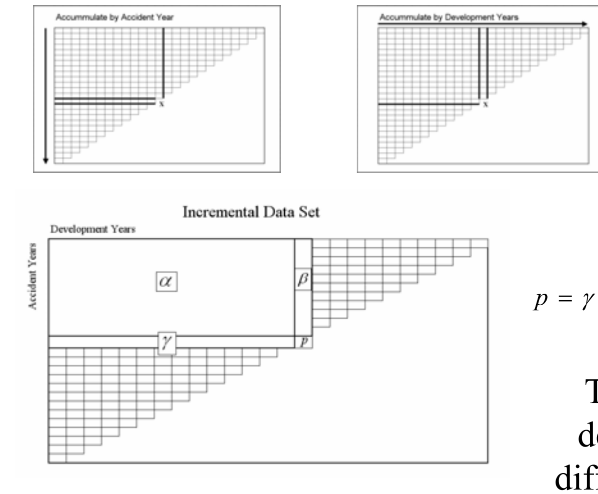
Condition 3: Non-constant trend

Mack=Chain Ladder (volume weighted average) treats accident years like development years. Can cumulate across or down. Does not matter!



Only Mean Answers are identical

Mack does not distinguish between accident years and development years



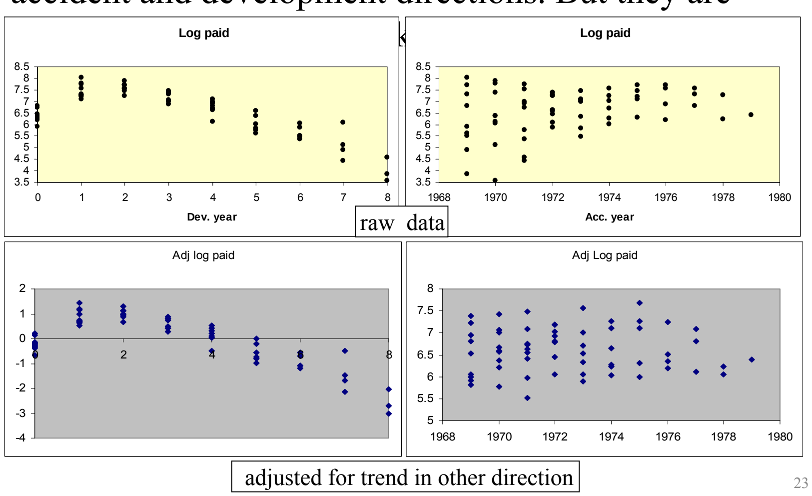
$$p = \gamma\left(\frac{\alpha+\beta}{\alpha} - 1\right) = \frac{\gamma\beta}{\alpha}$$

$$p = \beta\left(\frac{\alpha+\gamma}{\alpha} - 1\right) = \frac{\beta\gamma}{\alpha}$$

The standard deviations are different because of different conditioning

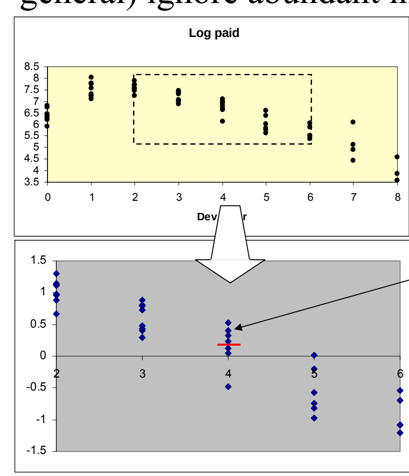
The Chain ladder (Volume weighted aveage) - Transpose Invariance property

Chain ladder does not distinguish between accident and development directions. But they are



Mack= The Chain Ladder (Volume Weighted Average)

Additionally, chain ladder (and ratio methods in general) ignore abundant information in nearby data.



- * If you left out a point, how would you guess what it was?

- observations at same dev. period *very* informative.

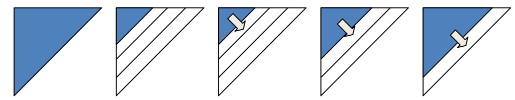
The quasi-Poisson GLM

- Model incrementals as quasi-Poisson (also called overdispersed Poisson), two-way cross-classification model with log-link.

- “Fit” doesn’t reproduce what you’d think of as chain ladder (expected value is not ratio times previous cumulative), but forecasts do.

One step ahead prediction errors

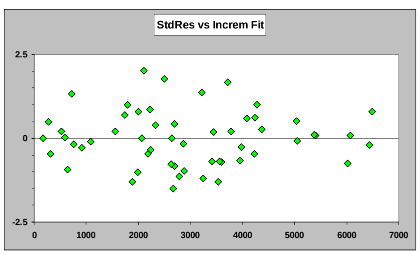
- - predictive behaviour of quasi-Poisson revealed by *one-step ahead prediction errors*.

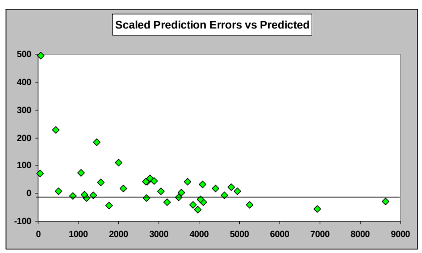


Leave data out and predict it →*validation*

- P.E. = (predicted – observed) (*vs* resid = fit – obs)
- can standardize by √(*predictive variance*)
- Two crucial plots:
- *prediction errors vs predicted* (*~ resids vs fit, better)*
- P.E.s vs CY (*res vs CY* can show some issues)

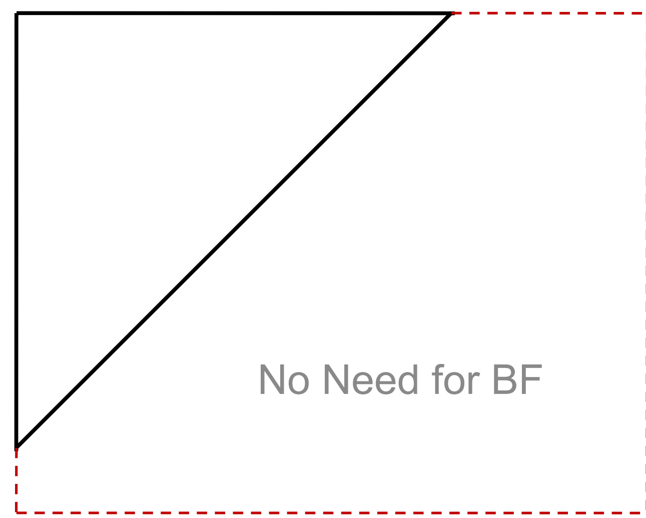
One step ahead prediction errors



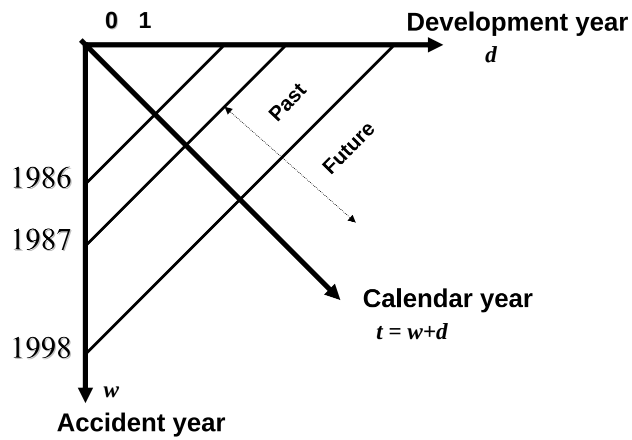


Res vs fit looks fine - BUT - *PEs vs pred* shows problem!

-Predictive behaviour revealed:
 – underpredicts small, overpredicts large



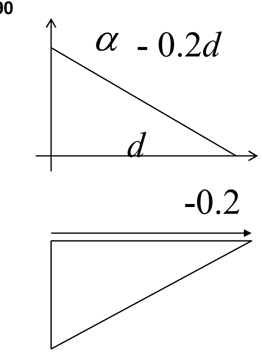
Trends occur in three directions:



M3IR5 Data

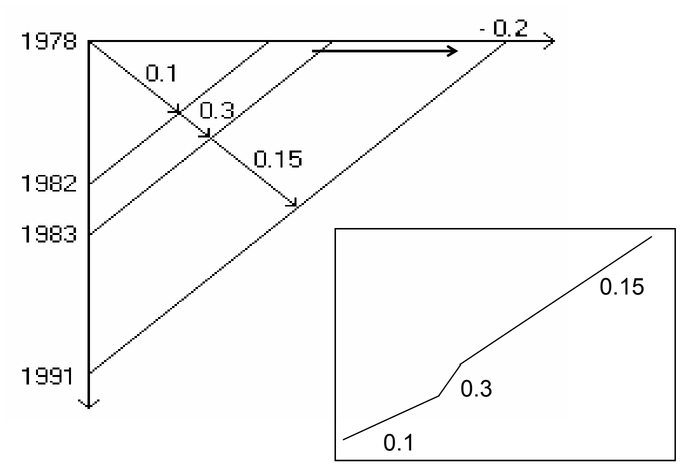
0	1	2	3	4	5	6	7	8	9	10	11	12	13
100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080	9072	7427
100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080	9072	
100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534	11080		
100000	81873	67032	54881	44933	36788	30119	24660	20190	16530	13534			
100000	81873	67032	54881	44933	36788	30119	24660	20190	16530				
100000	81873	67032	54881	44933	36788	30119	24660	20190					
100000	81873	67032	54881	44933	36788	30119	24660						
100000	81873	67032	54881	44933	36788	30119							
100000	81873	67032	54881	44933	36788								
100000	81873	67032	54881	44933									
100000	81873	67032	54881										
100000	81873	67032											
100000	81873												
100000													

alpha = 11.513

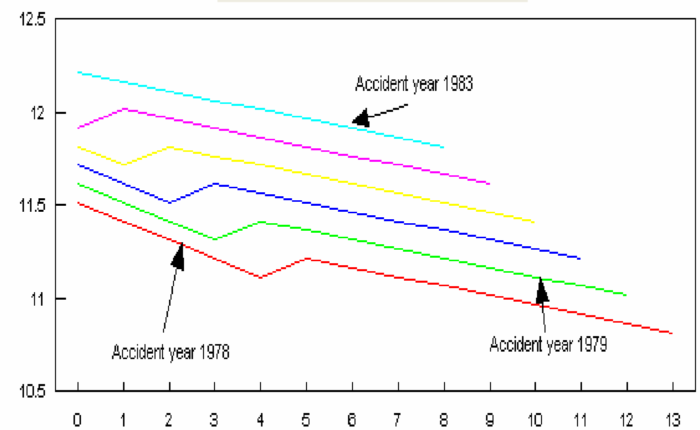


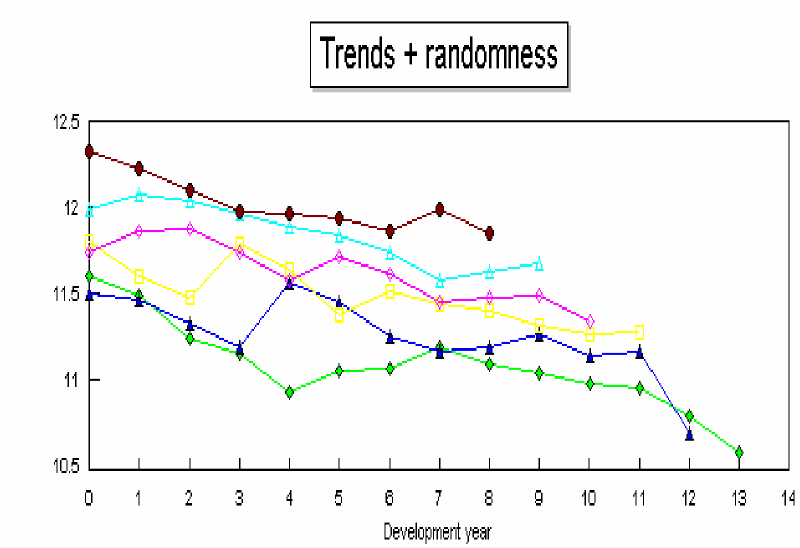
PAID LOSS = EXP(alpha - 0.2d)

Axiomatic Properties of Trends

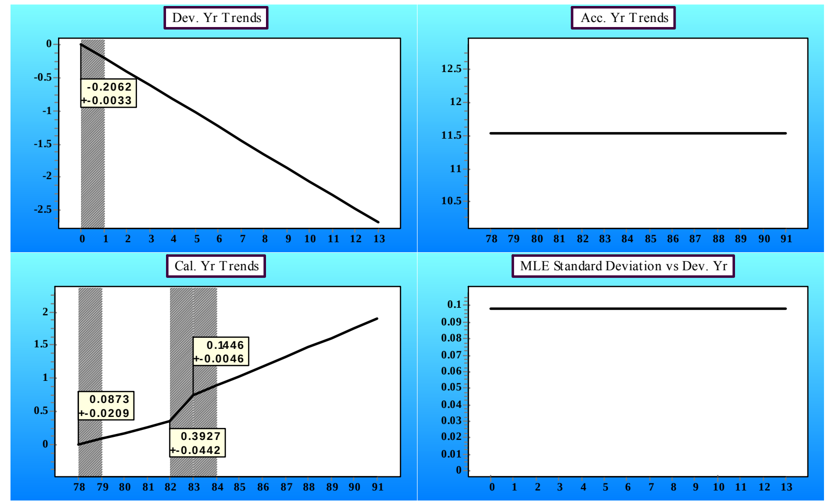


Resultant development year trends

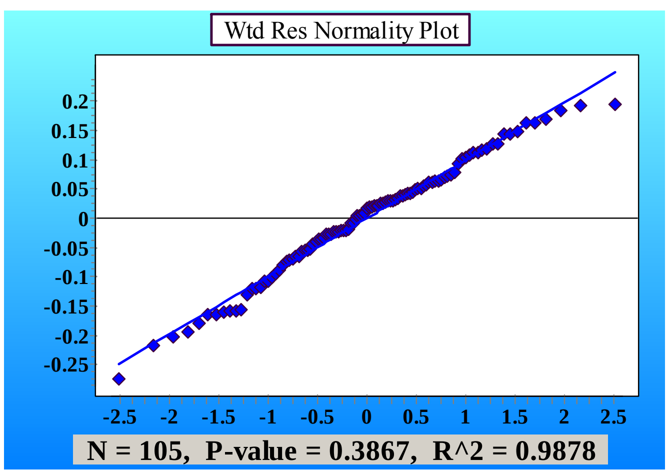




MODEL DISPLAYS. Graph bottom represents process variability



Normal distribution about trend structure- integral part of model



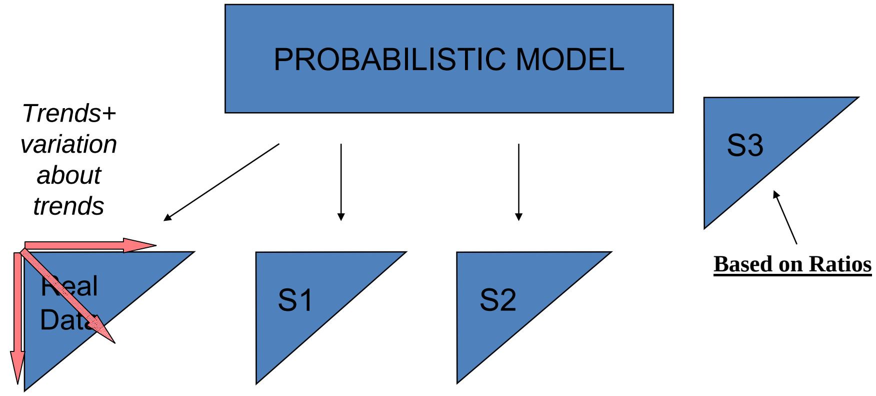
Forecast lognormals for each cell
All assumptions are explicit. Process variability and parameter uncertainty included

1984	205,644	220,996	169,549	166,858	15,289
1985	224,587	211,182	198,582	186,737	175,603
	221,660	247,187	207,918	18,780	17,816
1986	259,547	244,060	229,502	215,816	202,951
	220,334	234,427	23,094	21,896	20,799
1987	299,956	282,062	265,241	249,428	234,563
	271,278	28,430	26,939	25,576	24,325
1988	346,664	325,989	306,553	288,281	271,105
	35,037	33,181	31,483	29,927	28,496
1989	400,654	376,764	354,306	333,193	313,345
	40,913	38,797	36,858	35,076	33,433
1990	463,061	435,456	409,506	385,110	362,175
	47,859	45,440	43,218	41,171	39,280
1991	535,200	503,303	473,316	445,126	418,623
	56,078	53,304	50,750	48,391	46,206
	1995	1996	1997	1998	1999
al. Per.	2,506,808	2,278,761	2,052,087	1,824,784	1,594,672
Total	122,636	119,405	115,402	110,321	103,865

Simulate from forecast correlated lognormals
Percentiles (Quantiles) and V@R statistics
All assumptions are explicit. Process variability and parameter uncertainty included

Quantile Statistics and Value

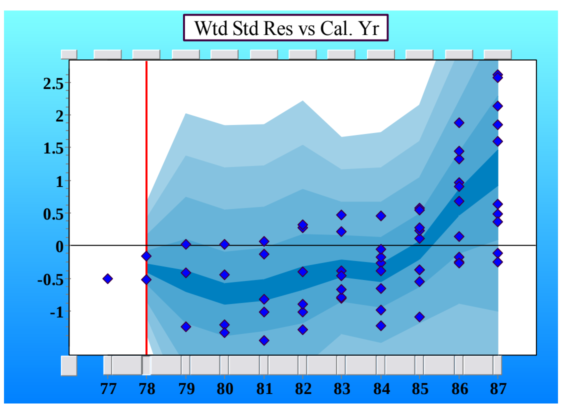
%	Sample Quantile	Sample # S.D.'s	Sample V-a-R	Kernel Quantile	Kernel # S.D.'s	Kernel V-a-R
99.995	26.970	3.820	3.544	27.145	4.008	3.718
99.99	26.937	3.783	3.510	27.065	3.922	3.639
99.98	26.866	3.707	3.439	26.970	3.820	3.544
99.97	26.803	3.640	3.377	26.904	3.748	3.477
99.96	26.773	3.607	3.347	26.850	3.690	3.423
99.95	26.755	3.587	3.328	26.802	3.639	3.376
99.94	26.749	3.581	3.323	26.759	3.592	3.333
99.93	26.703	3.532	3.277	26.719	3.549	3.293
99.92	26.691	3.519	3.265	26.682	3.508	3.255
99.91	26.587	3.406	3.160	26.646	3.469	3.219
99.9	26.567	3.385	3.141	26.611	3.432	3.185
99.8	26.299	3.096	2.872	26.353	3.154	2.927
99.7	26.152	2.937	2.725	26.201	2.991	2.775
99.6	26.049	2.827	2.623	26.096	2.877	2.670



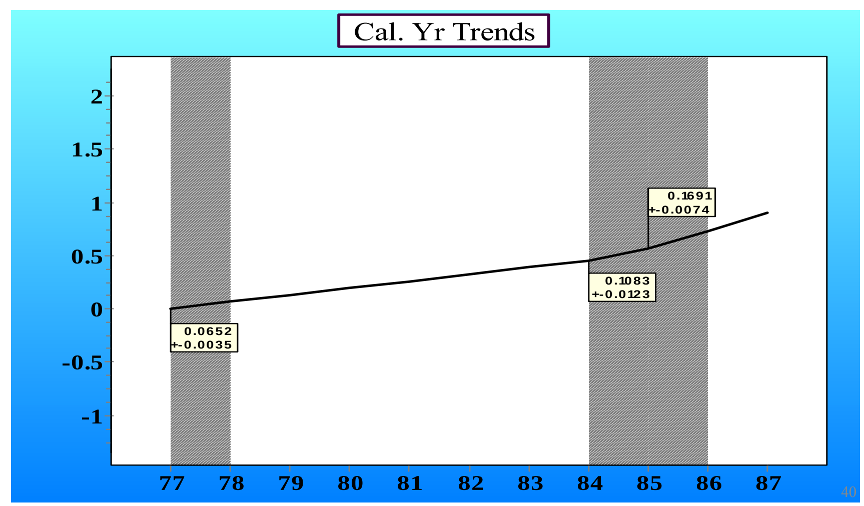
Simulated triangles cannot be distinguished from real data – similar trends, trend changes in same periods, same amount of random variation about trends

Models project past volatility into the future

TG ABC
PTF-Calendar period residuals adjusted for zero trend only

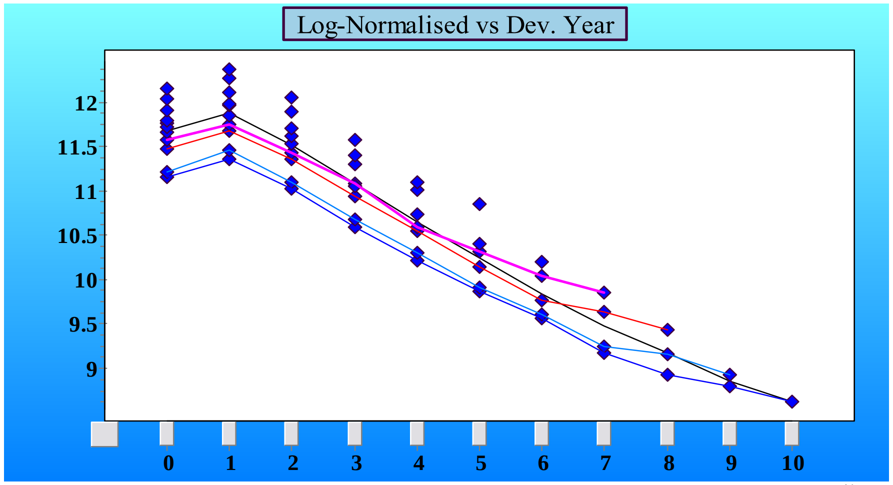


TG ABC
PTF-Calendar Year Trends. Have control on future assumptions

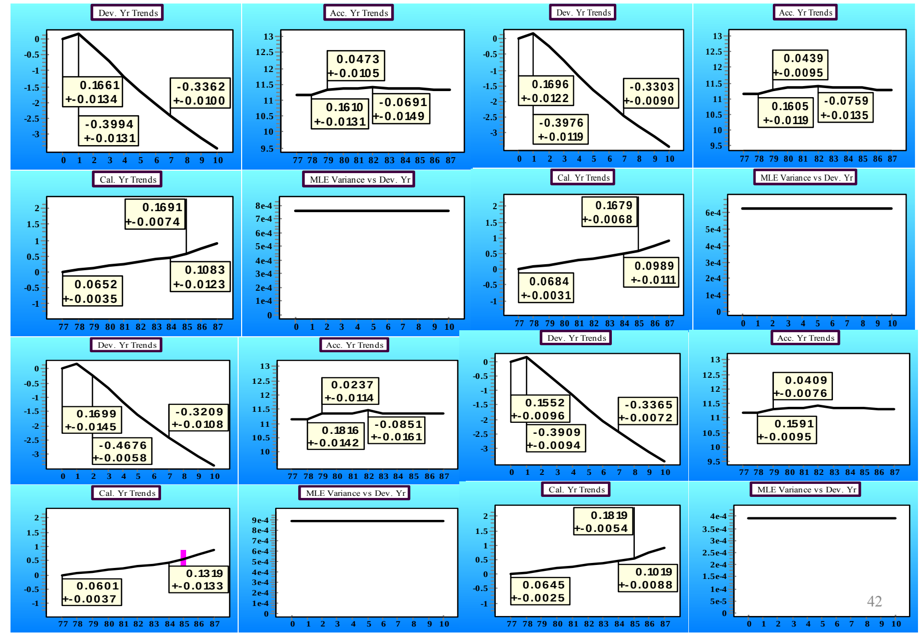


TG ABC

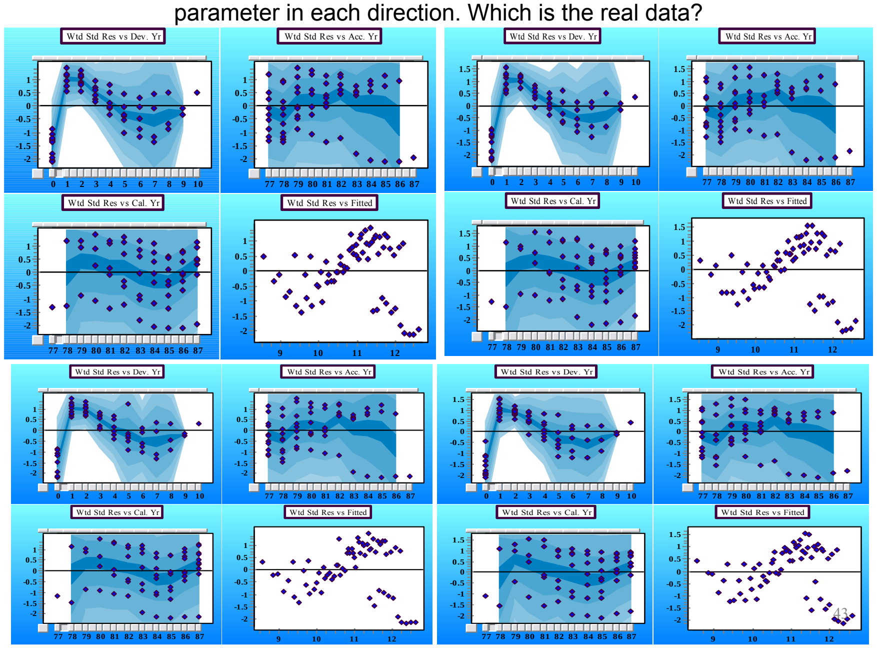
As you move down the accident years the “kick-up” is one development period earlier. Real data satisfies axiomatic trend properties



TG ABC: Three simulated from data model. Which is real data?



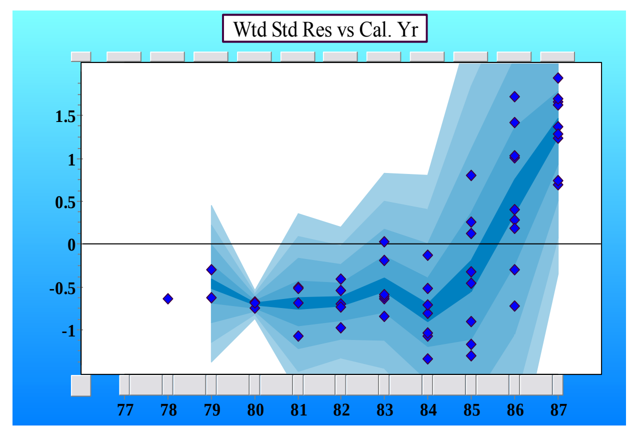
TG ABC: Three simulated, one real. Residuals of fitting only one parameter in each direction. Which is the real data?



TG ABC

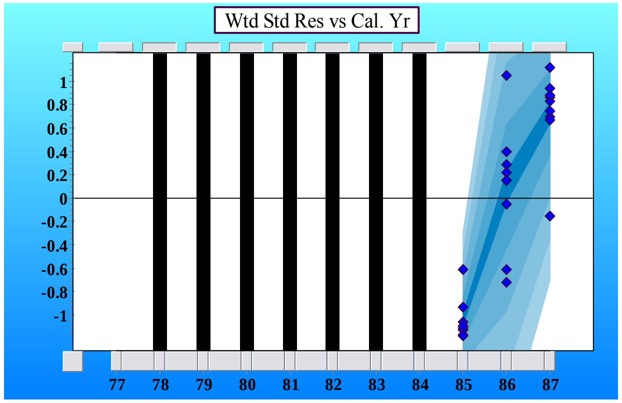
ELRF- Mack (volume weighted average link ratios) Residuals versus calendar year. Cannot capture calendar year trend structure.

No control on assumptions going forward



TG ABC

ELRF- Mack applied to last three years. No control on future assumptions



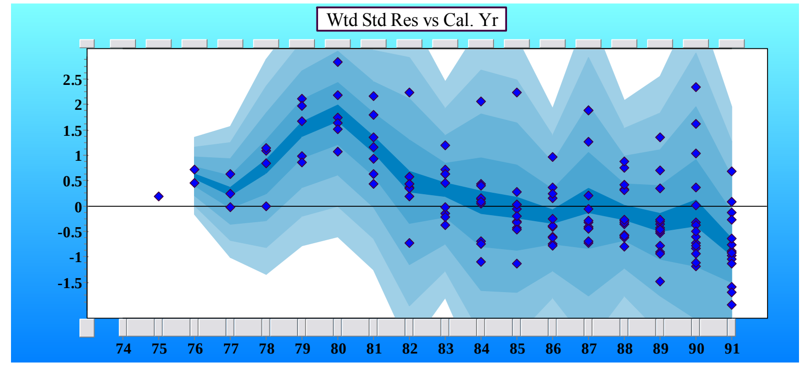
TG LR HIGH

ELRF- Mack

Trend estimated by method >> Trend in data

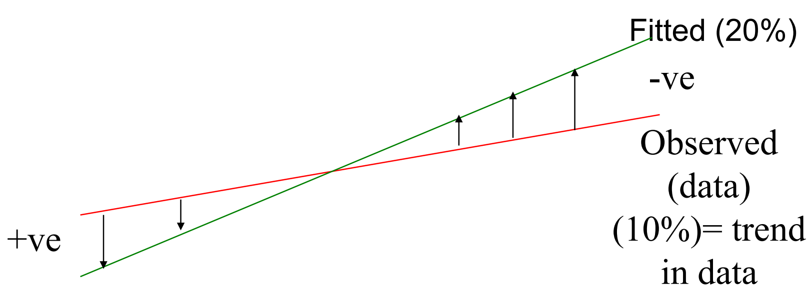
Mack= 896,133T +_104,117T

Arithmetic averages= 1,167,464T +- 234,466T



TG LR HIGH

Residuals= Trend in data- Trend estimated by method



Residual = Data trend –Fitted Trend

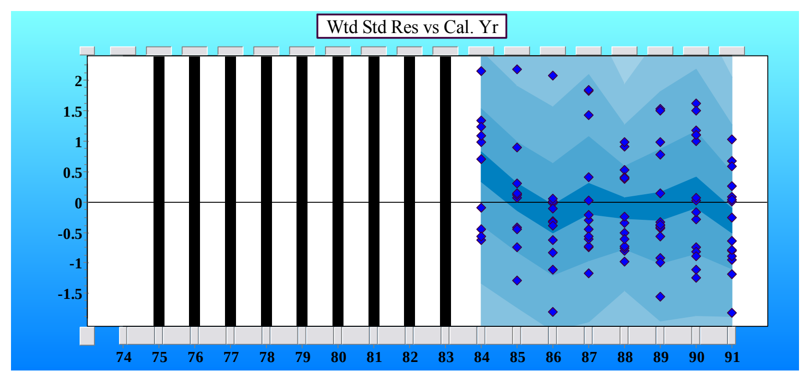
TG LR HIGH

Trend in data close to trend in method. Best ELRF model, link ratios =1, and still not very good. Trend assumption of method not known

468,439T+- 47,346T

Very different answer!

Mack= 896,133T +_104,117T

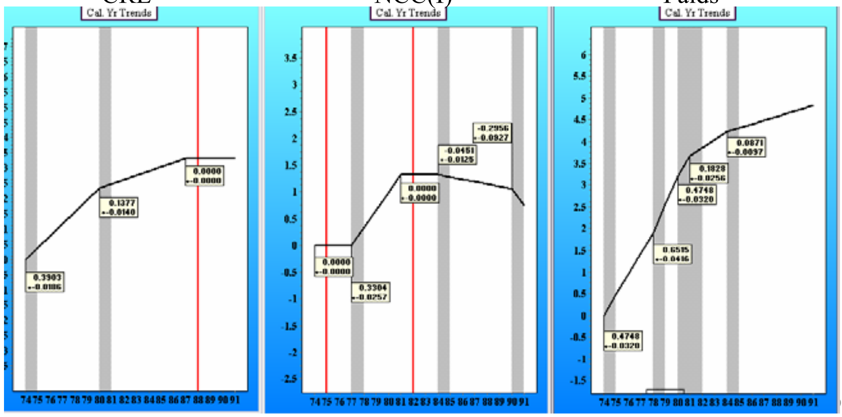


TG LR High

PTF calendar year structure for CREs, Number of claims closed and Paids

Based on information from CRE and NCC(I) if anything paid trend in future likely to come down further. If continue with 8.71%+_0.97% obtain 593,506T+_ 42,191T. If revert to (extremely unlikely) 18.28%+_ 2.56% obtain 751,912T+_79,509T. If assume 47.48%+_3.2% for one year, thereafter 18.28%+_ 2.56% (completely unreasonable) obtain 1,007,497T+_112,234T. Note Mack and arithmetic average give 896,133T +_104,117T and 1,167,464T +- 234,466T respectively. Important to monitor trends in each array.

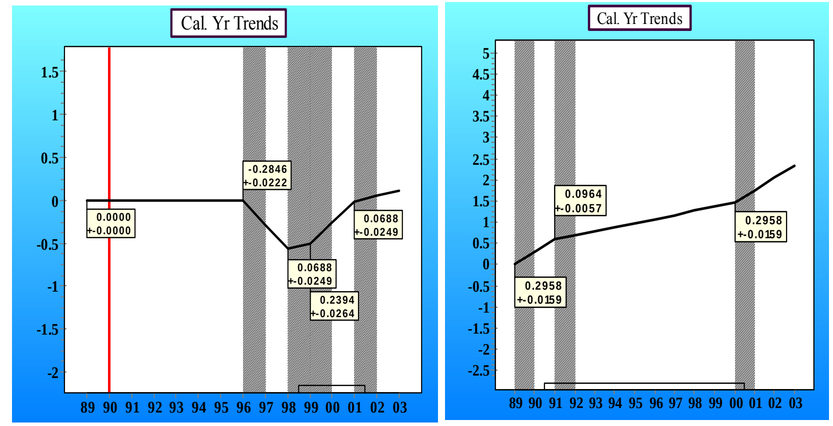
CRE NCC(I) Paids



COMPANY XYZ

CREs versus Paids

When was the company sold?



PTF: Forecast means and SDs based on easily interpretable assumptions. Trend assumption can be monitored 664,747+_85,384. Can also revert to longer term trend based on explanation for recent high trend

Comparisons | Summary Graphs | Forecast Sett
Yrs | Cal. Yrs | Observed vs Mean Estimate | Incurred Losses

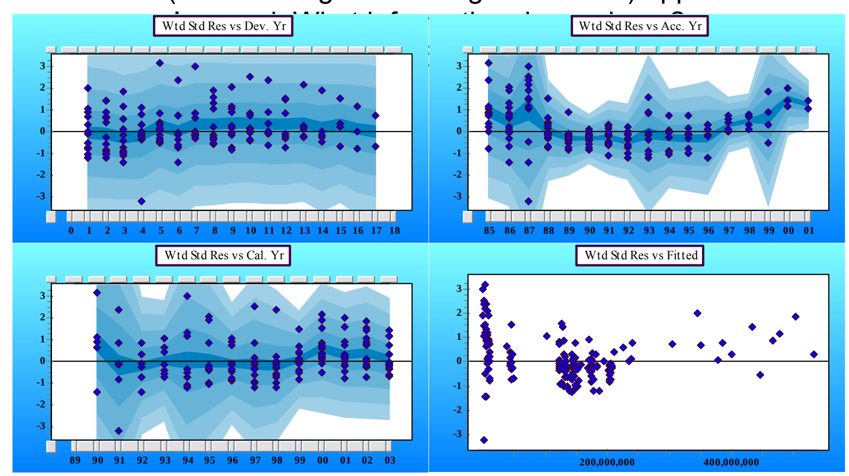
Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1990	3,737	177,556	555	0.15	0.00
1991	4,440	147,839	634	0.14	0.00
1992	5,550	138,313	789	0.14	0.01
1993	6,524	133,067	923	0.14	0.01
1994	9,758	134,195	1,390	0.14	0.01
1995	14,422	163,394	2,095	0.15	0.01
1996	21,371	199,307	3,172	0.15	0.02
1997	41,073	274,809	6,209	0.15	0.02
1998	104,330	478,365	15,508	0.15	0.03
1999	163,136	644,208	23,955	0.15	0.04
2000	195,443	621,764	28,108	0.14	0.05
2001	88,239	212,708	12,157	0.14	0.06
Total	664,747	3,803,196	85,384	0.13	0.02

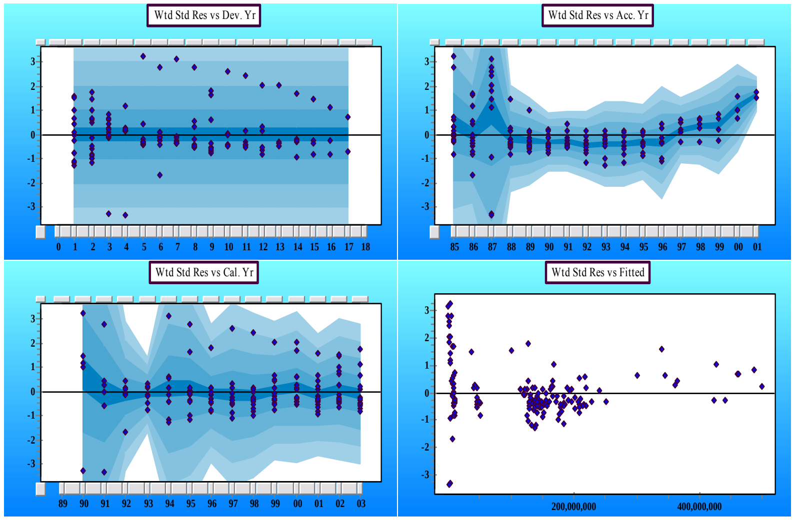
1 Unit = $1,000

COMPANY XYZ

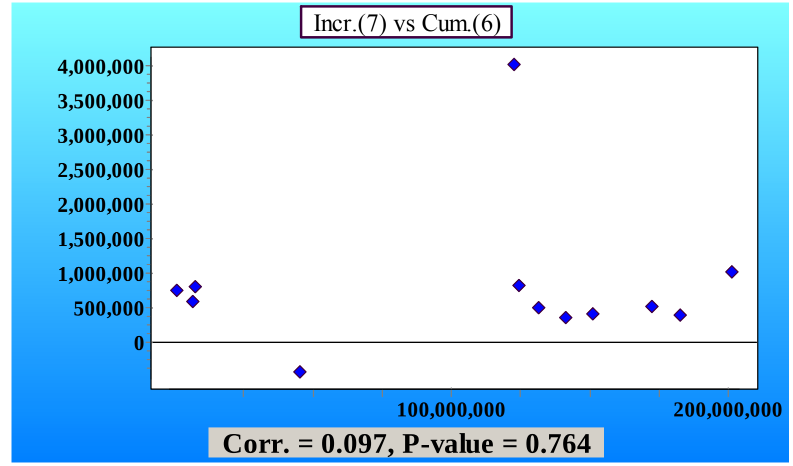
Mack (volume weighted average link ratios) applied to



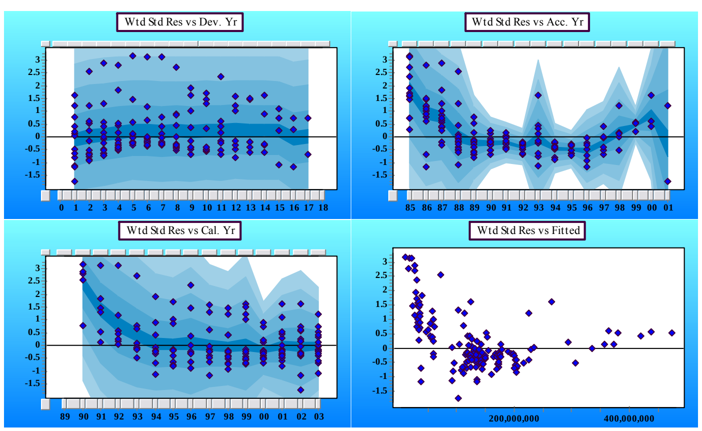
COMPANY XYZ
Arithmetic Averages applied to IL(C).
Reserves=2,100,714T +_1,071,680T
Mack =697,371T+_ 234,088T



COMPANY XYZ
Incurred data: No predictive power 6-7



COMPANY XYZ
Mack applied to PL(C) is 427,024T+_53,715T (Residuals below)
Arithmetic Average PL(C) is 711,144T+_233,066T
Mack IL(C) 697,371T+_234,088T
Arithmetic average to IL(C) 2,100,714T +_1,071,680T
Munich Chain Ladder?
PTF 664,747+_ 85,384 (Explicit assumptions)



Mack PL(C) 427,024+_53,715. Note inconsistent CVs

PTF 664,747+_85,384 explicit easily interpretable assumptions. Can revert to longer term trend and can

WC EBI 1985 - 2003:PL(C):ELRF[1]:Forecast Summaries

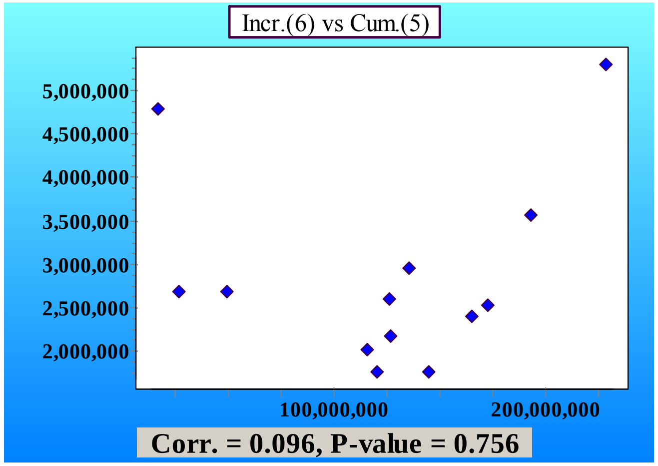
Acc. Yrs | Cal. Yrs | (%) Differences | Comparisons | Summary Graphs | BF & ELR

Accident Yr Summary

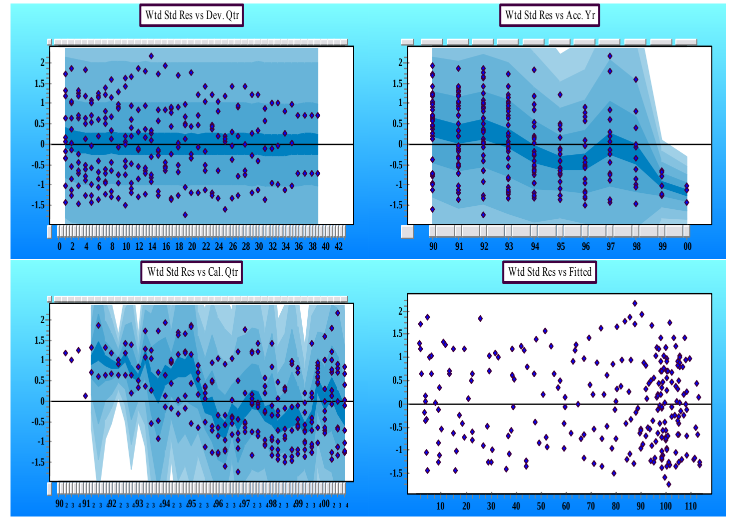
Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1991	8,022	151,421	1,261	0.16	0.01
1992	8,264	141,026	1,436	0.17	0.01
1993	8,737	135,280	1,650	0.19	0.01
1994	9,584	134,021	1,901	0.20	0.01
1995	13,142	162,114	2,806	0.21	0.02
1996	17,794	195,730	3,865	0.22	0.02
1997	27,401	261,137	6,535	0.24	0.03
1998	53,455	427,490	12,151	0.23	0.03
1999	89,976	571,049	23,699	0.26	0.04
2000	113,002	539,322	28,573	0.25	0.05
2001	52,474	176,943	18,355	0.35	0.10
Total	427,024	3,565,474	53,715	0.13	0.02

1 Unit = $1,000

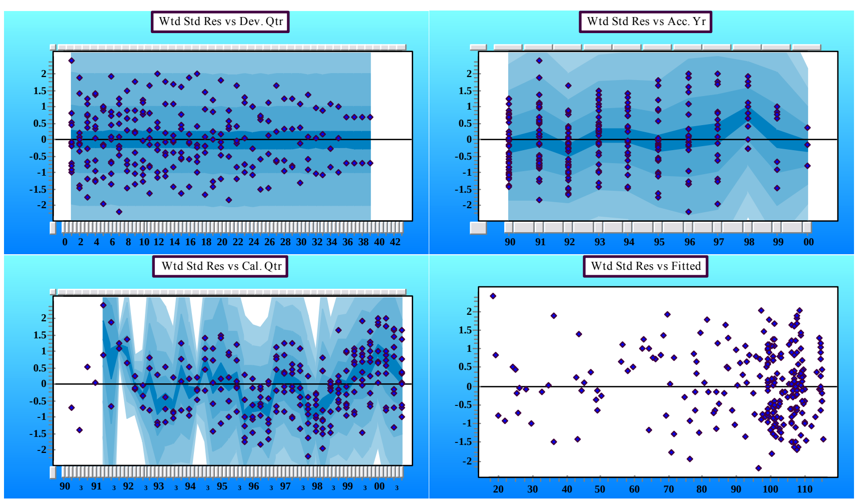
COMPANY XYZ PL(C)
No predictive power 5-6



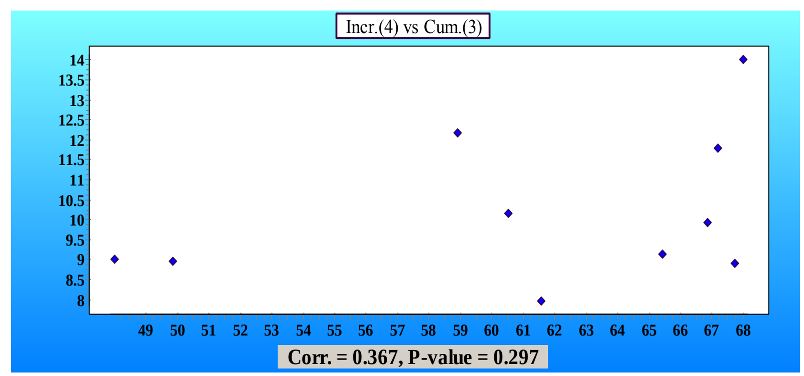
Seg 1
Mack PL(C)= 400,996T+_ 22,009T



Seg 1
Mack IL(C) 339,031T+_10,600T
Mack PL(C) 400,996T+_22,009T
Do you want to use Munich Chain Ladder?



IL(C) 3-4 no predictive power



Reserve Increases from year to year- Debunking a Myth

Here is a simple example that illustrates the main ideas that reserve increases do not represent under reserving. Indeed, they are necessary in order to manintain consisent estimates of prior year ultimates as the company writes new underwriting (accident) years).

On a logarithmic scale the data were generated as follows

Y(w,d) = 10 - 0.3*d +0.05 (w+d-1) where w is the accident year 1,...,7 and d is the devlopment year 0,..., 5.

The numbers down each column increase by 0.05 on a log scale (approximately 5% annual).
The numbers along each row decrease by 0.25 (=-0.3+0.05) on a log scale
We have assumed that the paid losses run-off after five years. Even if this is the case for 1999, this may not be the case for subsequent accident years especially if inflation is 'high'

Reserves and ultimates as at year end 2004

Accident Period vs Development Period									
Cal. Per. Total		0	1	2	3	4	5	Reserve	Ultimate
1999	22,026	22,026	17,154	13,360	10,405	8,103	6,311	0	77,359
2000	40,310	23,156	18,034	14,045	10,938	8,519	6,634	6,634	81,325
2001	55,736	24,343	18,958	14,765	11,499	8,955	6,974	15,930	85,494
2002	68,999	25,591	19,930	15,522	12,088	9,414	7,332	28,835	89,878
2003	80,639	26,903	20,952	16,318	12,708	9,897	7,708	46,631	94,486
2004	91,085	28,283	22,026	17,154	13,360	10,405	8,103	71,048	99,331
Total Fitted/Paid			2005	2006	2007	2008	2009	Total Reserve	Total Ultimate
Cal. Per. Total	358,796		66,022	46,251	30,589	18,112	8,103	169,078	527,873
1 Unit = $1									

Reserves and ultimates as at year end 2005

Accident Period vs Development Period									
Cal. Per. Total		0	1	2	3	4	5	Reserve	Ultimate
1999	22,026	22,026	17,154	13,360	10,405	8,103	6,311	0	77,359
2000	40,310	23,156	18,034	14,045	10,938	8,519	6,634	0	81,325
2001	55,736	24,343	18,958	14,765	11,499	8,955	6,974	6,974	85,495
2002	68,999	25,591	19,930	15,522	12,088	9,414	7,332	16,746	89,878
2003	80,639	26,903	20,952	16,318	12,708	9,897	7,708	30,313	94,486
2004	91,085	28,283	22,026	17,154	13,360	10,405	8,103	49,022	99,331
2005	95,755	29,733	23,156	18,034	14,045	10,938	8,519	74,691	104,424
Total Fitted/Paid			2006	2007	2008	2009	2010	Total Reserve	Total Ultimate
Cal. Per. Total	454,560		69,407	48,623	32,157	19,041	8,519	177,746	632,298
1 Unit = $1									

Reserve and ultimate as at year end 2004

Accident Year	Mean Reserve	Ultimate
1999	0	77,359
2000	6,634	81,325
2001	15,930	85,494
2002	28,835	89,878
2003	46,631	94,486
2004	71,048	99,331
Total	169,078	527,873

Reserve and ultimate as at year end 2005

Accident Year	Mean Reserve	Ultimate
1999	0	77,359
2000	0	81,325
2001	6,974	85,495
2002	16,746	89,878
2003	30,313	94,486
2004	49,022	99,331
2005	74,691	104,424
Total	177,746	632,298

Ratio of year t ultimate to year t-1
1.051267467
1.051275745
1.051266156
1.051269499
1.051277438
1.051273016
1.051266279
Ratio of Reserves

N.B.

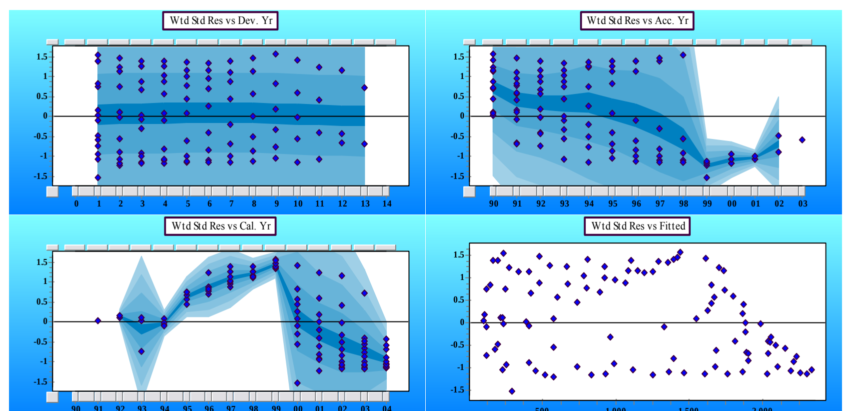
1. Estimtes of ultimate losses by accident year (1999- 2004) remain the same on update at end of 2005
2. The ratio of ultimate for year t to year t-1 is 1.05
3. Increase in total reserves from 2004 to 2005 is 1.05

Data from Murphy et al "*Manually Adjustable Link Ratio Model for Reserving*"

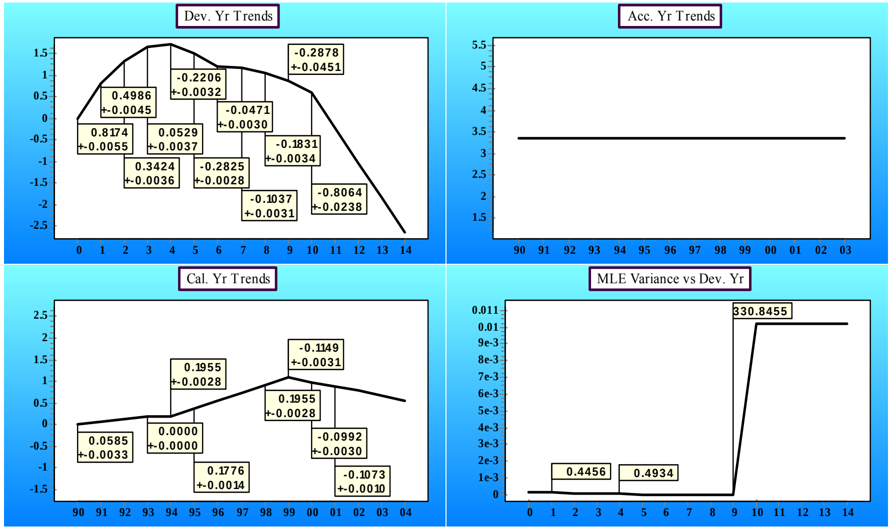
Mack (volume weighted averages) standardized residuals. Murphy only shows residuals versus accident year. Reserve too large by a factor of **two**

Reserve is 10,437 +_ 936

Data unlikely to be real. See subsequent slides



Data from Murphy et al “*Manually Adjustable Link Ratio Model for reserving*” PTF designed model. Process Variability almost zero.



Data from Murphy et al “*Manually Adjustable Link Ratio Model for Reserving*” PTF Forecast table. Note mean fitted (black) very close to observed (blue). An absolute perfect fit, one of the reasons it is not likely to be real data

Murphy:PL(I):PTF[M4-1]:Reserve Forecast Table

Accident Period vs Development Period

	Cal. Per. Total	0	1	2	3	4	5	6	7
1998	2,229	70	194	285	363	344	248	168	144
	2,233	70	192	288	367	345	248	169	1
1999	2,915	86	173	258	326	309	222	151	129
	2,911	88	173	257	328	308	225	1	1
2000	2,737	76	157	232	293	277	200	135	116
	2,733	76	159	231	289	278	1	1	1
2001	2,534	69	141	208	263	249	180	122	104
	2,536	68	139	204	262	2	1	1	1
2002	2,299	62	126	187	236	224	161	109	94
	2,294	58	127	187	2	2	1	1	1
2003	2,074	56	113	168	212	201	145	98	84
	2,061	53	114	1	2	2	1	1	1
	Total Fitted/Paid		2004	2005	2006	2007	2008	2009	2010
Cal. Per.	21,855		1,816	1,529	1,223	908	635	440	307
Total	21,821		1,816	12	11	10	9	8	7

1 Unit = $1

Data from Murphy et al “*Manually Adjustable Link Ratio Model for Reserving*” PTF Accident year Summary Reserve 5,461+_45.
Why is Mack 10,437+_936 more than double??

Murphy:PL(I):PTF[M4-1]:Reserve Forecast Summaries

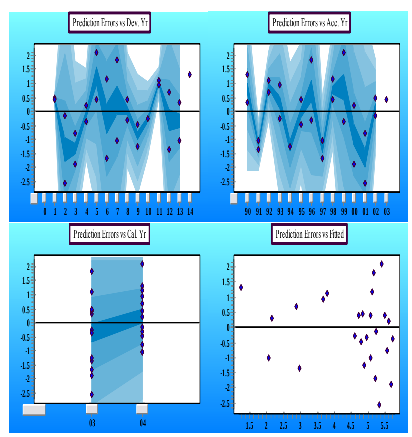
Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1995	134	2,399	10	0.08	0.00
1996	229	2,392	9	0.04	0.00
1997	335	2,273	8	0.02	0.00
1998	445	2,124	8	0.02	0.00
1999	550	1,929	7	0.01	0.00
2000	694	1,727	7	0.01	0.00
2001	872	1,545	7	0.01	0.00
2002	1,020	1,392	8	0.01	0.01
2003	1,084	1,251	8	0.01	0.01
Total	5,461	27,282	45	0.01	0.00

1 Unit = $1

Data from Murphy et al “*Manually Adjustable Link Ratio Model for Reserving*”
PTF model validation

Prediction errors two calendar years removed. That is at end 2002



Reserve forecasts beyond 2004 as at end 2002, 2003 and 2004. Extremely stable!