

# Section 1 – Introduction



## Instructional Approach to Mack and Bootstrap Models

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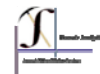


## Overview

- Notation
- Mack Model
- Bootstrap Model (Parametric)



Page 1.2



# Section 1 – Introduction



## Notation

- $w$  – denotes an Accident (or Policy) period (“when”).
- $d$  – denotes a development age (“delay”).
- $k$  – denotes a diagonal or constant  $w + d$ .
- $c(w,d)$  – denotes cumulative losses for accident (or policy) period  $w$ , and development age  $d$ .
- $c(w,n)$  – denotes ultimate losses for accident (or policy) period  $w$  [also denoted as  $U(w)$ ].
- $R(w,d)$  – denotes future development for accident (or policy) period  $w$ , and development age  $d$ . Note:  
 $R(w,d) = U(w) - c(w,d)$



Page 1.3



## Notation

- $q(w,d)$  – denotes incremental losses for accident (or policy) period  $w$ , and development age  $d$ .
- $f(d)$  – denotes the factor applied to  $c(w,d)$  to estimate  $q(w,d+1)$ .
- $F(d)$  – denotes the factor applied to  $c(w,d)$  to estimate  $c(w,d+1)$ . It is also known as the  $d$  to  $d+1$  age-to-age factor.



Page 1.4



# Section 1 – Introduction



## Notation

$G(w)$  – denotes a factor relating to accident (or policy) year  $w$ .

$h(w+d)$  – denotes a factor relating to diagonal  $k$  (or calendar year) in which  $w+d$  is constant.

$e(w,d)$  – denotes the mean of zero random fluctuation which occurs in cell  $w, d$ .



Page 1.5



## Notation

$E(x)$  – denotes the expectation of the random variable  $x$ . (Also  $\mu_x$ )

$Var(x)$  – denotes the variance of the random variable  $x$ . (Also  $\sigma_x^2$  or  $Sigma_x^2$ )

W – Weight

N – Total number of Accident (Policy) periods

n – Total number of Development periods


$\hat{x}$  – Estimate of  $x$ .



Page 1.6



# Section 1 – Introduction



## Notation Example

**Cumulative Losses**

| AY   | 1     | 2     | 3 |
|------|-------|-------|---|
| 1981 | 5,012 | 8,269 |   |
| 1982 | 106   | 4,285 |   |
| 1983 | 3,410 |       |   |


**Notation**

| AY   | 1           | 2           | 3           |
|------|-------------|-------------|-------------|
| 1981 | $c(1981,1)$ | $c(1981,2)$ | $c(1981,3)$ |
| 1982 | $c(1982,1)$ | $c(1982,2)$ |             |
| 1983 | $c(1983,1)$ |             |             |


Can be Year (i.e., 1981) or Index (i.e., 1)


$w$

$d$



Page 1.7





## Notational Example


**Incremental Losses**

| AY   | 1     | 2     | 3     |
|------|-------|-------|-------|
| 1981 | 5,012 | 3,257 | 2,638 |
| 1982 | 106   | 4,179 |       |
| 1983 | 3,410 |       |       |


**Notation**

| AY   | 1           | 2           | 3           |
|------|-------------|-------------|-------------|
| 1981 | $q(1981,1)$ | $q(1981,2)$ | $q(1981,3)$ |
| 1982 | $q(1982,1)$ | $q(1982,2)$ |             |
| 1983 | $q(1983,1)$ |             |             |

Calendar Effect denoted  $b(1982+1)$  or  $b(1983)$  or  $b(3)$



Page 1.8



# Section 1 – Introduction



## Notation Example

|      |        |       | Notation |             |             |
|------|--------|-------|----------|-------------|-------------|
| AY   | 1      | 2     | AY       | 1           | 2           |
| 1981 | 1.650  | 1.319 | 1981     | $F(1981,1)$ | $F(1981,2)$ |
| 1982 | 40.425 |       | 1982     | $F(1982,1)$ |             |
|      | 21.037 | 1.319 | Mean     | $F(1)$      | $F(2)$      |



## Basic Models

- And Now for Something Completely Different...



Mack Model



Bootstrap Model (Parametric)



# Section 1 – Introduction



## Questions on the Models?



Page I.11



## Suggested Bibliography

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Page I.12

