Bootstrap Model

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Bootstrap Overview

- Assume a model for loss development (the model assumes Chain Ladder)
  - Need some GLM assumptions to compute residual
- Use model to calculate expected or "fitted" values triangle
- Compute incremental residuals
- Bootstrap residuals with replacement and calculate sample historical triangle
- Compute ultimates for sample triangle
- Add process variance to projected incremental values
- Repeat many times...

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"Fitted" Incremental Triangle

- Work Backwards from observations on diagonal to create estimated cumulative triangle
  - $\hat{c}(w,n\text{-}1) = c(w,n)/F(n\text{-}1)$
  - $\hat{c}(w,n\text{-}2) = \hat{c}(w,n\text{-}1)/F(n\text{-}2)$
  - Fill in all cumulative entries on triangle
- Compute estimated or "fitted" incremental triangle

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Exercises

- Compute fitted incremental triangle from Mack data
  - Use weighted average loss development factors
  - Compute fitted cumulative triangle
  - Compute fitted incremental triangle

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Standardized Residuals

- In "Diagnostics" section, we used standardized residual:

$$z = \frac{x_i - \mu}{\sigma}$$

- More general Pearson Residual used with GLM models:

$$r = \frac{x - \mu}{\sqrt{Var(\mu)}}$$

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Exercise

- Compute Pearson Residuals from Mack incremental data
  - Assume Var(x) = E(x) = fitted incremental value

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Pearson Residuals

- If data assumed Normally distributed, Pearson Residual = standardized residual
- If data assumed Poisson, then:

$$Var(\mu) = \mu, \text{ so } r = \frac{x_i - \mu}{\sqrt{\mu}}$$

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Over-Dispersed Poisson

- Often Poisson distributed: Var > Mean
- One of the "Over-Dispersed Poisson" models uses the constant φ to inflate Variance:

$$Var(\mu) = \varphi\mu, \text{ and Scaled Pearson Residual is } \frac{x_i - \mu}{\sqrt{\varphi\mu}}$$

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Scale Parameter

- Using the Chi-Squared statistic:
  - N = sample size, p = # parameters

$$\varphi = X^2 = \sum_N \frac{[x_i - E(x_i)]^2}{(N-p)E(x_i)}$$

- Scaled Residual is: $$r_i = \frac{x_i - \mathrm{E}(x_i)}{\sqrt{\varphi E(x_i)}}$$

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Poisson Triangles

- The Poisson has been a useful Parametric assumption in modeling loss development triangles
- The “semi” Parametric bootstrap does not require a distribution assumption but
  - It uses a Pearson Residual
  - The Scaled Pearson Residual follows the Poisson assumption

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Triangle of Residuals

- Using actual and "fitted" incremental triangles, compute Unscaled Pearson Residuals:

$$e_U(w,d) = [q(w,d) - \hat{q}(w,d)] / \sqrt{\hat{q}(w,d)}$$

- Calculate Degrees of Freedom Adjustment:

$$df\Delta = \sqrt{\frac{N}{N-p}}$$

- Calculate Scaled Pearson Residuals:

$$e_S(w,d) = e_U(w,d) \times df\Delta$$

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Scale Parameter

- Use Unscaled Pearson Residuals and Degrees of Freedom to calculate the Scale Parameter:

$$\varphi = \frac{\sum e_U(w,d)^2}{N-p}$$

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Exercise

- Using Mack data and results of prior exercises
  1. Compute triangle of square of Unscaled Pearson Residuals
  2. Compute the triangle's degrees of freedom
  3. Using 1. and 2. compute the Scale Parameter for Mack data

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Bootstrap Residuals

- For each cell in the triangle, randomly select a Scaled Pearson Residual (with replacement)
- Transform residual into an incremental value for the triangle

$$q_S(w,d) = \hat{q}(w,d) + [e_S(w,d) \times \sqrt{\hat{q}(w,d)}]$$

- Calculate cumulative sample triangle
- Compute age-to-age factors

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Exercise

- Create a table of Scaled Pearson Residuals, using results of previous exercise
- Simulate a bootstrap triangle of residuals
- Create a triangle of incremental values from bootstrapped residuals
- Compute a cumulative triangle
- Compute age-to-age factors and weighted average age-to-age factors

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Process Variance

- Use Age-to-Age factors to compute ultimate for sample data
- Calculate incremental values for completed triangle
- Use the Gamma distribution to simulate random incremental values with:
  - Mean = sample incremental
  - Variance = sample incremental x Scale Parameter

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Distribution of Estimates

- Add incremental values after process variance to get ultimate and unpaid estimates
- Sum the unpaid amounts to get total unpaid
- Repeat many times…

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Distribution of Estimates

- A "trick" is needed to easily compute a distribution of unpaid amounts
- Use the Data Table (menu) function of Excel
- ={Table(,input cell=a constant)}

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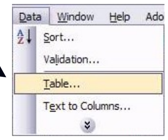
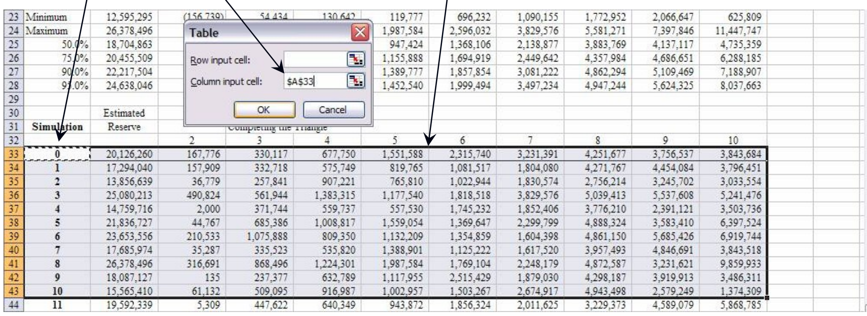
Data Table Function

Select Range and then Start with Data Menu



Set Column Input to Zero cell



23	Minimum	12,595,295	(156,739)	54,434	130,642	119,777	696,232	1,090,155	1,772,952	2,066,647	625,809
24	Maximum	26,378,496				1,987,584	2,596,032	3,829,576	5,581,271	7,397,846	11,447,747
25	50.0%	18,704,863				947,424	1,368,106	2,138,877	3,883,769	4,137,117	4,735,359
26	75.0%	20,455,509				1,155,888	1,694,919	2,449,642	4,357,984	4,686,651	6,288,185
27	90.0%	22,217,504				1,389,777	1,857,854	3,081,222	4,862,294	5,109,469	7,188,907
28	95.0%	24,638,046				1,452,540	1,999,494	3,497,234	4,947,244	5,624,325	8,037,663
29											
30		Estimated									
31	Simulation	Reserve		Completing the Triangle							
32			2	3	4	5	6	7	8	9	10
33	0	20,126,260	167,776	330,117	677,750	1,551,588	2,315,740	3,231,391	4,251,677	3,756,537	3,843,684
34	1	17,294,040	157,909	332,718	575,749	819,765	1,081,517	1,804,080	4,271,767	4,454,084	3,796,451
35	2	13,856,639	36,779	257,841	907,221	765,810	1,022,944	1,830,574	2,756,214	3,245,702	3,033,554
36	3	25,080,213	490,824	561,944	1,383,315	1,177,540	1,818,518	3,829,576	5,039,413	5,537,608	5,241,476
37	4	14,759,716	2,000	371,744	559,737	557,530	1,745,232	1,852,406	3,776,210	2,391,121	3,503,736
38	5	21,836,727	44,767	685,386	1,008,817	1,559,054	1,369,647	2,299,799	4,888,324	3,583,410	6,397,524
39	6	23,653,556	210,533	1,075,888	809,350	1,132,209	1,354,859	1,604,398	4,861,150	5,685,426	6,919,744
40	7	17,685,974	35,287	335,523	535,820	1,388,901	1,125,222	1,617,520	3,957,493	4,846,691	3,843,518
41	8	26,378,496	316,691	868,496	1,224,301	1,987,584	1,769,104	2,248,179	4,872,587	3,231,621	9,859,933
42	9	18,087,127	135	237,377	632,789	1,117,955	2,515,429	1,879,030	4,298,187	3,919,913	3,486,311
43	10	15,565,410	61,132	509,095	916,987	1,002,957	1,503,267	2,674,917	4,943,498	2,579,249	1,374,309
44	11	19,592,339	5,309	447,622	640,349	943,872	1,856,324	2,011,625	3,229,373	4,589,079	5,868,785

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Exercise

- Review bootstrap calculation in Bootstrap spreadsheet
- Where is the calculation of
  - The fitted cumulative triangle?
  - The fitted incremental triangle?
  - The residuals?
- What formula is used to resample residuals?
- Where is estimation of bootstrapped unpaid?
- Paste value a new triangle into the Data sheet and run a new model for 100-250 simulations

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GLM Residuals

- With GLM, Unscaled Pearson Residuals need another adjustment to standardize:

$$r_{ij}^{P*} = \frac{r_{ij}^{P}}{\sqrt{1-h_{ij}}}$$

- where h_{ij} is the corresponding diagonal of:

$$H = X(X^T W X)^{-1} X^T W$$

X = Design Matrix

W = Weight Matrix

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Questions on Bootstrap Model?

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