<u>Mack made easy</u> by Thomas Mack

$c(w,d)$ = cumulative losses already known, $1 \le w+d \le N+1$,

$F_d = c(\le, d+1) / c(<, d)$, $1 \le d \le N\text{-}1$, ATA factors,

$$\text{where } c(\le, d+1) := \sum_{w=1}^{N-d} c(w, d+1) \ , \ c(<, d) := \sum_{w=1}^{N-d} c(w, d),$$

$$\sigma_d^2 = \frac{1}{N-d-1} \sum_{w=1}^{N-d} c(w,d) \left(\frac{c(w,d+1)}{c(w,d)} - F_d \right)^2 \ , 1 \le d \le N\text{-}2,$$

$$\sigma_{N-1}^2 := \min\{\sigma_d^2 \mid 1 \le d \le N-2\} \ ,$$ Variance parameters.

<u>Recursive formula for prediction:</u> $(w+d \ge N+1)$

$\hat{c}(w, d+1) = \hat{c}(w,d) \cdot F_d$ starting with $\hat{c}(w, N+1\text{-}w) = c(w, N+1\text{-}w)$.

Recursion for the prediction variance pv of accident year w:

$$\mathrm{pv}\big(\hat{c}(w, d+1)\big) = \mathrm{pv}\big(\hat{c}(w,d)\big)\cdot F_d^2 + \big(\hat{c}(w,d)\big)^2\left(\frac{\sigma_d^2}{\hat{c}(w,d)} + \frac{\sigma_d^2}{c(<,d)}\right)$$

starting with $\mathrm{pv}\big(\hat{c}(w, N+1-w)\big) = 0$.

Recursion for the prediction variance of the total o/s losses:

$$\mathrm{pv}\big(\hat{c}(>, d+1)\big) = \mathrm{pv}\big(\hat{c}(>,d)\big)\cdot F_d^2 + \big(\hat{c}(\geq,d)\big)^2\left(\frac{\sigma_d^2}{\hat{c}(\geq,d)} + \frac{\sigma_d^2}{c(<,d)}\right)$$

with $\hat{c}(>,d) := \sum_{w=N+2-d}^{N} \hat{c}(w,d)$, $\hat{c}(\geq,d) := \sum_{w=N+1-d}^{N} \hat{c}(w,d)$

starting with $\mathrm{pv}\big(\hat{c}(>, 1)\big) = 0$.

How to use parameter variance and prediction variance?

Prediction variance:

- To fit a distribution for the reserve or the ultimate losses,
- To be used for calculation of the premium loading,
- To be used for risk modeling.

Parameter variance (omitting the first term in the large bracket):

- To construct a confidence interval for the best estimate,
- To assess the significance of the difference to other estimates.

Th. Mack's comments on the bootstrap approach shown before

The basis of the bootstrap procedure is the following

Theorem: In case of

- a full data triangle (i.e. no missing values, no trapezoid)
- with positive and independent increments (i.e. no incurreds)
- which follow a Poisson distribution,

the maximum likelihood estimate of the ultimate loss amount turns out to be equal to the chain ladder estimate.

But this is not a chain ladder model (not even for full triangle) because:

- For incomplete triangles, the estimated ultimates are different.
- The same holds if the weights in F(d) are changed.
- Chain ladder works for negative increments, too.

- At chain ladder, the increments are not independent.
- The residuals are different: CL calculates fitted values from previous amounts, i.e. $\hat{c}(w,d) = c(w,d-1) \cdot F(d)$, and not backwards.

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As a consequence, the prediction variances are always different, i.e. the bootstrap procedure shown does never yield the prediction variance of chain ladder (but of a different method with same estimated ultimates in some cases). They may or may not be close together.

Generally, a bootstrap approach to loss reserving contains at least as many assumptions than just to fit a lognormal distribution using the Mack chain ladder variance.