The Closure-Based Regression Method

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ABSTRACT

This paper outlines a powerful simple regression methodology using closure ratio as the independent variable and the cumulative net paid loss & ALAE development factor as the dependent variable.

The method is similar to the Berquist-Sherman Adjusted Paid Loss Development Method (Adjusted Paid Loss Method) in that closure replaces age in the development of losses. However, the Closure-Based Regression Method (Closure Method) directly analyzes the relationship between claim closure level and paid LDF with a stochastic rather than a deterministic approach.

The method, in addition to its simplicity, has three favorable characteristics. First, from a practical review, historical evidence demonstrates that it has proven to be very accurate when accident year exposures are fully earned, especially for the Bodily Injury coverage. Secondly, the method eliminates error introduced by chain-ladder approaches and interpolations required in an adjusted paid loss methodology. Finally, the method is visually compelling. The actuary often can see as well as statistically measure a dramatically improved fit as the independent variable is changed from age to closure level. The improved fit results from the belief that a closure-based triangle is inherently superior to an age-based triangle for paid loss development methods. Age is only a weak surrogate for closure.

KEYWORDS

Simple regression; closure ratio

1. Introduction

The traditional paid loss triangle tracks accident year losses by age of development. However, the development of paid losses is more closely related to the rate at which the claims department settles claims in that accident year than it is to the age of the accident year. Age is only a weak surrogate for settlement rate. If there is a change in settlement rate from one accident year to the next a paid loss development method which adjusts for closure rate is in order.

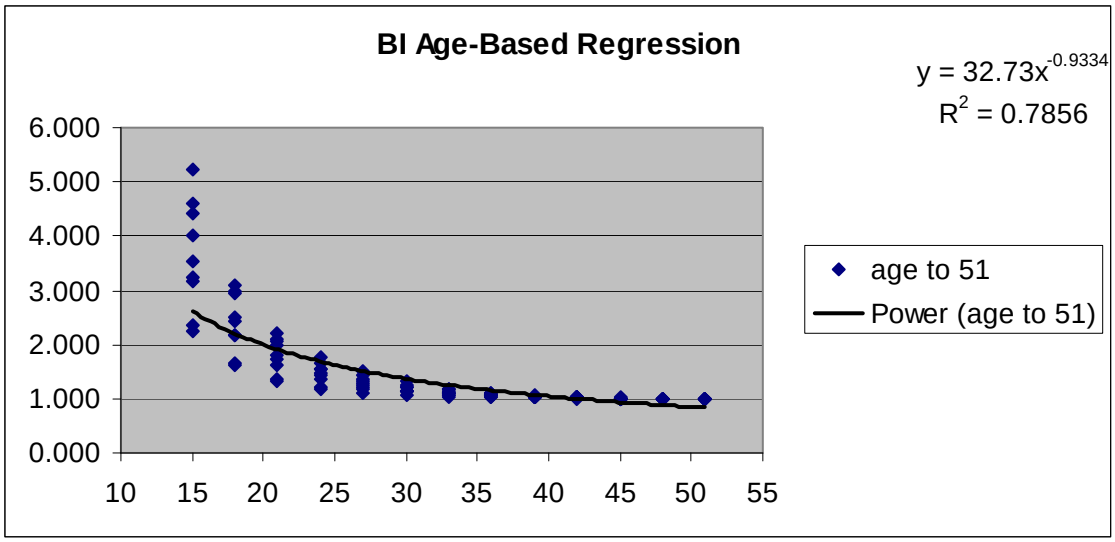
To verify that settlement rate should be used for paid loss development a comparison of simple regressions was made. A simple regression of closure ratio to LDF was compared to a simple regression of age to LDF for a non-standard personal automobile business.

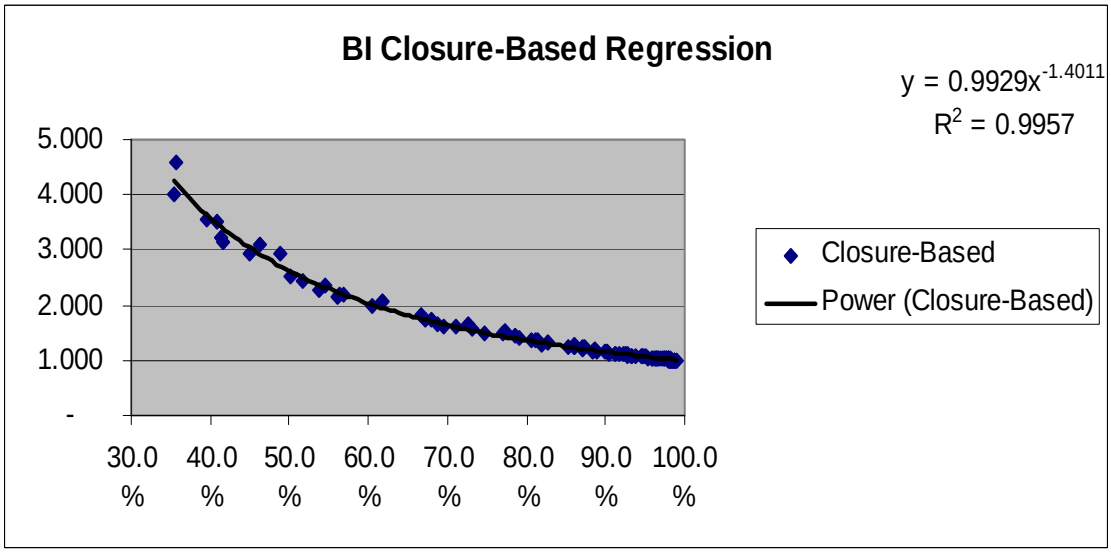
Closure ratio is defined as CWA / (CWA + Open) where CWA is closed claims with amount (payment). The LDF is defined as the cumulative net paid loss & ALAE development from the closure ratio (or age) of the given data point to a 99% closure ratio (or age of 51 months). Closure ratio (or age) is the independent variable while LDF is the dependent variable.

The graphs below represent Bodily Injury data for a non-standard personal automobile insurer for a large book of business. The graphs visually illustrate the greatly improved predictability that a closure-based analysis offers over a traditional age-based analysis for this business. The net paid loss & ALAE development factor (LDF) is on the y-axis. The accident year age is on the x-axis for the upper graph (closure ratio is on the x-axis for the lower graph). The LDF is the dependent variable on both graphs.

The age-based graph fits the age of all data points to their associated LDF. The LDF represents the development from the amount of cumulative net paid loss & ALAE at the age of the data point for that accident year to the amount of cumulative net paid loss & ALAE at age 51 for that accident year.

The closure-based graph fits the closure ratio of all data points to their associated LDF. This LDF represents the development from the amount of cumulative net paid loss & ALAE at the closure ratio of the data point for that accident year to the amount of cumulative net paid loss & ALAE at a 99% closure ratio for that accident year. Development to 99% was selected to maximize the amount of data to retrospectively evaluate while leaving as little tail development as possible. The comparison of the regressions begins at a closure ratio of 35% or at an age of 15. Likewise, the comparison of regressions ends at a closure ratio of 99% or at an age of 51. The range of age 15 to age 51 best represents the ages where the closure ratio range is 35% closed to 99% closed.


BI Age-Based Regression
y = 32.73x^-0.9334
R2 = 0.7856
6.000
5.000
4.000
3.000
2.000
1.000
0.000
10
15
20
25
30
35
40
45
50
55
age to 51
Power (age to 51)


BI Closure-Based Regression
y = 0.9929x^-1.4011
R2 = 0.9957
5.000
4.000
3.000
2.000
1.000
-
30.0 %
40.0 %
50.0 %
60.0 %
70.0 %
80.0 %
90.0 %
100.0 %
Closure-Based
Power (Closure-Based)

At first glance the upper graph would suggest extreme randomness in LDF's at low ages of development. The second graph illustrates that, in fact, there is not much randomness exhibited. Rather, a proper predictive variable, closure ratio, explains nearly all of the difference we see in the various levels of LDF. In other words, age appears to be a poor surrogate for closure. The r^2 of .9929 in the lower regression illustrates the tremendous predictive power of the closure ratio if it is not already obvious from a visual inspection. While a power curve is not a good curve selection for the age-based triangle and may not be the optimal fit for the closure-based triangle, the visual evidence of the superior fit of the closure ratio regression is clear. This visual is truly amazing. The first time the regressions were performed we were quite surprised to see such an improvement in fit. In particular, there is nearly a perfect fit for Bodily Injury which is the most difficult coverage to accurately predict for personal lines non-standard automobile.

After an analysis of the visual evidence above it is apparent that an adjustment to the age-based triangle is in order. The visual and statistical evidence gives the actuary confidence to place weight on an adjusted method over a traditional age-based development method.

The next focus is on minimizing the error introduced when applying an adjusted paid loss methodology. In the traditional Adjusted Paid Loss Method error occurs in converting an aged-based triangle into a closure-based triangle. Transforming an age-based triangle into a closure-based triangle requires interpolation at many data points along the triangle which introduces error. Additional compounding error is introduced in selecting age-to-age factors during the chain ladder process.

The Closure Method by contrast directly evaluates the relationship between closure ratio and paid loss development. The compounding error introduced in a chain ladder approach is eliminated. The Closure Method still introduces some error in the selection of the regression equation (equations if two or more fitted curves are used). There also is a slight amount of error in interpolating to get the ending closure point cumulative net paid loss & ALAE. This should be very slight, however at high closure ratios. The error in the tail assumption is the same whether the Closure Method or a chain ladder approach is used.

It should be noted that the Adjusted Paid Loss Method still has some advantages despite the errors introduced in the mechanical operation of the method. These will be reviewed later in the paper.

2. Closure-Based Regression Methodology Overview

Appendix A, Exhibit 2 graphs the regressions by coverage from the most recent actuarial evaluation for closure ratios of 35% and greater. The tight fit appears to be unique to Bodily Injury. However, the reason the Bodily Injury fit is tight is the fact that at 35% all exposures are fully earned for Bodily Injury. Using accident year data prior to age 12, when exposures are fully earned, distorts the correct projection of LDF. Appendix A, Exhibit 3 graphs the regressions by coverage from the most recent actuarial evaluation for closure ratios with age 12 months and higher. (The Physical Damage coverages were not illustrated because they are nearly 100% closed at age 12.) This not only is a fairer comparison, but it is the only way to effectively employ the method. In other words, although the fit becomes tighter as the closure ratio advances, a rather tight fit is evidenced for all coverages once the accident period exposures are fully earned. The significance of the Appendix A, Exhibit 3 graphs is that if an accident period exposures are fully earned, a very accurate ultimate net loss & ALAE estimate can be made at relatively low closure ratios (35% for this non-standard personal automobile Bodily Injury business).

There were three pleasant surprises at the conclusion of our initial single curve analysis. The first surprise was that Bodily Injury was found to be the coverage where the relationship was strongest at a low level of closure. This was exciting because Bodily Injury is a difficult coverage to accurately predict ultimate loss due to the long tail of this coverage.

The second surprise was that a remarkably good fit was determined by using only one independent variable. It certainly is possible that an even closer fit may result when one introduces other explanatory variables. However, the improvement in fit (if found) would need to be weighed against the principle of parsimony.

Finally, we discovered that the method is very accurate when the accident period exposures are fully earned. As stated above, this means that the Bodily Injury coverage ultimate net paid loss & ALAE estimate is fairly certain around an age of 12 months or so if there is a closure ratio of 35% or higher.

Note that with accident quarter triangles exposures are fully developed at age 3. In addition, the regression becomes stronger through an increase in the amount of data points regressed. An accident quarter approach can be particularly helpful in getting an accurate accident quarter projection on quick-settling lines such as Collision much sooner than with a traditional approach. One needs to review the regression fits to determine when a closure ratio is high enough to begin applying the method on an accident quarter basis.

3. The Closure-Based Regression Method

Before the method is begun a review of the overall fit of closure ratio to LDF is compared to the overall fit of accident year age to LDF. Graphs similar to the graphs on page 2 should be produced. The closure graph on page 2 illustrates that a single fitted curve for the Bodily Injury business produces an r^2 of .9957. This is exciting from an explanatory level, especially as it relates to an aged based r^2 of .7856 (using simple power curve fits). A single curve fit such as this will provide the actuary with a starting point of reference in regard to the expected improvement and resulting confidence to place in an adjusted paid method.

A single curve fit is not necessarily the best solution available, however. Fitting splines across sections of the scattered data points, particularly around areas of inflection on the original fitted curve may produce more accurate LDF's even though the r^2's of the individual splines may not be as high. Too many splines, however, may cause one to lose information about the shape of the curve and may lead to over-fitting. Currently, the data is fit with two curves (splines). The first power curve fits data from % closed to 80% closed. The second power curve fits data from 80% closed to 99% closed. A tail factor is used to project from 99% closed to ultimate.

While actuarial triangles are used to organize data for the regression analysis there is nothing that requires that the data be organized in triangular format for the purpose of reviewing statistics to select age to age LDF's. In other words, this is not a typical triangle-based development method. Cumulative net paid loss & ALAE by accident year and cumulative closure ratios by accident year are required, however. Note that for regression analysis the more data points the better, even if collected on a monthly or shorter time span. In addition, the benefits of an accident quarter approach have been outlined earlier.

The method begins by constructing triangles of cumulative closure ratio and cumulative net paid loss & ALAE to organize data for the regression analysis. Exhibit 1 is a simple calculation of the closure ratio using the traditional age-based triangles of cumulative closed claims with amount (closed claims with payment) and open claims. Exhibit 2 is the traditional age-based cumulative net paid loss & ALAE triangle.

Exhibit 3 uses the net paid loss & ALAE and closure ratio information in Exhibit 1 and Exhibit 2 to perform linear interpolation to calculate the expected cumulative net paid loss & ALAE at an 80% closure ratio. To illustrate, the 2003 accident year expected cumulative net paid loss & ALAE at an 80% closure ratio of $23,354,111 is found in Exhibit 3. $23,335,756

cumulative net paid loss & ALAE aligns with the data point where there is 79.95% closure. Likewise, $25,477,508 cumulative net paid loss & ALAE aligns with the data point where there is 85.24% closure. Simple linear interpolation yields expected cumulative net paid loss & ALAE of $23,354,111 at a closure ratio of 80%. Exponential interpolation would improve the interpolated estimate somewhat. In fact, an iterative process using a fitted curve from the method to perform the interpolation would be the most exact way to interpolate to the 80% closure ratio expected net paid loss & ALAE.

Exhibit 4 calculates the expected cumulative net paid loss & ALAE at a 99% closure ratio along with the tail estimate for a 99% closure ratio. The 99% tail factor in Exhibit 4 is based on an incurred loss development approach for accident years where the ultimate loss is not much in doubt. In Exhibit 4 a tail factor of 1.016 is eventually selected.

Exhibit 5 is the net paid loss & ALAE LDF from % closed to 80% closed. To illustrate, the first LDF listed for the 2003 accident year in Exhibit 5 is 277.754. This represents net paid loss & ALAE development from a 7.84% closure ratio to an 80.00% closure ratio. From Exhibit 1 for accident year 2003 at age 3 we see that the closure ratio is 7.84%. At this age and associated closure ratio for this accident year the cumulative net paid loss & ALAE is $84,082 from Exhibit 2. The expected cumulative net paid loss & ALAE for accident year 2003 at 80% closed is $23,354,111. Therefore, the associated LDF for a closure ratio of 7.84% is 23,354,111 / 84,082 = 277.754. Likewise, Exhibit 6 is the net paid loss & ALAE LDF from % closed to 99% closed.

Exhibit 7 organizes each data point's closure ratio and associated LDF. The first regression utilizes closure ratios starting at the % closed for that data point and truncating at an 80% closure ratio. Here we are regressing % closed to the % closed to 80% closed LDF. The second regression utilizes closure ratios starting at 80% closed and ending at 99% closed. Here we are regressing % closed to the % closed to 99% closed LDF. Note that in both lists of data points in Exhibit 7 the earlier data set calculated (7.84%; 277.754) has been discarded because it is less than age 12 when exposures were fully earned. All data points less than age 12 are discarded. We chose a 99% closure as a balance between maximizing data and reducing the amount of the tail.

Exhibit 8 shows the simple regression curve fits using a power curve in Excel. Exploration in regard to the number of splines to use as well as to different types of curve fits will again lead to incremental improvements in the fit. Certainly using special statistical software outside of Excel would lead to a more sophisticated analysis.

Exhibit 9 is the resulting Closure-Based Regression Method exhibit. This represents the

actuarial analysis as of 6/30/05 for the Bodily Injury coverage.

Column (1) is the cumulative closure ratio for the accident year. These are taken directly from Exhibit 1.

Column (2) is the LDF from the cumulative closure ratio for the accident year to a closure ratio of 80%. For accident years 2003 and prior the LDF defaults to 1.000 since these years are greater than 80% closed. Accident year 2005 is not calculated because the exposures are not yet fully earned. Accident year 2004 has an LDF of 1.609. This is calculated as .7271x (.5669)^(-1.3997)=1.609 by inserting the closure ratio of 56.69% into the regression equation from Exhibit 8.

Column (3) is the LDF from the "advanced" closure ratio for the accident year to a closure ratio of 99%. For accident years 2003 and prior the "advanced" closure ratio is the original closure ratio from column (1). To illustrate, accident year 2003 has a closure ratio of 89.45%. The LDF from 89.45% to 99.00% is 1.165. This is calculated as .9825x(.8945)^(-1.5244)=1.165 by inserting the closure ratio of 89.45% into the regression equation from Exhibit 8. For accident year 2004 the "advanced" closure ratio is 80.00% since we have already "advanced" the 56.69% closure ratio to 80.00% closed when we apply the 1.609 LDF. To illustrate, accident year 2004 now has a closure ratio of 80.00%. The LDF from 80.00% to 99.00% is 1.381. This is calculated as .9825x(.8000)^(-1.5244)=1.381 by inserting the closure ratio of 80.00% into the regression equation from Exhibit 8. Accident years 2001 and prior have an LDF that defaults to 1.000 since the closure ratio is already greater than 99%.

Column (4) is the tail LDF that depends on how far "advanced" the closure ratio is. For accident years 2001 and prior the "advanced" closure ratio is the original closure ratio from column (1) since these years are greater than 99% closed. For accident years 2001 and prior the tail LDF is taken from the tail LDF used in the Adjusted Paid Loss Development Method since there has been nothing learned from regression analysis for these closure ratios greater than 99%. Accident years 2002 through 2004 have an "advanced" closure ratio of 99%. The tail LDF from 99% closed to ultimate is 1.016. This factor was iteratively derived. The final iteration is shown in Exhibit 4 with final ultimate net paid loss & ALAE figures. The ultimate net paid loss & ALAE divided by the expected cumulative net paid loss & ALAE at 99% closed provides each of the seven estimates for the 99% closed to ultimate tail. The selection was made judgmentally considering these past seven 99% closure to 100% closure tails. A separate regression of the 99% closure to 100% closure could be made as well to improve upon the tail estimate.

Column (5) is the cumulative LDF [column (2) x column (3) x column (4)]. The cumulative LDF represents the net paid loss & ALAE development from the closure ratio shown

in column (1) to 100% closed.

Column (6) is the cumulative net paid loss & ALAE for the accident year taken directly from Exhibit 2.

Column (7) is the resulting ultimate net paid loss & ALAE for the accident year. [column (6) x column (5)].

Exhibit 10 is included to illustrate a projection on current year at age 12.

4. Results and Discussion

Analyzing the relationship of closure ratio and LDF as well as the relationship of age and LDF will provide visual evidence for the expected improvement offered by a closure-based analysis. We have experienced tremendous improvement in fits when we replace age with closure. The improved fit has been experienced across many companies and includes non-standard personal automobile, standard personal automobile and commercial automobile lines. The visual and statistical evidence produced by these regressions will give the actuary confidence when pursuing an adjusted paid loss approach.

Both the Closure-Based Regression Method and the Adjusted Paid Loss Development Method show significant improvement over the traditional age-based Paid Loss Development Method when claim settlement rates are changing. As expected, the Closure-Based Regression Method and the Adjusted Paid Loss Development Method generally move in the same direction with each new evaluation.

When closure ratios are fairly consistent in time a traditional paid loss development method that relies on statistics from this relatively consistent period in its age to age LDF analysis may produce reasonable results. Even in this case, however, a consistent closure ratio may only occur for a few years. Reliance on such thin data will certainly cause the actuary to be very reactive and lose the credibility provided in larger sets of data. We have, however, experienced large shifts in closure ratio over time with a resulting large inconsistency exhibited by the traditional Paid Loss Development Method.

The Adjusted Paid Loss Method transforms an age-based triangle into a closure-based triangle by use of interpolation across closure ratios. It is also a chain ladder approach. Both of these mechanics add error to projections. However, the Adjusted Paid Loss Method has advantages over the Closure-Based Regression Method. An analysis of historical triangle data can reveal information regarding the impact of various internal and external factors along with

points in time where their influence was heavier or lighter. The actuary can then judgmentally select LDF's that reflect this knowledge. In addition, the Closure-Based Regression Method does not use current accident years in fitting the regressions while the Adjusted Paid Loss Method works from the closure ratios exhibited in the last diagonal of the triangle. It is recommended that both methods be utilized and information from both be used in the selection of ultimates.

5. Conclusions

The Closure-Based Regression Method is a simple but powerful and visually compelling method. Retrospectively, it has proven to be very accurate for accident periods where exposures are fully earned. The fit of the development curves makes it clear as to why the method has proven to be so accurate in retrospective tests. The pure analysis of the relationship between claim closure level and LDF produces a very accurate paid loss development curve.

It is important to understand that while the Closure Method fit may be strong this does not preclude the curve from shifting in time. A traditional Berquist-Sherman approach should be used in conjunction with this method. Continual regression updates should be made with new data as well. Analysis of shifts in the curve by accident year should be undertaken. This being said, we believe that the Closure Method provides a powerful tool for the actuary to more confidently select ultimates as well as explain the effects of changing settlement rates to management.

There is little that can be done to leverage paid loss further given the high r^2 and the principle of parsimony. Although the closure fit is nearly perfect in some cases at an r^2 of .99+ it still is a method that leverages paid losses. LDF's, therefore, are still high in the early closure periods and small errors leverage to larger changes in ultimate loss. Regression methods that incorporate case reserves are an obvious consideration. Regressions involving incurred losses and closure ratios have shown a deterioration in r^2, however. It is believed that incurred losses represent a mixed-bag of information. The paid loss portion is obviously highly correlated with settlement rates but the case reserve portion will have a relationship to the age of the accident period.

No attempt has been made to date to test the method more universally beyond the personal automobile and commercial automobile lines. It can be stated that homogeneous, high frequency, low severity and low limits lines will perform best. The resulting closure ratio to LDF fits will be stronger. These lines will also have less dependence on case reserves and less case reserve level volatility.

6. References

Berquist, James R., and Sherman, Richard E.,"Loss Reserve Adequacy Testing: A Comprehensive Systematic Approach," Proceedings of the Casualty Actuarial Society Casualty Actuarial Society - Arlington,Virginia1977:LXVII 123-184

Closure Ratio
CWA / (CWA + Open)

Exhibit 1

	3	6	9	12	15	18	21	24	27	30	33	36	39
1993	4.01%	12.87%	21.80%	34.78%	54.43%	71.19%	82.61%	88.70%	92.80%	94.74%	96.35%	97.09%	97.76%
1994	9.25%	19.78%	29.38%	39.54%	53.67%	68.76%	81.04%	87.26%	89.96%	92.38%	94.69%	96.25%	97.02%
1995	7.45%	16.27%	26.07%	30.13%	41.74%	50.24%	60.50%	73.06%	81.30%	86.09%	90.49%	93.32%	95.82%
1996	3.47%	7.30%	13.13%	21.88%	35.33%	51.63%	66.61%	78.45%	86.11%	90.10%	92.85%	95.29%	96.41%
1997	5.99%	9.87%	17.96%	27.70%	41.43%	56.38%	67.95%	77.10%	82.63%	87.11%	91.31%	93.88%	95.88%
1998	6.74%	13.53%	17.86%	24.50%	35.65%	48.91%	61.68%	72.63%	79.11%	85.29%	89.01%	92.48%	95.11%
1999	4.47%	7.56%	14.45%	21.95%	32.50%	44.99%	56.79%	67.07%	74.56%	81.91%	88.25%	93.25%	96.58%
2000	5.91%	9.70%	14.88%	21.14%	32.33%	46.15%	61.84%	77.22%	86.10%	91.31%	94.51%	96.51%	97.84%
2001	5.17%	9.98%	16.92%	27.77%	40.94%	56.01%	69.56%	80.55%	87.07%	91.86%	94.45%	96.30%	97.48%
2002	5.64%	12.07%	18.41%	26.46%	39.10%	53.16%	65.78%	74.93%	82.28%	87.45%	91.37%	94.07%	95.71%
2003	7.84%	14.01%	21.53%	30.79%	45.57%	60.12%	71.03%	79.95%	85.24%	89.45%			
2004	6.68%	13.01%	21.34%	31.33%	43.26%	56.69%							
2005	6.41%	14.80%											

Closure Ratio
CWA / (CWA + Open)

	42	45	48	51	54	57	60	63	66	69	72	75	78
1993	98.15%	98.50%	99.15%	99.44%	99.51%	99.64%	99.77%	99.84%	99.84%	99.87%	99.90%	99.93%	99.93%
1994	97.85%	98.12%	98.75%	99.16%	99.53%	99.59%	99.69%	99.76%	99.83%	99.86%	99.97%	99.93%	99.97%
1995	97.59%	97.86%	98.16%	99.02%	99.28%	99.58%	99.66%	99.81%	99.81%	99.81%	99.81%	99.81%	99.85%
1996	97.31%	98.16%	98.62%	99.17%	99.20%	99.35%	99.45%	99.57%	99.66%	99.85%	99.97%	99.97%	99.97%
1997	97.33%	98.10%	98.33%	98.62%	99.15%	99.31%	99.57%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
1998	96.67%	97.98%	98.83%	99.37%	99.62%	99.64%	99.70%	99.72%	99.76%	99.94%	99.91%	99.97%	99.97%
1999	98.16%	99.03%	99.33%	99.59%	99.66%	99.70%	99.76%	99.80%	99.81%	99.86%	99.91%	99.93%	99.95%
2000	98.55%	99.06%	99.16%	99.51%	99.58%	99.65%	99.82%	99.83%	99.86%				
2001	98.27%	98.76%	99.25%	99.33%	99.56%								
2002	96.75%												

Closure Ratio
CWA / (CWA + Open)

	81	84	87	90	93	96	99	102	105	108	111	114	117
1993	99.97%	99.97%	99.97%	99.97%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
1994	99.97%	99.97%	99.97%	99.97%	99.97%	99.97%	99.97%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
1995	99.92%	99.92%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
1996	99.97%	99.97%	99.94%	99.97%	99.97%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	
1997	100.00%	99.98%	99.98%	100.00%	100.00%	100.00%	100.00%	100.00%					
1998	99.98%	99.98%	99.98%	99.98%									

Closure Ratio
CWA / (CWA + Open)

	120	123	126	129	132	135	138	141	144	147	150
1993	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%
1994	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%	100.00%				
1995	100.00%	100.00%	100.00%								

Net Paid Loss & ALAE

Exhibit 2

	3	6	9	12	15	18	21	24	27	30
1993	23,850	440,003	1,751,097	4,116,136	8,007,513	11,831,534	14,278,225	15,964,509	16,972,591	17,611,193
1994	91,158	864,423	2,311,886	5,194,806	8,138,088	11,220,809	13,671,712	15,040,857	15,717,248	16,392,829
1995	50,309	659,033	2,048,652	3,575,265	5,255,877	6,593,127	8,383,804	10,599,555	12,270,046	13,496,678
1996	8,759	166,574	802,108	2,395,691	4,695,914	7,698,792	10,443,855	13,009,514	15,069,204	16,353,709
1997	45,212	523,236	2,174,441	5,399,948	9,905,231	14,650,839	18,434,660	21,862,315	24,265,740	26,549,256
1998	72,687	541,156	1,655,582	4,395,460	8,315,417	13,052,565	18,438,977	23,245,442	26,921,907	30,469,043
1999	51,284	620,722	2,788,247	6,315,290	11,032,613	16,376,044	22,027,130	27,600,893	32,175,666	37,193,603
2000	49,795	510,115	1,879,673	4,394,381	8,213,158	13,818,971	20,718,279	28,087,482	33,234,257	37,530,775
2001	51,761	428,182	1,943,984	5,249,365	10,140,817	16,544,171	21,805,864	26,100,833	29,055,668	31,570,067
2002	71,883	622,682	2,480,476	5,631,654	10,285,303	15,869,288	21,187,513	25,809,388	29,432,258	32,323,392
2003	84,082	660,018	2,143,074	6,018,885	10,645,835	15,437,555	19,464,779	23,335,756	25,477,508	27,340,774
2004	104,833	752,085	2,643,954	5,872,516	9,367,849	13,256,219				
2005	56,502	621,000								

Net Paid Loss & ALAE

	33	36	39	42	45	48	51	54	57	60
1993	18,145,935	18,412,185	18,599,225	18,741,034	18,834,504	18,915,326	18,967,459	18,996,352	19,050,533	19,078,453
1994	16,952,098	17,564,927	17,867,592	18,114,938	18,249,237	18,346,070	18,405,621	18,422,544	18,425,305	18,456,108
1995	14,572,453	15,308,523	15,844,818	16,247,001	16,361,430	16,450,178	16,552,265	16,639,047	16,729,845	16,740,362
1996	17,207,743	17,887,704	18,276,301	18,501,618	18,698,400	18,745,079	18,841,748	18,897,751	18,940,734	18,987,524
1997	28,654,286	29,809,650	30,568,954	31,196,263	31,625,567	31,833,820	31,975,727	32,129,382	32,274,646	32,357,515
1998	33,014,258	34,859,098	36,209,913	37,021,153	37,616,331	38,092,555	38,387,968	38,524,498	38,566,044	38,597,300
1999	41,381,319	44,479,311	46,746,502	47,740,544	48,220,503	48,539,328	48,748,788	48,833,489	48,954,043	49,024,243
2000	39,510,184	40,848,765	41,818,055	42,340,356	42,648,087	42,824,268	42,954,016	43,054,964	43,102,420	43,193,467
2001	33,007,076	33,981,768	34,549,553	35,104,880	35,384,769	35,607,655	35,716,493	35,783,290		
2002	34,517,041	36,105,569	36,897,480	37,396,259						

Net Paid Loss & ALAE

	63	66	69	72	75	78	81	84	87	90
1993	19,082,369	19,089,861	19,107,056	19,115,565	19,117,441	19,120,137	19,131,581	19,134,472	19,135,588	19,136,313
1994	18,463,166	18,465,864	18,469,158	18,499,305	18,498,019	18,501,301	18,501,301	18,501,301	18,501,301	18,501,301
1995	16,771,895	16,780,070	16,778,792	16,775,710	16,776,170	16,775,406	16,783,796	16,785,812	16,798,273	16,798,490
1996	19,004,406	19,045,673	19,066,056	19,089,768	19,092,758	19,093,003	19,091,813	19,092,168	19,093,407	19,101,396
1997	32,410,993	32,423,724	32,427,203	32,426,483	32,427,056	32,429,432	32,429,494	32,429,758	32,428,019	32,428,319
1998	38,612,777	38,640,427	38,683,117	38,690,362	38,698,432	38,699,871	38,699,996	38,703,929	38,704,824	38,704,779
1999	49,069,529	49,087,172	49,110,490	49,135,304	49,159,716	49,192,079				

2000	43,209,409	43,227,069

Net Paid Loss & ALAE

	93	96	99	102	105	108	111	114	117	120
1993	19,144,923	19,145,048	19,145,048	19,145,048	19,127,253	19,127,253	19,127,253	19,127,253	19,127,253	19,127,253
1994	18,501,301	18,501,892	18,511,745	18,516,810	18,516,810	18,516,810	18,516,810	18,516,810	18,516,810	18,516,810
1995	16,798,495	16,798,495	16,798,495	16,798,495	16,798,495	16,798,495	16,798,495	16,798,495	16,798,495	16,798,495
1996	19,102,220	19,119,323	19,119,505	19,119,505	19,119,505	19,119,505	19,119,505	19,119,479		
1997	32,428,319	32,428,319	32,428,319	32,428,319						

Net Paid Loss & ALAE

	123	126	129	132	135	138	141	144	147	150
1993	19,127,253	19,127,253	19,127,253	19,127,253	19,127,253	19,127,253	19,099,702	19,099,702	19,099,702	19,099,702
1994	18,516,810	18,516,810	18,516,810	18,516,810	18,516,810	18,516,810				
1995	16,799,150	16,799,150								

Estimated Net Paid Loss & ALAE Exhibit 3

										80%
1993	71.19%	82.61%	11,831,534	14,278,225	80.00%	0.088	0.114	77.2%	22.8%	13,719,250
1994	68.76%	81.04%	11,220,809	13,671,712	80.00%	0.112	0.123	91.6%	8.4%	13,464,777
1995	73.06%	81.30%	10,599,555	12,270,046	80.00%	0.069	0.082	84.2%	15.8%	12,006,002
1996	78.45%	86.11%	13,009,514	15,069,204	80.00%	0.016	0.077	20.3%	79.7%	13,427,078
1997	77.10%	82.63%	21,862,315	24,265,740	80.00%	0.029	0.055	52.4%	47.6%	23,121,585
1998	79.11%	85.29%	26,921,907	30,469,043	80.00%	0.009	0.062	14.4%	85.6%	27,434,387
1999	74.56%	81.91%	32,175,666	37,193,603	80.00%	0.054	0.073	74.1%	25.9%	35,891,725
2000	77.22%	86.10%	28,087,482	33,234,257	80.00%	0.028	0.089	31.3%	68.7%	29,698,193
2001	69.56%	80.55%	21,805,864	26,100,833	80.00%	0.104	0.110	95.0%	5.0%	25,886,454
2002	74.93%	82.28%	25,809,388	29,432,258	80.00%	0.051	0.074	69.0%	31.0%	28,309,584
2003	79.95%	85.24%	23,335,756	25,477,508	80.00%	0.000	0.053	0.9%	99.1%	23,354,111

Estimated Net Paid Loss & ALAE

Exhibit 4

											99%	Ultimate	99% to ult tail
1993	98.50%	99.15%	18,834,504	18,915,326	99.00%	0.005	0.006	76.9%	23.1%	18,896,622	19,099,702	1.011
1994	98.75%	99.16%	18,346,070	18,405,621	99.00%	0.002	0.004	61.3%	38.7%	18,382,576	18,516,810	1.007
1995	98.16%	99.02%	16,450,178	16,552,265	99.00%	0.008	0.009	98.2%	1.8%	16,550,423	16,799,150	1.015
1996	98.62%	99.17%	18,745,079	18,841,748	99.00%	0.004	0.006	69.5%	30.5%	18,812,221	19,119,709	1.016
1997	98.62%	99.15%	31,975,727	32,129,382	99.00%	0.004	0.005	71.3%	28.7%	32,085,267	32,428,694	1.011
1998	98.83%	99.37%	38,092,555	38,387,968	99.00%	0.002	0.005	31.0%	69.0%	38,184,180	38,715,487	1.014
1999	98.16%	99.03%	47,740,544	48,220,503	99.00%	0.008	0.009	96.2%	3.8%	48,202,331	49,212,336	1.021
2000	98.55%	99.06%	42,340,356	42,648,087	99.00%	0.005	0.005	87.5%	12.5%	42,609,579		
2001	98.76%	99.25%	35,384,769	35,607,655	99.00%	0.002	0.005	49.8%	50.2%	35,495,854		
											selected tail		1.016

Net Paid Loss & ALAE
Closure Point to 80% Closed LDF

Exhibit 5

	3	6	9	12	15	18	21	24	27	30
1993	575.231	31.180	7.835	3.333	1.713	1.160	N/A	N/A	N/A	N/A
1994	147.708	15.577	5.824	2.592	1.655	1.200	N/A	N/A	N/A	N/A
1995	238.645	18.218	5.860	3.358	2.284	1.821	1.432	1.133	N/A	N/A
1996	1,532.946	80.607	16.740	5.605	2.859	1.744	1.286	1.032	N/A	N/A
1997	511.404	44.190	10.633	4.282	2.334	1.578	1.254	1.058	N/A	N/A
1998	377.432	50.696	16.571	6.242	3.299	2.102	1.488	1.180	1.019	N/A
1999	699.862	57.823	12.873	5.683	3.253	2.192	1.629	1.300	1.115	N/A
2000	596.409	58.219	15.800	6.758	3.616	2.149	1.433	1.057	N/A	N/A
2001	500.115	60.457	13.316	4.931	2.553	1.565	1.187	N/A	N/A	N/A
2002	393.829	45.464	11.413	5.027	2.752	1.784	1.336	1.097	N/A	N/A
2003	277.754	35.384	10.897	3.880	2.194	1.513	1.200	1.001	N/A	N/A

Net Paid Loss & ALAE Closure Point to 99% Closed LDF Exhibit 6

	3	6	9	12	15	18	21	24	27
1993	792.311	42.947	10.791	4.591	2.360	1.597	1.323	1.184	1.113
1994	201.656	21.266	7.951	3.539	2.259	1.638	1.345	1.222	1.170
1995	328.975	25.113	8.079	4.629	3.149	2.510	1.974	1.561	1.349
1996	2,147.759	112.936	23.453	7.853	4.006	2.444	1.801	1.446	1.248
1997	709.663	61.321	14.756	5.942	3.239	2.190	1.740	1.468	1.322
1998	525.323	70.560	23.064	8.687	4.592	2.925	2.071	1.643	1.418
1999	939.910	77.655	17.288	7.633	4.369	2.943	2.188	1.746	1.498
2000	855.700	83.529	22.669	9.696	5.188	3.083	2.057	1.517	1.282
2001	685.764	82.899	18.259	6.762	3.500	2.146	1.628	1.360	1.222

	30	33	36	39	42	45	48	51	54
1993	1.073	1.041	1.026	1.016	1.008	1.003	N/A	N/A	N/A
1994	1.121	1.084	1.047	1.029	1.015	1.007	1.002	N/A	N/A
1995	1.226	1.136	1.081	1.045	1.019	1.012	1.006	N/A	N/A
1996	1.150	1.093	1.052	1.029	1.017	1.006	1.004	N/A	N/A
1997	1.209	1.120	1.076	1.050	1.028	1.015	1.008	1.003	N/A
1998	1.253	1.157	1.095	1.055	1.031	1.015	1.002	N/A	N/A
1999	1.296	1.165	1.084	1.031	1.010	N/A	N/A	N/A	N/A
2000	1.135	1.078	1.043	1.019	1.006	N/A	N/A	N/A	N/A
2001	1.124	1.075	1.045	1.027	1.011	1.003	N/A	N/A	N/A

Regression Analysis

Exhibit 7

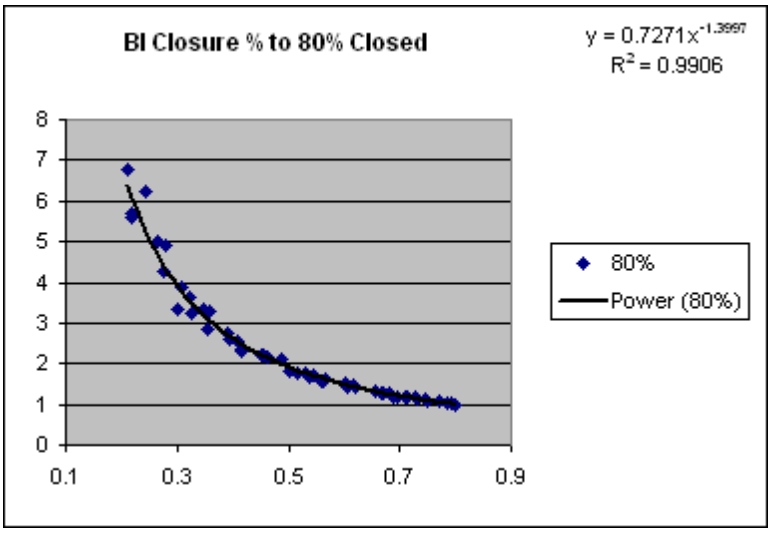
% Closed to 80% Regression		80% to 99% Regression	
Closure Ratio	LDF	Closure Ratio	LDF
21.14%	6.758	80.5%	1.360
21.88%	5.605	81.0%	1.345
21.95%	5.683	81.3%	1.349
24.50%	6.242	81.9%	1.296
26.46%	5.027	82.6%	1.323
27.70%	4.282	82.6%	1.322
27.77%	4.931	85.3%	1.253
30.13%	3.358	86.1%	1.226
30.79%	3.880	86.1%	1.282
32.33%	3.616	86.1%	1.248
32.50%	3.253	87.1%	1.222
34.78%	3.333	87.1%	1.209
35.33%	2.859	87.3%	1.222
35.65%	3.299	88.3%	1.165
39.10%	2.752	88.7%	1.184
39.54%	2.592	89.0%	1.157
40.94%	2.553	90.0%	1.170
41.43%	2.334	90.1%	1.150
41.74%	2.284	90.5%	1.136
44.99%	2.192	91.3%	1.135
45.57%	2.194	91.3%	1.120
46.15%	2.149	91.9%	1.124
48.91%	2.102	92.4%	1.121
50.24%	1.821	92.5%	1.095
51.63%	1.744	92.8%	1.113
53.16%	1.784	92.9%	1.093

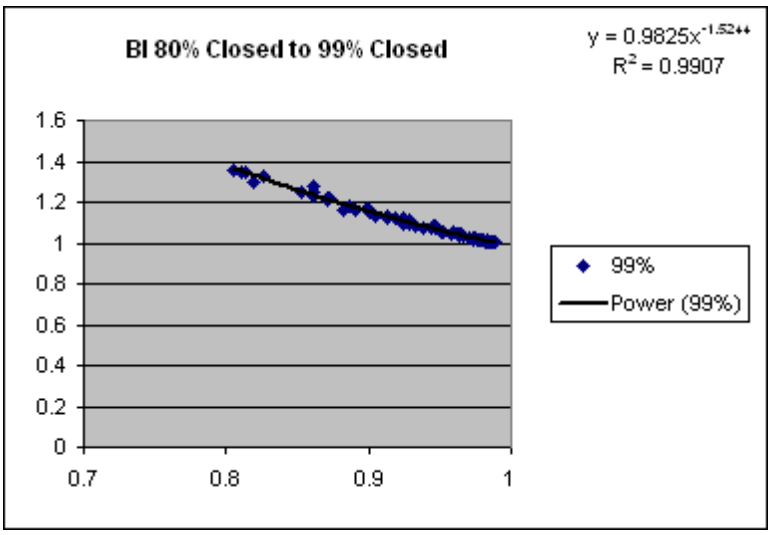
% Closed to 80% Regression	
Closure Ratio	LDF
53.67%	1.655
54.43%	1.713
56.01%	1.565
56.38%	1.578
56.79%	1.629
60.12%	1.513
60.50%	1.432
61.68%	1.488
61.84%	1.433
65.78%	1.336
66.61%	1.286
67.07%	1.300
67.95%	1.254
68.76%	1.200
69.56%	1.187
71.03%	1.200
71.19%	1.160
72.63%	1.180
73.06%	1.133
74.56%	1.115
74.93%	1.097
77.10%	1.058
77.22%	1.057
78.45%	1.032
79.11%	1.019
79.95%	1.001

80% to 99% Regression	
Closure Ratio	LDF
93.2%	1.084
93.3%	1.081
93.9%	1.076
94.4%	1.075
94.5%	1.078
94.7%	1.084
94.7%	1.073
95.1%	1.055
95.3%	1.052
95.8%	1.045
95.9%	1.050
96.3%	1.047
96.3%	1.045
96.3%	1.041
96.4%	1.029
96.5%	1.043
96.6%	1.031
96.7%	1.031
97.0%	1.029
97.1%	1.026
97.3%	1.017
97.3%	1.028
97.5%	1.027
97.6%	1.019
97.8%	1.016
97.8%	1.019

80% to 99% Regression	
Closure Ratio	LDF
97.9%	1.015
97.9%	1.012
98.0%	1.015
98.1%	1.015
98.1%	1.007
98.1%	1.008
98.2%	1.006
98.2%	1.010
98.2%	1.006
98.3%	1.011
98.3%	1.008
98.5%	1.003
98.5%	1.006
98.6%	1.004
98.8%	1.002
98.8%	1.003
98.8%	1.002

Regression Analysis **Exhibit 8**





Bodily Injury
Closure-Based Regression Method
As Of 6/30/05

Exhibit 9

Accident Year	Closure Ratio	Indicated LDF From Point To 80% Closed	Indicated LDF To 99% Closed	Tail	LDF	Cumulative Net Paid Loss & ALAE	Ultimate Net Paid Loss & ALAE
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1993	100.00%	1.000	1.000	1.000	1.000	19,099,702	19,099,702
1994	100.00%	1.000	1.000	1.000	1.000	18,516,810	18,516,810
1995	100.00%	1.000	1.000	1.000	1.000	16,799,150	16,799,150
1996	100.00%	1.000	1.000	1.000	1.000	19,119,479	19,119,709
1997	100.00%	1.000	1.000	1.000	1.000	32,428,319	32,428,694
1998	99.98%	1.000	1.000	1.000	1.000	38,704,779	38,715,487
1999	99.95%	1.000	1.000	1.000	1.000	49,192,079	49,212,336
2000	99.86%	1.000	1.000	1.002	1.002	43,227,069	43,325,885
2001	99.56%	1.000	1.000	1.007	1.007	35,783,290	36,031,280
2002	96.75%	1.000	1.033	1.016	1.050	37,396,259	39,260,406
2003	89.45%	1.000	1.165	1.016	1.183	27,340,774	32,350,451
2004	56.69%	1.609	1.381	1.016	2.257	13,256,219	29,920,994
2005	14.80%	N/A	N/A	N/A	N/A	N/A	N/A

Bodily Injury
Closure-Based Regression Method
As Of 12/31/04

Exhibit 10

Calendar Accident Year	Closure Ratio	Indicated LDF From Point To 80% Closed	Indicated LDF To 99% Closed	Tail	LDF	Cumulative Net Paid Loss & ALAE	Ultimate Net Paid Loss & ALAE
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
1993	100.00%	1.000	1.000	1.000	1.000	19,099,702	19,099,702
1994	100.00%	1.000	1.000	1.000	1.000	18,516,810	18,516,810
1995	100.00%	1.000	1.000	1.000	1.000	16,798,495	16,798,495
1996	100.00%	1.000	1.000	1.000	1.000	19,119,505	19,119,505
1997	100.00%	1.000	1.000	1.000	1.000	32,428,319	32,437,317
1998	99.98%	1.000	1.000	1.001	1.001	38,703,929	38,736,432
1999	99.91%	1.000	1.000	1.001	1.001	49,135,304	49,187,057
2000	99.82%	1.000	1.000	1.003	1.003	43,193,467	43,337,380
2001	99.25%	1.000	1.000	1.014	1.014	35,607,655	36,101,300
2002	94.07%	1.000	1.078	1.014	1.094	36,105,569	39,482,369
2003	79.95%	1.000	1.381	1.014	1.400	23,335,756	32,667,960
2004	31.33%	3.686	1.381	1.014	5.159	5,872,516	30,298,806

Appendix A
Exhibit 1

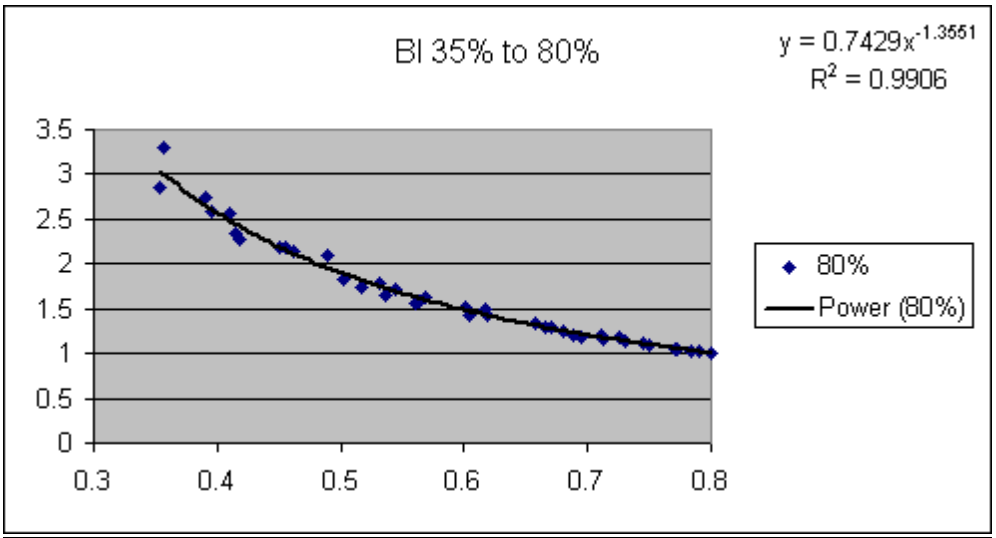
r^2 By Coverage

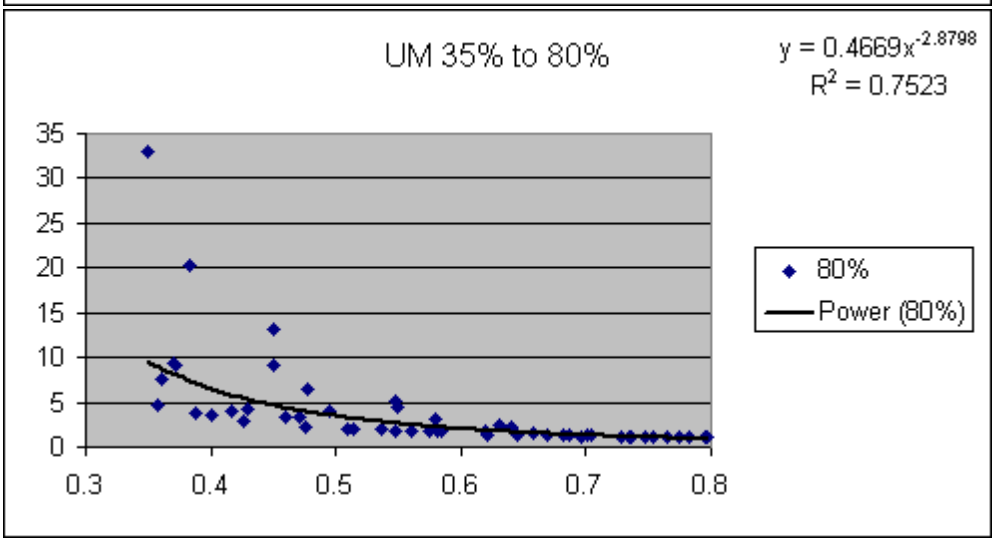
Using a single curve to 99% closure

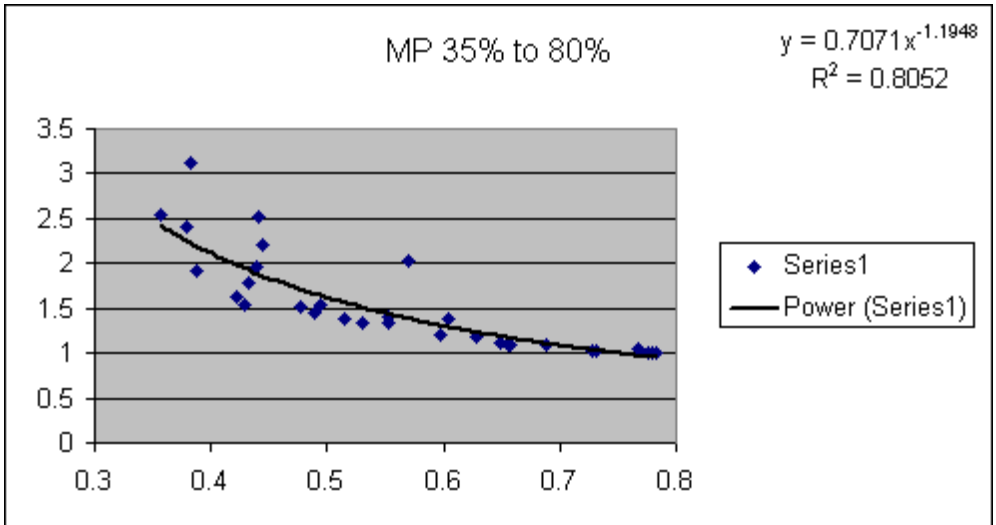
BI	UM	MP	PD
0.9952	0.9798	0.9207	0.9775

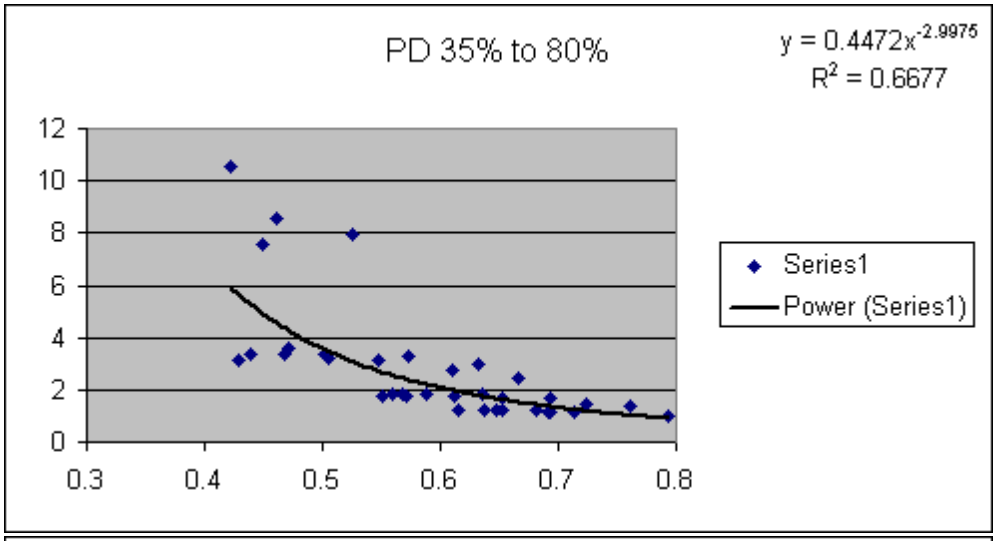
Appendix A
Exhibit 2

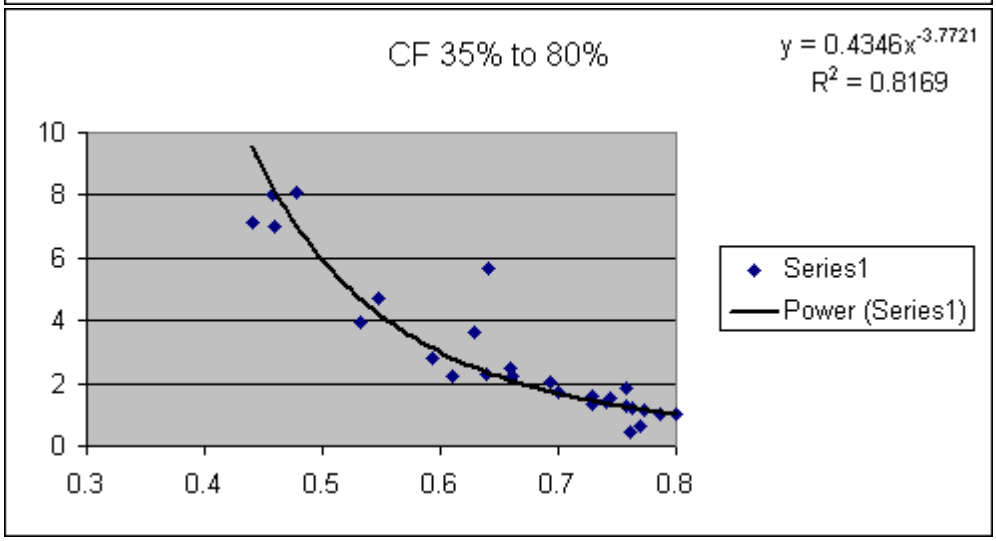
Simple Regression 35% to 80% Closure

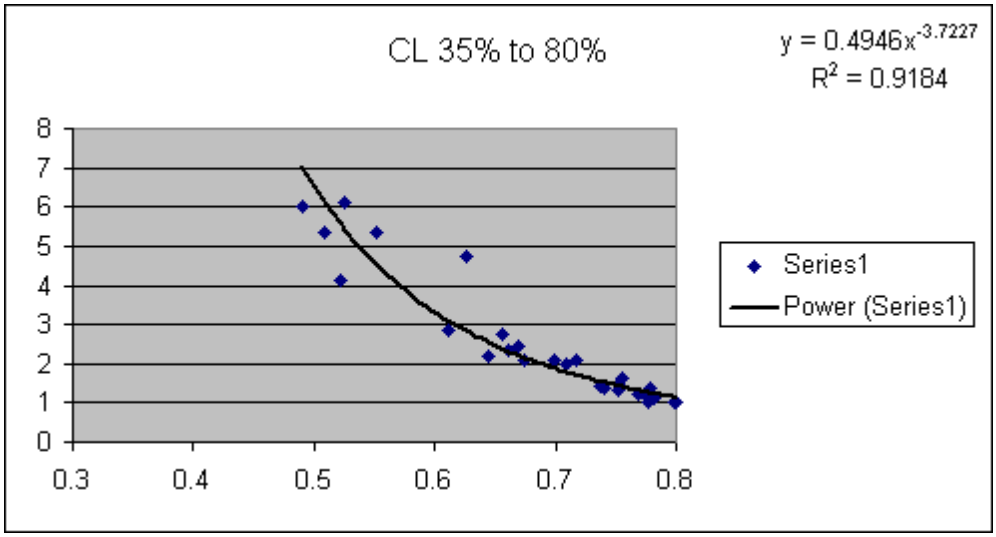




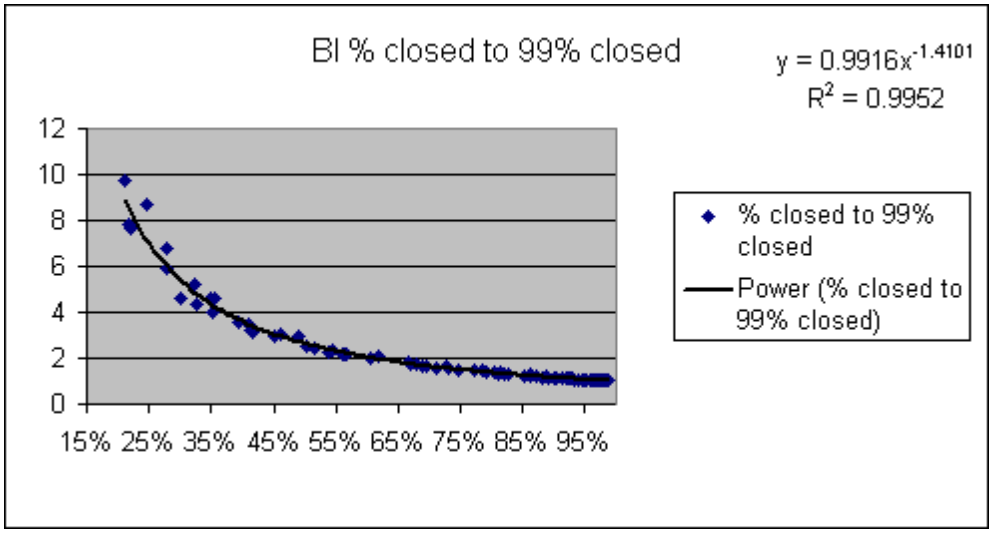


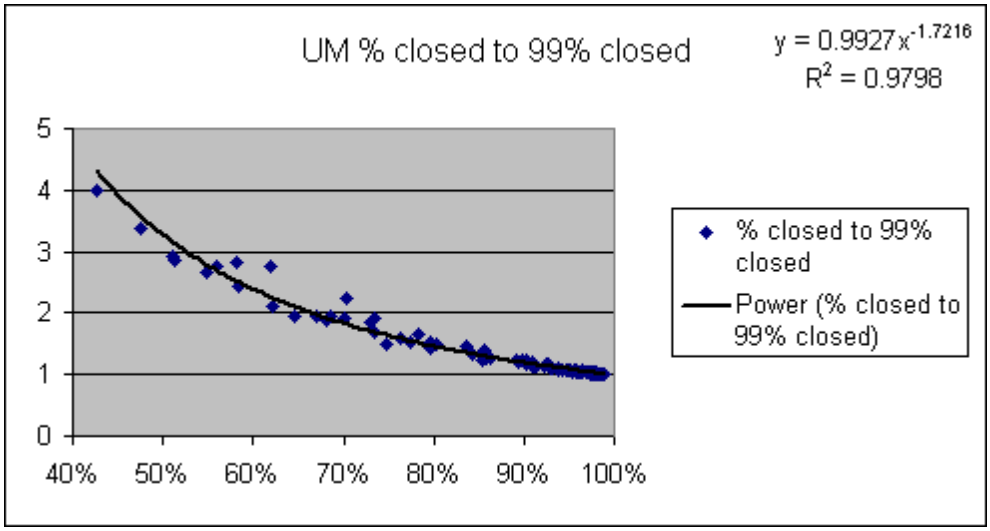

PD 35% to 80%
y = 0.4472x-2.9975
R2 = 0.6677
12
10
8
6
4
2
0
0.3
0.4
0.5
0.6
0.7
0.8
Series1
Power (Series1)

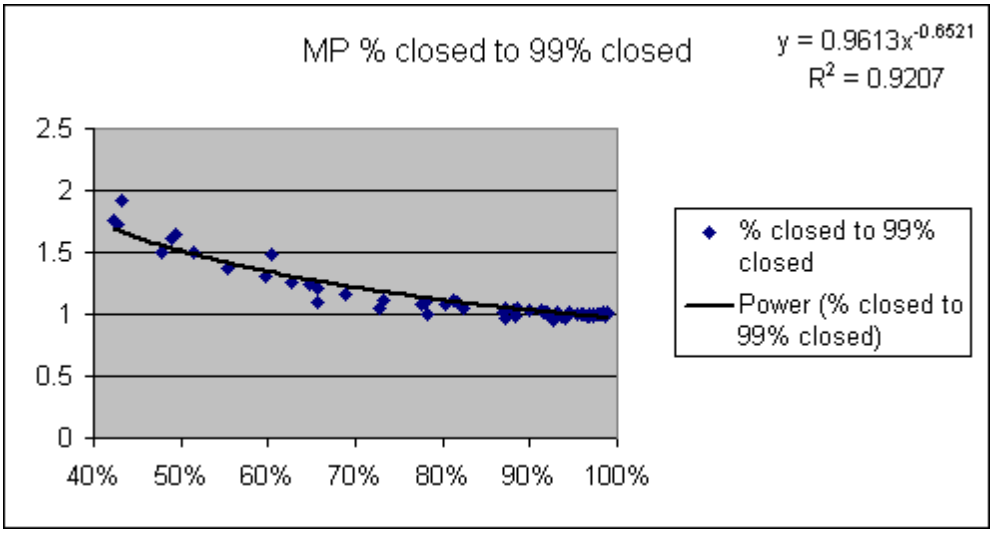

CF 35% to 80%
y = 0.4346x-3.7721
R2 = 0.8169
10
8
6
4
2
0
0.3
0.4
0.5
0.6
0.7
0.8
Series1
Power (Series1)

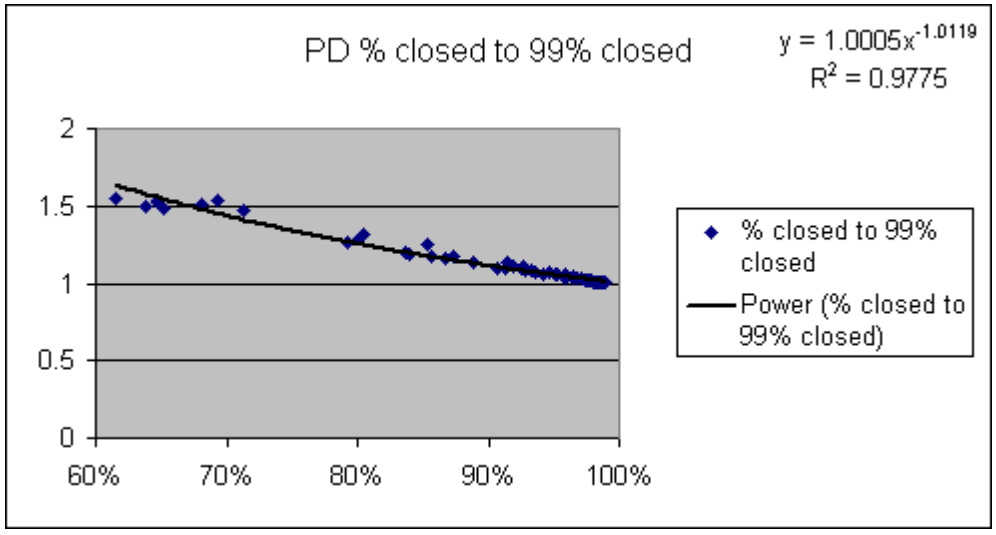

CL 35% to 80%
y = 0.4946x-3.7227
R2 = 0.9184
8
7
6
5
4
3
2
1
0
0.3
0.4
0.5
0.6
0.7
0.8
Series1
Power (Series1)

Appendix A
Exhibit 3
Simple Regression % Closed to 99% Closure
Using Age 12 or Greater






MP % closed to 99% closed
y = 0.9613x^-0.6521
R² = 0.9207
2.5
2
1.5
1
0.5
0
40%
50%
60%
70%
80%
90%
100%
% closed to 99% closed
Power (% closed to 99% closed)


PD % closed to 99% closed
y = 1.0005x^-1.0119
R² = 0.9775
2
1.5
1
0.5
0
60%
70%
80%
90%
100%
% closed to 99% closed
Power (% closed to 99% closed)