Multiple LOBs, Segments and Layers

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More information will be available at

CABINET ROOM

6:30pm-11:30pm

Thursday September 18th

Insureware
Software Solutions and eConsulting for P&C Insurance

MULTIPLE TRIANGLE MODELLING (or MPTF)

APPLICATIONS

- MULTIPLE LINES OF BUSINESS- DIVERSIFICATION?
- MULTIPLE SEGMENTS
  - MEDICAL VERSUS INDEMNITY
  - SAME LINE, DIFFERENT STATES
  - GROSS VERSUS NET OF REINSURANCE
  - LAYERS INCLUDING HIGH SEVERITY/LOW FREQUENCY
- CREDIBILITY MODELLING
  - ONLY A FEW YEARS OF DATA AVAILABLE
  - HIGH PROCESS VARIABILITY LEADS TO IMPRECISE ESTIMATES OF TRENDS

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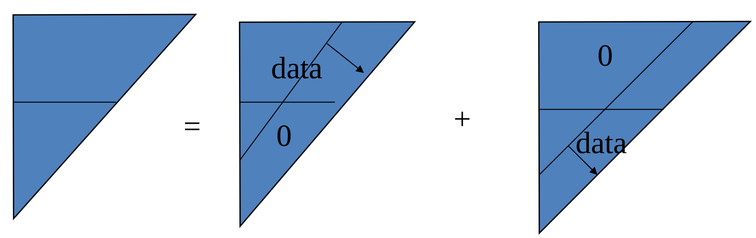
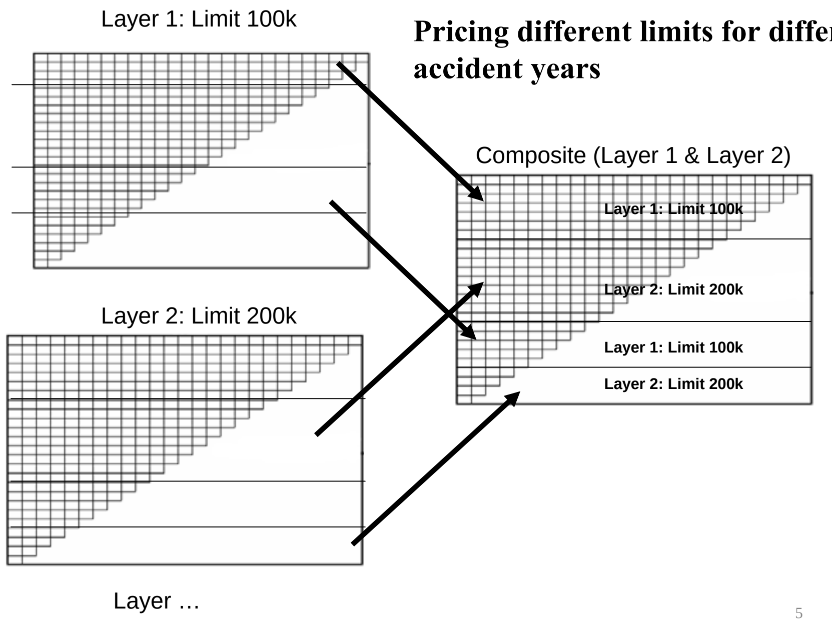
Breaking up a triangle due to change in mix of risks



1. Change of mix of business
2. Different development and/or inflation
3. Different process variability

Pricing different limits for different accident years



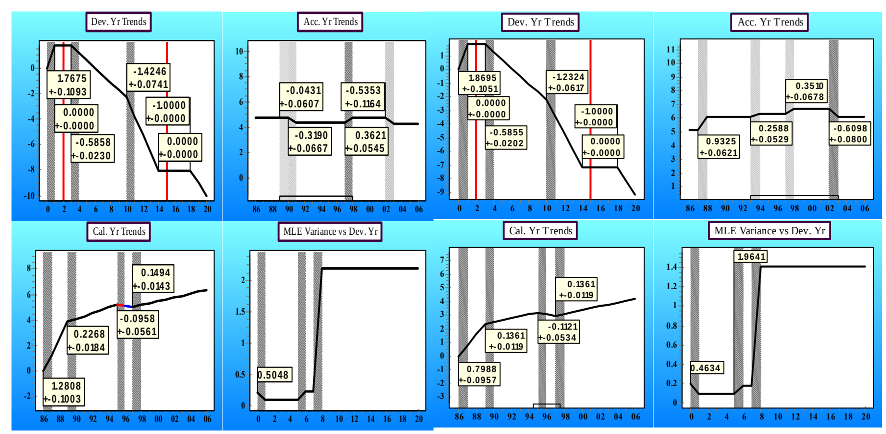
BENEFITS

- **One composite model for all LOBs!**
- Level of Diversification- optimal risk capital allocation by LOB and calendar year
- Reserve and underwriting risk charge
- Combined reserve and underwriting risk charge is not additive
- **No two companies are the same in respect of volatility and correlations**

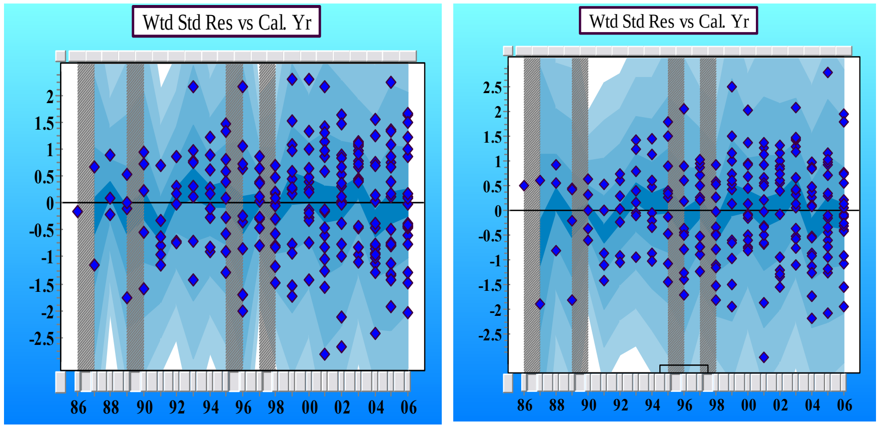
Mack and Bootstrap

- You cannot measure process correlation unless the model captures the trend structure in the data (correctly)
- Mack induces spurious correlations
- In respect of risk charges (Economic Capital) it is the calendar year relationships and the calendar year liability stream that are important

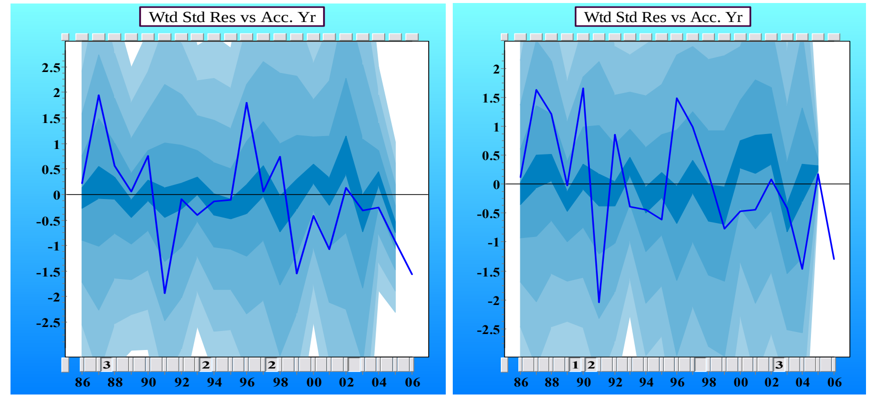
When do two LOBs (LOB A & LOB B) have common drivers?
TWO LOBs "same" trend structure



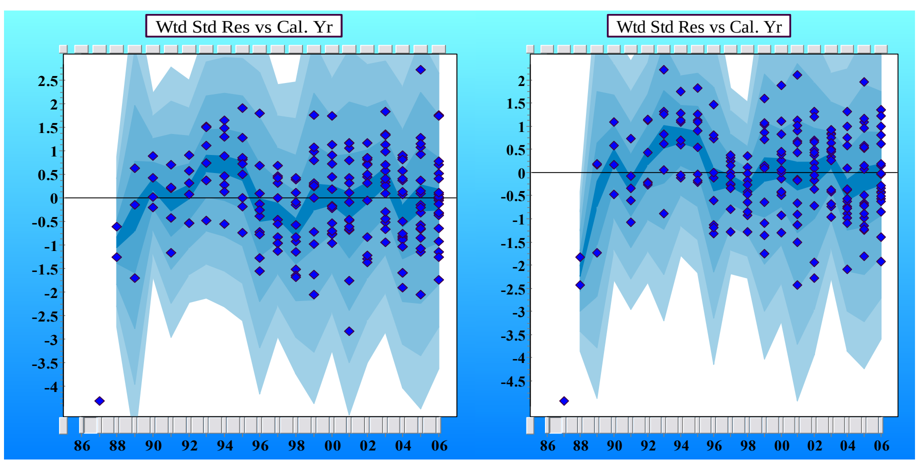
When do two LOBs (LOB A & LOB B) have common drivers?
TWO LOBs "same" trend structure and high process correlation



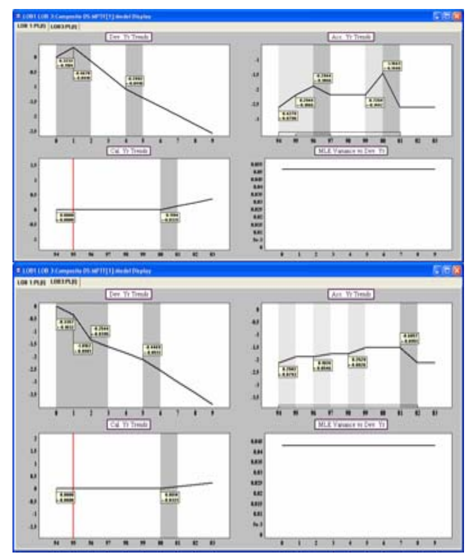
When do two LOBs (LOB A & LOB B) have common drivers?
TWO LOBs "same" trend structure and high process correlation
Trace of calendar year 2006 versus accident years.
Note high process correlation (of 0.85).



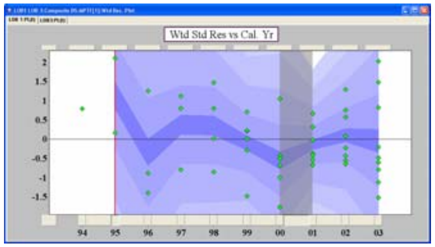
When do two LOBs (LOB A & LOB B) have common drivers?
TWO LOBs "same" trend structure and high process correlation
(Data adjusted for development and accident period trends only)

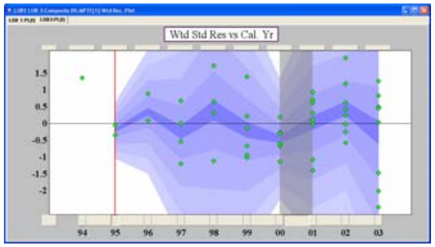


Model Displays for LOB1 and LOB3
Calendar year trend change in 01 for each LOB



LOB1 and LOB3 Weighted Residual Plots for Calendar Year. Note some process correlation of 0.35





Pictures shown above correspond to two linear models, which described by the following equations

$$\begin{aligned} \mathbf{y}_1 &= \mathbf{X}_1\boldsymbol{\beta}_1 + \boldsymbol{\varepsilon}_1, \\ \mathbf{y}_2 &= \mathbf{X}_2\boldsymbol{\beta}_2 + \boldsymbol{\varepsilon}_2, \end{aligned} \qquad (1)$$

$$E\boldsymbol{\varepsilon}_i = \mathbf{0},\ \ i = 1, 2;\ \ E(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2^{\mathrm{T}}) = \mathrm{cov}(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2) = \mathbf{C};\quad corr(\boldsymbol{\varepsilon}_1, \boldsymbol{\varepsilon}_2) = \mathbf{R}$$

Without loss of sense and generality two models in (1) could be considered as one linear model:

$$\begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_2 \end{pmatrix} \begin{pmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{pmatrix} + \begin{pmatrix} \boldsymbol{\varepsilon}_1 \\ \boldsymbol{\varepsilon}_2 \end{pmatrix} \qquad (2)$$

Which could be rewritten as

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

For illustration of the most simple case we suppose that size of vectors *y* in models (1) are the same and equal to *n*, also we suppose that

$$E(\boldsymbol{\varepsilon}_i, \boldsymbol{\varepsilon}_i^{\mathrm{T}}) = \mathrm{var}(\boldsymbol{\varepsilon}_i) = \mathbf{I}_n \sigma_i^2,\ \ i = 1, 2; \qquad \mathbf{C} = \mathbf{I}_n \sigma_{12}$$

In this case

$$\mathrm{var}(\boldsymbol{\varepsilon}) = \boldsymbol{\Sigma} = \begin{pmatrix} \mathbf{I}_n\sigma_1^2 & \mathbf{I}_n\sigma_{12} \\ \mathbf{I}_n\sigma_{12} & \mathbf{I}_n\sigma_2^2 \end{pmatrix}$$

For example, when *n* = 3

$$\boldsymbol{\Sigma} = \begin{pmatrix} \sigma_1^2 & 0 & 0 & \sigma_{12} & 0 & 0 \\ 0 & \sigma_1^2 & 0 & 0 & \sigma_{12} & 0 \\ 0 & 0 & \sigma_1^2 & 0 & 0 & \sigma_{12} \\ \sigma_{12} & 0 & 0 & \sigma_2^2 & 0 & 0 \\ 0 & \sigma_{12} & 0 & 0 & \sigma_2^2 & 0 \\ 0 & 0 & \sigma_{12} & 0 & 0 & \sigma_2^2 \end{pmatrix}$$

There is a big difference between linear models in (1) and linear model (2), as in (1) we consider models separately and could not use additional information, from dependency of these models, what we can do in model (2). To extract this additional information we need to use proper methods to estimate vector of parameters $\boldsymbol{\beta}$. The estimation

$$\hat{\boldsymbol{\beta}} = (\mathbf{X}^{\mathbf{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{T}}\mathbf{y}$$

which derived by ordinary least square (OLS) method, does not provide any advantage, as covariance matrix $\boldsymbol{\Sigma}$ is not participating in estimations. Only general least square (GLS) estimation

$$\tilde{\boldsymbol{\beta}} = (\mathbf{X}^{\mathbf{T}}\boldsymbol{\Sigma}^{-1}\mathbf{X})^{-1}\mathbf{X}^{\mathbf{T}}\boldsymbol{\Sigma}^{-1}\mathbf{y}$$

could help to achieve better results.

However, it is necessary immediately to underline that we do not know elements of the matrix $\boldsymbol{\Sigma}$ and we have to estimate them as well. So, practically, we should build iterative process of estimations

$$\tilde{\boldsymbol{\beta}}^{(m)}, \ \tilde{\boldsymbol{\Sigma}}^{(m)}$$

and this process will stop, when we reach estimations with satisfactory statistical properties.

There are some cases, when model (2) provides the same results as models in (1). They are:

1. Design matrices in (1) have the same structure (they are the same or proportional to each other).
2. Models in (1) are non-correlated, another words

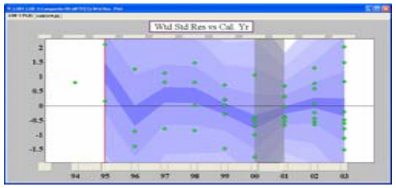
$$\sigma_{12} = 0$$

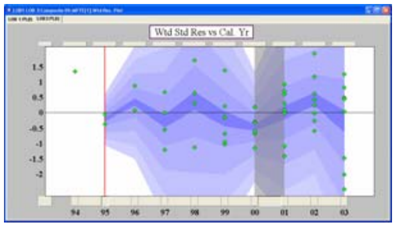
However in situation when two models in (1) have common regressors model (2) again will have advantages in spite of the same structure of design matrices.

Clusters of LOBs

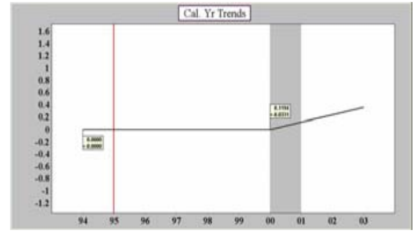
- For 40 LOBs there are 780 pair wise correlations
- Set up clusters
- Zero correlations between clusters

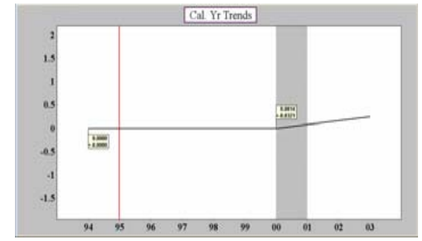
Weighted Residual Plots for LOB1 and LOB3 versus Calendar Years
What does correlation mean? (Regression of one set of residuals against the other)

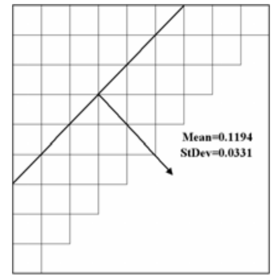


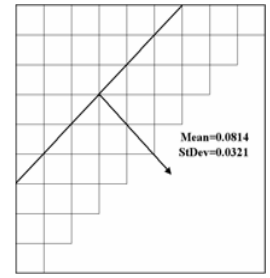


Model Displays for LOB1 and LOB3 for Calendar Years









Model for individual iota parameters

$$\hat{\iota}_1 \sim N(\mu_1, \quad \sigma_1^2); \qquad \hat{\mu}_1 = 0.1194; \quad \hat{\sigma}_1 = 0.0331$$
$$\hat{\iota}_2 \sim N(\mu_2, \quad \sigma_2^2); \qquad \hat{\mu}_2 = 0.0814; \quad \hat{\sigma}_2 = 0.0321$$

$$\begin{pmatrix} \iota_1 \\ \iota_2 \end{pmatrix} \sim N(\boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad \hat{\boldsymbol{\mu}} = \begin{pmatrix} 0.1194 \\ 0.0814 \end{pmatrix}, \quad \hat{\boldsymbol{\Sigma}} = \begin{pmatrix} 0.001097 & 0.000344 \\ 0.000344 & 0.001027 \end{pmatrix}$$

$$\rho = corr(\iota_1, \iota_2), \qquad \hat{\rho} = 0.359013$$

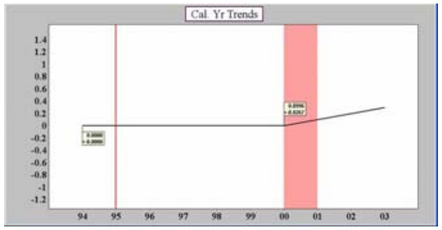
There are two types of correlations involved in calculations of reserve distributions.

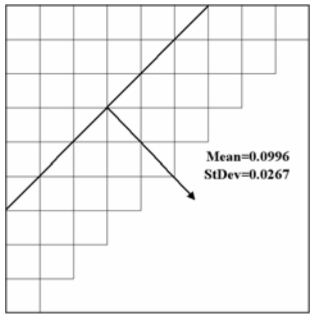
Weighted Residual Correlations, that is process correlation between datasets:
0.359013 – is weighted residual correlation between datasets LOB1 and LOB3;

Correlations in parameter estimates:
0.324188 – is correlation between iota parameters in LOB1 and LOB3.

These two types of correlations induce correlations between triangle cells and within triangle cells. These induce reserve correlations between accident years, calendar years and aggregates.

Common iota parameter in both triangles





$$\iota \sim N(\mu, \quad \sigma^2); \quad \hat{\mu} = 0.0996; \quad \hat{\sigma} = 0.0267$$

Two effects:

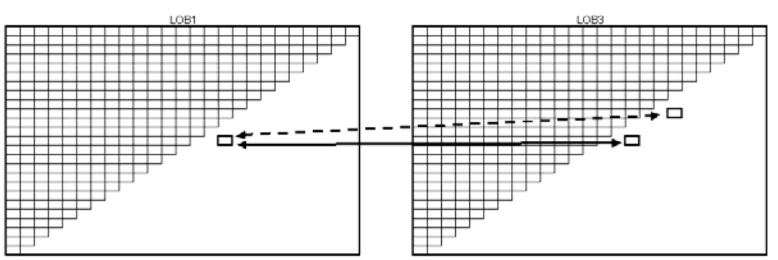
Same parameter for each LOB increases correlations and CV of aggregates

Single parameter with lower CV reduces CV of aggregates

Forecasted reserve distributions by accident year, calendar year and total are correlated

⟷ Indicates dependency through residuals' and parameters' correlations

⇠ ⇢ Indicates dependency through parameter estimate correlations only



Dependency of aggregates in aggregate table

In each forecast cell and in aggregates by accident year and calendar year (and total)

Var(Aggregate) >> *Var*(LOB1) + *Var*(LOB2).

Correlation between reserve distributions is 0.833812

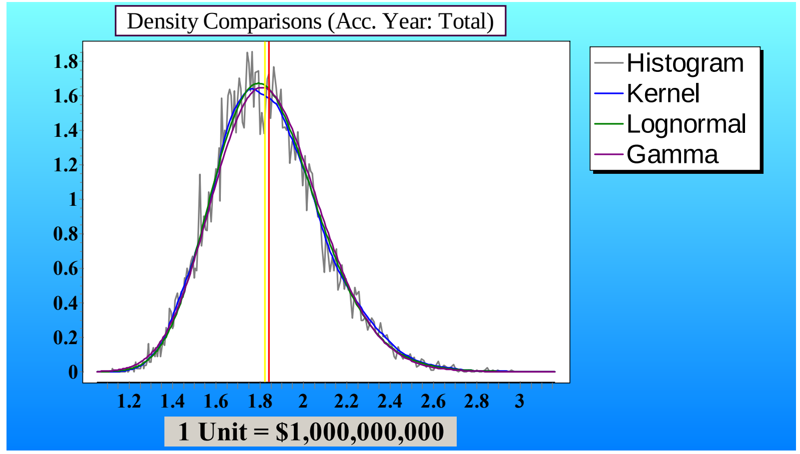
Forecast tables- means and standard deviations

Aggregate | LOB 1:PL(I) | LOB3:PL(I)

Accident Period vs Development P

	Cal. Per. Total	0	1	2	3	4	
1999	556,988	189,228	159,750	78,696	60,749	47,388	3
	528,693	185,925	145,365	79,106	69,145	58,323	
2000	589,958	210,160	200,323	100,868	77,243	59,794	5
	563,241	182,549	177,216	121,784	110,365	12,139	1
2001	664,716	214,273	184,441	83,467	66,778	53,839	4
	583,994	157,342	159,261	77,836	13,890	11,695	1
2002	650,118	169,078	157,737	77,789	60,160	47,013	3
	652,459	149,481	158,285	16,266	13,044	10,698	
2003	745,017	271,726	255,924	127,389	98,226	76,543	6
	655,937	152,377	55,024	28,026	22,533	18,488	1
	Total Fitted/Paid		2004	2005	2006	2007	
Cal. Per.	5,182,400		577,384	373,171	281,102	209,809	15
Total	5,035,182		70,955	47,090	40,275	34,059	2

Simulations from lognormals correlated within LOB and between LOBs to find distribution of aggregate of LOB 1 and LOB 3



Percentiles and V@R for aggregate based on explicit assumptions. Can also compute by calendar year and accident year

LOB 1 LOB 3:Composite DS:MPTF[optimal-1]:Reserve PALD Summary

All Statistics | Acc Year: Total | Simulated Values | Quantile Summary

Diagnostics | Distributions | Quantiles and V-a-R

Quantile Statistics and Value at R

%	Sample			Kernel		
	Quantile	# S.D.'s	V-a-R	Quantile	# S.D.'s	V-a-R
99.995	2.848	4.509	1.010	2.925	4.856	1.088
99.99	2.767	4.148	0.929	2.855	4.543	1.018
99.98	2.766	4.145	0.929	2.781	4.214	0.944
99.97	2.714	3.914	0.877	2.746	4.055	0.909
99.96	2.693	3.818	0.855	2.721	3.945	0.884
99.95	2.668	3.710	0.831	2.702	3.859	0.865
99.94	2.662	3.682	0.825	2.686	3.789	0.849
99.93	2.653	3.641	0.816	2.673	3.730	0.836
99.92	2.642	3.591	0.805	2.661	3.678	0.824
99.91	2.638	3.574	0.801	2.651	3.633	0.814
99.9	2.622	3.503	0.785	2.642	3.592	0.805
99.8	2.562	3.236	0.725	2.581	3.319	0.744
99.7	2.531	3.098	0.694	2.544	3.153	0.706

Risk Capital Allocation by LOB based on variance covariance formulae

LOB 1 LOB 3:Composite DS:MPTF[optimal-1]:Reserve Forecast Summaries

Aggregate | LOB 1:PL(I) | LOB3:PL(I)

Acc. Yrs | Cal. Yrs | Observed vs Mean Estimate | Los
(%) Differences | Comparisons | Summary Graphs | Corre

By Datasets | By Periods

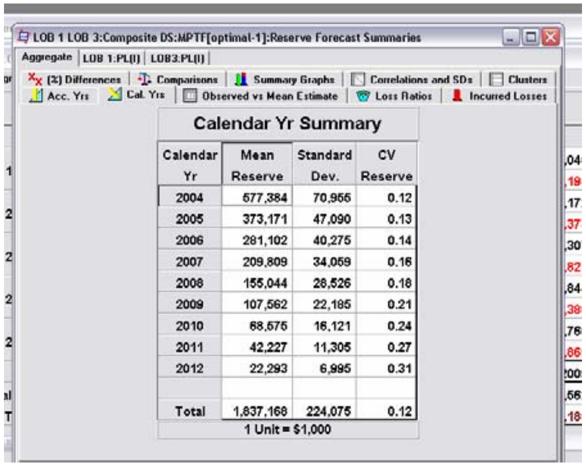
Correlations | Standard Deviations | Correlations (logs) | Capital Allocation

Totals
Acc. Yrs
Cal. Yrs

Capital Allocation (Totals)

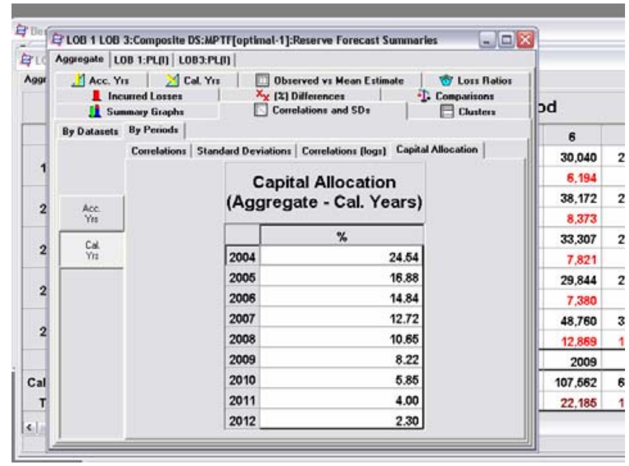
	%
LOB 1:PL(I)	37.63
LOB3:PL(I)	62.37

Calendar year liability stream based on explicit assumptions



Calendar Yr	Mean Reserve	Standard Dev.	CV Reserve
2004	677,384	70,955	0.12
2005	373,171	47,090	0.13
2006	281,102	40,275	0.14
2007	209,809	34,059	0.16
2008	155,044	28,526	0.18
2009	107,562	22,185	0.21
2010	68,675	16,121	0.24
2011	42,227	11,305	0.27
2012	22,293	6,995	0.31
Total	1,837,168	224,075	0.12

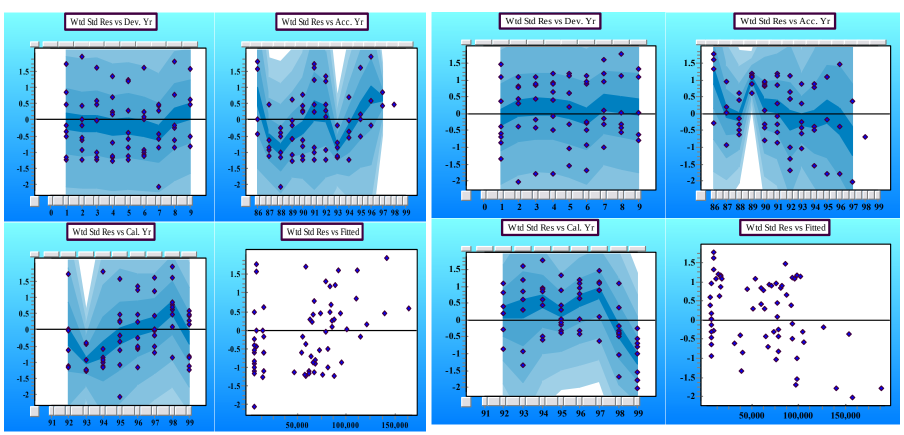
Risk Capital allocation by calendar year for the aggregate reserves of LOB1 and LOB3



	%
2004	24.54
2005	16.88
2006	14.84
2007	12.72
2008	10.65
2009	8.22
2010	5.85
2011	4.00
2012	2.30

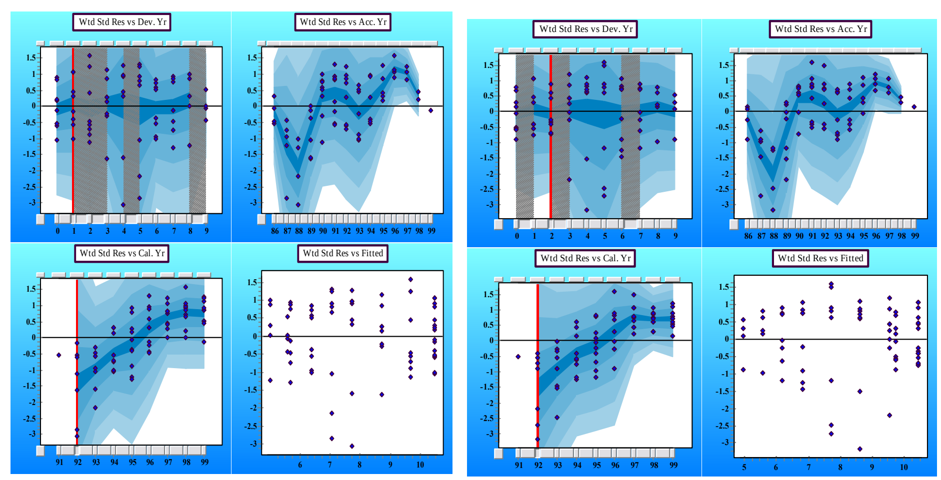
SAD and SAM Two WC segments

Mack residuals for each. Note remaining structure and have no idea how these two segments maybe related

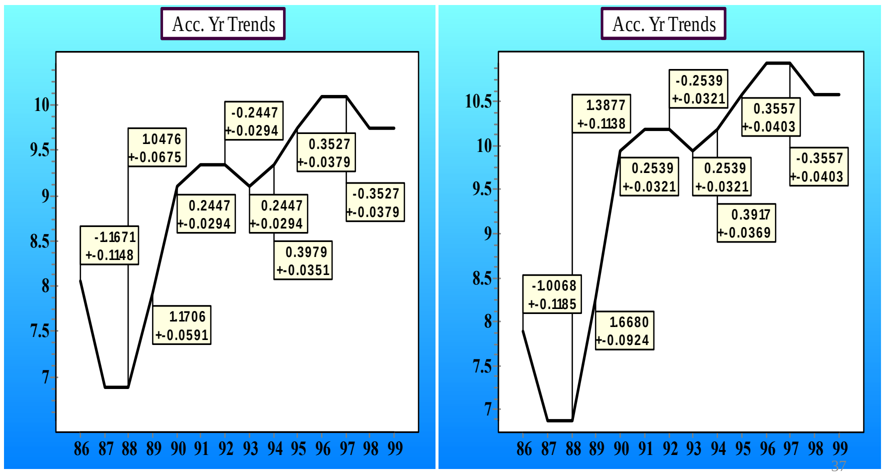


SAD and SAM- WC segments (PTF)

Removal of Development Yr trends only. Note accident year and calendar year similarities in trend structure. Have an immediate idea of how the segments maybe related



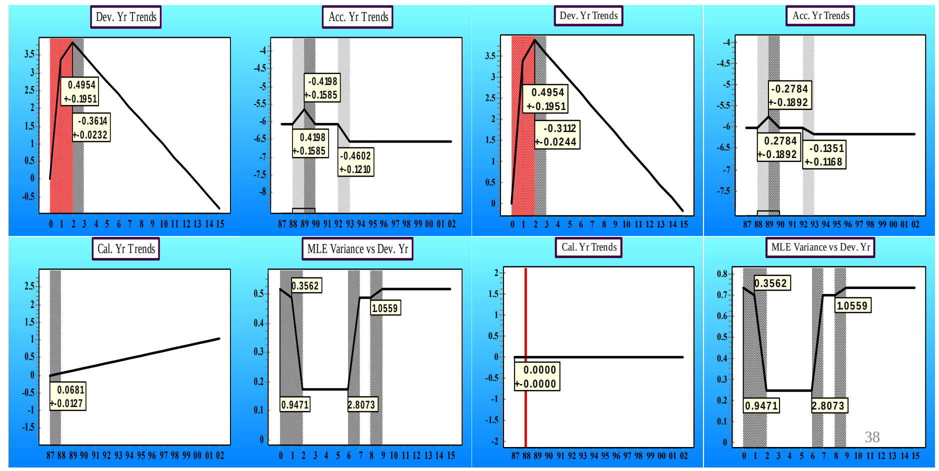
SAD and SAM- WC segments (PTF)
Strong similarities in accident year trend structure



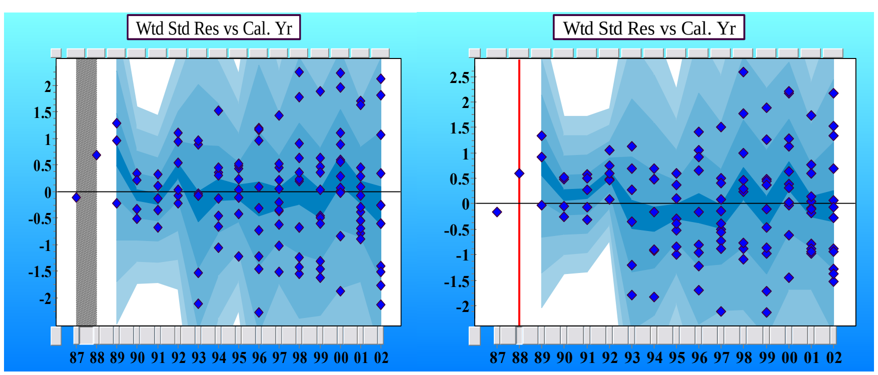
Gross versus Net of Reinsurance

Is your outward reinsurance program optimal?

Model Displays- same trend structure. Have common drivers. Net zero calendar year trend and gross 6.81%+_1.27%. Net has higher process variance!



Gross and Net of Reinsurance
Weighted Residuals versus Calendar year. Note high process correlations (common drivers)



Weighted Residual Covariances Between Datasets

	FAC ENG G:Paid Incremental	FAC ENG N:Paid Incremental
FAC ENG G:Paid Incremental	0.174511	0.174520
FAC ENG N:Paid Incremental	0.174520	0.248382

Weighted Residual Correlations Between Datasets

	FAC ENG G:Paid Incremental	FAC ENG N:Paid Incremental
FAC ENG G:Paid Incremental	1	0.838250
FAC ENG N:Paid Incremental	0.838250	1

Note CV of gross reserves < CV of net reserves
Because Net data has higher process variance!!

Comp N & G:Composite DS:MPTF[optimal-2]:Reserve Forecast S...

Aggregate | FAC ENG G:Paid Incremental | FAC ENG N:Paid Incremental

(%) Differences | Comparisons | Summary Graphs | Forecast Settings | Correlations and SDs | Acc. Yrs | Cal. Yrs | Observed vs Mean Estimate | Loss Ratios

Accident Yr Summary

Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.	CV Reserve	CV Ultimate
1992	904	15,839	394	0.44	0.02
1993	562	8,104	234	0.42	0.03
1994	433	3,596	170	0.39	0.05
1995	616	3,662	228	0.37	0.06
1996	883	4,962	313	0.35	0.06
1997	3,176	11,604	945	0.30	0.08
1998	4,434	10,642	1,140	0.26	0.11
1999	6,746	17,310	1,554	0.23	0.09
2000	25,186	35,132	5,397	0.21	0.15
2001	57,933	60,632	11,968	0.21	0.20
2002	0	0	0	0.22	0.22
Total	101,705	216,641	15,450	0.15	0.07

Comp N & G:Composite DS:MPTF[optimal-1]:Reserve

Aggregate | FAC ENG G:Paid Incremental | FAC ENG N:Paid

(%) Differences | Summary Graphs | Forecast Settings | Acc. Yrs | Cal. Yrs | Observed vs Mean Esti

Accident Yr Summary

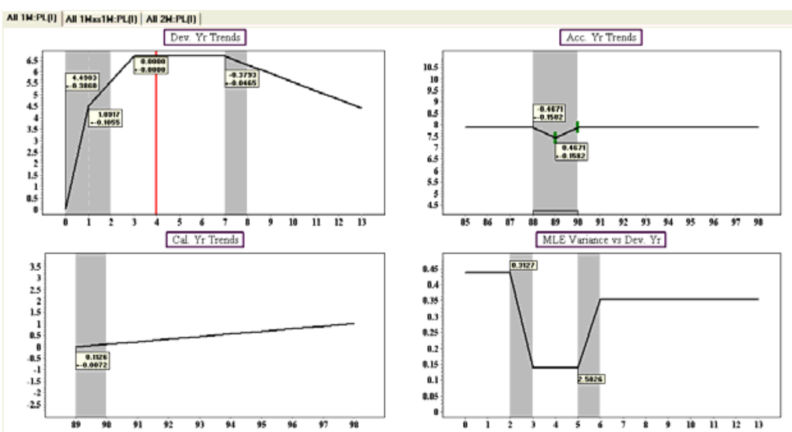
Acc. Yr	Mean Reserve	Mean Ultimate	Standard Dev.
1992	550	10,791	303
1993	448	5,782	237
1994	327	3,159	164
1995	440	2,159	208
1996	597	4,353	270
1997	1,999	6,517	750
1998	2,610	7,400	832
1999	3,728	11,084	1,041
2000	13,109	19,452	3,321
2001	28,464	29,820	6,776
2002	0	0	0
Total	52,821	134,323	8,725

MODELING LAYERS

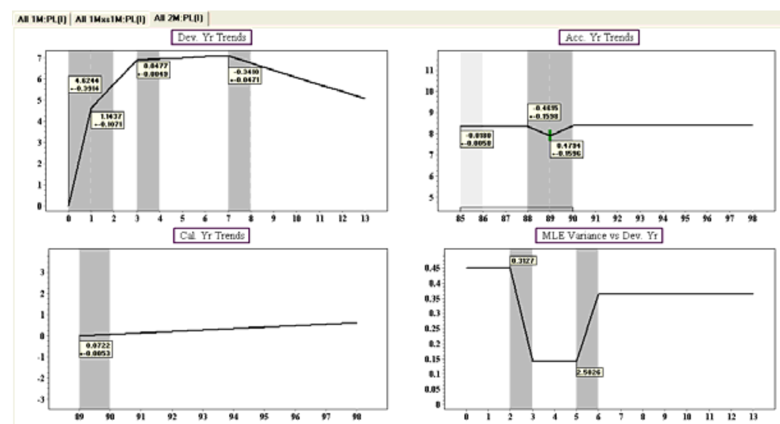
- Similar Structure
- Highly Correlated
- Surprise Finding!

CV of reserves limited to $1M is the same as CV of reserves limited to $2M !

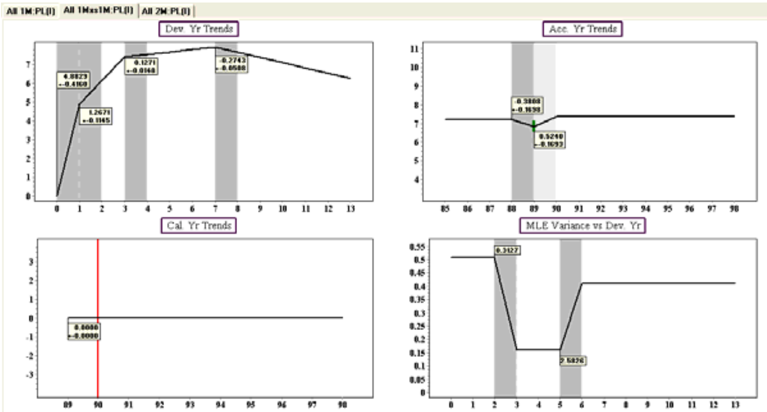
Model Display for All 1M: PL(I)



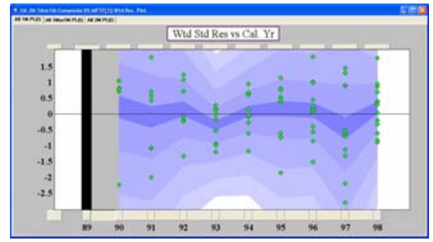
Model Display for All 2M: PL(I)

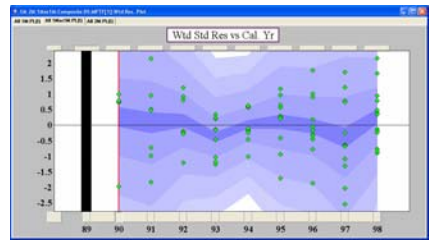


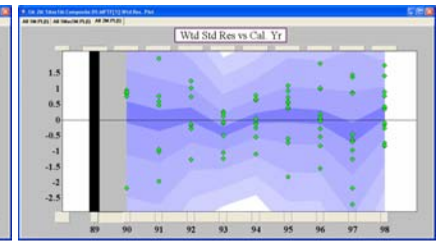
Model Display for All 1Mxs1M: PL(I)



Note that All 1Mxs1M has zero inflation, and All 2M has lower inflation than All 1M, and distributions of parameters going forward are correlated







Residual displays vs calendar years show high correlations between three triangles

.

Compare Accident Year Summary

Aggregate | All 2M:PL(I)

Comparisons | Summary Graphs | Correlations and SDs | Clusters
Acc. Years | Cal. Years | Loss Ratios | (%) Errors

Accident Year Summary

Acc. Year	Mean		Standard	CV	
	Reserve	Ultimate	Dev.	Reserve	Ultimate
1985	0	48,940	0	****	0.00
1986	1,731	46,283	1,274	0.74	0.03
1987	4,268	57,251	2,312	0.54	0.04
1988	7,944	56,959	3,592	0.45	0.06
1989	8,439	49,038	3,616	0.43	0.07
1990	21,169	70,392	7,486	0.35	0.11
1991	32,145	74,080	10,359	0.32	0.14
1992	47,762	89,314	14,284	0.30	0.16
1993	63,937	90,534	17,645	0.28	0.19
1994	79,474	97,967	19,623	0.25	0.20
1995	95,679	109,426	21,712	0.23	0.20
1996	112,626	118,155	23,938	0.21	0.20
1997	124,732	127,021	26,080	0.21	0.21
1998	135,287	135,753	28,250	0.21	0.21
Total	735,192	1,171,112	84,443	0.11	0.07

1 Unit = $1,000

Aggregate | All 1M:PL(I) | All 1Mxs1M:PL(I)

Comparisons | Summary Graphs | Correlations and SDs | Clusters
Acc. Years | Cal. Years | Loss Ratios | (%) Errors

Accident Year Summary

Acc. Year	Mean		Standard	CV	
	Reserve	Ultimate	Dev.	Reserve	Ultimate
1985	0	48,940	0	****	0.00
1986	1,755	46,307	1,319	0.75	0.03
1987	4,255	57,238	2,347	0.55	0.04
1988	7,812	56,827	3,585	0.46	0.06
1989	8,498	49,097	3,692	0.43	0.08
1990	21,373	70,596	7,661	0.36	0.11
1991	32,176	74,111	10,484	0.33	0.14
1992	47,539	89,092	14,342	0.30	0.16
1993	63,459	90,056	17,622	0.28	0.20
1994	78,862	97,356	19,542	0.25	0.20
1995	95,235	108,983	21,639	0.23	0.20
1996	112,800	118,329	23,962	0.21	0.20
1997	125,981	128,270	26,308	0.21	0.21
1998	138,137	138,603	28,812	0.21	0.21
Total	737,883	1,173,804	84,854	0.11	0.07

1 Unit = $1,000

Consistent forecasts based on composite model.

If we compare forecast by accident year for Limited 1M and limited 2M t is easy to see that CV is the same.

Aggregate | All 1M:PL(I)

Comparisons | Summary Graphs | Correlations and SDs | Clusters
Acc. Years | Cal. Years | Loss Ratios | (%) Errors

Accident Year Summary

Acc. Year	Mean		Standard	CV	
	Reserve	Ultimate	Dev.	Reserve	Ultimate
1985	0	22,453,799	0	****	0.00
1986	828,038	22,089,949	599,031	0.72	0.03
1987	2,124,068	28,497,418	1,131,771	0.53	0.04
1988	4,112,417	29,396,971	1,829,332	0.44	0.06
1989	4,516,936	26,252,822	1,906,865	0.42	0.07
1990	11,639,942	39,028,980	4,053,276	0.35	0.10
1991	18,378,282	43,010,289	5,836,036	0.32	0.14
1992	28,391,320	53,745,298	8,373,832	0.29	0.16
1993	39,601,636	56,981,763	10,802,788	0.27	0.19
1994	51,349,621	63,560,581	12,561,474	0.24	0.20
1995	64,503,904	73,624,625	14,547,167	0.23	0.20
1996	79,233,602	83,196,467	16,801,628	0.21	0.20
1997	91,459,618	92,716,710	19,166,452	0.21	0.21
1998	103,342,254	103,348,904	21,737,628	0.21	0.21
Total	499,481,639	737,904,575	59,445,317	0.12	0.08

1 Unit = $1

Aggregate | All 2M:PL(I)

Comparisons | Summary Graphs | Correlations and SDs | Clusters
Acc. Years | Cal. Years | Loss Ratios | (%) Errors

Accident Year Summary

Acc. Year	Mean		Standard	CV	
	Reserve	Ultimate	Dev.	Reserve	Ultimate
1985	0	48,940	0	****	0.00
1986	1,731	46,283	1,274	0.74	0.03
1987	4,268	57,251	2,312	0.54	0.04
1988	7,944	56,959	3,592	0.45	0.06
1989	8,439	49,038	3,616	0.43	0.07
1990	21,169	70,392	7,486	0.35	0.11
1991	32,145	74,080	10,359	0.32	0.14
1992	47,762	89,314	14,284	0.30	0.16
1993	63,937	90,534	17,645	0.28	0.19
1994	79,474	97,967	19,623	0.25	0.20
1995	95,679	109,426	21,712	0.23	0.20
1996	112,626	118,155	23,938	0.21	0.20
1997	124,732	127,021	26,080	0.21	0.21
1998	135,287	135,753	28,250	0.21	0.21
Total	735,192	1,171,112	84,443	0.11	0.07

1 Unit = $1,000