Evaluating Risk Transfer:
The Bootstrap Model And Other Techniques

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Outline

- Loss Distribution

    - Parameterized

        - Loss Ratio

        - Frequency/severity

    - Bootstrap Model

- Bootstrap Model

    - Reserve Variability

    - Prospective (i.e., Risk Transfer) Application

- Examples

- Considerations in Selection of Loss Distribution

- Other Applications

- Questions

Loss Distribution

- Parameterized – Loss Ratio

    – E.g., lognormal

    – Most common method

- Parameterized – Frequency/Severity

    – Frequency: Poisson, binomial, negative binomial, normal, etc.

    – Severity: Lognormal, gamma, inverse Gaussian, etc.

    – Typically developed on a ground-up basis

- Bootstrap model

    – Used frequently for

        • Reserve variability

        • Capital modeling, etc.

    – Can also provide a prospective loss distribution

Bootstrap Model

- Main application is reserve variability
  - Usually a retrospective model
  - Can be prospective
- Basis of model
  - Uses entire triangle
  - Calculates "scaled Pearson residual" for each accident year/ development period
  - Assumption is that these are independent and identically distributed
 - Residuals then used to simulate triangle "as it could have been"

Bootstrap Model (cont.)

- Versions of model

  - Original based on paid chain ladder only (given in England/Verrall paper)

  - Multiple papers since then (England/Verrall, Pinheiro, etc.)

  - More sophisticated models now exist

    - Incurred chain ladder

    - Bornhuetter-Ferguson

      - Paid and incurred

      - Uses lognormal or other distribution for a priori

    - Cape Cod

  - Can weight methods together

  - Multiple lines of business correlated, etc.

- Typical uses

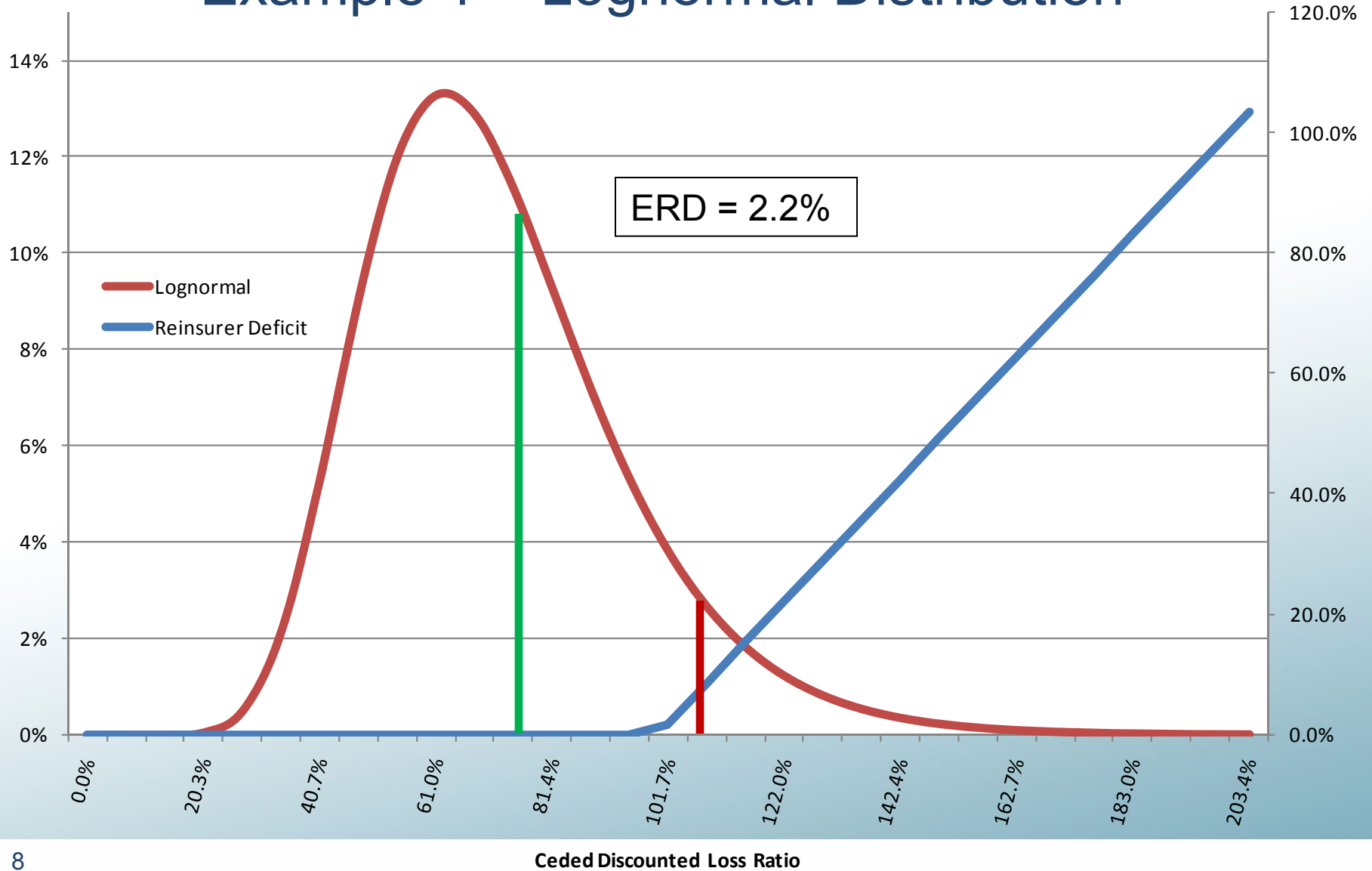 - Reserve distribution

  - Capital requirements

Example 1

- Proposed Reinsurance Treaty

  - Per occurrence excess of loss

  - No swing rating, corridors, etc.

- Parameter Method Assumptions

  - Discounted ceded loss ratio lognormally distributed

    - Mean of 75%

    - CV of 0.3

Example 1 – Lognormal Distribution

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Example 1 – Lognormal Distribution

ERD = 2.2%
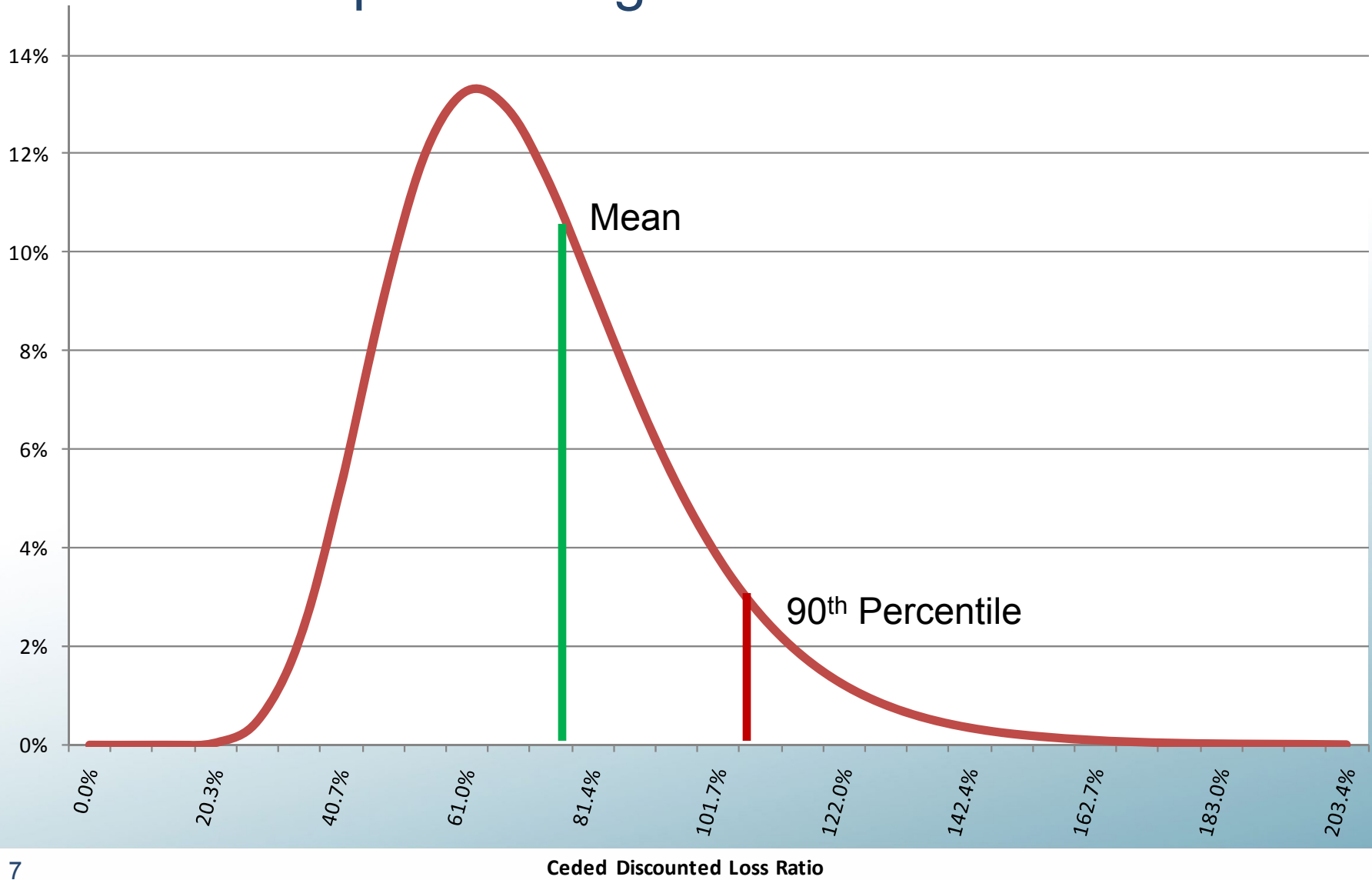
Ceded Discounted Loss Ratio

Example 1 – Lognormal Distribution

EXPECTED REINSURER DEFICIT			
Cumulative Probability	Discounted Loss Ratio	Discounted Profit / (Loss)	Reinsurer Deficit
90%	104.6%	-4.6%	4.6%
91%	106.5%	-6.5%	6.5%
92%	108.5%	-8.5%	8.5%
93%	110.8%	-10.8%	10.8%
94%	113.4%	-13.4%	13.4%
95%	116.4%	-16.4%	16.4%
96%	120.1%	-20.1%	20.1%
97%	124.8%	-24.8%	24.8%
98%	131.3%	-31.3%	31.3%
99%	142.2%	-42.2%	42.2%
Avg of Above	xxx	xxx	1.8%
Expected Value	xxx	xxx	2.2%

Example 1 – Lognormal Distribution

- Average of scenario reinsurer deficits

  - Will understate expected value

  - If sufficient points are included, will approximate the expectation

- Expected Reinsurer Deficit

  - Under parameterized distribution, can be calculated directly from parameters

  - E.g., lognormal:

    - ERD = $\exp(\mu + \sigma^2/2) \times \Phi[(\mu + \sigma^2) / \sigma] - \Phi(\mu/ \sigma^2)$

Example 1 – Parameter Assumptions

- Mean

    – Developed from cedant data

- Coefficient of Variation

    – Developed loss ratios will typically understate this

        • Inherently "expected value" estimates

        • Small sample

        • Includes volatility due in part to market forces (could overstate CV)

Bootstrap Model Requirements

- Minimum

    – Paid triangle

- Also helpful

    – Incurred triangle

    – Premium/exposure

    – A Priori loss distribution (for Bornhuetter-Ferguson)

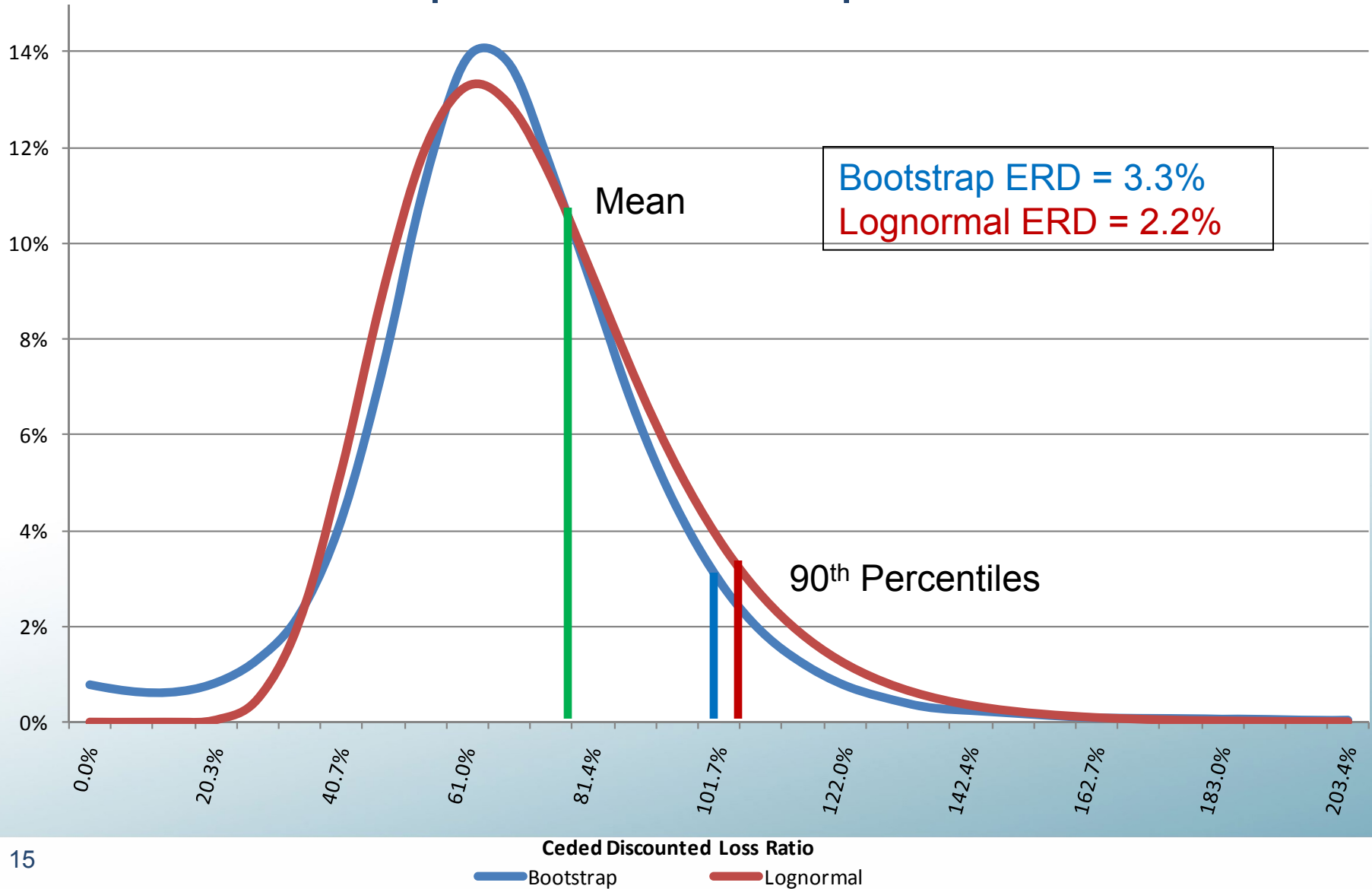   – Loss trends / on-level factors (for Cape Cod)

Bootstrap Model Variance

- **Parameter Variance**

  - "Recreates" paid/incurred triangles as they could have been

  - Develops unpaids from these using standard development methods

    - Chain ladder

    - Bornhuetter-Ferguson

    - Cape Cod

- **Process Variance**

  - Simulates incremental unpaids

    - Typically uses Gamma (proxy for overdispersed Poisson)
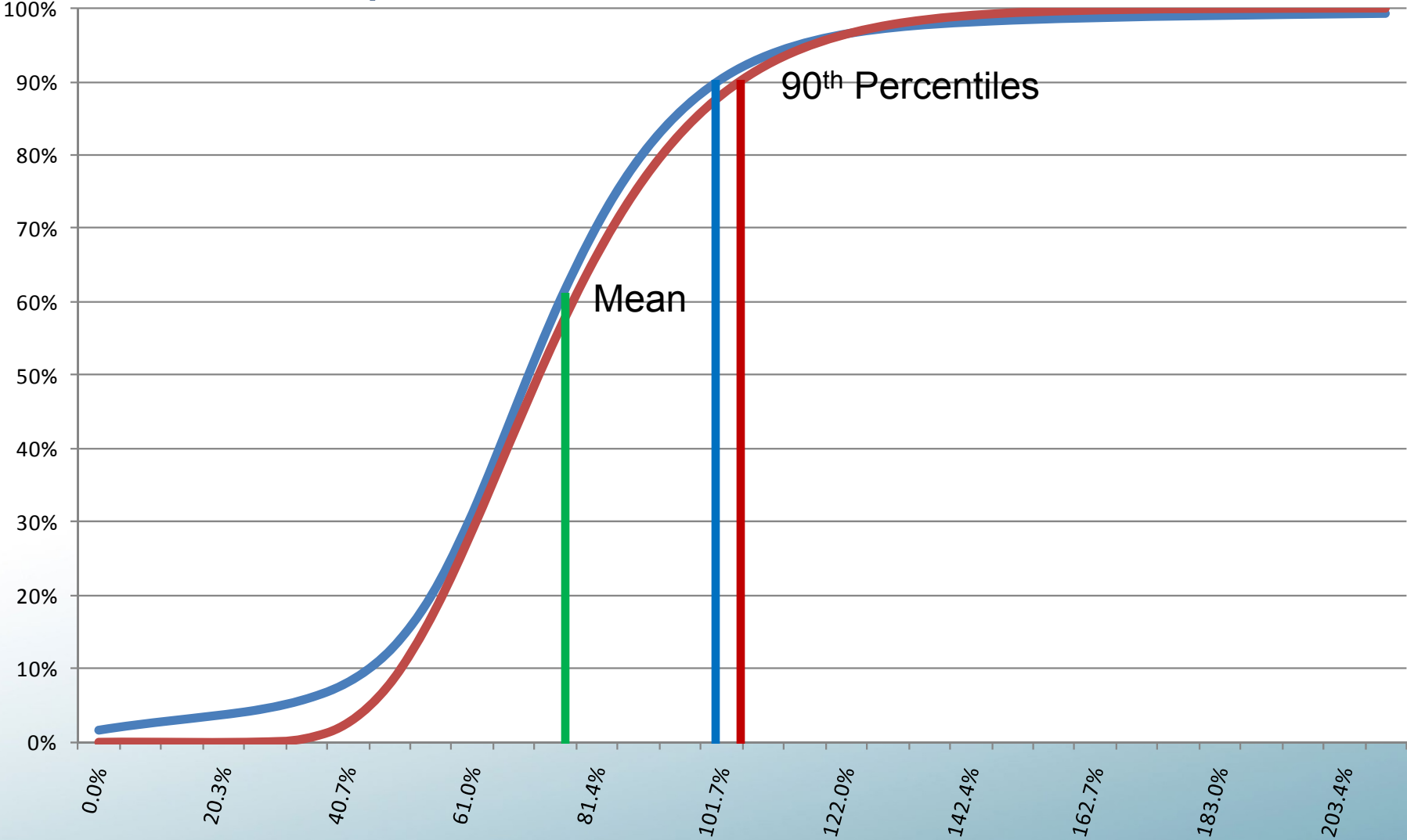
    - Lognormal, etc. also an option

Bootstrap Model Variance (cont.)

Process Variance

**Process &
Parameter Variance
(Simulated)**

**Parameter
Variance**

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(Simulated)**

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Process Variance
(Simulated)**
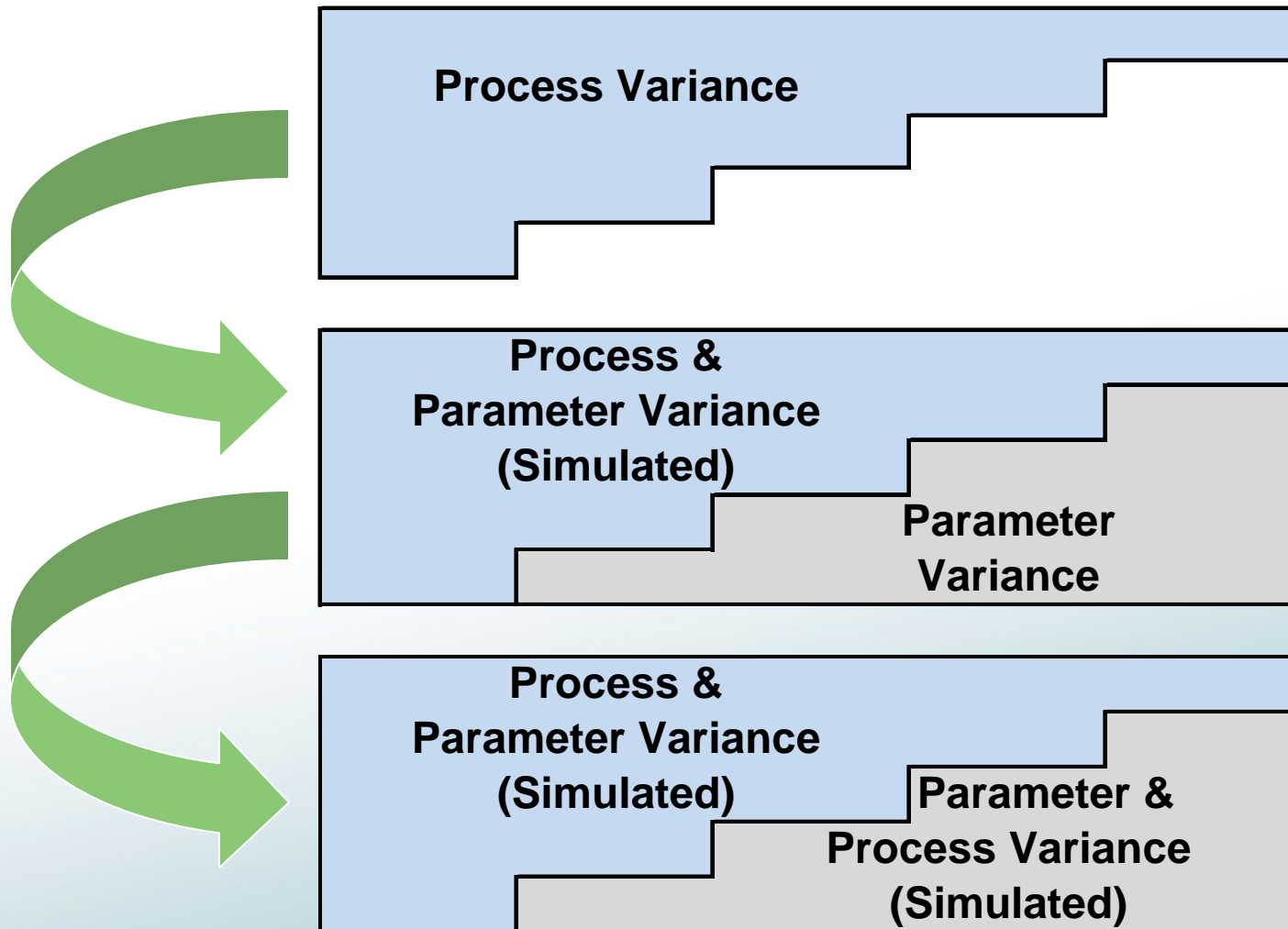
Example 1 – Bootstrap Model

Bootstrap ERD = 3.3%
Lognormal ERD = 2.2%

Mean

90th Percentiles

Ceded Discounted Loss Ratio

Bootstrap ——— Lognormal
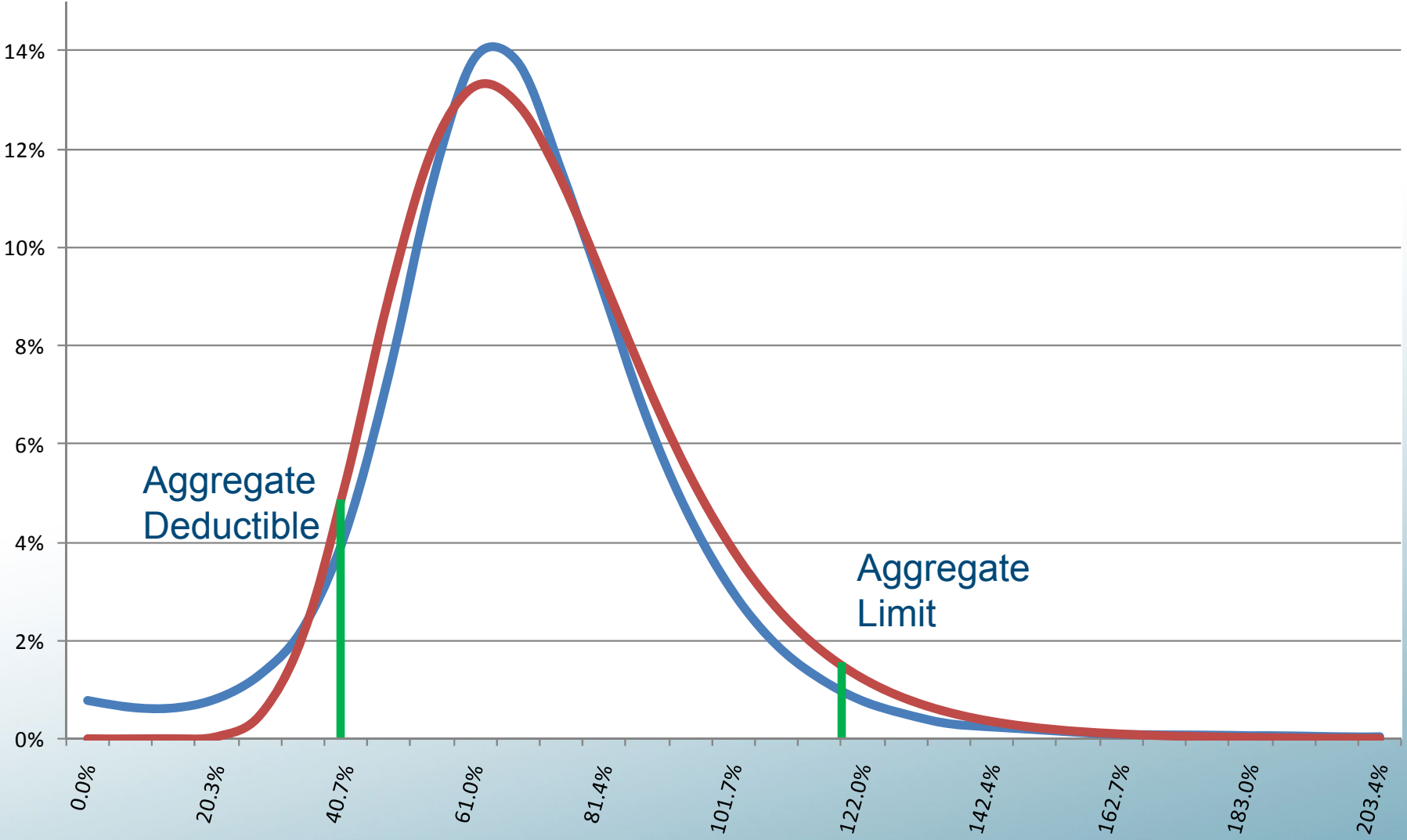
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Example 1 – Cumulative Distributions

Example 2

- Proposed Reinsurance Treaty

    – Per occurrence excess of loss (as Example 1)

    – Aggregate deductible of $20,000,000

    – Aggregate limit of $40,000,000

    – Ceded premium of $25 million (half of Example 1)

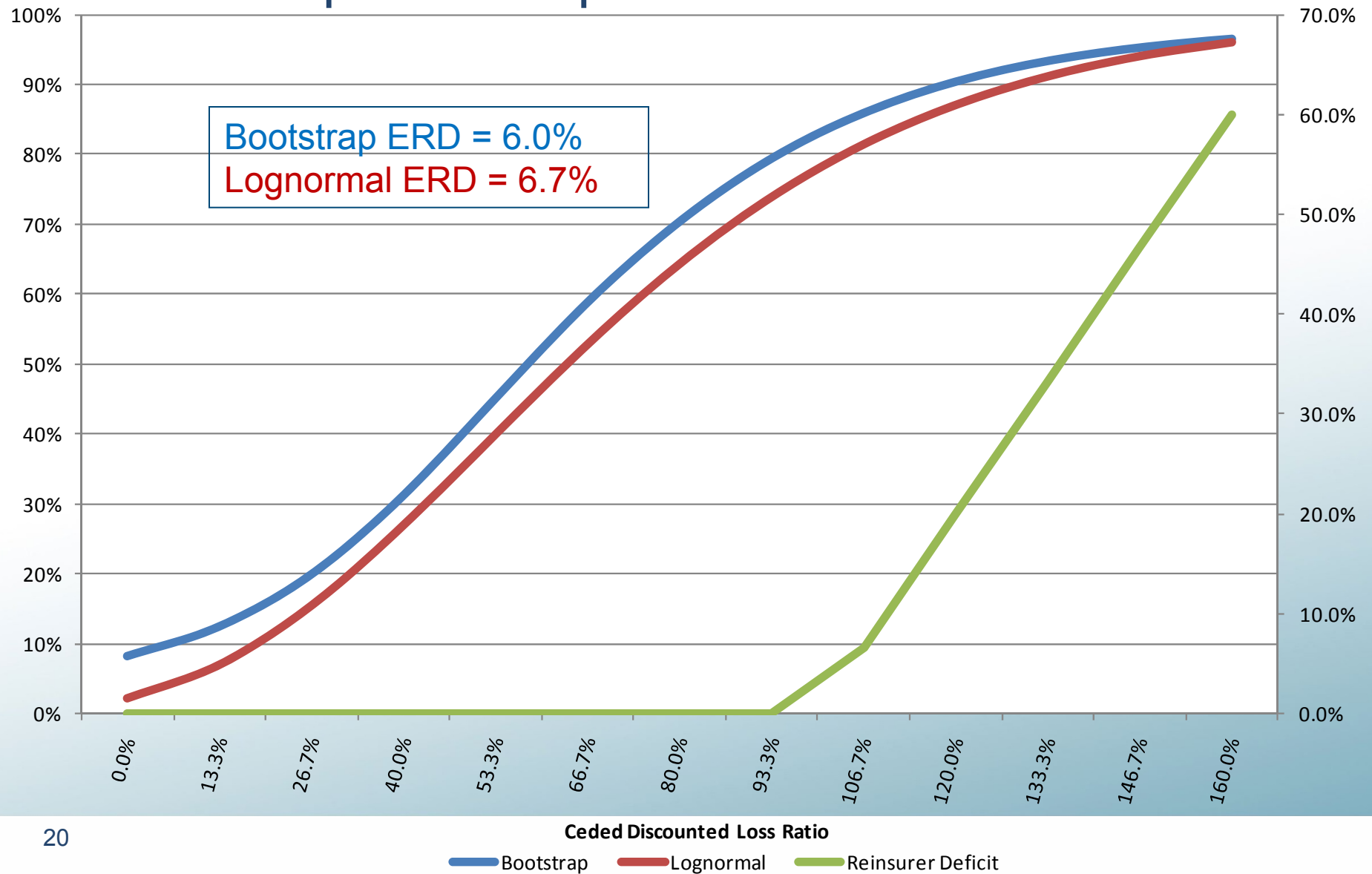   – Other assumptions the same

Example 2 – Effect on Distribution

Example 2 – Cumulative Distribution

Ceded Discounted Loss Ratio

— Bootstrap   — Lognormal

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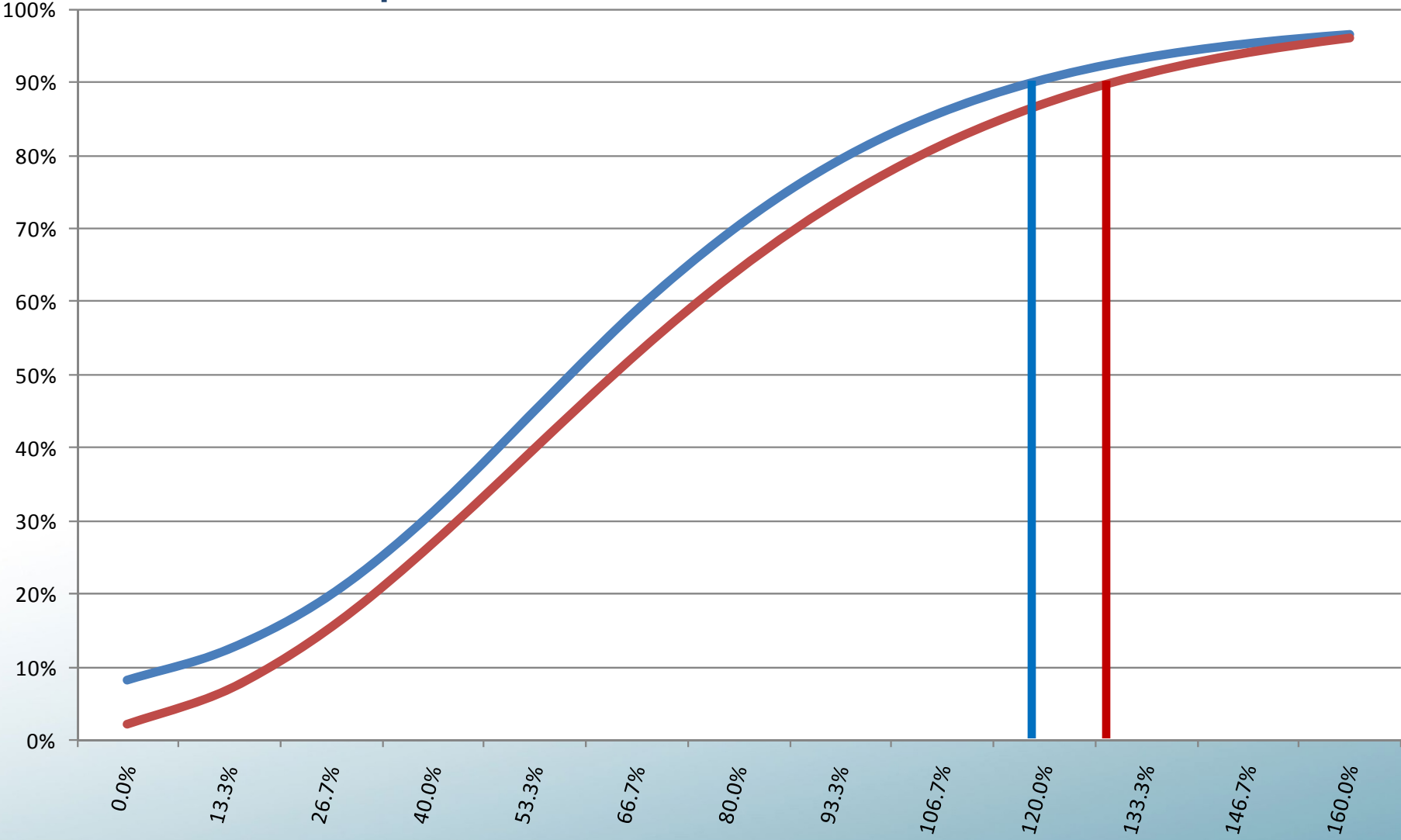
Example 2 – Expected Reinsurer Deficit

Bootstrap ERD = 6.0%
Lognormal ERD = 6.7%

Ceded Discounted Loss Ratio

Bootstrap   Lognormal   Reinsurer Deficit

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Example 2 – Bootstrap Model

- Triangles of paid/incurred losses

  – Historical years restated under proposed treaty terms

  – Losses stated prior to aggregates

  – Include parameter & process variance in all triangle cells

- Resulting distribution

  – Gross of aggregate deductible/limit

  – Can adjust each simulated loss scenario for these
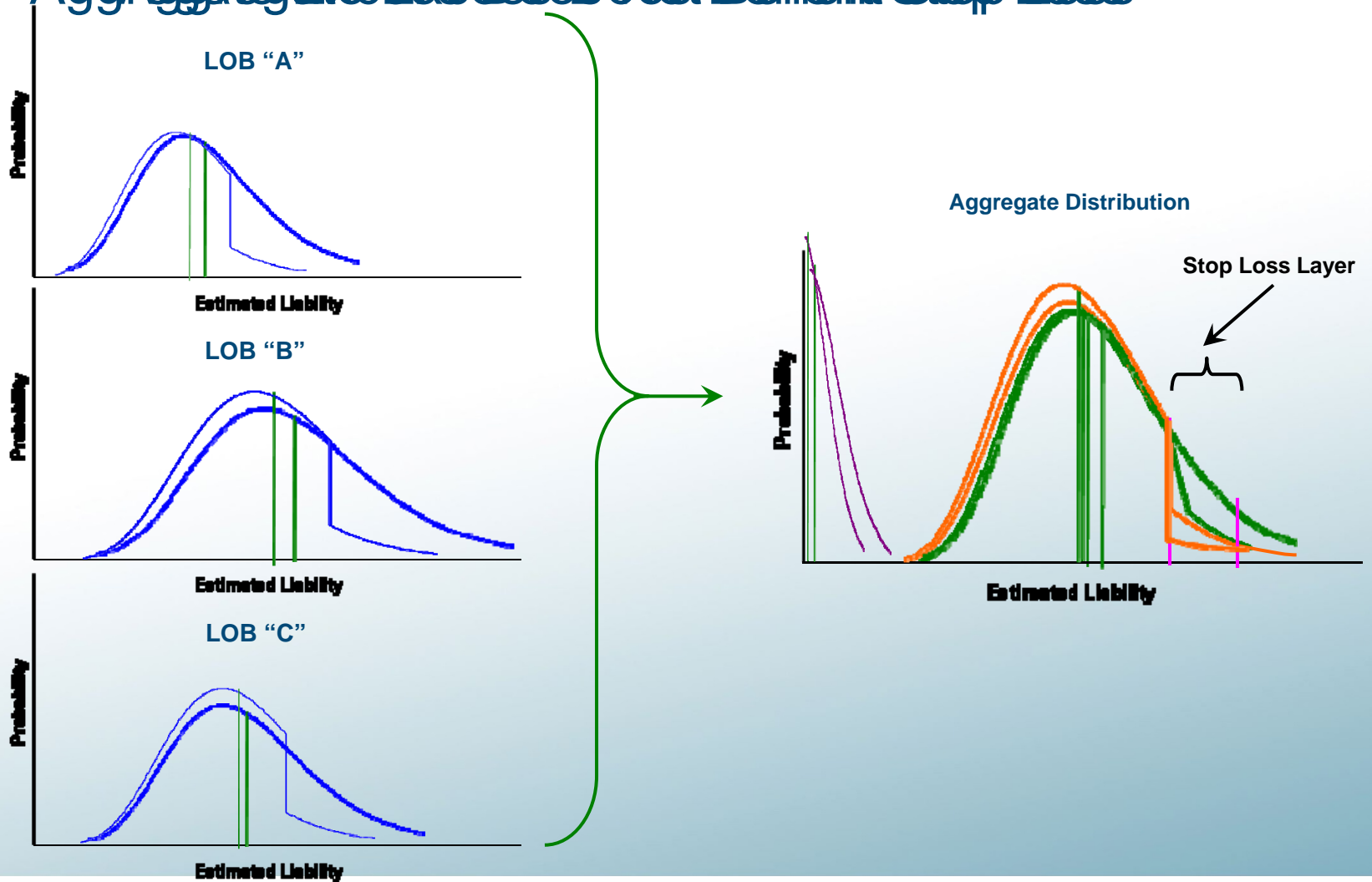
Example 3 – Aggregate Excess
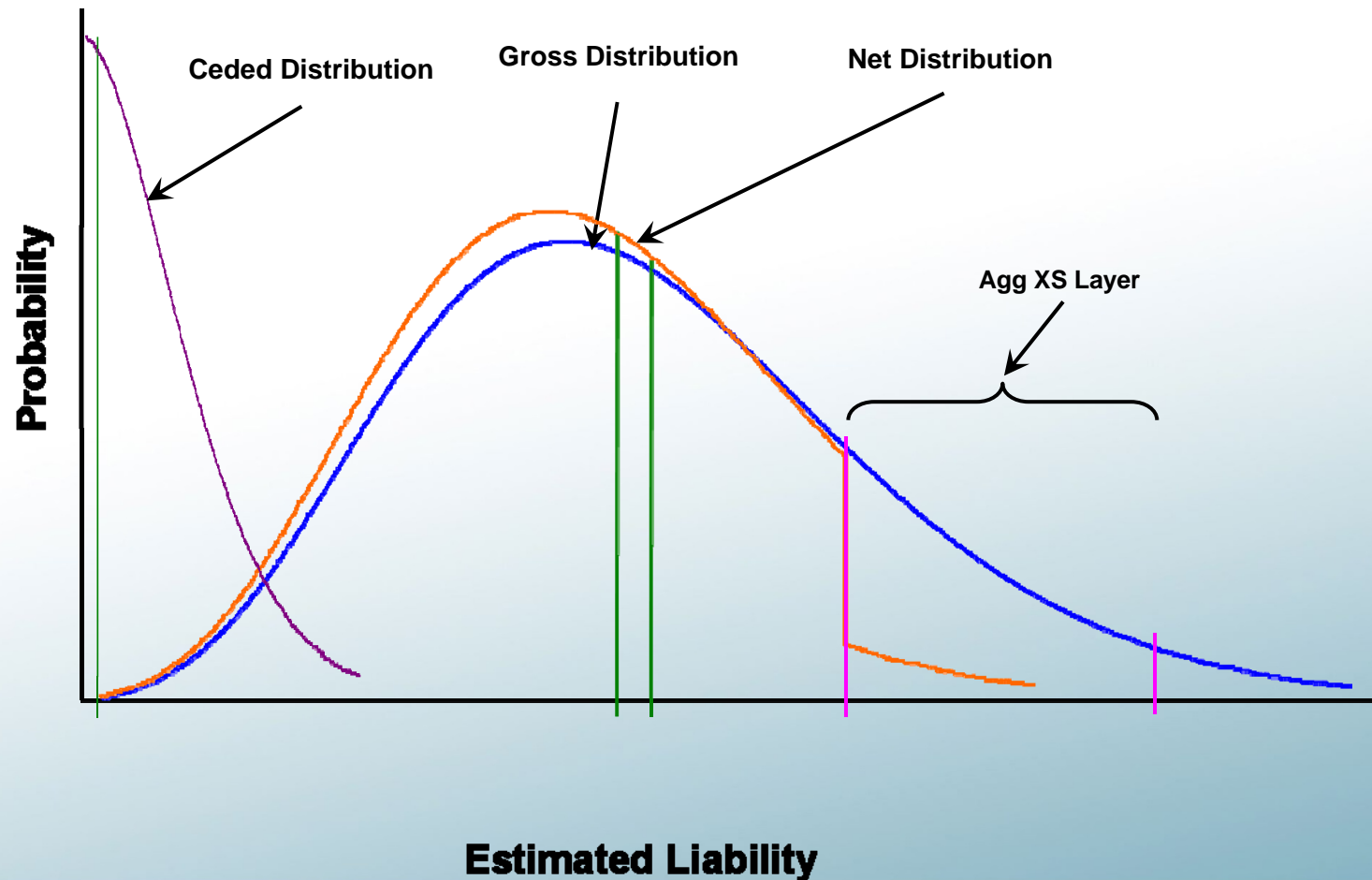
Example 4 – Quota Share Reinsurance
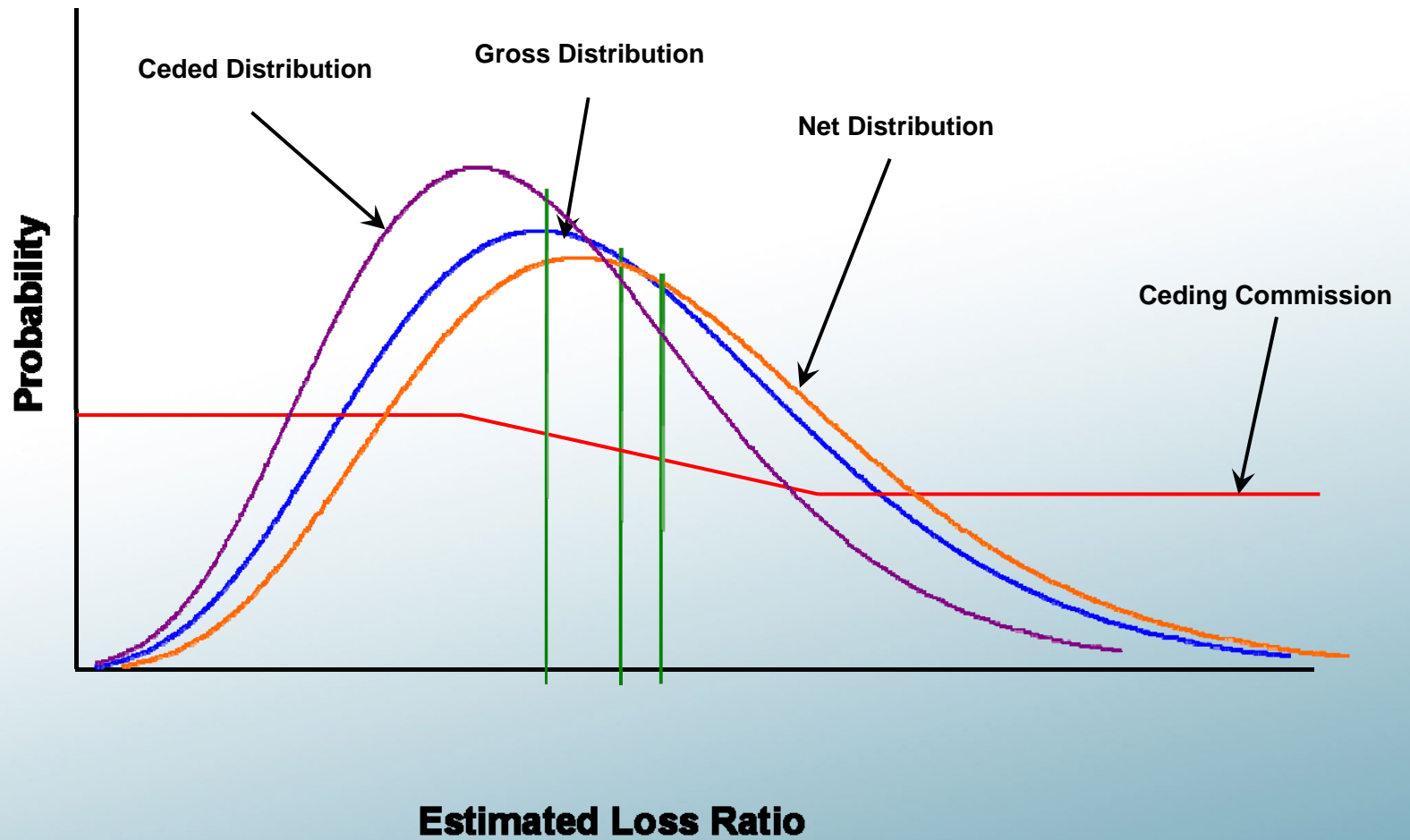
Example 5 – Stop Loss Reinsurance

Aggregate Excess Does Not Benefit Stop Loss

LOB "A"

LOB "B"

LOB "C"

Aggregate Distribution

Stop Loss Layer

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Considerations in Selection of Loss Distribution

- Parameterized loss ratio distribution

  - May be the simplest formulaically

  - Difficult to estimate variance

- Compound frequency/severity distribution

  - May be easier to estimate variance for these components

  - Computationally more time-consuming

  - May require simulation

- Bootstrap model

  - Requires historical data

  - Does not require variance or correlation assumptions

  - May require working with numerous simulated scenarios

  - May allow parameter estimation for parameterized distributions

What If There's No Risk Transfer?

- To Account For As Reinsurance
  - Aggregate Cover
    - Increase aggregate limit
    - Decrease aggregate deductible
    - Decrease ceded premium
  - Quota Share
    - Increase loss ratio cap
    - Decrease ceded premium at higher percentiles
      - Greater provisional ceding commission
      - Lesser swing range
- Use Deposit Accounting
  - Only option if treaty already in effect

A Note on Other Applications

- Reserving for aggregate deductibles / limits

  - Example:

    - Per occurrence excess of loss treaty

    - Developed accident year losses of $45 million

    - $50 million aggregate limit

  - IBNR indications

    - Judgmental provision, e.g.:

      - 30% likelihood of losses exceeding aggregate

      - $10 million expected value loss in excess of aggregate (if exceeded)

      - Implies $3 million increase in net reserve

    - Parameterized distribution

      - Should incorporate data to date

    - Bootstrap model

      - Losses would most likely be stated gross of aggregate limit

Questions

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