What are the business applications of creating a distribution of possible outcomes? Why should management care about distributions?

- Reserve distributions can be used to apply appropriate relative risk margins to products. This charges the products with the proper cost of doing business and supports sound decision-making
- Developing reserve distributions is a critical step in capital allocation and developing target ROE
- A general understanding of reserve distributions lends management insight into the uncertainty and risk associated with various products. This knowledge can be an aid in making strategic decisions.

Other Practical Applications

- Mark had a slide on potential business applications of the concept of stochastic distributions. Wherever variability in performance is critical, a stochastic distribution of a performance metric will add insight and improve predictability of results. Here are some practical examples.
- Example: Measuring expected performance of Profit Centers
  - Can use aggregate distributions to measure underwriting performance of Profit Centers. This would be another dimension in the segmentation of the business. Profit Centers could refer to Marketing regions, Operational Divisions, individual Product segments

Other Practical Applications

- Example: Compare Plan vs. Actual Performance
  - Companies establish Profit & Growth Plans and monitor the actual results against plan. When does a deviation warrant action? If the decision-makers have a good understanding of the underlying variability in the results, it will enable them to discriminate a "symptomatic" deviation from one that is merely "noise", with a reasonable level of confidence. Without this ability, one may introduce instability into an otherwise stable environment.

How do you know if a distribution is reasonable?

- Results should be tempered with direct knowledge of the line of business.
  - Do the results of the model square with our subjective understanding of the business being assessed?

- Historical data is not the end of the story when it comes to understanding future uncertainty.

Example: Is the risk associated with future D&O emergence reflected in historical triangles?

- External factors, such as legal climate and economic conditions, are constantly having an impact on uncertainty in claim emergence.
- The answer to whether or not your distribution is reasonable does not lie in the data, but in understanding external trends and uncertainty in the current business climate.
- Letting data flow through a model without adding critical thought to the process can create a misleading product.

Example: General Liability

General Liability

AY	Earned Premium	Incremental Incurred Loss+ALAE				
		12	24	36	48	60
1994	5,979,188	869,555	313,826	519,376	1,097,046	191,884
1995	6,128,668	884,588	64,931	268,823	221,408	185,511
1996	6,281,884	918,632	262,406	230,749	82,320	306,586
1997	6,438,932	768,330	256,978	363,073	407,110	517,726
1998	6,599,905	864,012	333,624	413,250	551,129	239,721
1999	6,764,902	1,335,861	311,216	878,406	333,770	553,551
2000	6,934,025	850,238	793,467	651,906	953,594	369,193
2001	7,107,376	928,698	758,152	951,811	176,134	440,225
2002	7,285,060	1,128,123	786,819	609,582	621,397	404,138
2003	7,467,187	785,468	374,794	447,062	657,254	560,164
2004	7,653,866	647,389	347,513	490,828	750,708	560,752
2005	7,845,213	563,210	330,603	428,876	340,507	
2006	11,767,819	718,665	689,496	687,986		
2007	17,651,729	1,378,122	1,174,232			
2008	26,477,593	3,549,035				

Example: General Liability

- Changes in a book of business create additional uncertainty.
- Assessing this uncertainty necessitates understanding what gave rise to those changes.
  - Did the growth stem from a change in underwriting guidelines?
  - Did the company expand into new classes or markets?
- How have changes to the book affected the uncertainty associated with that book?

Building aggregate reserve distributions

- The primary challenge in building an aggregate distribution is understanding the correlation between lines of business.

- The aggregate distribution depends much more on the correlations between lines of business than it does on the individual lines' reserve distributions. Even with perfect knowledge of every line's distribution, we are still a long way from understanding our aggregate distribution.

Building aggregate reserve distributions

If we split our reserves into n (equal) pieces, the variance of our aggregate distribution is as follows:

$$Var(X_{agg}) = n^{-2} * (Var(x_1) + Var(x_2) + \ldots + Var(x_n) + \sum_{i=1 \text{ to } n}\sum_{j \neq i} Cov(x_i, x_j))$$

Or

$$Var(X_{agg}) = 1/n * \underline{\boldsymbol{\sigma^2}} + (n-1)/n * \underline{\boldsymbol{Cov}}$$

Where $\underline{\boldsymbol{\sigma^2}}$ is the average variance and $\underline{\boldsymbol{Cov}}$ is the average covariance.

As n gets large, the first term becomes negligible and the aggregate variance depends only on the covariance between elements.

Building aggregate reserve distributions

- In practice, correlation measurement is often done as an afterthought once the bulk of analysis has been done.

- In reality, it drives results much more than we often give it credit for.

How do we know if correlations are reasonable?

- Given the importance of measuring correlation, and the difficulty in truly understanding it, one must be particularly careful in selecting correlations:
  - Compare a variety of methods.
  - Are the selected correlations statistically significant? Can we make a case that the results are not spurious?
  - Do the selected correlations make intuitive sense? Do they resonate with your operational understanding of the business? Question any large (positive or negative) correlations that cannot be reconciled.

How do we know if correlations are reasonable?

- The aggregate distribution should be similar regardless of how we segment our data.
  - A reasonable test might involve running the model using aggregate data
    - If the distribution is similar to the one derived from segmented data we may feel more comfortable with our aggregate data and our correlation assumptions.

(On a contrary note, looking at data in aggregate may hide information that is transparent at a finer granularity)

What happens when the mean of a distribution does not equal a best estimate?

- The mean, or median of the distribution may not equal management's booked reserves or the actuary's best estimate for a number of reasons.

- Most variability models rely on simple paid or incurred loss data. Other information such as claim counts or knowledge of operational changes in claims provide additional insight into expected loss development. If this information is employed in setting reserves, the best estimate will justifiably differ from the modeled best estimate.

What happens when the mean of a distribution does not equal a best estimate?

- One might argue that there is less inherent variability about an actuary's best estimate, as the estimate derived from better/more complete data will, by its nature, have less uncertainty about it.
- Which set of input data will have more uncertainty about its conclusions?

Input to traditional reserve analysis	**Input to stochastic reserve model**
paid losses	Paid losses
Case reserves	Case reserves
A priori expected loss ratio	A priori expected loss ratio
Reported claim count	
Cwop count	
Open claim count	
Discussion with claims	

What happens when the mean of a distribution does not equal a best estimate?

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- A data point that appears as volatility in a development triangle may be easily explainable with knowledge from the claims department, and may even be predictable by the Actuary before it develops. This data point adds volatility in a model, but does not actually reflect any uncertainty in reserves.

Example: Auto Collision

Auto Collision		Incurred @ 3 mos	Development Factors 3-6	6-9	9-12
2007 Q1		4,666,500	1.16	1.07	1.02
2007 Q2		3,075,000	1.30	1.07	1.02
2007 Q3		3,297,000	1.31	1.07	1.02
2007 Q4		4,500,000	1.17	1.08	1.02
2008 Q1		4,523,083	1.15	1.09	1.02
2008 Q2		3,185,604	1.30	1.07	1.02
2008 Q3		2,970,741	1.30	1.07	1.02
2008 Q4		5,334,046	1.16	1.07	1.02
2009 Q1		4,810,461	1.14	1.07	1.02
2009 Q2		3,741,643	1.32	1.07	
2009 Q3		3,272,938	1.29		
2009 Q4		5,256,572			

Making Adjustments to data

- When is it appropriate to make adjustments to raw data prior to use in a model?

Example: Case Reserving Error

AY	Earned Premium	Incremental Incurred Loss+ALAE 12	24	36	48	60
2001	7,107,376	928,698	758,152	951,811	176,134	440,225
2002	7,285,060	1,128,123	786,819	609,582	3,121,397	(1,845,862)
2003	7,467,187	785,468	374,794	447,062	657,254	560,164
2004	7,653,866	647,389	347,513	490,828	750,708	560,752
2005	7,845,213	563,210	330,603	428,876	340,507	
2006	8,041,343	491,088	471,155	470,124		
2007	8,242,377	643,506	548,301			
2008	8,448,436	1,132,422				

Upon investigation, the fluctuation in AY 2002 was cause by an adjuster mistakenly setting up a reserve for 2,500,000 instead of 250,000 and subsequently settling the claim for 250,000.

Should this data be included or not? Reserving errors do happen, will continue to happen, and can have a real impact on earnings statements.

This volatility, however, adds no additional uncertainty to the company's ultimate cash flows.

Making Adjustments to data

- If an adjustment adds fidelity to the model then it is appropriate.

One must take care, however.

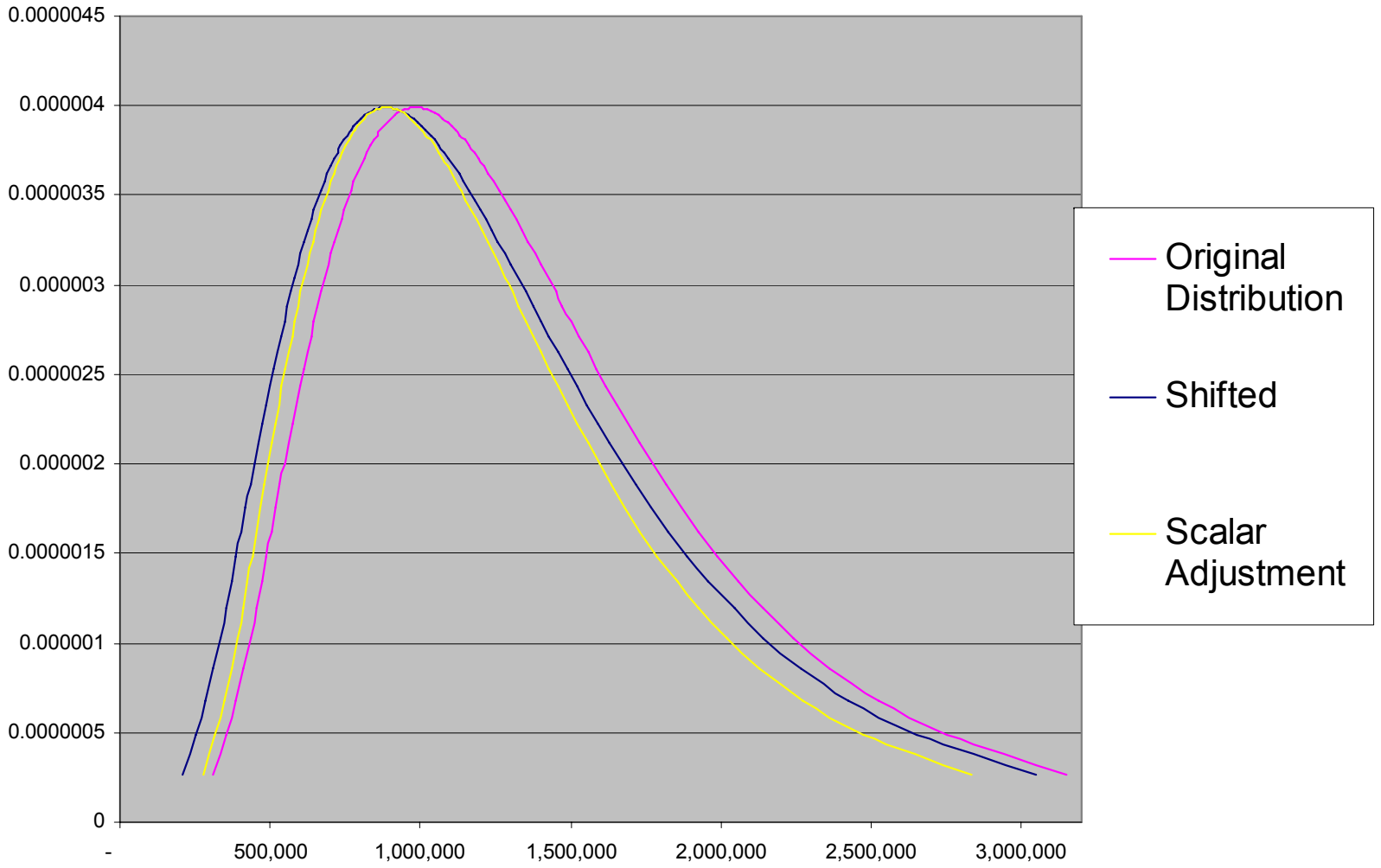
- If a user is allowed to manipulate a model too much, the outcome will merely reflect the user's prior expectations.
- On the other hand, a model can never reflect all of the complexities that exist in the real world. A user must be able to adjust for these complexities in order to make any model useful.

When management wants a distribution around their booked reserve, is it OK to re-center the best estimate distribution?

- If management recognizes that the booked reserve is not a true mean estimate of liabilities, then it may be more appropriate to show the actuary's modeled distribution with the booked reserve as a point estimate wherever it falls in the distribution.

- If management believes its estimate to be a mean estimate, it may be more appropriate to re-center the distribution. The question in the end, however, is whether or not the actuary feels that he/she is presenting a good faith estimate of the distribution of possible outcomes.

If a distribution is re-centered, is it more appropriate to make a scalar adjustment, or to shift the distribution additively?



At what level should reserve distributions be analyzed?

- What resolution is appropriate? By major line? By coverage?
- Should gross losses and salvage & subrogation be analyzed separately, or does it make sense to look at losses net of salvage & subrogation?
- Should LAE reserves be included with loss, or assessed separately?
- Should direct, ceded, and assumed losses be analyzed separately or will net loss suffice?
- There is no “right” answer to this question. It all depends on the resources that the company wishes to commit to assessing reserve variability and the relative value that is added by further resolution of the data.

At what level should reserve distributions be analyzed?

- Aside from cost/benefit considerations, there are other components that must be considered in deciding at what level the data will be analyzed.

- Theoretically, regardless of how the data is sliced, the aggregate distribution should be the same. In practice, however, this is not the case.

At what level should reserve distributions be analyzed?

- The Actuary should test and understand how the model responds to analysis at various resolutions. If there is a large discrepancy in results, the difference should be understood.

- The lower the resolution, the more correlations will need to be selected. If we have 10 lines of business, and we analyze loss, LAE, and S&S, on a direct, ceded, and assumed basis, we will have to develop 4,005 correlation estimates. Given the degree to which correlations drive the aggregate distribution and the difficulty in measuring correlation, this may be undesirable.

At what level should reserve distributions be analyzed?

# of data elements	# of correlation estimates
1	-
5	10
10	45
25	300
50	1,225
75	2,775
100	4,950
250	31,125

How many years of historical data are appropriate for analysis?

- Again, there is no objectively correct answer.
- Like many actuarial problems, we must weigh stability of results against responsiveness to change.
- These considerations are difficult enough in traditional actuarial analysis. Things are made much more difficult by the fact that we are dealing with abstract questions.

How many years of historical data are appropriate for analysis?

What is the "Variability" we are attempting to measure?

- Are systematic changes in development patterns over time part of the variability we are attempting to measure?
- What is the appropriate variability about our estimate at $?_x$

	Incurred @ 12	LDF's 12-24	24-36	36-48
1989	5,172,820	2.323	1.778	1.251
1990	5,351,613	2.280	1.817	1.241
1991	5,536,587	2.240	1.838	1.253
1992	5,727,953	2.219	1.844	1.240
1993	5,925,934	2.146	1.871	1.298
1994	6,130,758	2.164	1.883	1.307
1995	6,342,662	2.143	1.857	1.299
1996	6,561,889	2.124	1.876	1.300
1997	6,788,694	2.097	1.910	1.298
1998	7,023,339	2.081	1.940	1.264
1999	7,266,094	2.074	1.959	1.296
2000	7,517,239	2.074	1.990	1.269
2001	7,777,065	2.069	2.004	1.270
2002	8,045,871	2.031	2.028	1.267
2003	8,323,969	1.982	2.054	1.260
2004	8,611,678	1.948	2.060	1.292
2005	8,909,332	1.908	2.046	1.338
2006	9,217,275	1.888	2.093	?
2007	9,535,860	1.853	?	?
2008	9,865,458	$?_x$	?	?
	σ_{20yr}	0.131	0.096	0.027
	σ_{15yr}	0.100	0.084	0.025
	σ_{10yr}	0.087	0.059	0.024
	σ_{5yr}	0.051	0.024	0.031

Conclusion

- In theory,
  - stochastic reserve models provide us an objective way to assess uncertainty about our estimates

Conclusion

- In practice,
  - Every model requires countless subjective decisions to be made by the user.
    - Whether to include/exclude outlying data
    - At what granularity to analyze data
    - How many years of data to use
    - How to incorporate operational knowledge of business
    - Other subjective adjustments
- The effect of these decisions is often ambiguous.

Conclusion

- As actuaries take on more theoretical business applications, the implications of their work will necessarily become more abstract.
- The challenge, whether using subjective or objective methods, is to remain as faithful as possible to the operational realities of insurance and to create a product that adds value to its users.