Casualty Loss Reserve Seminar
Technical Provisions in Solvency II
What EU Insurers Could Do with Schedule P

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DATA • ANALYTICS • DECISION SUPPORT

ISO Innovative Analytics

Three CLRS Call Papers and two ASTIN Colloquium Papers on Stochastic Loss Reserving

- **Overall Objectives**
  - Treat as a statistical problem
  - Use realistic distributions in fitting model
    - Various versions of the collective risk model
  - Use Bayesian methodology
    - Compelling case in “Thinking Outside the Triangle”
  - Aggressive retrospective testing on subsequent data
    - How good are statistical models in predicting ultimates?
  - Apply results to insurer risk management
    - Given the limits of our ability to predict ultimate losses (whatever the are), how do we manage insurer risk?
    - IAA and Solvency II

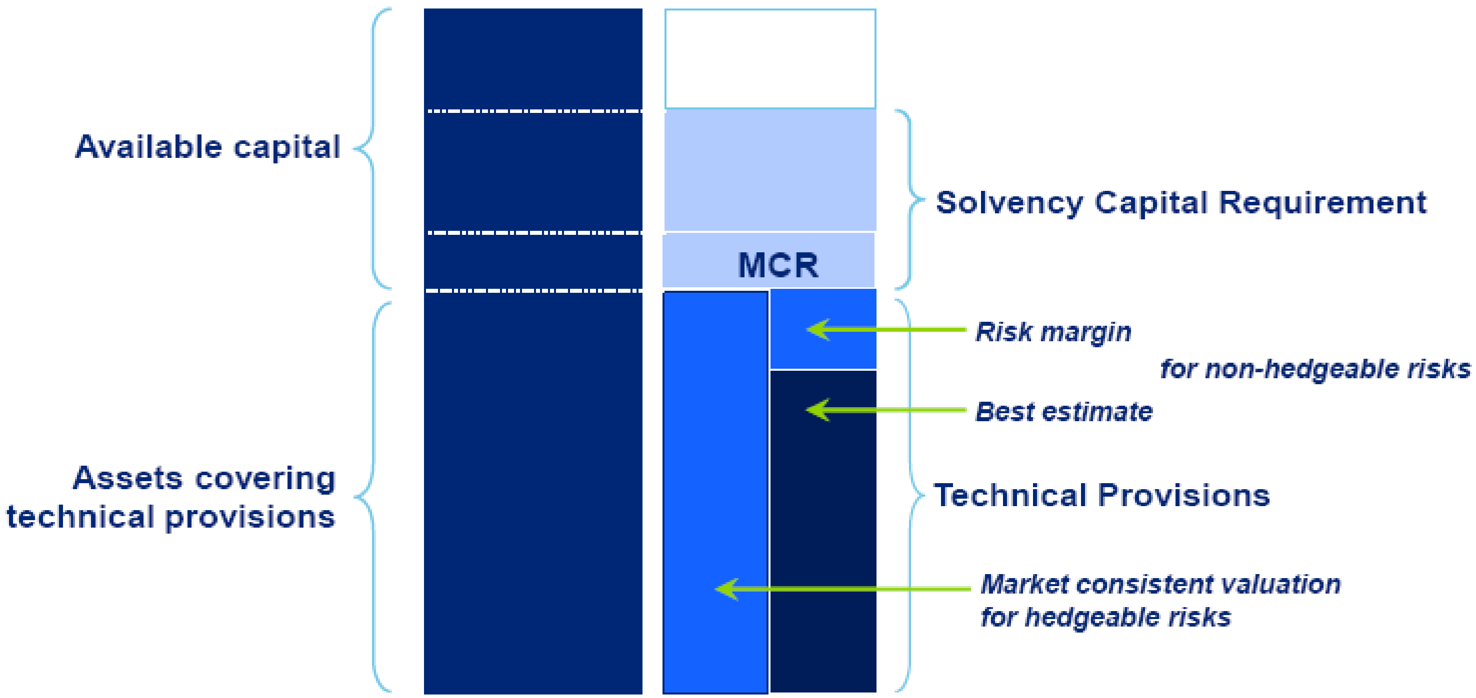
Outline of Presentation

- **Technical Provisions in Solvency II**
  - As described in the EU Framework Directive
- **American NAIC Schedule P Loss Triangles**
- **How to use data in Schedule P**
  - Calculate best estimate
  - Compare predictions against experience
  - Calculate risk margin (Market Value Margin)

Background

- **Solvency II adopted by European Parliament**
  - Originally effective October 31, 2012
- **Objectives include:**
  - Increased focus on effective risk management, control and governance
  - Market consistent valuation of assets & liabilities
  - Increased disclosure and transparency

Focus of the Paper
Technical Provisions



Conceptually in line with IFRS

Principles Underlying Technical Provisions Described in Articles 76-83 of EU Framework Directive

EUROPEAN UNION

THE EUROPEAN PARLIAMENT	THE COUNCIL
	Brussels, 19 October 2009 (OR. en)
2007/0143 (COD)	PE-CONS 3643/1/09 REV 1
	SURE 15 ECOFIN 349 CODEC 693

LEGISLATIVE ACTS AND OTHER INSTRUMENTS

Subject:	DIRECTIVE OF THE EUROPEAN PARLIAMENT AND OF THE COUNCIL on the taking-up and pursuit of the business of Insurance and Reinsurance (Solvency II) (recast)

Clip from Article 77
Calculation of technical provisions

1. The value of technical provisions shall be equal to the sum of a best estimate and a risk margin as set out in paragraphs 2 and 3.

2. The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

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The best estimate shall be calculated gross, without deduction of the amounts recoverable from reinsurance contracts and special purpose vehicles. Those amounts shall be calculated separately, in accordance with Article 81.

Clip from Article 77
Calculation of technical provisions

4. Insurance and reinsurance undertakings shall value the best estimate and the risk margin separately.

- - - - - - - - - -

5. Where insurance and reinsurance undertakings value the best estimate and the risk margin separately, the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.

Clip from Article 80
Segmentation

Insurance and reinsurance undertakings shall segment their insurance and reinsurance obligations into homogeneous risk groups, and as a minimum by lines of business, when calculating their technical provisions.

Clip from Article 83
Comparisons against experience

Insurance and reinsurance undertakings shall have processes and procedures in place to ensure that best estimates, and the assumptions underlying the calculation of best estimates, are regularly compared against experience.

Where the comparison identifies systematic deviation between experience and the best estimate calculations of insurance or reinsurance undertakings, the undertaking concerned shall make appropriate adjustments to the actuarial methods being used and/or the assumptions being made.

Schedule P Loss Triangles

- **Part of American NAIC Annual Statement**
  - All American insurers must submit to regulators
  - Data is available to the public
    - At a price (academic discounts available)
    - http://www.naic.org/store_financial_home.htm
  - Multiples lines of business
  - Paid and incurred loss triangles

From Data Reported in Schedule P
Assemble a Triangle of Incremental Paid Losses

- **Include Earned Premium**
- **This data is for a real insurer**
- **Commercial Auto Liability**

Insurer #1 in Paper

AY	Premium	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_{10}
1	29,701	5,234	5,172	3,708	1,783	923	537	175	145	8	0
2	27,526	5,234	5,683	4,392	2,134	1,377	673	155	81	47	-
3	30,750	5,702	5,865	7,966	2,472	NA	143	152	73	-	-
4	35,814	6,349	4,611	3,959	2,522	1,924	622	206	-	-	-
5	42,277	8,377	6,890	4,055	3,795	1,292	1,422	-	-	-	-
6	50,088	9,291	13,836	12,441	4,086	2,293	-	-	-	-	-
7	56,921	12,029	12,462	8,369	7,034	-	-	-	-	-	-
8	61,406	13,119	12,618	9,117	-	-	-	-	-	-	-
9	67,983	15,860	14,893	-	-	-	-	-	-	-	-
10	73,359	16,498	-	-	-	-	-	-	-	-	-

Amounts in Thousands

Using Schedule P Data

- **We of course have the reported earned premium and incremental paid loss**
- **But we also have similar data for every American insurer for each line of business**
  - ***A great source of "prior information" that can be used in a Bayesian analysis***
  - Use Schedule P data for 50 large insurers to create 5000 "benchmark scenarios" for each line of insurance.
    - Select 100 parameter sets from each of the 50 large insurers with likelihood "close" to the maximum likelihood.
    - Fit a gamma distribution to the combined set for each parameter.

Loss Model

- Expected Loss

$$\mu_{AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} \cdot t^{AY+Lag-1}$$

- Variance of Loss

$$Var\left[X_{AY,Lag}\right] = \mu_{AY,Lag} \cdot \tau_{Lag} \cdot \left(1 + 1/\alpha\right) + c \cdot \mu_{AY,Lag}^{2}$$

$$\tau_{Lag} = Sev \cdot \left(1 - \left(1 - \frac{Lag}{10}\right)^{3}\right) \text{ for } Lag = 1,2 \ldots,10.$$

- $\{ELR_{AY}\}$,$\{Dev_{Lag}\}$, *t*, *c*, and *Sev* are unknown parameters,

Tweedie Model of Losses in Each (*AY,Lag*) Cell

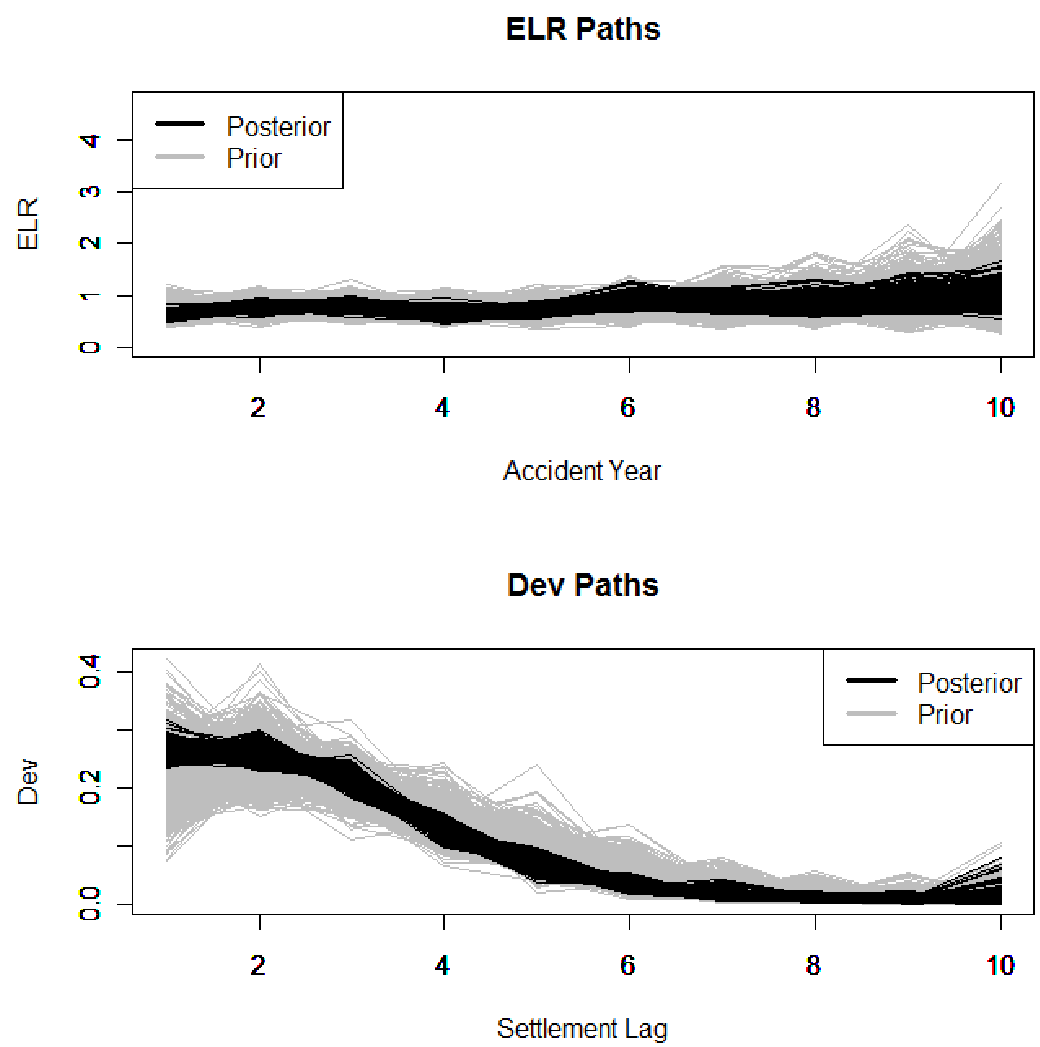
Tweedie - Compound Poisson with Gamma Severity

$$\mu_{AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} \cdot t^{AY+Lag-1}, p = \frac{\alpha+2}{\alpha+1}$$

$$\phi_{AY,Lag} \cdot \mu_{AY,Lag}^{p} = \mu_{AY,Lag} \cdot \tau_{Lag} \cdot \left(1+1/\alpha\right) + c \cdot \mu_{AY,Lag}^{2}$$

- Pick a parameter set $\{ELR_{AY}\}$, $\{Dev_{Lag}\}$, t, c, *Sev*
- Translate parameters into Tweedie parameters
  - $\mu_{AY,Lag}$, p and $\phi_{AY,Lag}$
- Calculate sample parameter sets from posterior distribution with Metropolis-Hastings Algorithm
  - Generate thousands of parameter sets
  - Thinning - Keep a sample of 500 parameter sets

Graphical Representation of Metropolis-Hastings Sample



Note that the posteriors are tighter, showing how the data narrows the range of results.

"Information Reduces Uncertainty"

Claude Shannon

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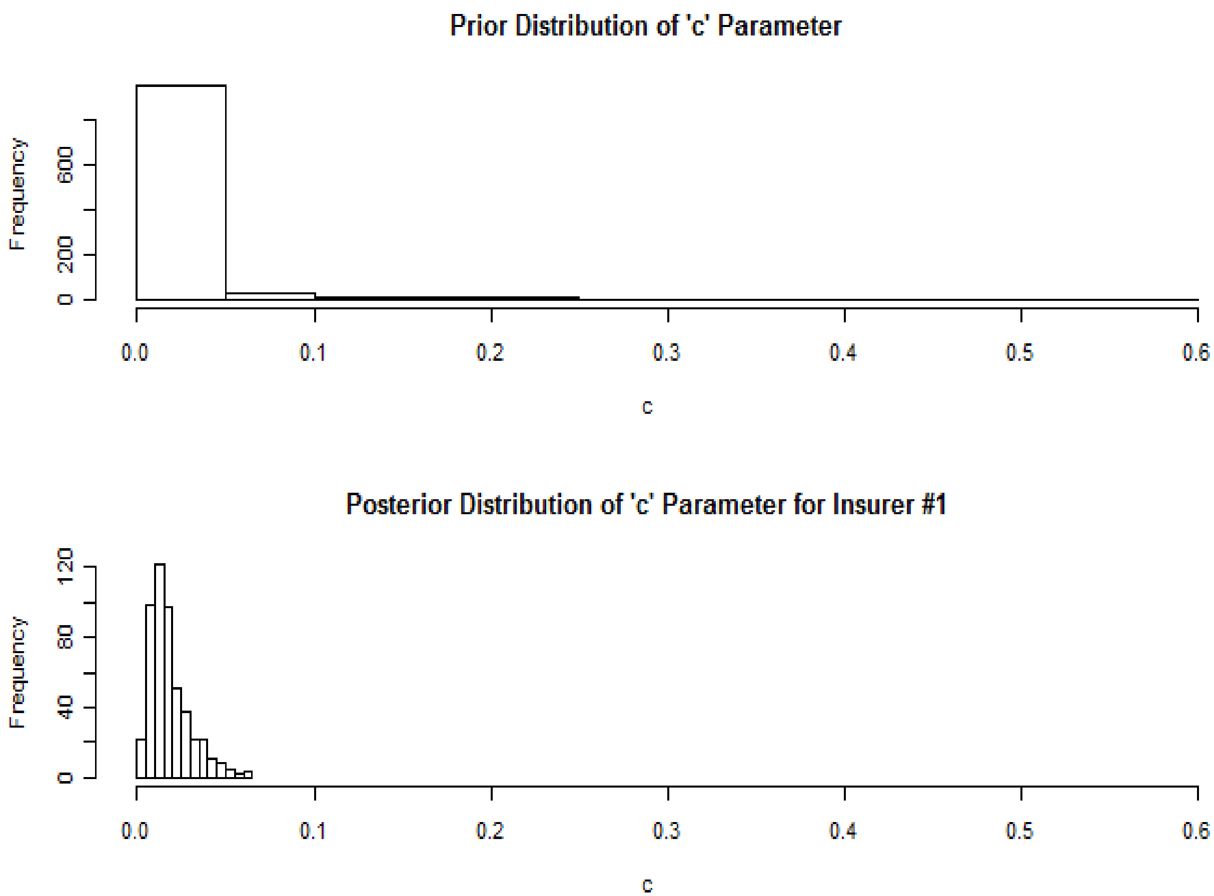

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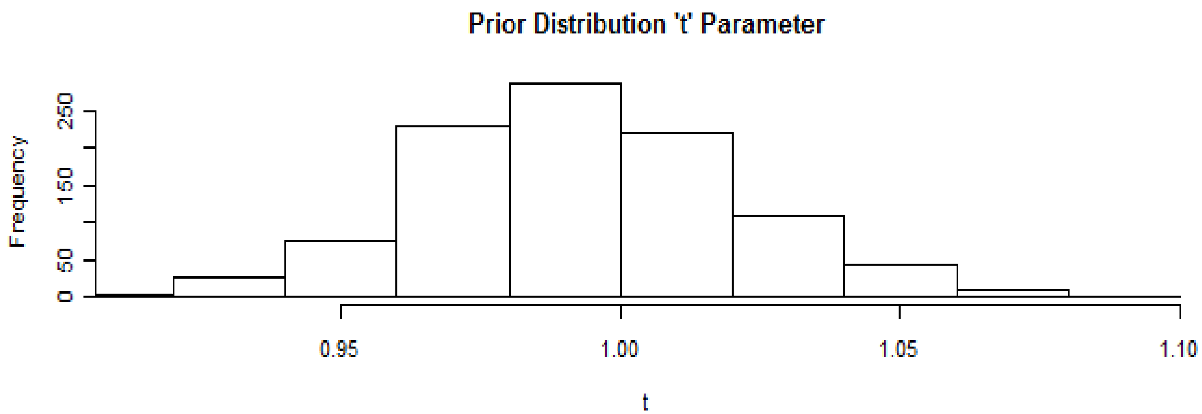
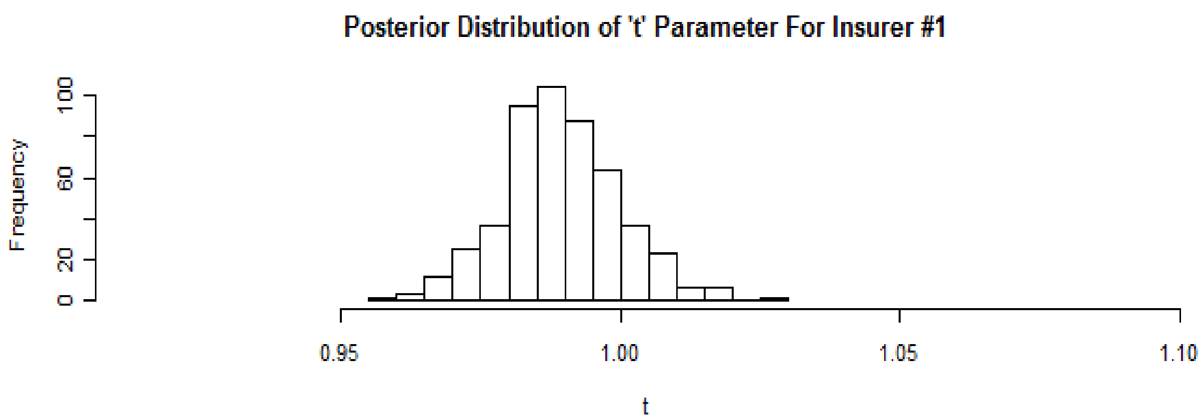
Graphical Representation of Metropolis-Hastings Sample





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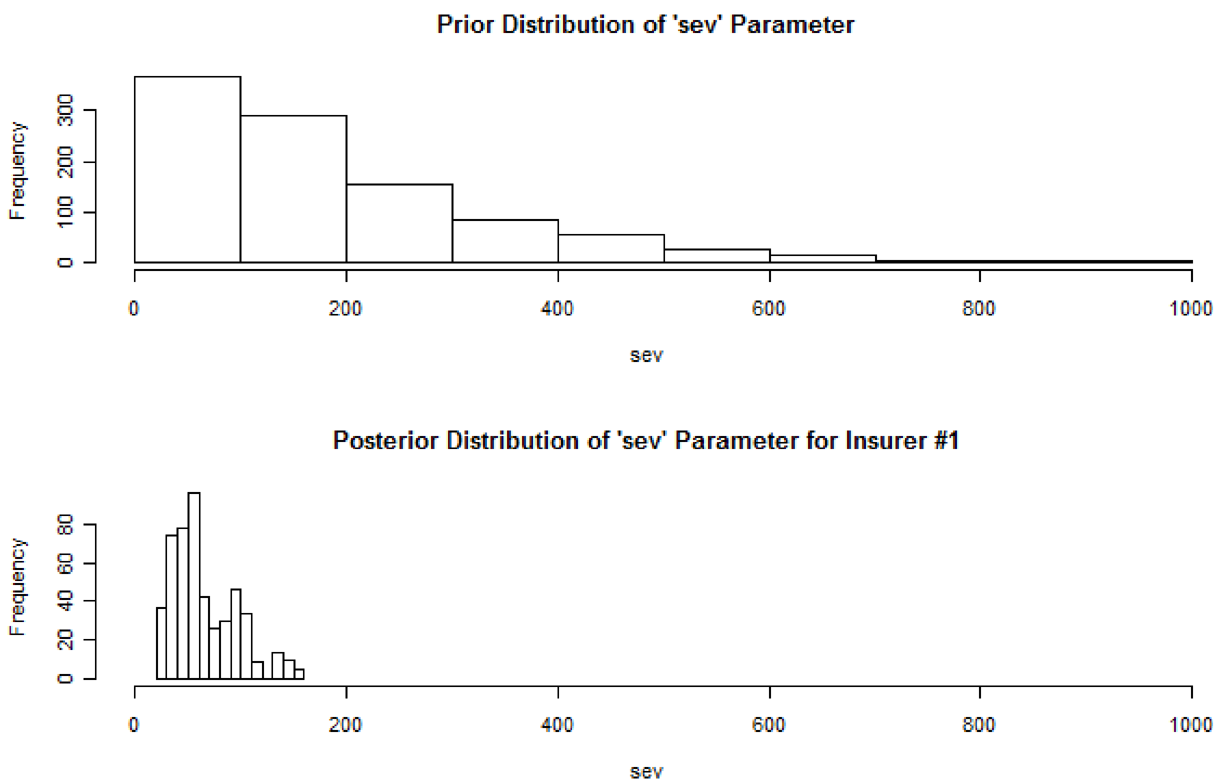

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Calculating the Best Estimate
From Article 77 – Framework Directive

2. The best estimate shall correspond to the probability-weighted average of future cash-flows, taking account of the time value of money (expected present value of future cash-flows), using the relevant risk-free interest rate term structure.

- Expected loss for *n*th Metropolis-Hastings scenario

$$\mu_{n,AY,Lag} = Premium_{AY} \cdot ELR_{AY} \cdot Dev_{Lag} \cdot t^{AY+Lag-1}$$

- Best Estimate $i = 4\%$

$$\sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} \left(\frac{1}{500} \sum_{n=1}^{500} \mu_{n,AY,Lag} \right) \cdot \frac{1}{(1+i)^{AY+Lag-11.5}} = 91{,}220$$

Comparisons against experience

Insurance and reinsurance undertakings shall have processes and procedures in place to ensure that best estimates, and the assumptions underlying the calculation of best estimates, are regularly compared against experience.

Where the comparison identifies systematic deviation between experience and the best estimate calculations of insurance or reinsurance undertakings, the undertaking concerned shall make appropriate adjustments to the actuarial methods being used and/or the assumptions being made.

Distribution of Reserve Outcomes

Incremental Paid Losses

AY	Premium	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_{10}
1	29,701	5,234	5,172	3,708	1,783	923	537	175	145	8	0
2	27,526	5,234	5,683	4,392	2,134	1,377	673	155	81	47	$X_{2,10}$
3	30,750	5,702	5,865	7,966	2,472	NA	143	152	73	$X_{3,9}$	$X_{3,10}$
4	35,814	6,349	4,611	3,959	2,522	1,924	622	206	$X_{4,8}$	$X_{4,9}$	$X_{4,10}$
5	42,277	8,377	6,890	4,055	3,795	1,292	1,422	$X_{5,7}$	$X_{5,8}$	$X_{5,9}$	$X_{5,10}$
6	50,088	9,291	13,836	12,441	4,086	2,293	$X_{6,6}$	$X_{6,7}$	$X_{6,8}$	$X_{6,9}$	$X_{6,10}$
7	56,921	12,029	12,462	8,369	7,034	$X_{7,5}$	$X_{7,6}$	$X_{7,7}$	$X_{7,8}$	$X_{7,9}$	$X_{7,10}$
8	61,406	13,119	12,618	9,117	$X_{8,4}$	$X_{8,5}$	$X_{8,6}$	$X_{8,7}$	$X_{8,8}$	$X_{8,9}$	$X_{8,10}$
9	67,983	15,860	14,893	$X_{9,3}$	$X_{9,4}$	$X_{9,5}$	$X_{9,6}$	$X_{9,7}$	$X_{9,8}$	$X_{9,9}$	$X_{9,10}$
10	73,359	16,498	$X_{10,2}$	$X_{10,3}$	$X_{10,4}$	$X_{10,5}$	$X_{10,6}$	$X_{10,7}$	$X_{10,8}$	$X_{10,9}$	$X_{10,10}$

Predictive Distribution
of Reserve Outcomes – 1 Year

$$R_1 = \sum_{AY=2}^{10} X_{AY,12-AY}$$

Distribution of Reserve Outcomes in Next Diagonal

1. **Select a random parameter set from the list**

$$\left\{sev_n, t_n, c_n, \left\{ELR_{n,AY}\right\}, \left\{Dev_{n,Lag}\right\}\right\}$$

2. **For each (*AY,Lag*) cell in next calendar year (*AY* = 2,…,10, *Lag* = 12 – *AY*):**
   - Calculate $\mu_{AY,Lag}$ from Equation 1.
   - Calculate $\phi_{AY,Lag}$ from Equation 6.
   - Select a random loss $X_{AY,Lag}$ from a Tweedie distribution with parameters p = 1.67, $\mu_{AY,Lag}$, and $\phi_{AY,Lag}$.

3. **Set** $X = \sum_{AY=2}^{10} X_{AY,12-AY}$

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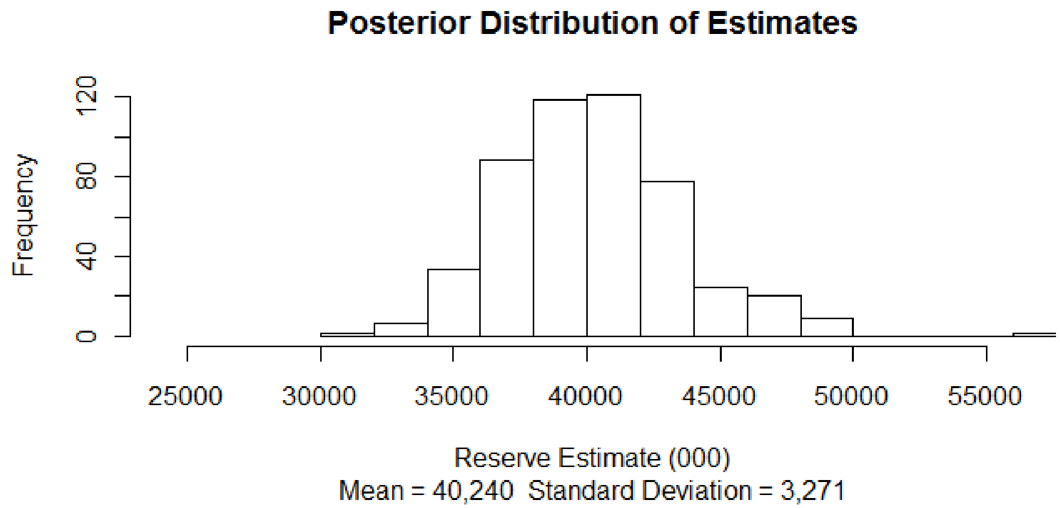
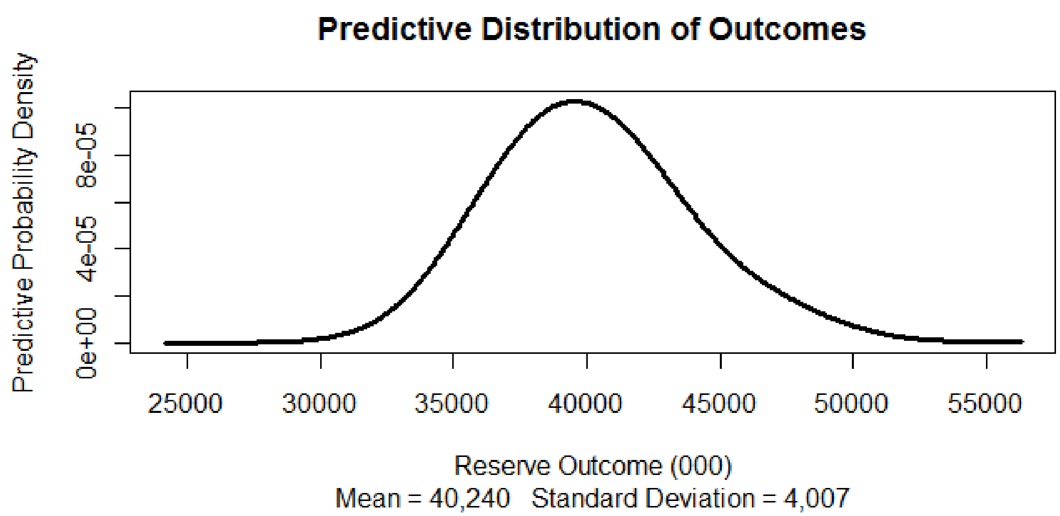
Distribution of Reserve Estimates and Outcomes for Insurer #1

Histogram of $\{\mu_{AY,Lag}\}$ from Metropolis-Hastings output



Can be done by Simulations or FFT

I used FFT



Where the Actual Data Lies on the Predictive Distribution

Insurer	Actual $\sum_{AY=2}^{10} x_{AY,12-AY}$	Expected $\sum_{AY=2}^{10} \mu_{AY,12-AY}$	Ratio $\frac{\text{Actual}}{\text{Expected}}$	p-value
1	41,403	40,240	102.89%	0.6408
2	11,082	13,089	84.67%	0.1080
3	46,735	57,389	81.44%	0.0019
4	102,257	212,926	48.02%	0.0000

Insurers 3 and 4 Need
Close Examination

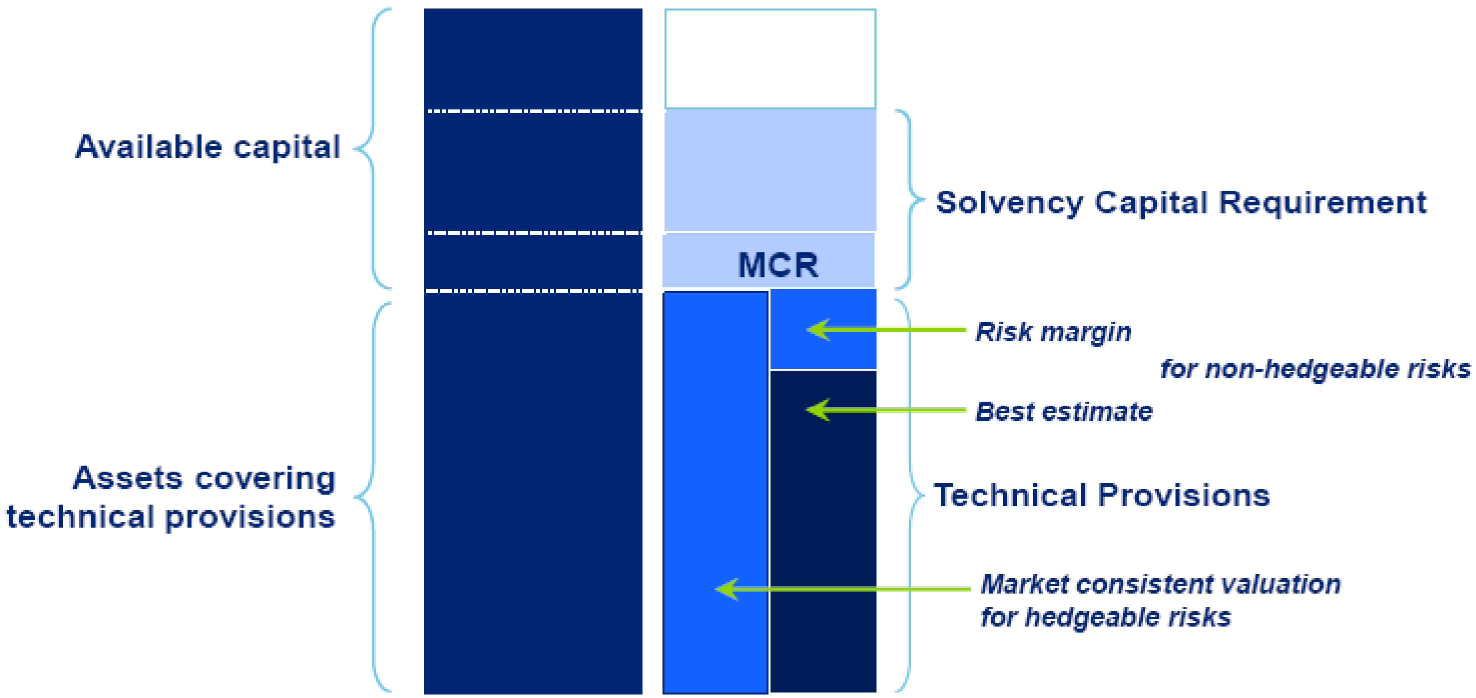
ISO

Technical Provisions, Risk Margins and the IAA

- International Association of Insurance Supervisors (IAIS) requested help from the International Actuarial Association (IAA) to work on the issues of risk based capital and risk margins for loss reserves.
- Available on IIA Website

  http://www.actuaries.org/LIBRARY/Papers/IAA_Measurement_of_Liabilities_2009-public.pdf
- Refer to risk margin as Market Value Margin (MVM)

Focus of Presentation
Technical Provisions



Conceptually in line with IFRS

IAIS – Properties of Risk Margins

1. The less that is known about the current estimate and its trend; the higher the risk margins should be.
2. Risks with low frequency and high severity will have higher risk margins than risks with high frequency and low severity.
3. For similar risks, contracts that persist over a longer timeframe will have higher risk margins than those of shorter duration.
4. Risks with a wide probability distribution will have higher risk margins than those risks with a narrower distribution.
5. To the extent that emerging experience reduces uncertainty, risk margins will decrease, and vice versa.

Possibilities

- **Undiscounted reserves**
  – Only satisfy Property 3
- **Percentile method**
  – Does not satisfy Property 3
- **Cost of Capital Method**
  – Satisfies all properties
- **So – What is the Cost of Capital Method?**

What is Capital?

- **Sufficient for time horizon of one year**
  - Controversial – Many prefer a longer time horizon
- **Capital = TVaR@99% – Expected Loss**
  - Calculate for each future payment year
    - i.e. sub diagonal
  - Remember the time value of money
- **FFT methods on Tweedie distributions allow for reliable calculation of TVaR**

Statistics of Interest for Risk Margin Distribution of Reserve Outcomes

Incremental Paid Losses

AY	*Premium*	Lag_1	Lag_2	Lag_3	Lag_4	Lag_5	Lag_6	Lag_7	Lag_8	Lag_9	Lag_{10}
1	29,701	5,234	5,172	3,708	1,783	923	537	175	145	8	0
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Predictive Distribution of Reserve Outcomes – 1 Year

$$R_1 = \sum_{AY=2}^{10} X_{AY,12-AY}$$

Predictive Distribution of Reserve Outcomes – 10 Year

$$R_{10} = \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}$$

Statistics of Interest for Risk Margins
Predictive Distributions of Reserve Outcomes

- Simulation
  - Randomly select $\{ELR_i\}$,$\{Dev_j\}$, t, c, *Sev* from the posterior
  - Simulate $R_{10} = \sum_{AY=2}^{10} \sum_{Lag=12-AY}^{10} X_{AY,Lag}$ where $X_{AY,Lag}$ ~ Tweedie
- Simulate R_1 Similarly
- Use the Fast Fourier Transform
  - Faster, more accurate, but uses some math

Calculating Capital Needs in the Future
1 Year Time Horizon - Discount @ 4%

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
t	L_t^{Nom}	ΔL_t^{Nom}	L_t^{Disc}	TVaR_t^{Nom}	$\Delta\text{TVaR}_t^{Nom}$	TVaR_t^{Disc}	C_t
0	40,375	13,882	37,526	52,875	15,933	48,415	10,889
1	26,493	12,004	24,870	36,942	15,641	34,103	9,233
2	14,490	6,867	13,624	21,301	8,603	19,516	5,893
3	7,622	3,661	7,165	12,698	4,741	11,524	4,358
4	3,962	1,919	3,719	7,957	2,606	7,150	3,432
5	2,042	766	1,910	5,352	834	4,779	2,869
6	1,276	484	1,205	4,517	230	4,119	2,914
7	792	341	760	4,287	190	4,050	3,290
8	451	451	442	4,097	4,097	4,017	3,575

Calculating Capital Needs in the Future
10 Year Time Horizon - Discount @ 4%

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
t	L_t^{Nom}	ΔL_t^{Nom}	L_t^{Disc}	TVaR_t^{Nom}	$\Delta\text{TVaR}_t^{Nom}$	TVaR_t^{Disc}	C_t
0	97,503	40,375	91,220	128,894	48,491	118,529	27,309
1	57,128	26,493	53,695	80,403	31,742	73,819	20,124
2	30,635	14,490	28,824	48,661	17,133	44,401	15,576
3	16,145	7,622	15,201	31,528	9,412	28,705	13,504
4	8,523	3,962	8,035	22,116	6,225	20,255	12,219
5	4,561	2,042	4,317	15,891	4,321	14,717	10,400
6	2,519	1,276	2,407	11,570	3,673	10,899	8,493
7	1,243	792	1,202	7,898	3,801	7,590	6,388
8	451	451	442	4,097	4,097	4,017	3,575

Explanation of C_t Calculation (Documentation)

(1) The time, t, after the liability is set.

(2) The expected value of all future payments, $L_t^{Nom} = \sum_{AY=2+t}^{10} \sum_{Lag=12+t-AY}^{10} \mu_{AY,Lag}$.

(3) $\Delta L_t^{Nom} = L_t^{Nom} - L_{t+1}^{Nom}$.

(4) The discounted liability, $L_t^{Disc} = \sum_{k=t}^{8} \frac{\Delta L_k^{Nom}}{(1+i)^{k-t+0.5}}$.

(5) The Tail-Value-at-Risk, i.e., the conditional expected value of the random loss,

$\sum_{AY=2+t}^{10} \sum_{Lag=12+t-AY}^{10} X_{AY,Lag}$, given that the loss exceeds the 99th percentile.

(6) $\Delta \text{TVaR}_t^{Nom} = \text{TVaR}_t^{Nom} - \text{TVaR}_{t+1}^{Nom}$.

(7) The discounted $\text{TVaR}_t^{Disc} = \sum_{k=t}^{8} \frac{\Delta \text{TVaR}_k^{Nom}}{(1+i)^{k-t+0.5}}$.

(8) The needed capital at time t is expected to be $C_t = \text{TVaR}_t^{Disc} - L_t^{Disc}$.

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Risk Margin Version 1 – Capital Cash Flow (CCF)

- **C_t = required capital at time *t* for one year.**
  - TVaR@99% - Expected Loss (discounted)
- ***i* = risk-free rate of return**
- ***r* = risky rate of return due to insurer's investors**
- **$MVM_{CCF} = C_0$ – PV(Released Capital @ rate *r*)**

$$MVM_{CCF} = C_0 - \sum_{t=0}^{\infty} \frac{C_t \cdot (1+i) - C_{t+1}}{(1+r)^{t+1}}$$

$$MVM_{CCF} = (r-i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t+1}}$$

After some algebra

Versions 2 and 3

- **Capital Cash Flow (CCF)**

$$MVM_{CCF} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+r)^{t+1}}$$

- **Swiss Solvency Test (SST)**
  - Starts at *t* = 1. Ignores capital raised in first year.
  - Discounts at rate *i* instead of rate *r*.

$$MVM_{SST} = (r - i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

- **Solvency II/QIS4 (SII)**
  - Starts at *t* = 0

$$MVM_{SII} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

All three versions satisfy the IAIS criteria.

Rationale Behind MVM_{SST}

$$MVM_{SST} = (r - i) \cdot \sum_{t=1}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

"The risk margin can be expressed as the expected present value of the cost of capital necessary to buffer the nonhedgeable risk of insurance liabilities during the entire lifetime of the insurance liabilities."

Rationale Behind MVM_{SII}

$$MVM_{SII} = (r - i) \cdot \sum_{t=0}^{\infty} \frac{C_t}{(1+i)^{t+1}}$$

Both Solvency II and SST require capital to cover risk over a one year time horizon. SST says that you don't need a risk margin to cover the first year. Solvency II says you do.

The Results

r = 10% i =4%   Best Estimate = 91,220

Time Horizon	MVM_{CCF}	%	MVM_{SST}	%	MVM_{QIS4}	%
1	1,994	2.2%	1,854	2.0%	2,411	2.6%
10	5,082	5.6%	4,736	5.2%	6,129	6.7%

Summary

- **Used Bayesian analysis and likelihood of a triangle of data based on the Tweedie model to calculate posterior probabilities of scenarios**
- **Used posterior probability of scenarios to calculate**
  - Current Estimate
  - Compare Actual with Prediction
  - Risk margin