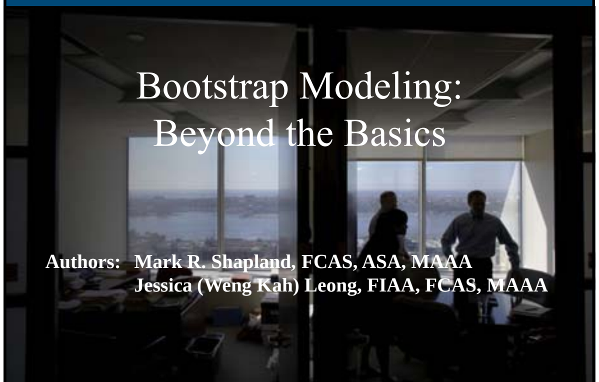
Bootstrap Modeling: Beyond the Basics



Casualty Loss Reserve Seminar
September 20 – 21, 2010

Milliman

Paper Outline

- Introduction
- Notation
- Basic ODP Model / GLM Framework
- Generalizing the GLM Framework
- Practical Data Issues / Algorithm Enhancements
- Model Diagnostics
- Using Multiple Models
- Aggregation Issues / Model Uses
- Testing & Future Research

2 Milliman

Basic ODP Model / GLM Framework

- Start with a triangle of cumulative data:

w \ *d*	1	2	3	…	n-1	n
1	c(1,1)	c(1,2)	c(1,3)	…	c(1,n-1)	c(1,n)
2	c(2,1)	c(2,2)	c(2,3)	…	c(2,n-1)	
3	c(3,1)	c(3,2)	c(3,3)	…		
…	…	…				
n-1	c(n-1,1)	c(n-1,2)				
n	c(n,1)					

- For GLM, we will use the incremental data:

w \ *d*	1	2	3	…	n-1	n
1	q(1,1)	q(1,2)	q(1,3)	…	q(1,n-1)	q(1,n)
2	q(2,1)	q(2,2)	q(2,3)	…	q(2,n-1)	
3	q(3,1)	q(3,2)	q(3,3)	…		
…	…	…				
n-1	q(n-1,1)	q(n-1,2)				
n	q(n,1)					

3 Milliman

Basic ODP Model / GLM Framework

- The GLM formulation is as follows:

$$E[q(w,d)] = m_{w,d}$$
$$Var[q(w,d)] = \phi E[q(w,d)] = \phi m_{w,d}^{z}$$
$$\ln[m_{w,d}] = \eta_{w,d}$$

$\eta_{w,d} = c + \alpha_w + \beta_d$, where: $w = 1, 2, \dots, n$; $d = 1, 2, \dots, n$; and $\alpha_1 = \beta_1 = 0$

$z = 0(Normal),\ 1(Poisson),$
$2(Gamma),\ or\ 3(Inverse\,Gaussian)$
$\phi = Scale\ Parameter$

- Alternatively:

$\eta_{w,d} = \alpha_w + \beta_d$, where: $w = 1, 2, \dots, n$ and $d = 2, 3, \dots, n$

4

Milliman

Basic ODP Model / GLM Framework

- Let's consider a simple example:

	1	2	3
1	$q(1,1)$	$q(1,2)$	$q(1,3)$
2	$q(2,1)$	$q(2,2)$	
3	$q(3,1)$		

- Transforming to a log scale:

	1	2	3
1	$\ln[q(1,1)]$	$\ln[q(1,2)]$	$\ln[q(1,3)]$
2	$\ln[q(2,1)]$	$\ln[q(2,2)]$	
3	$\ln[q(3,1)]$		

- Specify a system of equations with vectors α_w and β_d :

$$\ln[q(1,1)] = 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 0\beta_2 + 0\beta_3$$
$$\ln[q(2,1)] = 0\alpha_1 + 1\alpha_2 + 0\alpha_3 + 0\beta_2 + 0\beta_3$$
$$\ln[q(3,1)] = 0\alpha_1 + 0\alpha_2 + 1\alpha_3 + 0\beta_2 + 0\beta_3$$
$$\ln[q(1,2)] = 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 1\beta_2 + 0\beta_3$$
$$\ln[q(2,2)] = 0\alpha_1 + 1\alpha_2 + 0\alpha_3 + 1\beta_2 + 0\beta_3$$
$$\ln[q(1,3)] = 1\alpha_1 + 0\alpha_2 + 0\alpha_3 + 1\beta_2 + 1\beta_3$$

5

Milliman

Basic ODP Model / GLM Framework

- Converting to matrix notation we have:

$$Y = X \times A$$

- Where:

$$Y = \begin{bmatrix} \ln[q(1,1)] & 0 & 0 & 0 & 0 & 0 \\ 0 & \ln[q(2,1)] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ln[q(3,1)] & 0 & 0 & 0 \\ 0 & 0 & 0 & \ln[q(1,2)] & 0 & 0 \\ 0 & 0 & 0 & 0 & \ln[q(2,2)] & 0 \\ 0 & 0 & 0 & 0 & 0 & \ln[q(1,3)] \end{bmatrix},$$

$$X = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \text{ and } A = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \beta_2 \\ \beta_3 \end{bmatrix}$$

6

Milliman

Basic ODP Model / GLM Framework

- Solving for the parameters in A that minimize the difference between Y and W, where:

$$W = \begin{bmatrix} \ln[m_{1,1}] & 0 & 0 & 0 & 0 & 0 \\ 0 & \ln[m_{2,1}] & 0 & 0 & 0 & 0 \\ 0 & 0 & \ln[m_{3,1}] & 0 & 0 & 0 \\ 0 & 0 & 0 & \ln[m_{1,2}] & 0 & 0 \\ 0 & 0 & 0 & 0 & \ln[m_{2,2}] & 0 \\ 0 & 0 & 0 & 0 & 0 & \ln[m_{1,3}] \end{bmatrix}$$

- X = Design Matrix, and W = Weight Matrix
- Then we have:

$$\ln[m_{1,1}] = \eta_{1,1} = \alpha_1$$
$$\ln[m_{2,1}] = \eta_{2,1} = \alpha_2$$
$$\ln[m_{3,1}] = \eta_{3,1} = \alpha_3$$
$$\ln[m_{1,2}] = \eta_{1,2} = \alpha_1 + \beta_2$$
$$\ln[m_{2,2}] = \eta_{2,2} = \alpha_2 + \beta_2$$
$$\ln[m_{1,3}] = \eta_{1,3} = \alpha_1 + \beta_2 + \beta_3$$

and

	1	2	3
1	$\ln[m_{1,1}]$	$\ln[m_{1,2}]$	$\ln[m_{1,3}]$
2	$\ln[m_{2,1}]$	$\ln[m_{2,2}]$	
3	$\ln[m_{3,1}]$		

Finally, exponentiating we get:

	1	2	3
1	$m_{1,1}$	$m_{1,2}$	$m_{1,3}$
2	$m_{2,1}$	$m_{2,2}$	
3	$m_{3,1}$		

7 Milliman

Basic ODP Model / GLM Framework

- Using this "GLM Framework" and setting z=1 (Poisson), the solution exactly replicates the "fitted" values using volume-weighted average age-to-age ratios!
- This is generally referred to as the Over-Dispersed Poisson (ODP) Bootstrap model.
- Instead of solving the GLM, we can simplify by using the volume-weighted average ratios.
- We refer to this as the "Simplified GLM"
- The "Simplified GLM" also improves issues with negative incremental values.

8 Milliman

Basic ODP Model / GLM Framework

- Using a model fit to the data, bootstrapping involves sampling the residuals with replacement, using:

$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}}$$

- Where: $N = Number\ of\ Cells\ in\ Triangle\ [e.g.,\ n \times (n+1) \div 2]$

  $p = Number\ of\ Parameters\ [e.g.,\ 2 \times n - 1]$

- From the sampled residuals and fitted incremental values, we can derive a sample triangle using:

$$q'(w,d) = r^{*} \times \sqrt{m_{w,d}^{z}} + m_{w,d}$$

9 Milliman

Bootstrap Modeling: Beyond the Basics

Basic ODP Model / GLM Framework

- However, in order to correct for a bias in (and standardize) the residuals, the GLM framework requires a hat matrix adjustment factor:

$$H = X\left(X^T W X\right)^{-1} X^T W$$

$$f_{w,d}^H = \sqrt{\frac{1}{1-H_{i,i}}}$$

Can approximate with Degrees of Freedom Adjustment Factor:

$$f^{DoF} = \sqrt{\frac{N}{N-p}}$$

- Thus:

$$r_{w,d}^H = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^z}} \times f_{w,d}^H$$

And Scaled Residuals:

$$r_{w,d}^S = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^z}} \times f^{DoF}$$

- Continuing the bootstrap process…

10

Milliman

Basic ODP Model / GLM Framework

- Each sample incremental triangle can be converted to cumulative values
- Sample age-to-age factors can be calculated (parameter risk)
- A point estimate can be calculated
- We can add process variance to the future incremental values (from the point estimate) using a Poisson (or Gamma) distribution assuming each incremental cell is the mean and the variance is the cell value times the scale parameter (i.e., to over-disperse the variance):

$$\phi = \frac{\sum r_{w,d}^2}{N-p}$$

- Repeat a significant number of iterations.

11

Milliman

Basic ODP Model / GLM Framework

For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Cumulative Data

	1	2	3	4	5	6
1	95	150	180	200	210	215
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	180			
5	130	170				
6	125					

1) Actual Cumulative Data

Cumulative Data

	1	2	3	4	5	6
1	95	150	180	200	210	215
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	180			
5	130	170				
6	125					
Factors:	1.429	1.151	1.128	1.037	1.024	

2) Avg. Age-to-Age Factors

12

Milliman

Bootstrap Modeling: Beyond the Basics

Basic ODP Model / GLM Framework

For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Fitted Cumulative Data

	1	2	3	4	5	6
1	109.16	155.94	179.45	202.50	210.00	215.00
2	109.16	155.94	179.45	202.50	210.00	
3	113.20	161.71	186.10	210.00		
4	109.49	156.41	180.00			
5	119.00	170.00				
6	125.00					
Factors:		1.429	1.429	1.429	1.429	1.429

3) "Fit" Cumulative Data

Fitted Incremental Data

	1	2	3	4	5	6
1	109.16	46.78	23.51	23.05	7.50	5.00
2	109.16	46.78	23.51	23.05	7.50	
3	113.20	48.51	24.39	23.90		
4	109.49	46.92	23.59			
5	119.00	51.00				
6	125.00					

4) "Fitted" Incremental Data

13 Milliman

Basic ODP Model / GLM Framework

For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Incremental Data

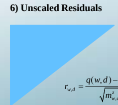
	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

5) Actual Incremental Data

Unscaled Pearson residuals

	1	2	3	4	5	6
1	-1.35	1.20	1.34	-0.64	0.91	0.00
2	0.08	0.47	-1.76	1.45	-0.91	
3	-0.77	1.65	0.12	-0.80		
4	1.00	-1.74	0.29			
5	1.01	-1.54				
6	0.00					

6) Unscaled Residuals



$$r_{w,d} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}}$$

14 Milliman

Basic ODP Model / GLM Framework

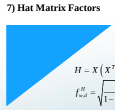
For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Hat Matrix Factors

	1	2	3	4	5	6
1	1.652	1.273	1.225	1.295	1.440	0.000
2	1.652	1.273	1.225	1.295	1.440	
3	1.683	1.283	1.235	1.309		
4	1.795	1.299	1.237			
5	2.057	1.347				
6	0.000					

7) Hat Matrix Factors



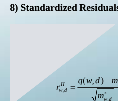
$$H = X\left(X^{T}WX\right)^{-1}X^{T}W$$

$$f_{w,d}^{H} = \sqrt{\frac{1}{1 - H_{i,i}}}$$

Standardized Pearson residuals

	1	2	3	4	5	6
1	-2.24	1.53	1.64	-0.82	1.31	0.00
2	0.13	0.60	-2.15	1.87	-1.31	
3	-1.30	2.12	0.15	-1.04		
4	1.80	-2.26	0.36			
5	2.07	-2.07				
6	0.00					

8) Standardized Residuals



$$r_{w,d}^{H} = \frac{q(w,d) - m_{w,d}}{\sqrt{m_{w,d}^{z}}} \times f_{w,d}^{H}$$

15 Milliman

Basic ODP Model / GLM Framework

For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Random Residuals

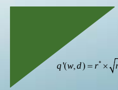
	1	2	3	4	5	6
1	-1.31	-2.07	0.36	-2.07	-2.07	1.64
2	-0.82	-0.82	-2.26	1.64	0.36	
3	2.07	1.87	1.87	-2.15		
4	-1.31	-0.82	1.80			
5	-0.82	1.80				
6	0.60					

9) Sample Random Residuals

Sample Incremental Triangle

	1	2	3	4	5	6
1	95.42	32.59	25.26	13.09	1.82	8.66
2	100.56	41.16	12.55	30.92	8.49	
3	135.27	61.57	33.64	13.38		
4	95.73	41.29	32.34			
5	110.03	63.88				
6	131.70					

10) Sample Incremental Data



$$q'(w,d) = r^* \times \sqrt{m^z_{w,d}} + m_{w,d}$$

16 Milliman

Basic ODP Model / GLM Framework

For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Sample Cumulative Triangle

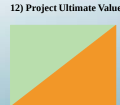
	1	2	3	4	5	6
1	95.42	128.01	153.27	166.36	168.18	176.84
2	100.56	141.72	154.27	185.19	193.68	
3	135.27	196.84	230.49	243.87		
4	95.73	137.02	169.37			
5	110.03	173.91				
6	131.70					
Factors:		1.448	1.172	1.107	1.029	1.052

11) Sample Age-to-Age Factors

Projected Cumulative Data

	1	2	3	4	5	6
1	95.42	128.01	153.27	166.36	168.18	176.84
2	100.56	141.72	154.27	185.19	193.68	203.65
3	135.27	196.84	230.49	243.87	251.02	263.95
4	95.73	137.02	169.37	187.43	192.93	202.87
5	110.03	173.91	203.81	225.55	232.16	244.13
6	131.70	190.67	223.46	247.30	254.55	267.66

12) Project Ultimate Values



17 Milliman

Basic ODP Model / GLM Framework

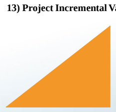
For the "Simplified GLM":

- Here's a simple example using a 6 x 6 triangle:

Projected Incremental Data

	1	2	3	4	5	6	Point Estimate
1							
2						9.98	9.98
3					7.15	12.93	20.08
4				18.07	5.49	9.94	33.50
5			29.91	21.74	6.61	11.96	70.22
6		58.98	32.79	23.84	7.25	13.11	135.97
							269.74

13) Project Incremental Values



Incremental Data w/ Process Variance

	1	2	3	4	5	6	Possible Outcome
1							
2						15.20	15.20
3					9.85	9.71	19.57
4				24.60	9.35	13.52	47.47
5			29.81	14.12	5.97	13.52	63.43
6		44.97	38.24	24.46	11.79	9.10	128.56
							274.23

14) Add Process Variance

$$\phi = \frac{\sum r^2_{w,d}}{N-p}$$

- Repeat steps 9 – 14, 10,000 times!

18 Milliman

Generalizing the GLM Framework

For the "Simplified GLM":

- Using Incurred data (instead of Paid) results in a distribution of IBNR
- We can adjust by converting Incurred ultimate values to a "random" paid development pattern
- We could also adjust the "calculate point estimate step" by switching to a Bornhuetter-Ferguson, Cape Cod or other methodology.

19

Generalizing the GLM Framework

For the "GLM Framework":

- We can abandon the need to have age-to-age factors:
- Here's a simple example using a 6 x 6 triangle:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					
	Factors:	1.429	1.151	1.128	1.037	1.024

Model Parameters

α_1	4.69
α_2	4.69
α_3	4.73
α_4	4.70
α_5	4.78
α_6	4.83
β_2	-0.85
β_3	-0.69
β_4	-0.02
β_5	-1.12
β_6	-0.41

Fitted Values

	1	2	3	4	5	6
1	109.16	46.78	23.51	23.05	7.50	5.00
2	109.16	46.78	23.51	23.05	7.50	
3	113.20	48.51	24.39	23.90		
4	109.49	46.92	23.59			
5	119.00	51.00				
6	125.00					

20

Generalizing the GLM Framework

For the "GLM Framework":

- We can abandon the need to have age-to-age factors:
- And use only one α_w or "level" parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.74
β_2	-0.87
β_3	-0.70
β_4	-0.02
β_5	-1.13
β_6	-0.41

Fitted Values

	1	2	3	4	5	6
1	114.17	48.00	23.75	23.33	7.50	5.00
2	114.17	48.00	23.75	23.33	7.50	
3	114.17	48.00	23.75	23.33		
4	114.17	48.00	23.75			
5	114.17	48.00				
6	114.17					

21

Generalizing the GLM Framework

For the “GLM Framework”:

- We can abandon the need to have age-to-age factors:
- Or use only one β_d or “development” parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.65
α_2	4.65
α_3	4.68
α_4	4.61
α_5	4.71
α_6	4.83
β_2	-0.65

Fitted Values

	1	2	3	4	5	6
1	104.61	54.76	28.66	15.00	7.85	4.11
2	104.17	54.53	28.54	14.94	7.82	
3	108.20	56.64	29.65	15.52		
4	100.14	52.42	27.44			
5	111.59	58.41				
6	125.00					

22

Milliman

Generalizing the GLM Framework

For the “GLM Framework”:

- We can abandon the need to have age-to-age factors:
- Or use only one α_w and one β_d parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.69
β_2	-0.66

Fitted Values

	1	2	3	4	5	6
1	108.53	55.93	28.83	14.86	7.66	3.95
2	108.53	55.93	28.83	14.86	7.66	
3	108.53	55.93	28.83	14.86		
4	108.53	55.93	28.83			
5	108.53	55.93				
6	108.53					

23

Milliman

Generalizing the GLM Framework

For the “GLM Framework”:

- We can abandon the need to have age-to-age factors:
- And add a calendar period parameter using:

$$\eta_{w,d} = \alpha_w + \beta_d + \gamma_k$$

Where: $w = 1, 2, \ldots, n$
$d = 2, 3, \ldots, n$
$k = 2, 3, \ldots, n$

24

Milliman

Bootstrap Modeling: Beyond the Basics

Generalizing the GLM Framework

For the “GLM Framework”:

- We can abandon the need to have age-to-age factors:
- And use only one α_w and one β_d and one γ_k parameter:

Incremental Data

	1	2	3	4	5	6
1	95	55	30	20	10	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	25			
5	130	40				
6	125					

Model Parameters

α_1	4.63
β_2	-0.67
γ_2	0.02

Fitted Values

	1	2	3	4	5	6
1	102.20	53.31	27.81	14.51	7.57	3.95
2	104.65	54.59	28.48	14.85	7.75	
3	107.16	55.90	29.16	15.21		
4	109.74	57.24	29.86			
5	112.37	58.62				
6	115.07					

25 Milliman

Practical Data Issues

- Negative Incremental Values
- Non-Zero Sum of Residuals
- Using an N-Year Weighted Average
- Missing Values
- Outliers
- Heteroscedasticity → **Problems with fit**
- Mis-shapen Data
- Exposure Adjustment
- Parametric Bootstrapping

26 Milliman

Practical Data Issues

1. Heteroscedasticity → Problems with fit

2. Mis-shapen Data

27 Milliman

Residuals

AY	1	2	3	4	5	6	7	8	9	10
2000	28	-28	18	-27	-11	0	7	-2	7	
2001	82	-83	16	-39	-15	31	-3	6	-7	
2002	48	-48	18	-55	8	-10	-3	-4		
2003	0	1	-15	26	-3	-34	-1			
2004	-139	138	-3	36	13	14				
2005	-35	36	-20	29	6					
2006	-35	35	-1	20						
2007	13	-12	-8							
2008	52	-52								
2009										

28

Milliman

Residuals

AY	1	2	3	4	5	6	7	8	9	10
2000	28	-28	18	-27	-11	0	7	-2	7	
2001	82	-83	16	-39	-15	31	-3	6	-7	
2002	48	-48	18	-55	8	-10	-3	-4		
2003	0	1	-15	26	-3	-34	-1			
2004	-139	138	-3	36	13	14				
2005	-35	36	-20	29	6					
2006	-35	35	-1	20						
2007	13	-12	-8							
2008	52	-52								
2009										

29

Milliman

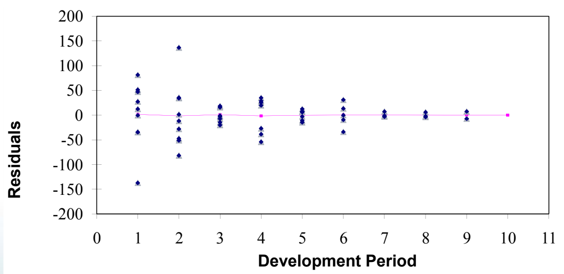
Residuals

AY	1	2	3	4	5	6	7	8	9	10
2000	28	-28	18	-27	-11	0	7	-2	7	
2001	82	-83	16	-39	-15	31	-3	6	-7	
2002	48	-48	18	-55	8	-10	-3	-4		
2003	0	1	-15	26	-3	-34	-1			
2004	-139	138	-3	36	13	14				
2005	-35	36	-20	29	6					
2006	-35	35	-1	20						
2007	13	-12	-8							
2008	52	-52								
2009										

30

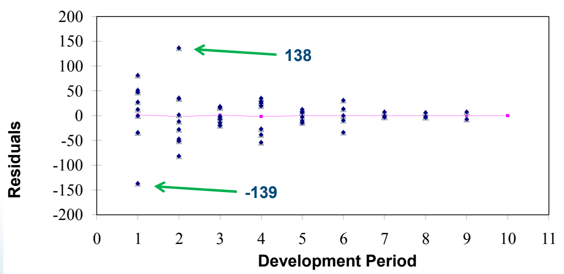
Milliman

Plot of Residuals against Development Period



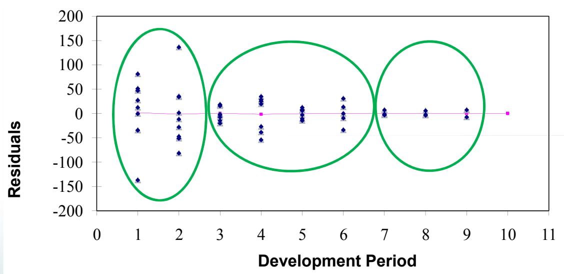
31 Milliman

Plot of Residuals against Development Period



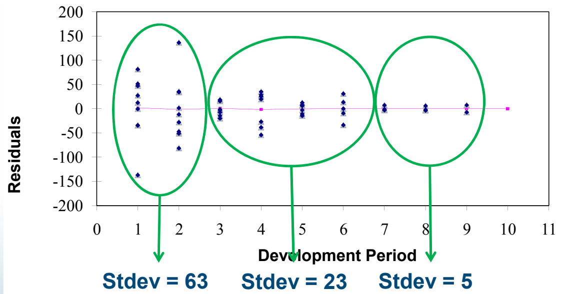
32 Milliman

Plot of Residuals against Development Period



33 Milliman

Plot of Residuals against Development Period



34

Milliman

Calculating Hetero-factors

$$\frac{\text{Standard deviation group 1}}{\text{Standard deviation group 2}} = \frac{63}{23}$$

= 2.8 = hetero-factor(d=3 to 6)

$$\frac{\text{Standard deviation group 1}}{\text{Standard deviation group 3}} = \frac{63}{5}$$

= 11.7 = hetero-factor(d=7 to 9)

35

Milliman

Residuals

AY	1	2	3	4	5	6	7	8	9	10
2000	28	-28	18	-27	-11	0	7	-2	7	
2001	82	-83	16	-39	-15	31	-3	6	-7	
2002	48	-48	18	-55	8	-10	-3	-4		
2003	0	1	-15	26	-3	-34	-1			
2004	-139	138	-3	36	13	14				
2005	-35	36	-20	29	6					
2006	-35	35	-1	20						
2007	13	-12	-8							
2008	52	-52								
2009										

x 2.8 (columns 3 to 6)

x 11.7 (columns 7 to 9)

36

Milliman

Bootstrap Modeling: Beyond the Basics

Hetero-adjusted Residuals

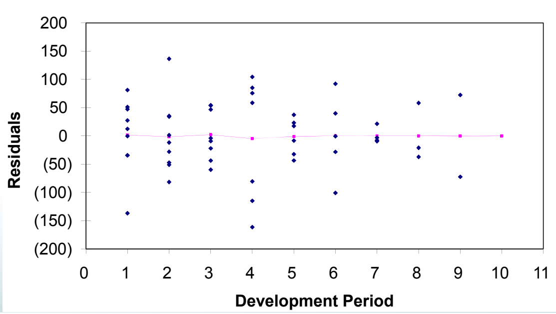
Residuals

AY	1	2	3	4	5	6	7	8	9	10
2000	28	-28	50	-75	-30	-1	84	-25	85	
2001	82	-83	43	-107	-41	86	-31	69	-85	
2002	48	-48	51	-150	22	-26	-37	-44		
2003	0	1	-41	70	-8	-94	-13			
2004	-139	138	-9	97	35	37				
2005	-35	36	-56	79	16					
2006	-35	35	-4	55						
2007	13	-12	-21							
2008	52	-52								
2009										

Stdev = 63 Stdev = 63 Stdev = 63

37 Milliman

Plot of Hetero-adjusted Residuals



38 Milliman

Formulas

residual = (actual – fitted) / √fitted

actual* = fitted + √fitted x residual

39 Milliman

Bootstrap Modeling: Beyond the Basics

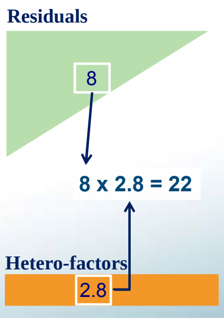
Formulas with Hetero-factors

$$\text{residual}(d) = \frac{\text{actual} - \text{fitted}}{\sqrt{\text{fitted}}} \times \text{hetero-factor}(d)$$

$$\text{actual}(d)^* = \text{fitted} + \frac{\sqrt{\text{fitted}} \times \text{residual}}{\text{hetero-factor}(d)^*}$$

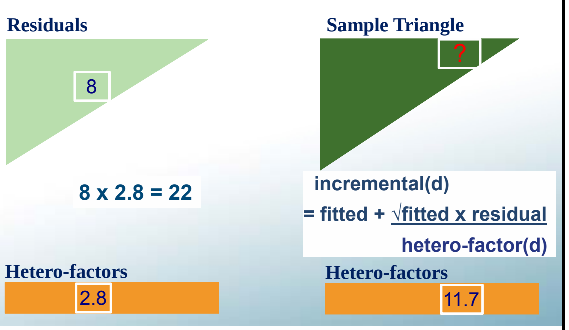
40 Milliman

Residuals

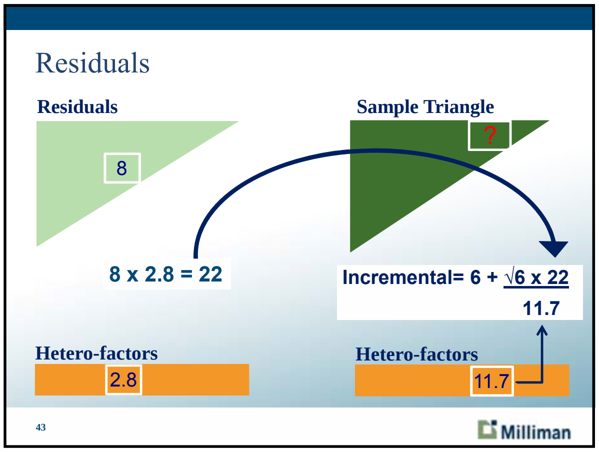


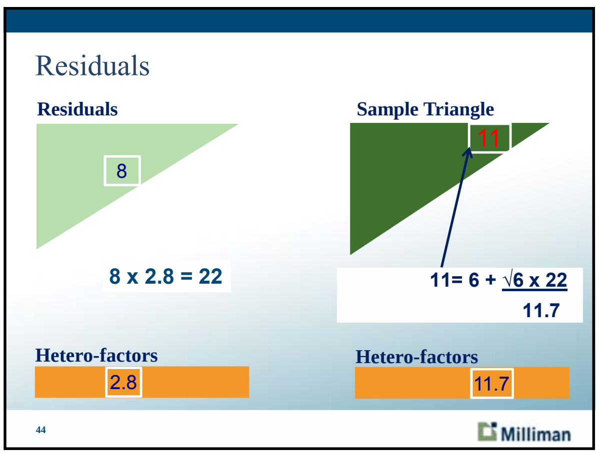
41 Milliman

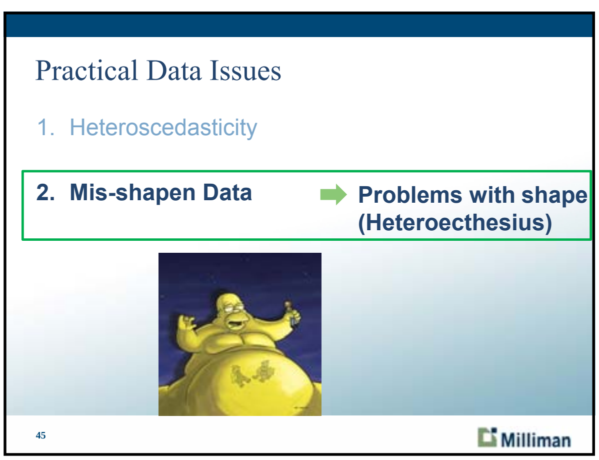
Residuals



42 Milliman


Residuals
Residuals
8
8 x 2.8 = 22
Sample Triangle
?
Incremental= 6 + √6 x 22
11.7
Hetero-factors
2.8
Hetero-factors
11.7
43
Milliman


Residuals
Residuals
8
8 x 2.8 = 22
Sample Triangle
11
11= 6 + √6 x 22
11.7
Hetero-factors
2.8
Hetero-factors
11.7
44
Milliman


Practical Data Issues
1. Heteroscedasticity
2. Mis-shapen Data
Problems with shape (Heteroecthesius)
45
Milliman

1. Problems with data: Mis-shapen

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

Triangle as of June 2009

46 Milliman

1. Problems with data: Mis-shapen

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

47 Milliman

1. Problems with data: Mis-shapen

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

48 Milliman

Bootstrap Modeling: Beyond the Basics

1. Problems with data: Mis-shapen

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

49 Milliman

Fitted triangle

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

Actual triangle as of June 2009

Fitted triangle as of December 2008

50 Milliman

Age-to-age factors: interpolate

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

12-24 24-36 36-48 48-60

51 Milliman

Interpolate for fitted triangle

6	←	18	←	30	←	42	←	54
6	←	18	←	30	←	42		
6	←	18	←	30				
6	←	18						
6								

12-24	24-36	36-48	48-60
6-18	18-30	30-42	42-54

52 Milliman

Actual data

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

53 Milliman

Solution

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

54 Milliman

Bootstrap Modeling: Beyond the Basics

Gross-up last diagonal

12	24	36	48	60
12	24	36	48	
12	24	36		
12	24			
12				

55

Milliman

Gross-up last diagonal

12	24	36	48	60
12	24	36	48	
12	24	36		
12	24			
12				

Fitted triangle as of December 2009

56

Milliman

Sample triangle

12	24	36	48	60
12	24	36	48	
12	24	36		
12	24			
12				

12-24 24-36 36-48 48-60

57

Milliman

Solution

12	24	36	48	54
12	24	36	42	
12	24	30		
12	18			
6				

12-24	24-36	36-48	48-60
6-18	18-30	30-42	42-54

58 Milliman

Solution

12		24		36		48		54
12		24		36		42	→	54
12		24		30	→	42	→	54
12		18	→	30	→	42	→	54
6	→	18	→	30	→	42	→	54

12-24	24-36	36-48	48-60
6-18	18-30	30-42	42-54

59 Milliman

Model Diagnostics

- Residual Graphs
- Normality Test
- Outliers
- Parameter Adjustment
- Model Results

60 Milliman

Summary

- GLM
- Practical data issues
- Model Diagnostics

61

Milliman