



# **Practical Solutions to Reserving Problems**

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**2010 CAS Casualty Loss Reserve Seminar**

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## Speakers and Topics

- Julie Joyce: Moderator
- Scott Kaminski: Minimum Bias LDFs
- Tom Toce: Deriving Tail Factors
- Susan Furray: A Hindsight Analysis of Common Reserving Methodologies

# What is Minimum Bias?

- An iterative method to determine relativities for multiple dimensions
- It is easily verifiable and auditable - forecast error equals zero
- Lacks a formal test of a variable's statistical significance
  - However, an informal test will be proposed

## What is Minimum Bias? (continued)

The minimum bias iterative formula reduces down to:

$$\text{Dim } x_G = \frac{\sum_G (\text{Pure premium})_{GT} (\text{Exposures})_{GT}}{\sum_G (\text{Base})_{GT} (\text{Dim } y_T)}$$

### Steps

- “Prime the pump”
- Repeat!

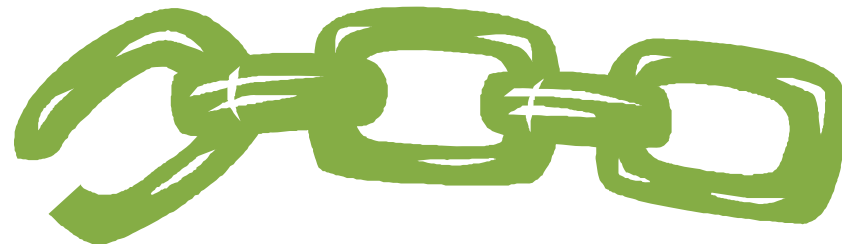


# Weighted Average Loss Development Factors

The weight assigned to each link ratio equals losses at age 3

| AY   | Age (Month) |       | Link Ratio | Weight | Diff. between Proj & Actual |
|------|-------------|-------|------------|--------|-----------------------------|
|      | 3           | 6     |            |        |                             |
| 2006 | 264         | 345   | 1.31       | 0.25   | 130                         |
| 2007 | 500         | 1,014 | 2.03       | 0.48   | (114)                       |
| 2008 | 274         | 509   | 1.86       | 0.26   | (16)                        |
|      | 1,038       | 1,868 |            | 1.00   | (0)                         |

Weighted average link ratio  
(also computed as 1,868/1,038) 1.80



## Minimum Bias LDFs

Let's revise our original formula:

$$\text{Dim } x_G = \frac{\sum_G (\text{Pure Premium})_{GT} (\text{Exposures})_{GT}}{\sum_G (\text{Base})(\text{Exposures})_{GT} (\text{Dim } y_T)}$$

Step 1: Replace (Pure Premium) with an incremental LDF

Step 2: Exposures is the “weight” assigned to an observation.

To compute the wtd. avg. LDF, (Exposures) = Losses

$$\text{Dim } x_G = \frac{\sum_G (\text{Age to age factor} - 1)_{GT} (\text{Losses})_{GT}}{\sum_G (\text{Losses})_{GT} (\text{Dim } y_T)}$$

# Minimum Bias LDF Model

First, update the model with loss data

| <u>AY</u> | <u>AQ</u> | <u>AGE</u> | <u>DIM1</u> | <u>DIM2</u> | <u>DIM3</u> | <u>DIM4</u> | <u>INCURREDS</u> |
|-----------|-----------|------------|-------------|-------------|-------------|-------------|------------------|
| 1991      | 1         | 3          | AUTO        | LIABILITY   | CA          | N           | 23,906           |
| 1991      | 1         | 3          | AUTO        | LIABILITY   | MN          | N           | 3,054            |
| 1991      | 1         | 3          | AUTO        | LIABILITY   | MN          | N           | 625              |
| 1991      | 1         | 3          | AUTO        | LIABILITY   | MN          | N           | 5,656            |
| 1991      | 1         | 3          | AUTO        | PROPERTY    | CA          | N           | 30,369           |
| 1991      | 1         | 3          | AUTO        | PROPERTY    | MN          | N           | 27,107           |
| 1991      | 1         | 3          | HOME        | LIABILITY   | MN          | N           | 4,649            |
| 1991      | 1         | 3          | HOME        | PROPERTY    | MN          | N           | 37,891           |
| 1991      | 2         | 3          | AUTO        | LIABILITY   | CA          | N           | 4,952            |



# Minimum Bias LDF Model

Second, make initial selections for each of the factors.

## INITIAL VALUES

| AGE | Factor | DIM1 | Factor | DIM2      | Factor | DIM3 | Factor | DIM4 | Factor |
|-----|--------|------|--------|-----------|--------|------|--------|------|--------|
| 3   | 1.00   | Auto | 1.10   | LIABILITY | 1.20   | MN   | 1.00   | N    | 1.00   |
| 6   | 0.70   | Home | 0.90   | PROPERTY  | 0.80   | CA   | 1.00   |      |        |
| 9   | 0.49   |      |        |           |        |      |        |      |        |
| 12  | 0.34   |      |        |           |        |      |        |      |        |
| 15  | 0.24   |      |        |           |        |      |        |      |        |
| 18  | 0.17   |      |        |           |        |      |        |      |        |
| 21  | 0.12   |      |        |           |        |      |        |      |        |
| 24  | 0.08   |      |        |           |        |      |        |      |        |
| 27  | 0.06   |      |        |           |        |      |        |      |        |

This is the incremental LDF  
(Age-to-Age LDF - 1.00).

## How does it work?

| AGE | Factor |
|-----|--------|
| 3   | 1.02   |
| 6   | 0.18   |
| 9   | 0.16   |
| 12  | 0.11   |
| 15  | 0.08   |
| 18  | 0.07   |
| 21  | 0.06   |
| 24  | 0.07   |
| 27  | 0.04   |

| DIM1 | Factor |
|------|--------|
| Auto | 1.11   |
| Home | 0.89   |

| DIM2      | Factor |
|-----------|--------|
| LIABILITY | 1.34   |
| PROPERT   | 0.61   |

| DIM3 | Factor |
|------|--------|
| MN   | 1.00   |
| CA   | 1.01   |

Our example will assume:

Age: 3

State: MN

Covg. type: Liability

Coverage: Auto

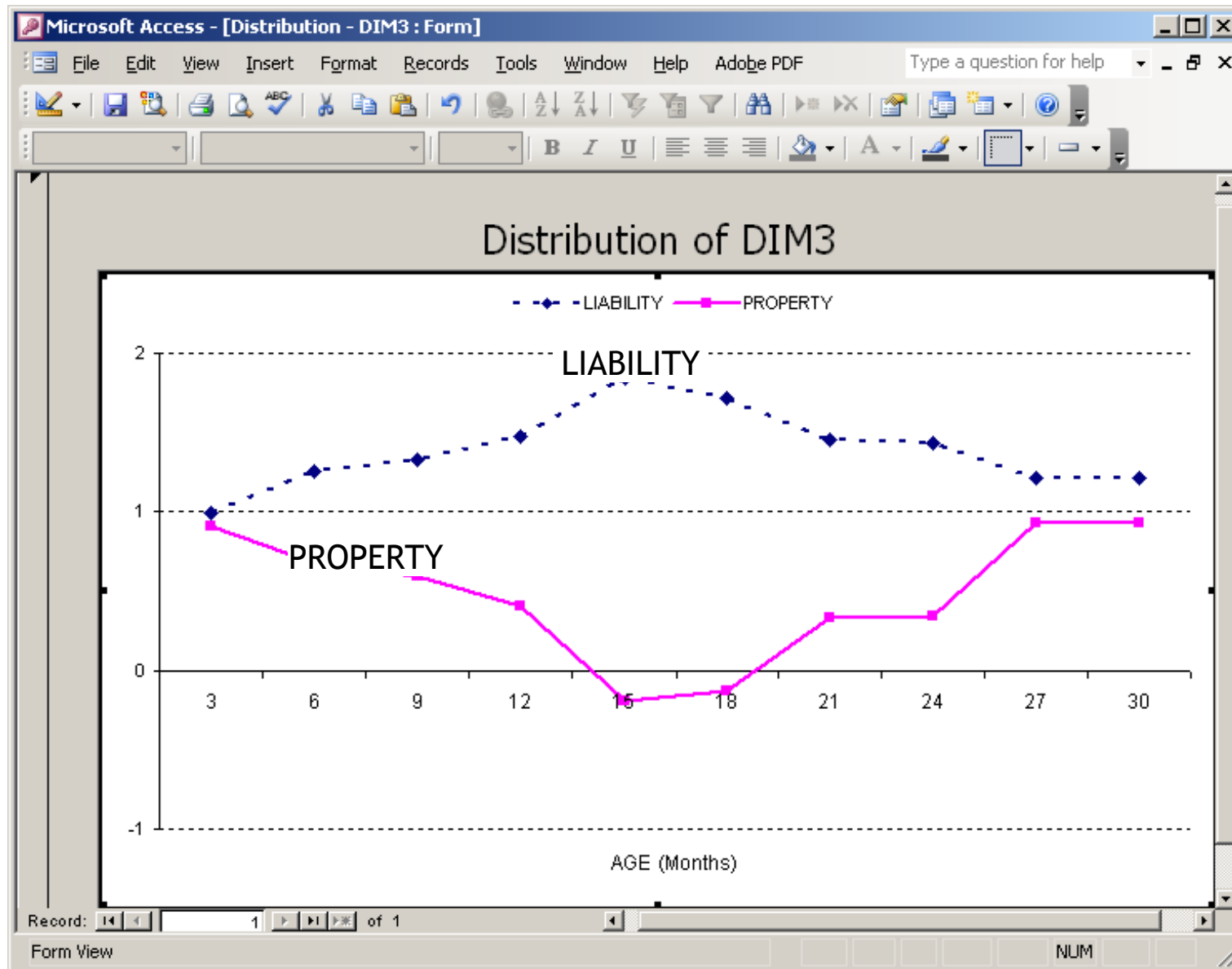
Multiplying all the factors equals:

$$1.02 \times 1.00 \times 1.34 \times 1.11 = 1.52$$

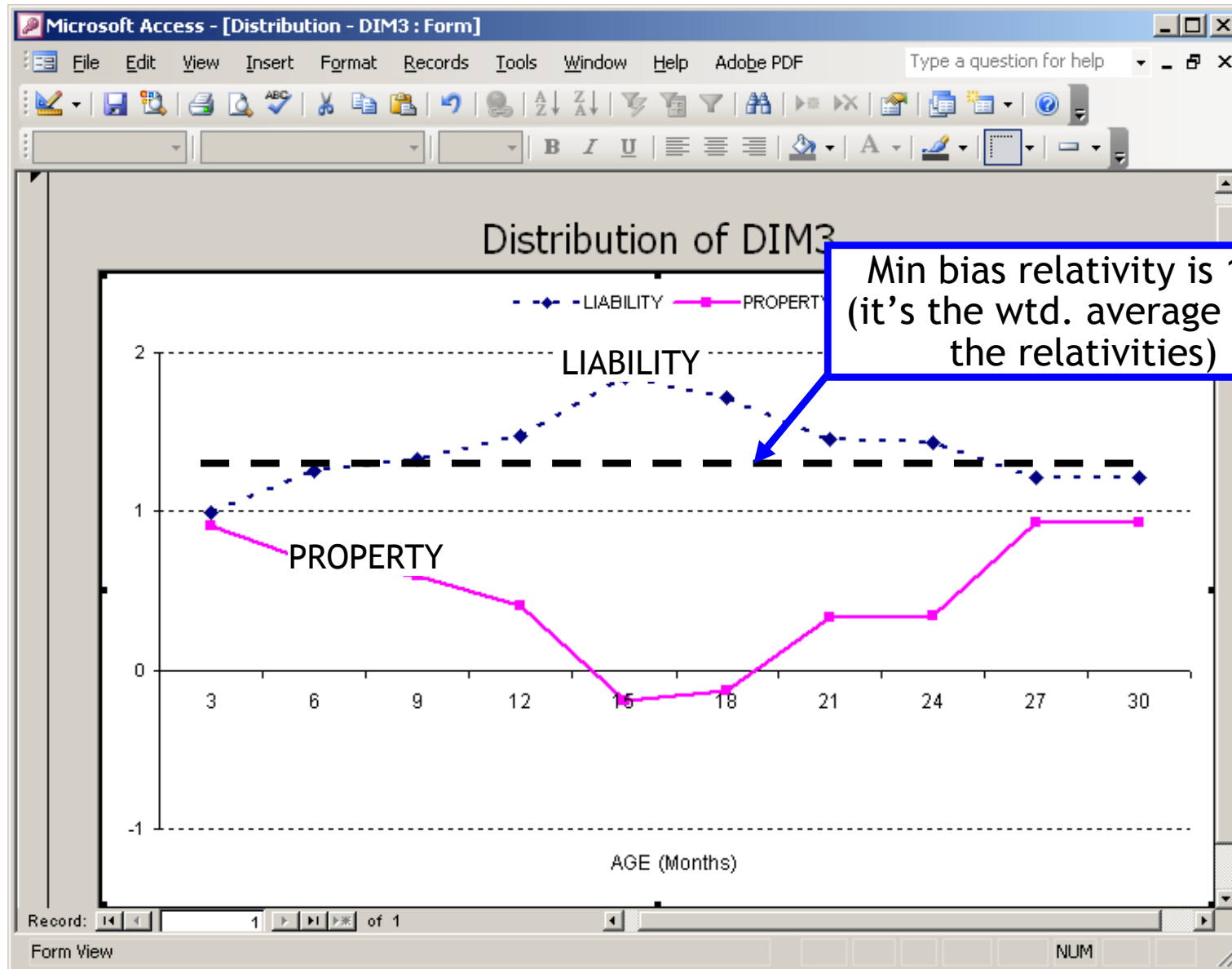
Since this is the incremental factor, we add one. The final minimum bias LDF is then 2.52.



# Distribution of Minimum Bias Factors



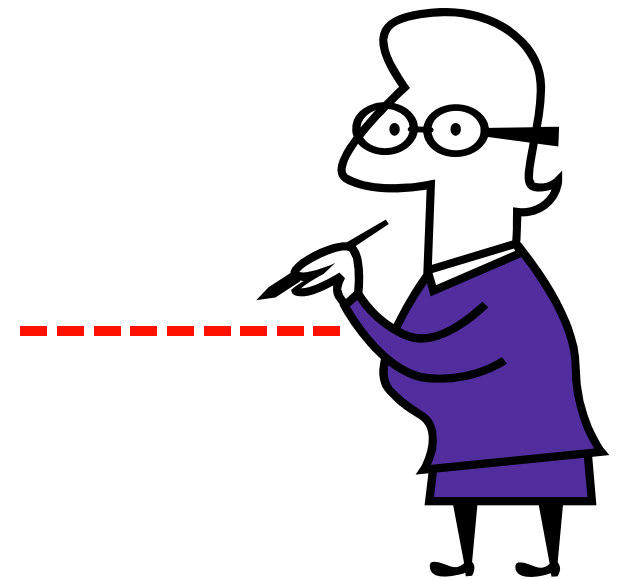
# Distribution of Minimum Bias Factors



Min bias relativity is 1.34  
(it's the wtd. average of all  
the relativities)

## Applications and Benefits

- A thoughtful way to reflect the unique development of a dimension which lacks credibility
- Allows you to create loss development patterns in line with the dimensions used in pricing
- Readily auditable and explainable





It assumes each dimension develops the same as the 'countrywide' curve.

Losses have to be greater than zero at each age

# References

Anderson, et. al., “A Practitioner’s Guide to Generalized Linear Models”

Bailey & Simon, “Two Studies in Automobile Insurance Ratemaking”

Berquist & Sherman, “Loss Reserve Adequacy Testing: A Comprehensive, Systematic Approach”

Feldblum & Brosius, “Minimum Bias Procedure: A Practitioner’s Guide”

Mildenhall, “A Systematic Relationship Between Minimum Bias and Generalized Linear Models”

Peck, “Discussion of A Simulation Test of Prediction Errors of Loss Reserve Estimation Techniques”

Taylor & McGuire, “Loss Reserving with GLMS: A Case Study”

These are all available via the CAS website.