Calculating reserve risk over a one-year time horizon

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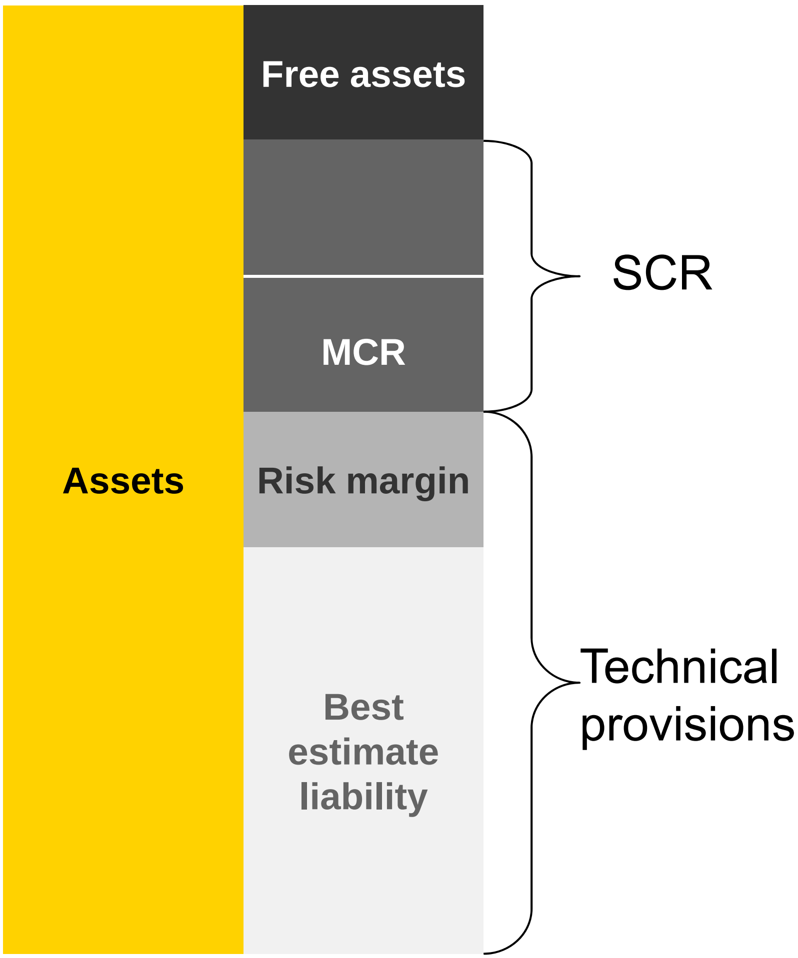
Items for discussion

- Background
- Solvency II and VaR
- What is a time horizon?
- Methods to measure reserve risk
- Concept and approach
- Model derivation
- Validation and results
- Observations

Background

- Emerging capital frameworks
  - Swiss solvency test
  - Solvency II (S-II)
  - Bermuda
- Market value-based frameworks are emerging as a regulatory paradigm
- New capital frameworks use VaR and one-year time horizon
- VaR and time horizon have their roots in daily mark-to-market asset trading frameworks

S-II view of market consistent balance sheet

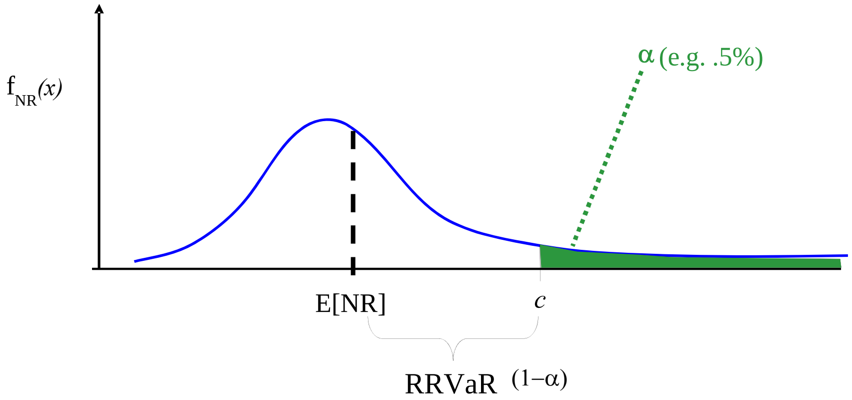


- Assets valued at market value
- Capital requirements
  - Solvency Capital Requirement (SCR) = first regulatory intervention point
  - Minimum Capital Requirement (MCR) = final regulatory intervention point
- Technical provisions
  - = best estimate of all future cash flows discounted at a risk-free interest rate
  - + risk margin for non-hedgeable risks

S-II and VaR

- S-II requires a VaR model for reserve risk ("RRVaR")

$$F_{NR}(x) = \Pr(NR \le x)$$
$$F_{NR}(c) = \Pr(NR \le c) = 1 - \alpha$$
$$RRVaR_{(1-\alpha)} = \left|c - E[NR]\right|$$
$$RRVaR_{(1-\alpha)} = EC_{(1-\alpha)}$$
Over a fixed annual interval, ≥ one - year where $c \ge E[NR]$



S-II and VaR

- S-II is a market value-based framework for capital adequacy analysis
- Key components of VaR
  - Definition of adverse event: loss in market value
  - Time horizon: one year
  - Risk metric: S-II picks 99.5 percentile or 1 in 200-year event
- VaR is at odds with traditional statutory insurance reporting
  - Book value of assets
  - No discounting or risk margins in loss reserves
  - Adverse events are changes in accounting values
  - Time horizons are “life of liabilities”
  - Reserve ranges are relative to best estimate and are not typically loss distributions

What is a time horizon?

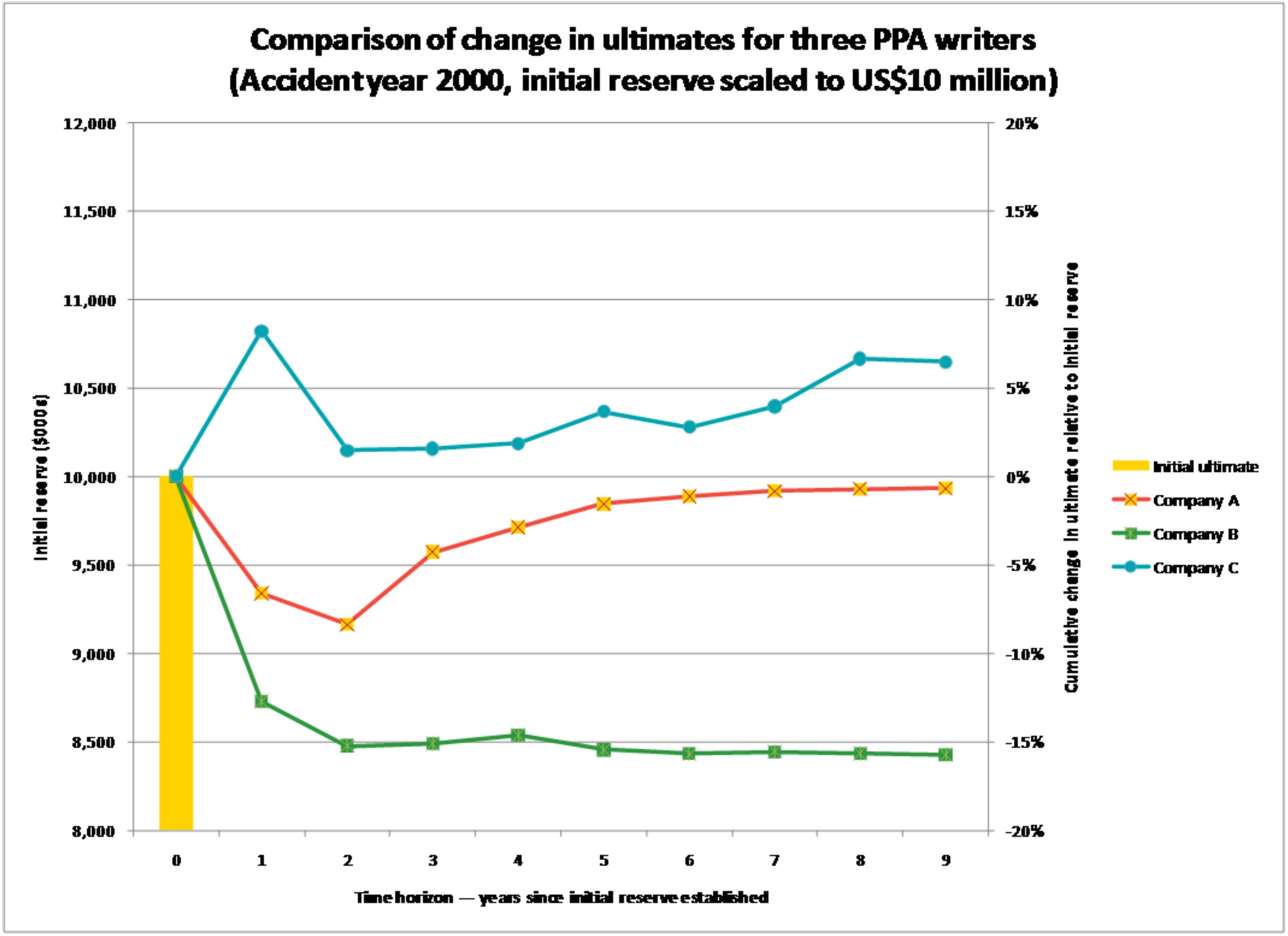
- Changing views of current and future conditions impact estimates of value

Factor influencing estimates	Market value of a simple corporate bond	Property-Casualty loss reserve
1	Bond structure	Loss experience
2	Inflation levels	Economic conditions
3	Interest rates	Legal environment
4	Credit spreads	Actuarial indications
5	Company-specific issues	Management judgment
…	…	…

What is a time horizon?

- Period of time over which adverse events can emerge
- Different than standard loss development
- Focus for reserve risk is on how the recorded reserve estimate can change over time
- How does the current assessment of future events change over time?
- Estimates can be impacted by many items over time:
  - **Emerging loss experience**
  - **Emerging economic conditions**
  - **Changing legal environment**
  - **Changing actuarial results (single or multiple methods)**
  - **Changing interpretation of actuarial results**
  - **Management interpretation of results**
  - **Interpretation of competitor actions and results**

What is a time horizon?



Potential methods to measuring reserve risk over a time horizon

1. Simulate variability of actuarial data triangles
   - How does the variability in losses impact indications?
   - Mack, Bootstrap, etc
2. Analyze variability in historic reserve estimates
   - Parameterize distributions based on financial statement history
   - Review historic outcomes of the reserving process rather than variability of inputs
   - A long-term, consistent financial statement history is required for meaningful analysis

Concept of historical outcomes-based reserve risk model

Characteristics of reserving process	Model requirements	Modeling technique
1. Multi-step decision process not solely reliant on actuarial indications	Focus on variability of outcomes, not variability of indications	Utilize historic variability in financial statement estimates
2. Prior changes in estimates impact likelihood of future changes	Incorporate changes to date in ultimate estimates	Regression to estimate future changes using cumulative changes as a predictor variable
3. Volume impacts variability of reserve estimates	Reflect volume-related impacts on variability of estimates	Utilized volume as a scaling factor in regression variables
4. Changes in estimates, which are sometimes extreme, occur regularly	Reasonable reflection of variability in estimates including tail events	Utilized by long-term historic changes from individual companies/groups for the entire P/C insurance industry
5. Reserves for adjacent AYs tend to have common assumptions and movement	Total reserves should be viewed as a portfolio of AYs at different levels of maturity	Correlate movement of AYs within portfolio and across time-steps

Approach to develop and apply RRVaR model

Data construction and analysis → **Fit and validate model inputs** → **Create simulation model** → **Produce and interpret results**

Data construction and analysis

1. Create dataset
2. Determine appropriate AY model structure

Fit and validate model inputs

1. Fit model parameters for each AY
2. Assess the model fit
3. Calculate AY correlation and calendar year (CY) correlation

Create simulation model

1. Build one-year simulation model
2. Build multiple-year simulation model using one-year model output as inputs

Produce and interpret results

1. Summarize results of model for time horizon of interest

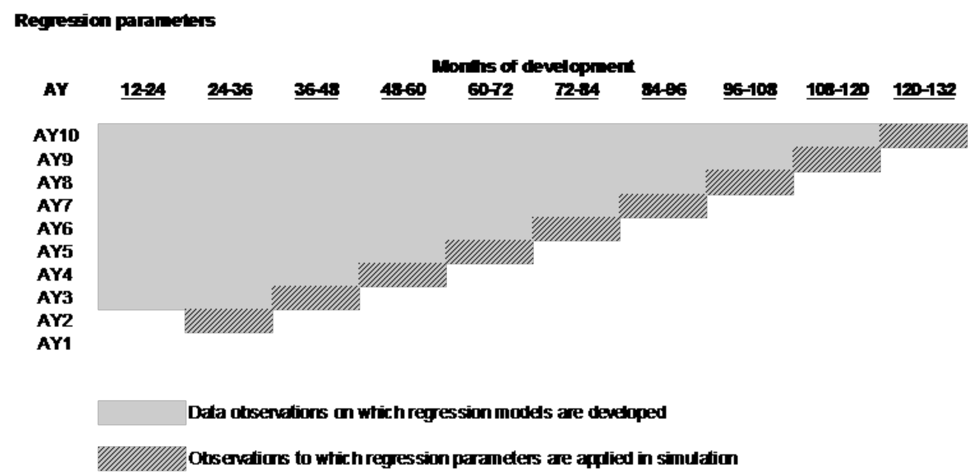
Data construction and analysis

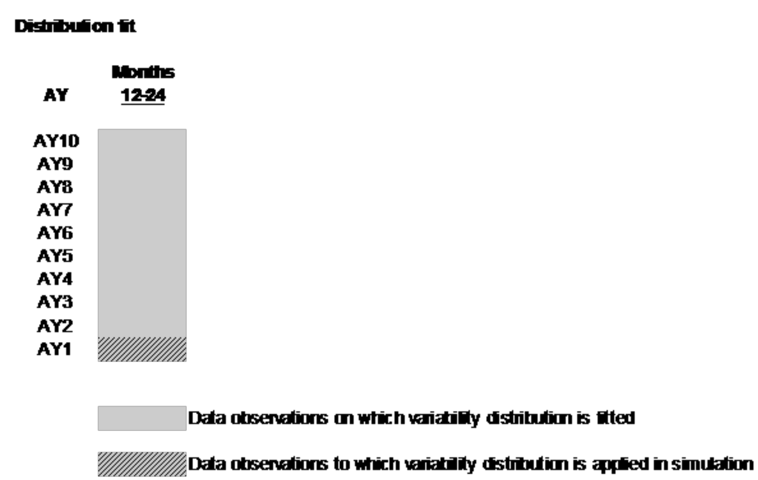
- Database construction from 1998–2009 annual statements (AS)
  - AYs back to 1990 for PPAL (Schedule P Pt B)
  - Data for individual companies aggregated to group level, using group definitions in 2009
  - Between 3,000 and 5,000 data points depending on AY maturity
  - Transformations of data variables necessary due to size–dependent variability
    - Variability decreases as the size of the company increases
    - Scaling factor (base-ten logarithm of AY initial ultimate) adjusts variables to common scale for analysis
- Variable analysis and regression performed using SAS
- SAS outputs used as inputs to simulation model to produce results for sample company

Use of data in model development and application

- The colored region of the triangle represents the values used to determine the model parameters used in the forecasted estimates.
- The shaded boxes of the triangle represent forecasted values using the parameters.
- The most recent year, not having a second evaluation, cannot be forecasted using the regression parameters.

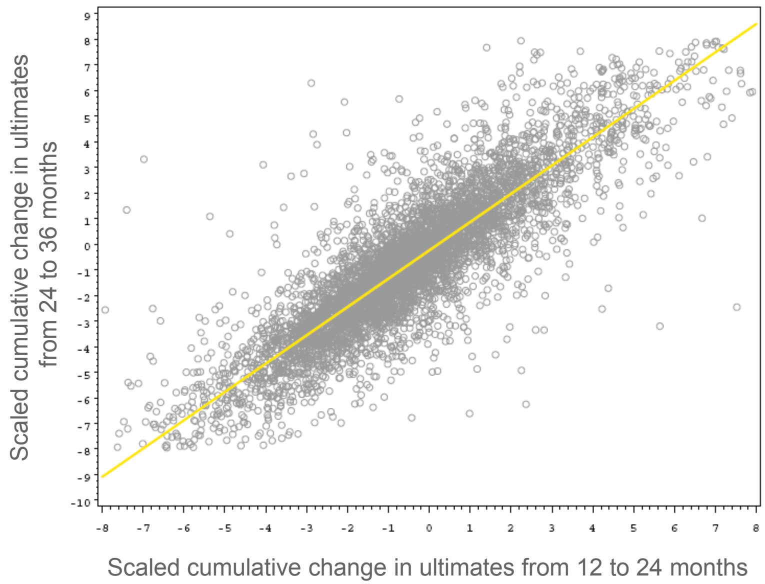
- A distribution can be used to estimate the change from 12 to 24 months.
- The colored column represents the values included in the distribution fit used to estimate the change from 12 to 24 months.
- The shaded box in the column represents the value generated using the distribution.





Fitting regression model for AY change in ultimate

- Regression model developed using AYs with at least two changes in ultimate (AY2 – AY10)



$$ChgUlt_k = CChgUlt_{k+1} - CChgUlt_k$$

$$= \left(\frac{CChgUlt_{k+1}}{Res_1}\cdot v^2 - \frac{CChgUlt_k}{Res_1}\cdot v^2\right)\cdot\frac{Res_1}{v^2}$$

$$= \left(\left(\beta_{0k} + \beta_{1k}\cdot\frac{CChgUlt_k}{Res_1}\cdot v^2 + \varepsilon_k\right) - \frac{CChgUlt_k}{Res_1}\cdot v^2\right)\cdot\frac{Res_1}{v^2}$$

$$= \left((\beta_{0k} + \varepsilon_k) + (\beta_{1k} - 1)\cdot\frac{CChgUlt_k}{Res_1}\cdot v^2\right)\cdot\frac{Res_1}{v^2}$$

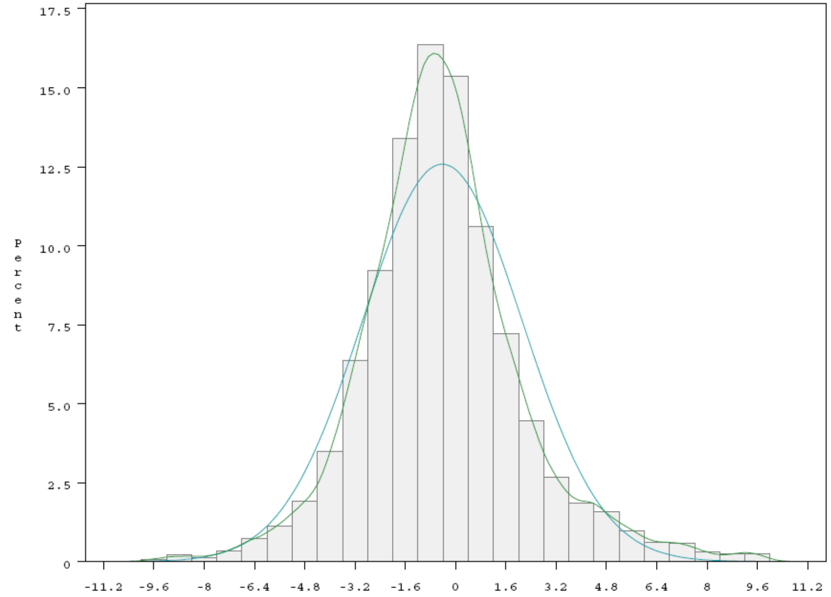
$$= (\beta_{0k} + \varepsilon_k)\cdot\frac{Res_1}{v^2} + (\beta_{1k} - 1)\cdot CChgUlt_k$$

$ChgUlt_k$	Change in Ultimate between k and k+1 periods
$CChgUlt_k$	Cumulative Change in Ultimate through period k
Ult_1	Initial Ultimate
Res_1	Initial Reserve (at time 1)
v	Scaling factor, equals $\log_{10}(Ult_1)$,
$\beta_{0k}, \beta_{1k}, \varepsilon_k$	Regression coefficients/terms, model at period k

- With the initial reserve and the current change in ultimate to date as input values, a linear regression model is used to estimate the cumulative change in ultimate after 1 additional year.

Fitting distribution for AY change in ultimate

- Estimate of reserve variability from 12 month evaluation to 24 month evaluation (AY1 only)



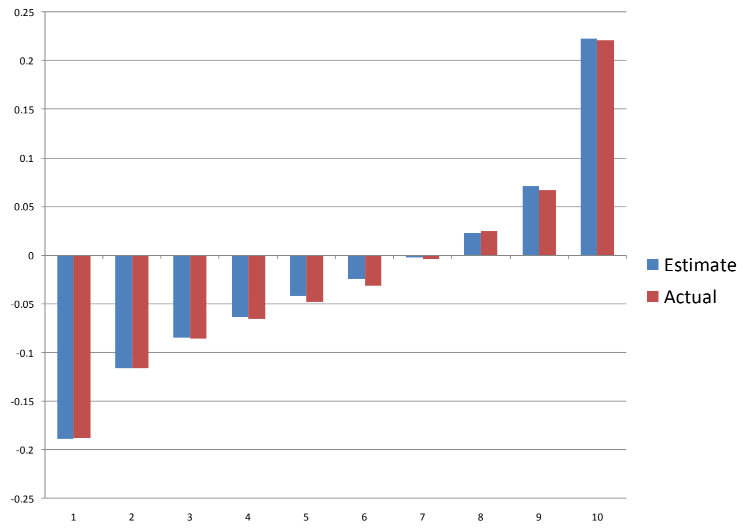
Distribution of scaled change in ultimate between 12 and 24 months

- The first change cannot be modeled using regression, because there is no prior change
- The data variable for the first period follows a positively-skewed, heavy-tailed distribution
- Sampled values can be used to provide an estimated change in ultimate between 12 and 24 months.

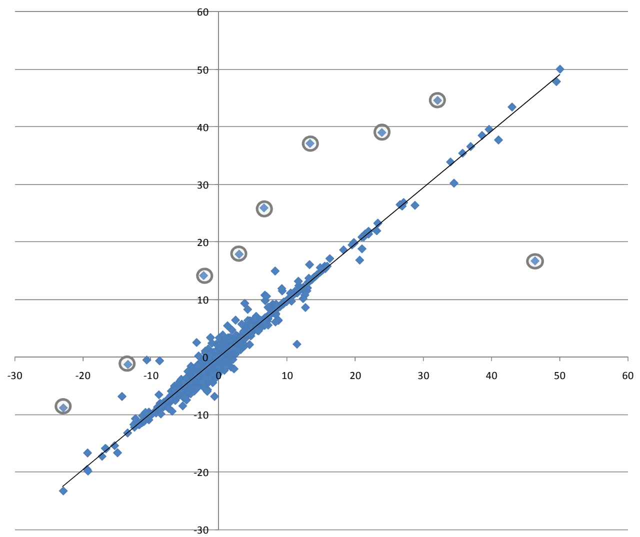
Validation of regression models using AS data as of 12/31/09

- Parameters from models fit on data from AS data for 2008 and earlier were applied to data from 2009 AS to test model "goodness of fit"

Lift chart



Scatter plot



- The model results fit the 2009 AS data.
- Most of the nine outliers in the scatter plot (circled observations) are related to very small premium volumes (<$400k).

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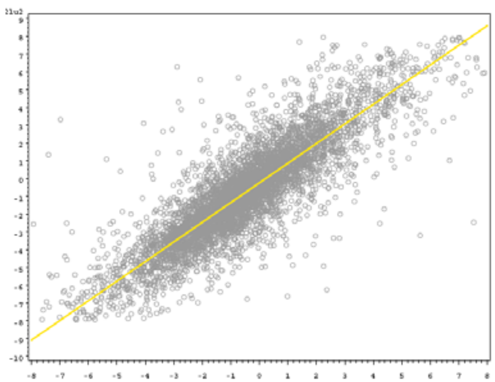
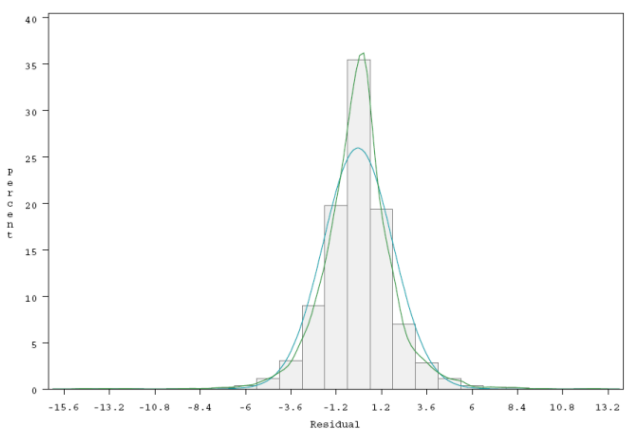
Steps required to produce a VaR distribution for one-year time horizon

Simulation model inputs for each AY

Step 1: Model parameters



Step 2: Error structure



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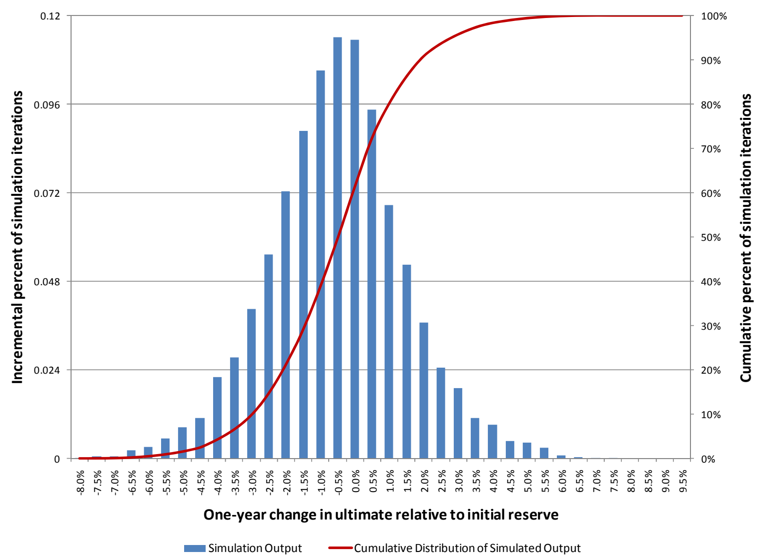
Step 3: Correlation between AYs

1.000	0.259	0.021	0.009	0.004	0.002	0.001	0.001
0.259	1.000	0.189	0.017	0.008	0.004	0.002	0.001
0.021	0.189	1.000	0.157	0.015	0.007	0.004	0.002
0.009	0.017	0.157	1.000	0.138	0.014	0.007	0.003
0.004	0.008	0.015	0.138	1.000	0.124	0.012	0.006
0.002	0.004	0.007	0.014	0.124	1.000	0.114	0.012
0.001	0.002	0.004	0.007	0.012	0.114	1.000	0.106
0.001	0.001	0.002	0.003	0.006	0.012	0.106	1.000

= Step 4: simulation of distribution of reserve variability over a one-year time horizon for aggregate of all AYs



Starting values for simulation model for one-year time horizon

Model inputs for sample company from 2009 AS Schedule P Part B

Ultimate LALAE

	Months of Development										(1)	(2)
AY	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**Ult_1**	**Ult_k**
AY10	2,921,547	2,916,396	2,918,810	2,912,157	2,904,075	2,907,743	2,910,087	2,909,578	2,909,421	2,909,352	2,921,547	2,909,352
AY9	2,966,618	2,970,120	2,929,217	2,913,977	2,910,557	2,913,386	2,914,298	2,912,542	2,909,683		2,966,618	2,909,683
AY8	3,297,394	3,318,102	3,266,505	3,217,454	3,217,982	3,221,398	3,218,589	3,212,429			3,297,394	3,212,429
AY7	3,737,269	3,685,571	3,595,945	3,562,582	3,561,283	3,556,815	3,551,908				3,737,269	3,551,908
AY6	4,043,762	3,874,490	3,822,898	3,802,818	3,782,879	3,771,351					4,043,762	3,771,351
AY5	4,189,683	4,102,969	4,063,952	4,027,908	3,989,882						4,189,683	3,989,882
AY4	4,236,348	4,228,745	4,189,401	4,142,613							4,236,348	4,142,613
AY3	4,427,012	4,428,524	4,385,835								4,427,012	4,385,835
AY2	4,479,146	4,393,712									4,479,146	4,393,712
AY1	4,701,999											

LALAE Reserves

	Months of Development										(3)	(4)
AY	**12**	**24**	**36**	**48**	**60**	**72**	**84**	**96**	**108**	**120**	**Res_1**	**Res_k**
AY10	1,687,692	842,653	445,555	224,878	104,093	55,959	31,100	19,615	13,447	10,885	1,687,692	10,885
AY9	1,748,209	903,367	473,258	230,047	107,899	57,736	34,063	21,129	12,669		1,748,209	12,669
AY8	1,943,544	1,027,575	537,600	246,851	116,537	61,938	36,204	19,700			1,943,544	19,700
AY7	2,223,703	1,119,374	562,778	264,358	124,052	59,725	33,193				2,223,703	33,193
AY6	2,376,720	1,122,653	578,801	286,602	123,625	58,669					2,376,720	58,669
AY5	2,403,811	1,181,797	620,390	291,771	119,508						2,403,811	119,508
AY4	2,374,435	1,200,057	617,918	286,192							2,374,435	286,192
AY3	2,459,209	1,217,840	638,668								2,459,209	638,668
AY2	2,520,669	1,207,478									2,520,669	1,207,478
AY1	2,666,377											

Notes:

(1) =Column 1 of triangle of Ultimate LALAE
(2) =Last diagonal of triangle of Ultimate LALAE
(3) =Column 1 of triangle of LALAE Reserves
(4) =Last diagonal of triangle of LALAE Reserves

Example of calculation within simulation model for one-year time horizon

- Using regression parameters, simulation model estimates the change in ultimate for AYs over one-year time horizon

$$ChgUlt_k = CChgUlt_{k+1} - CChgUlt_k$$
$$= (\beta_{0k} + \varepsilon_k) \cdot \frac{Res_1}{v^2} + (\beta_{1k} - 1) \cdot CChgUlt_k$$

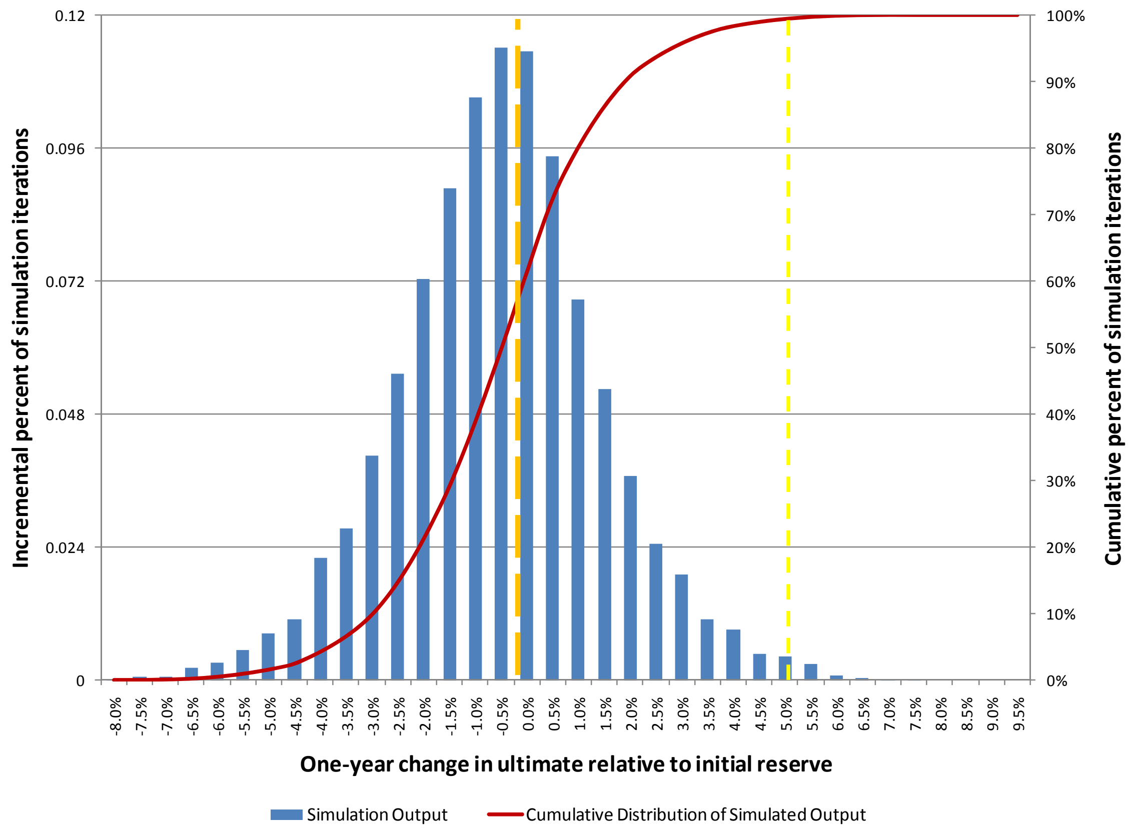
	(1)	(2)	(3)	(4)	(5)	(6) v	(7) β_0	(8) β_1	(9)
AY	Ult_1	Ult_k	Res_1	Res_k	$CChgUlt_k$	Scaling factor	Intercept	Slope	$ChgUlt_k$
AY10	2,921,547	2,909,352	1,687,692	10,885	-12,195	6.466	-0.01	1.00	-418
AY9	2,966,618	2,909,683	1,748,209	12,669	-56,936	6.472	-0.02	1.00	-642
AY8	3,297,394	3,212,429	1,943,544	19,700	-84,966	6.518	-0.04	1.00	-1,869
AY7	3,737,269	3,551,908	2,223,703	33,193	-185,361	6.573	-0.03	1.00	-1,796
AY6	4,043,762	3,771,351	2,376,720	58,669	-272,412	6.607	-0.02	1.01	-5,048
AY5	4,189,683	3,989,882	2,403,811	119,508	-199,801	6.622	-0.07	1.02	-7,217
AY4	4,236,348	4,142,613	2,374,435	286,192	-93,735	6.627	-0.10	1.03	-8,432
AY3	4,427,012	4,385,835	2,459,209	638,668	-41,178	6.646	-0.32	1.06	-20,482
AY2	4,479,146	4,393,712	2,520,669	1,207,478	-85,434	6.651	-0.35	1.10	-28,195

Notes:

(1) =Column 1 of triangle of Ultimate LALAE
(2) =Last diagonal of triangle of Ultimate LALAE
(3) =Column 1 of triangle of LALAE Reserves
(4) =Last diagonal of triangle of LALAE Reserves
(5) =(2) - (1)
(6) =Log10((1))
(7) =Intercept parameter for AY
(8) =Slope parameter for AY
*(9) = [(7)] * (3) / [(6) ^ 2] + [(8) - 1] * (5)*

- Simulation model creates correlated error distribution around AY model estimates to represent reserve variability

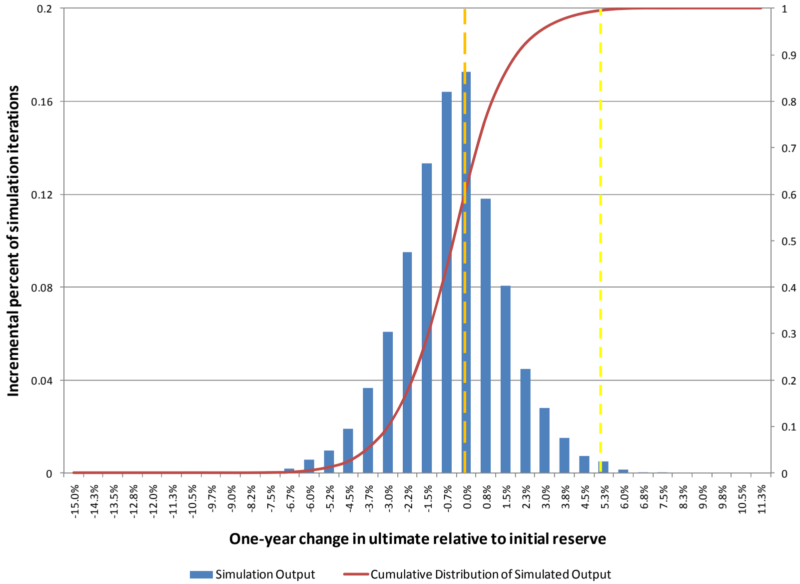
Example of output for simulation over one-year time horizon



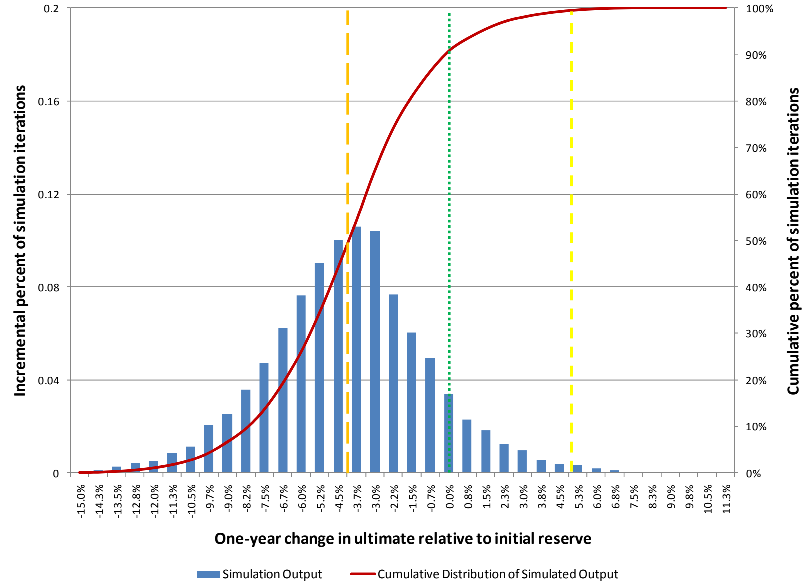
- The chart above shows a simulated output for a company over a one-year time horizon. The dashed yellow line represents the 99.5 percentile, a change in ultimate relative to current reserve of 5.1%. The dashed orange line, roughly 0%, represents the mean of the distribution.
- The model is sensitive to initial conditions, so this value will vary for different companies.

Comparison of simulation output for companies with different characteristics

Large, growing company with recent adverse development



Small, stable company with some favorable development

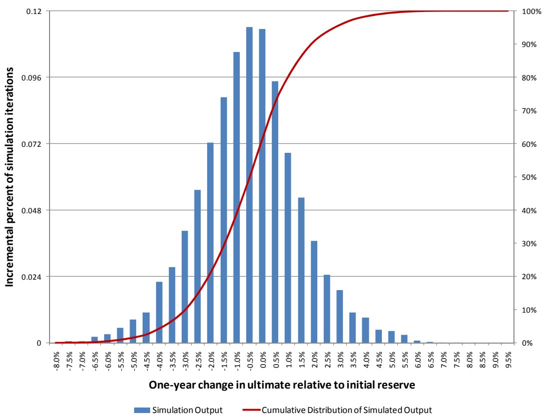


- Yellow dashed lines are 99.5 percentile of distribution; orange dashed lines represent the mean of the distribution.
- The means of the distributions are 0% and -4% for the large and small company, respectively (0% change is shown on the small company chart as the green dashed line).

Steps required to produce a VaR distribution for multiple-year time horizon

Step 1: Model output for each AY for each CY time step

1st future period



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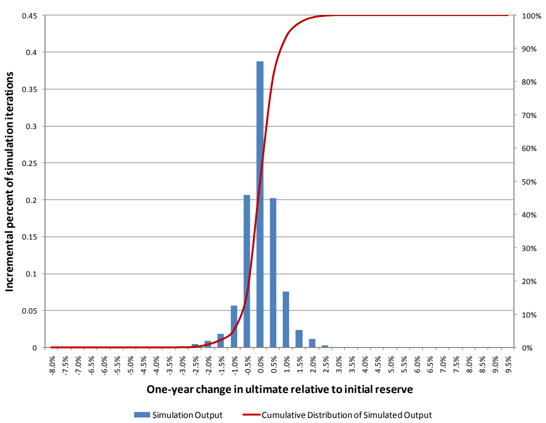
5th future period



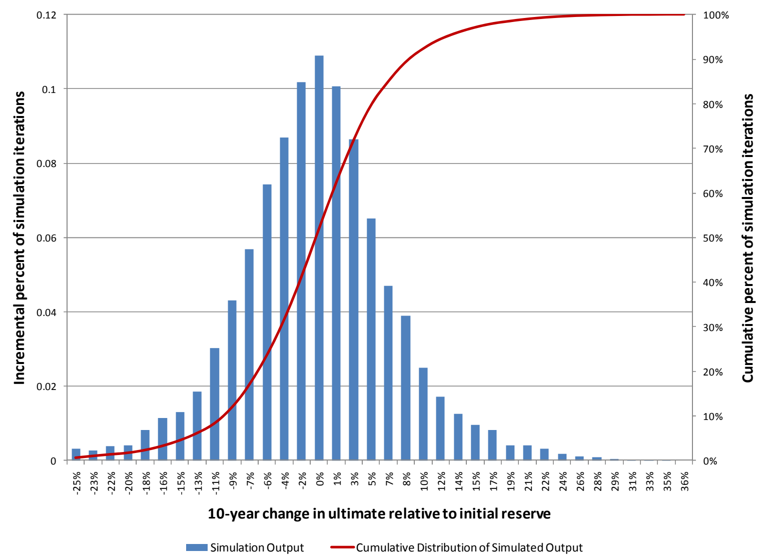
⋮

Step 2: Correlation between CYs

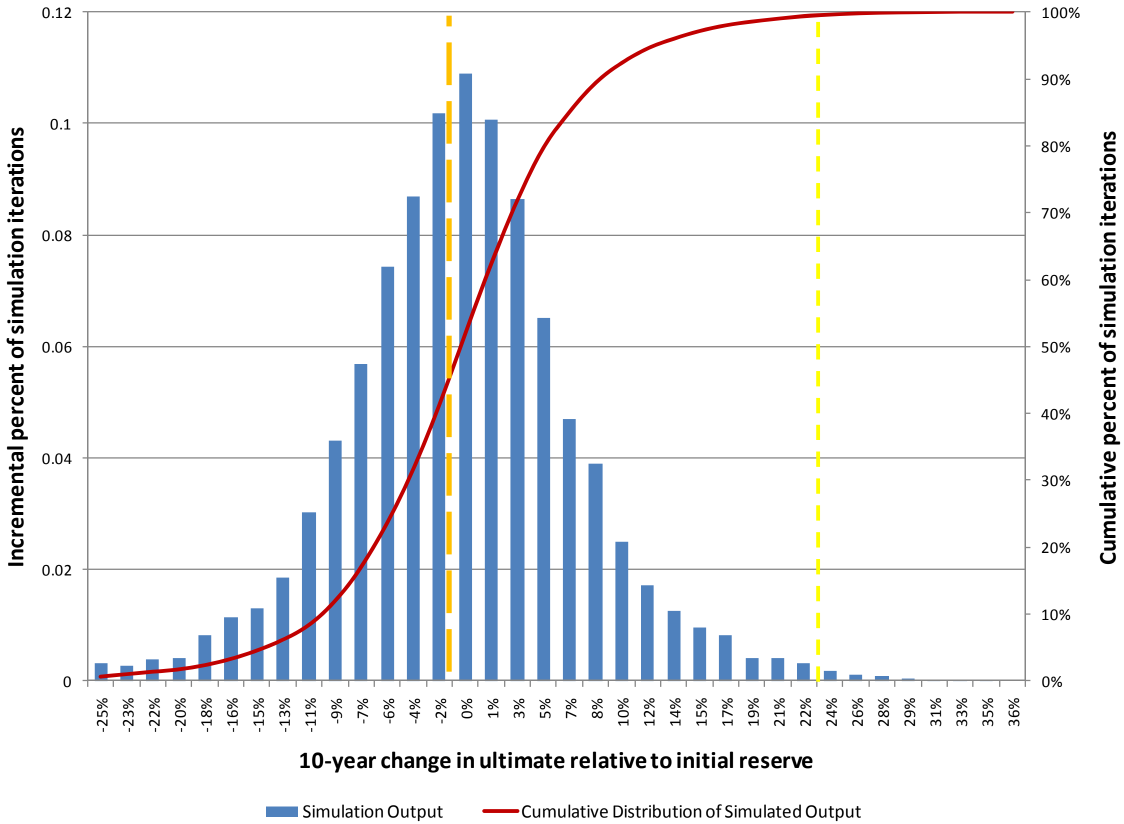
\+

1.000	0.411	0.260	0.168	0.117	0.093	0.070	0.058	…	0.042
0.411	1.000	0.411	0.260	0.168	0.117	0.093	0.070	…	0.055
0.260	0.411	1.000	0.411	0.260	0.168	0.117	0.093	…	0.058
0.168	0.260	0.411	1.000	0.411	0.260	0.168	0.117	…	0.070
0.117	0.168	0.260	0.411	1.000	0.411	0.260	0.168	…	0.093
0.093	0.117	0.168	0.260	0.411	1.000	0.411	0.260	…	0.117
0.070	0.093	0.117	0.168	0.260	0.411	1.000	0.411	…	0.168
0.058	0.070	0.093	0.117	0.168	0.260	0.411	1.000	…	0.260
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋱	⋮
0.042	0.055	0.058	0.070	0.093	0.117	0.168	0.260	…	1.000

= Step 3: simulation of distribution of reserve variability over a multiple-year time horizon



Example of output for simulation over multiple-year time horizon



- The chart above shows a simulated output for a company over a 10-year time horizon. The dashed yellow line represents the 99.5-percentile, a change in ultimate relative to current reserve of 23%. The dashed orange bar, roughly -1%, represents the mean of the distribution.

Observations

- Method provides a much needed framework
  - Capital analysis
  - Discrete time horizons
- Advantages of outcomes-based approach
  - Based and calibrated on historic events
  - Easily verifiable against industry results
  - Differentiates between companies based on
    - volume,
    - adverse development-to-date,
    - Relative dollar distribution of reserves by accident year
  - Can be used in situations where traditional loss development data isn't available