The Very Very Basic Guide to Reserve Variability and Ranges

Casualty Loss Reserve Seminar, Lake Buena Vista, FL

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TOWERS WATSON

Agenda

- The first half of the session will answer the question "why care about reserve variability?"

  - We will explore drivers internal and external to insurance companies

- The second half will provide an introduction to the terminology and basic concepts associated with the development of reserve ranges

- We plan to have time at the end for questions

Why Care about Reserve Variability?

There are a number of existing or growing forces driving the increased interest in reserve variability

- Insurance company management

- Regulatory authorities

- Future accounting concepts

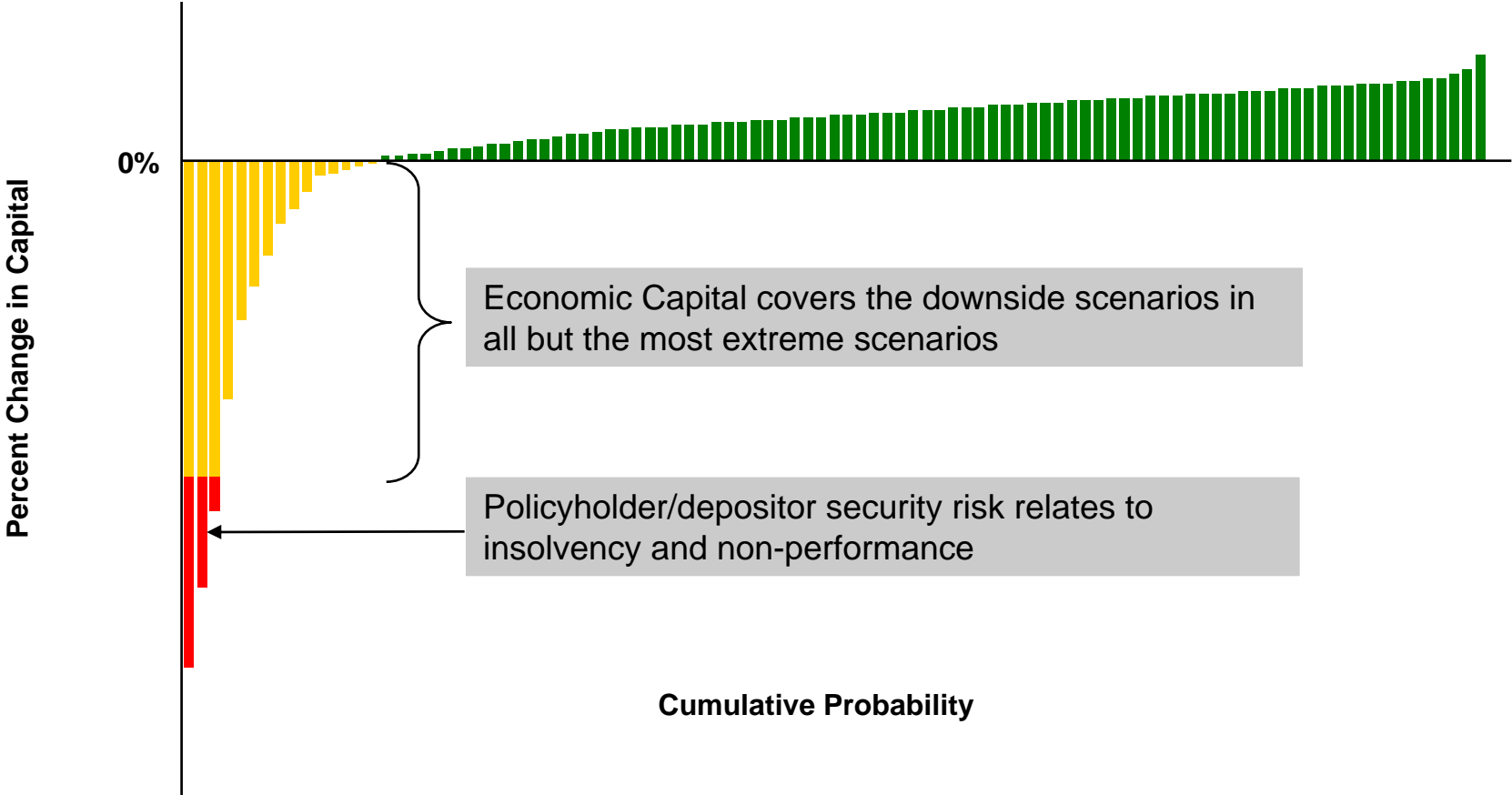- Rating agencies

- Actuarial best practices

Because the development of relevant and meaningful information about reserve variability takes time, companies should start climbing the learning curve today

The best reason to do this work is that information about reserve variability is useful in a business context

- Knowing the uncertainty of an estimate can improve operational decision making

  - Anticipating "negative surprises"

  - Rigorous range analysis allows assessment of the probability of "worse than expected" results

  - Allows for risk management interventions

- Effective capital management considers the uncertainty of the largest balance sheet entry

  - How to allocate surplus to line, branch? It should consider the riskiness of each level of the operation

- Asset management may be improved by gaining a better grasp of how shared economic driver variables (e.g., interest rates and inflation) affect both assets and liabilities

- Transparency related to the reporting of financial results is promoted by an understanding of the inherent volatility of the loss estimates

  - Transparency is the "watch word" of Boards of Directors and Wall Street analysts following the recent economic crisis

# Economic capital considers estimated variation in results

**Impact of Hypothetical Scenarios on Company Capital Structure
(One-year Time Horizon)**

Economic Capital covers the downside scenarios in all but the most extreme scenarios

Policyholder/depositor security risk relates to insolvency and non-performance

Cumulative Probability

Percent Change in Capital

0%

The SEC is pressuring registrants for more robust analysis and disclosure of potential variability of loss reserves

- Although the SEC's 10-K filing does not specifically require a company to disclose ranges of loss liability estimates, questions directed at P/C insurers have required them to begin to disclose their current practices. Similar to the 10-K itself, these additional disclosures are publicly available

  - Companies have responded cautiously

  - Current practices appear to vary widely

- Additional information sought by the SEC includes

  - Key assumptions underlying the methods used to determine reported reserves

  - Effects on reported reserves of reasonably likely changes to these assumptions

  - Description of methods used to calculate ranges of estimates

  - Retrospective tests of the quality of previous estimates and their influence on current selections

  - Rationale for selection of one method over another

- Given the current economic turmoil, it can be expected that the SEC will only intensify its demands for more transparency

Recent SEC inquiries have resulted in disclosure by publicly traded P/C insurers of current practices with respect to reserve variability analysis

A variety of approaches to range analysis
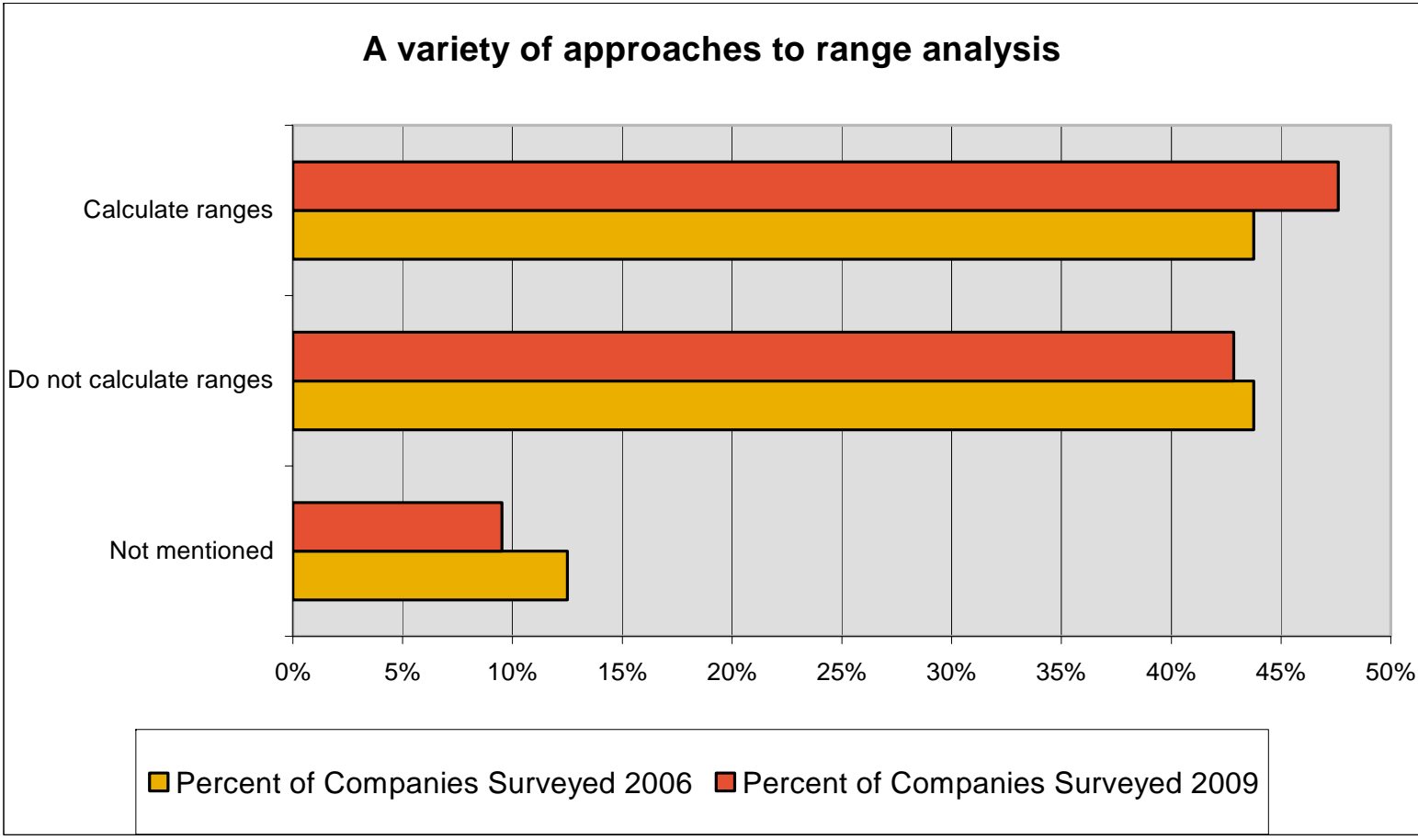
Source: Towers Watson analysis based on public SEC filings

Disclosures exhibit a great deal of variation in how companies present the results of the quantifications

- Some companies disclose ranges of actuarial central estimates derive from different estimation methods, while other provide ranges that attempt to capture the actual variability inherent in the reserves

- Some companies disclose the results of stochastic simulations of the unpaid claim liabilities at specified percentiles, while others provide judgmental "high" and "low" estimates derived from the simulated distributions

- Others do not publish an overall range, but instead provide scenario-specific outcomes based on deterministic changes in assumptions

- Ranges tend to be expressed without a time frame, although some companies do provide a risk horizon. In the absence of a specified time frame, readers should assume that risk horizon is on a run-off basis (i.e., ultimate value)

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The time horizon of the estimate (and the corresponding range) must be clearly understood
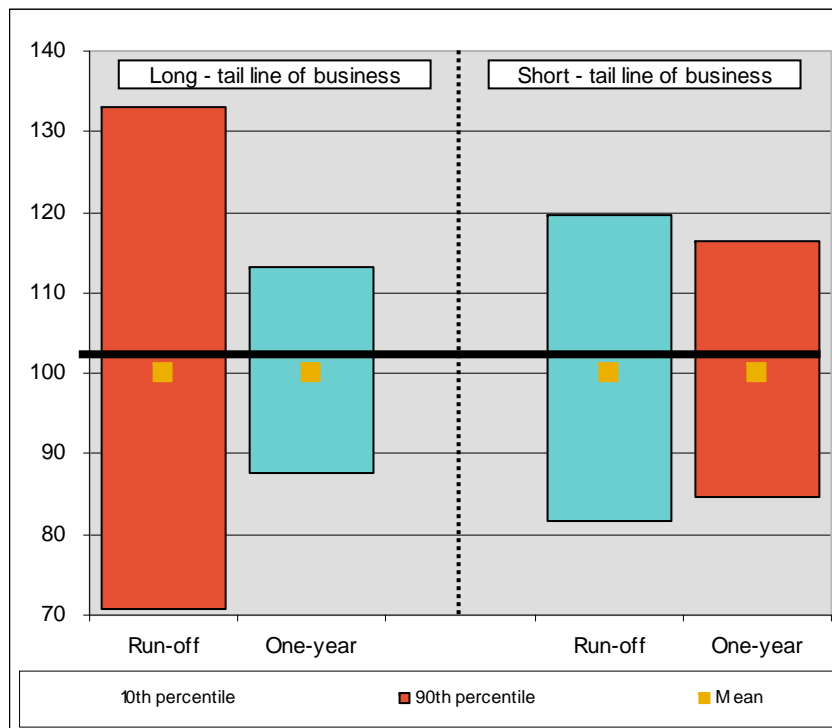

- **Run-off risk horizon:** What is the potential adverse variation in the ultimate cost of the claim liabilities from the current actuarial central estimate?

- **One-year risk horizon:** What is the potential adverse change in the actuarial central estimate of ultimate claim costs that could occur, with the benefit of one additional year of actual claim emergence and other relevant information?

- The one-year risk horizon requires that a company model how much of the uncertainty resolves during the upcoming year

Progress is being made by the accounting boards toward the objective of a global insurance accounting standard

- The International Accounting Standards Board (IASB) has been working on a standard for insurance accounting
  - One standard for life and p/c companies, including captives
  - Not applicable to self-insured liabilities
  - Joint discussions with FASB. Aim is to converge to one global standard
- The Exposure Draft (ED) was issued in late July, after months of delays
  - Much of the ED pertains to life insurance issues (e.g., unit linked contracts, participation features, minimum guarantees, etc.)
- Some objectives of the new standard include
  - Provide users of financial statements more useful information for economic decision-making
  - Eliminate inconsistencies and weaknesses in current practices
  - Provide comparability across entities, jurisdictions and capital markets
  - Be more principles-based than prescribed in approach

Presentation1

IASB/FASB Determination of Claim Liabilities – Building Block Approach

Liabilities for unpaid losses under the two approaches are as follows:

- Composite margin
- Residual margin
- Provision for risk
- Present value of unpaid losses

IASB   FASB
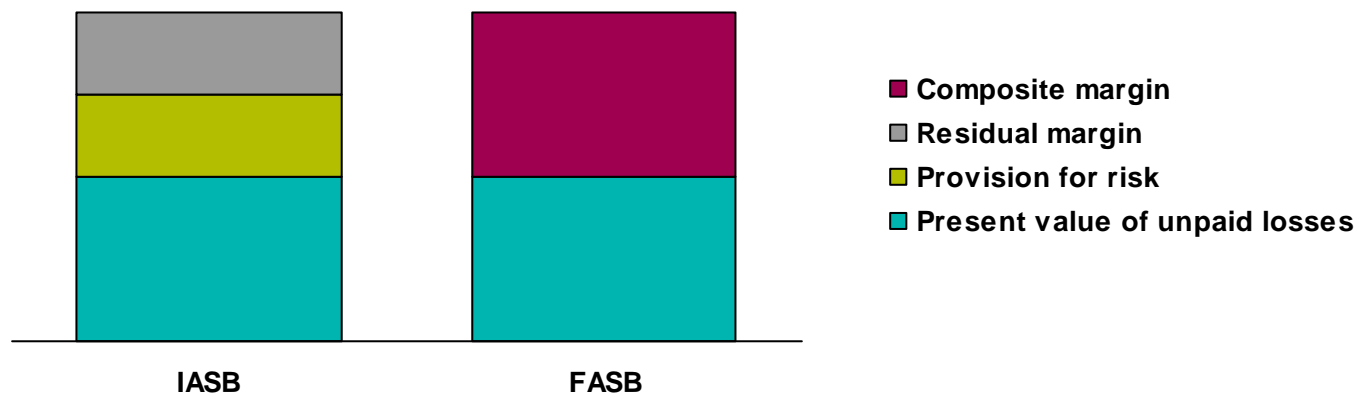
Definitions

Provision for risk – "Maximum amount the insurer would pay to be relieved of the risk…"

Residual margin – PV premiums less PV costs less risk margin; earned over policy period.

Composite margin – PV of premiums less PV costs; earned over policy period.

All of this is measured on a portfolio basis, but does not recognize diversification benefits across portfolios.

Despite its appearance, IASB specifically states that their method is not equal to fair value.

Variability of claim liabilities is accounted for by a risk margin under the IASB scheme

Three methods to calculate risk margins are permitted:

1. Confidence level – Margin can be based on specific confidence level. However, this method is not permitted if distribution is not statistically close to normal.

2. Conditional tail expectancy (CTE) – Based on probability weighted amounts beyond a certain confidence level (e.g., all amounts above 95%); also known as Tail Value at Risk.

3. Cost of capital – Based on cost of additional capital needed such that assets equal liabilities at a certain confidence level. Target confidence level in this approach is expected to be very high (e.g., 99.5%).

Calculations are done on a gross basis, with separate calculations/ margins for ceded losses.

The rating criteria used by the S&P and A.M. Best cite reserve variability as a consideration

- Standard & Poor's (S&P)

  - "P/C Criteria for Assessing Insurers/Reinsurers' Loss Reserve Adequacy"

    — States S&P "may calculate a range into which the level of adequacy will likely fall and quantify the possible effect on capital. The point estimate and the endpoints for the range of estimates are compared with existing capital to approximate the effect any potential reserve deficiency might have on the company's ability to meet its obligations"

    — Published May 18, 2009

  - It is anticipated that in the near term S&P will expect higher rated companies to stochastically model loss reserve distributions to gain more confidence in the loss reserve variability

  - Consideration of the variability inherent in the loss reserve is also consistent with their stated desire for insurers/reinsurers to develop robust enterprise risk management programs, including the quantification of economic capital

The rating criteria used by the S&P and A.M. Best cite reserve variability as a consideration
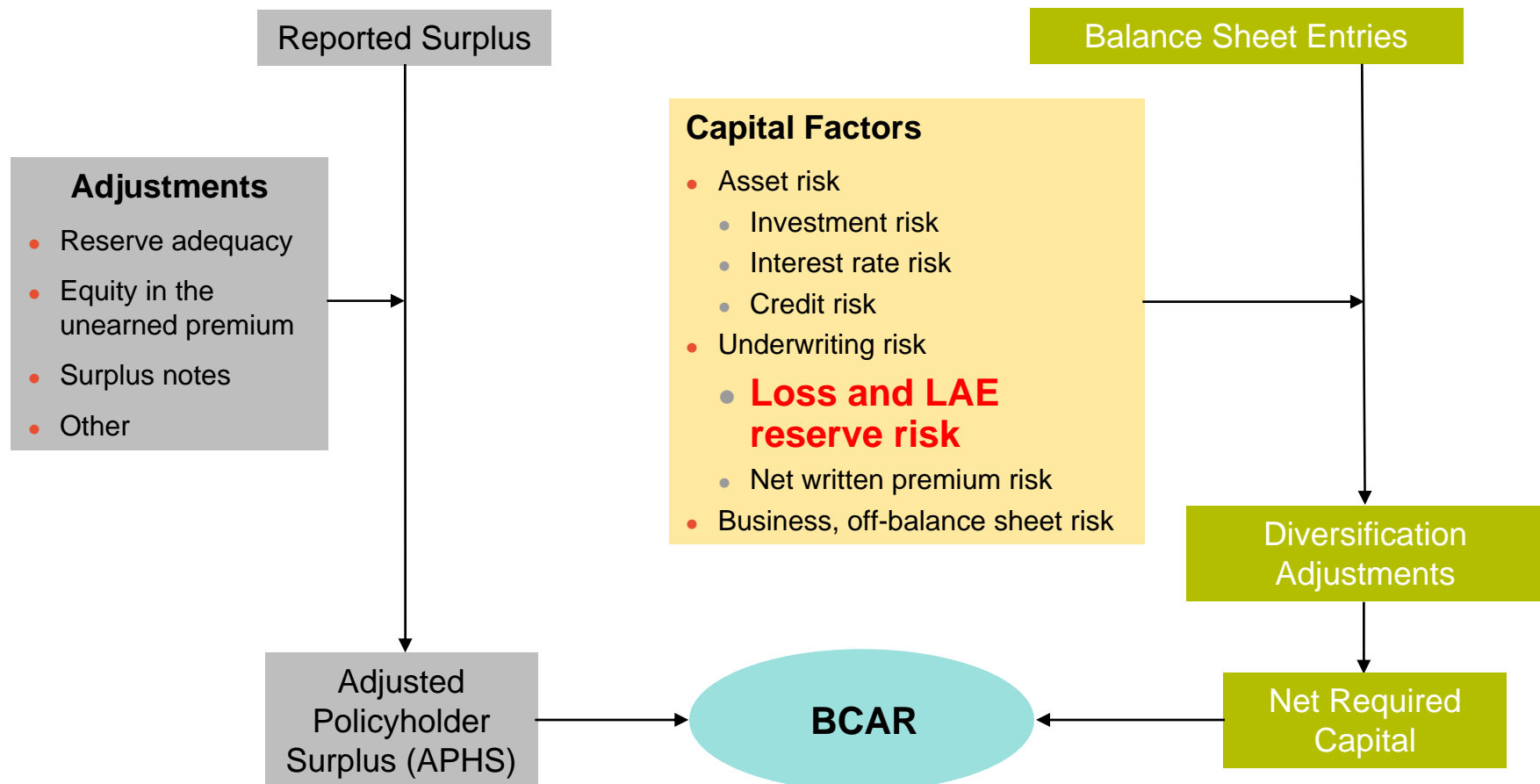
- A.M. Best
  - "An Explanation of Best's Rating System and Procedures"
    — States Best's evaluates "the degree of uncertainty in loss reserves. If the level of uncertainty exceeds any equity in the reserves, or is considered large in relation to net income and surplus, we will require a company to maintain a more conservative capital position…"
    — 2008 Edition
  - It is anticipated that A.M. Best will be slower to implement probabilistic reserve distributions.
    — Moreover, more robust reserve modeling will be a positive rating factor only if range estimates are reasonable relative to industry trends and historical experience
    — A.M. Best seems to value consistently favorable reserve trends without stochastic modeling over robust stochastic modeling while reporting adverse reserve development

For both S&P and A.M. Best, transparency, especially related to volatility of reserves and adverse reserve development, is a key to avoiding an increase in required capital

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A.M. Best's Capital Adequacy Ratio (BCAR) currently uses a factor-based approach to risk, but have announced their intention to build a stochastic capital adequacy model

Calculation of Adjusted Surplus

Calculation of Required Capital

Reported Surplus

Balance Sheet Entries

Adjustments

- Reserve adequacy
- Equity in the unearned premium
- Surplus notes
- Other

Capital Factors

- Asset risk
  - Investment risk
  - Interest rate risk
  - Credit risk
- Underwriting risk
- **Loss and LAE reserve risk**
  - Net written premium risk
- Business, off-balance sheet risk

Diversification Adjustments

Adjusted Policyholder Surplus (APHS)

BCAR

Net Required Capital

Actuarial organizations reflect the increasing sophistication of the discussion of the uncertainty surrounding loss liability estimates through published papers and continuing educational opportunities

- Examples of recent papers and articles

  - "P/C Actuarial Communication on Reserves Ranges and Variability of Unpaid Claim Estimates"

    — Issue Brief of the American Academy of Actuaries, published in September 2008

    — Written by the Committee on Property and Liability Financial Reporting (COPLFR)

    — Stated goal is to improve "casualty actuaries' communications with regard to ranges of unpaid claim estimates"

  - "Measurement of Liabilities for Insurance Contracts: Current Estimates and Risk Margins"

    — A Research Paper of the International Actuarial Association, published April 15, 2009

    — Written by the Ad Hoc Risk Margin Working Group

    — Stated purpose is to address those issues "that will help determine future practice for measuring liabilities for insurance contracts for both regulatory and general purpose financial reporting"

  - *Variance,* the peer-reviewed journal of the Casualty Actuarial Society, has in the last three years published 11 articles on topics related to reserve variability. Each of the six issues contained at least one article directly related to the topic

- There are numerous continuing educational opportunities related to reserve variability offered each year through the various actuarial organizations in the U.S. and elsewhere

Actuarial Standards of Practice (ASOP) cite the importance of considering the variability inherent in the loss estimates

Source	Topic	Considerations
ASOP 43: P/C Unpaid Claim Estimates	Uncertainty	• Consider uncertainty associated with unpaid claim estimate analysis; does not require or prohibit the measurement of this uncertainty • Consider the types, sources (model, parameter and process risks), and correlation of uncertainty; choosing appropriate methods, models and assumptions

ASOP 36: SAO Regarding P/C Loss & LAE Reserves	Uncertainty	• Actuarial estimates are inherently uncertain
		   • Dependent on future contingent events
		   • Future events/conditions often differ from the past
		   • Actual settlement amount for unpaid claims can differ from stated reserve amount

ASOP 9: Documentation and Disclosure in P/C Insurance Ratemaking, Loss Reserves and Valuations (Appendix 1, CAS Statement of Principles Regarding P&C Insurance Ratemaking)	Risk	• Charges to be reflected in the profit and contingency provision
		   • Rate should include charge for risk of random variation from expected costs
		   • Rate should include a charge for any systematic variation of the estimated costs from the expected costs

ASOP 9: Documentation and Disclosure in P/C Insurance Ratemaking, Loss Reserves and Valuations (Appendix 1, CAS Statement of Principles Regarding P&C L&LAE Reserves)	Uncertainty	• Due to uncertainty inherent in the estimation of the required reserve provision a range of reserves may be actuarially sound • The most appropriate reserve from within a range depends on both the relative likelihood of estimates within the range and the financial reporting context
ASOP 9: Documentation and Disclosure in P/C Insurance Ratemaking, Loss Reserves and Valuations (Appendix 3, CAS Statement of Principles Regarding P&C Valuaions)	Sensitivity testing	• Address the sensitivies of the appraisal value to changes in key assumptions • Consider whether the results reflect a reasonable range of variation in the key assumptions

• ASOP 36: Currently undergoing revisions

• ASOP 9: Repeal is pending revisions in ASOP 41(Actuarial Communications)

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Current practices related to statutory actuarial Opinions call for consideration of the variability of the reserve estimate

- The Property and Casualty Practice Note, prepared by the American Academy of Actuaries, related to Statements of Actuarial Opinion on P&C Loss Reserves as of December 31, 2009, provides information on current and emerging practices related to reserve variability, among other topics

- Opinion

  - ASOP 36 is cited as the source for the statement that a reserve makes a reasonable provision if it is within the range of reasonable estimates

  - It is suggested that, if a reserve estimate is subject to an unusually high degree of variability, the actuary may choose to provide comment on this within the Opinion

- Actuarial Opinion Summary (AOS)

  - The AOS makes provision for the actuary to compare the carried reserves to the range directly, instead of a point estimate

  - ASOP 36 is again cited as the source for various key definitions

Now is the time to stop being the proverbial ostrich and to start looking forward to the benefits of understanding the range of results

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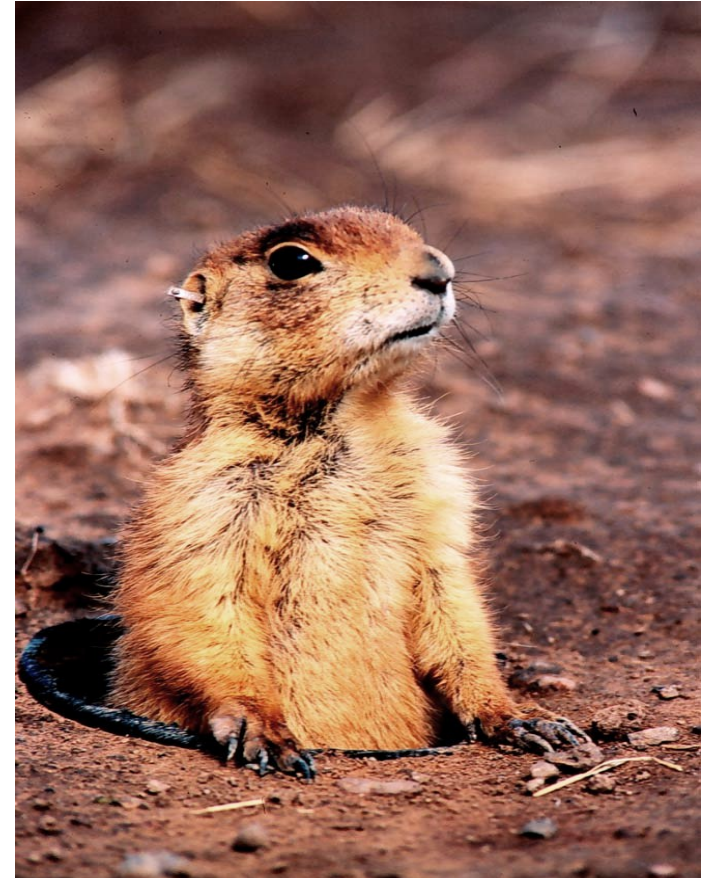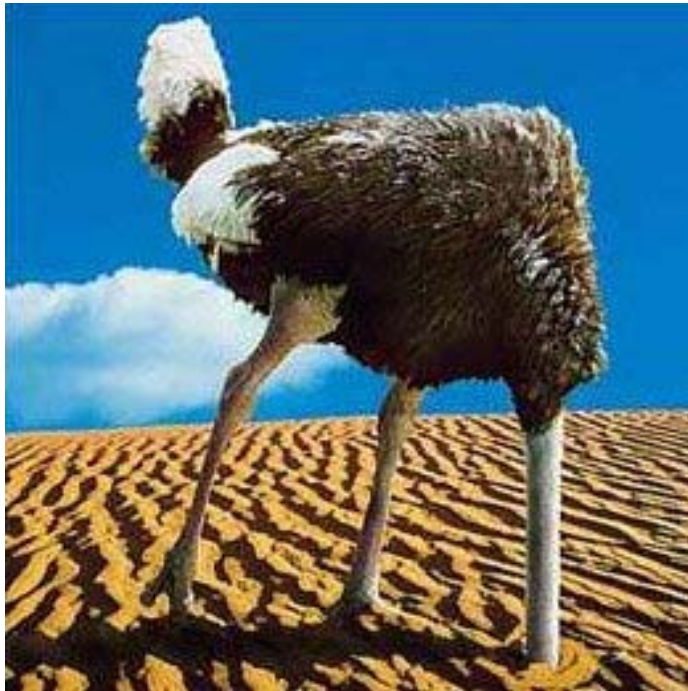
Background of Stochastic Techniques

Stochastic techniques consider the entire range of outcomes

The range of best estimates is likely to understate the range of actual outcomes

Important part of the process:

Understanding differences in methods

Best estimate Method #1

Best estimate Method #2
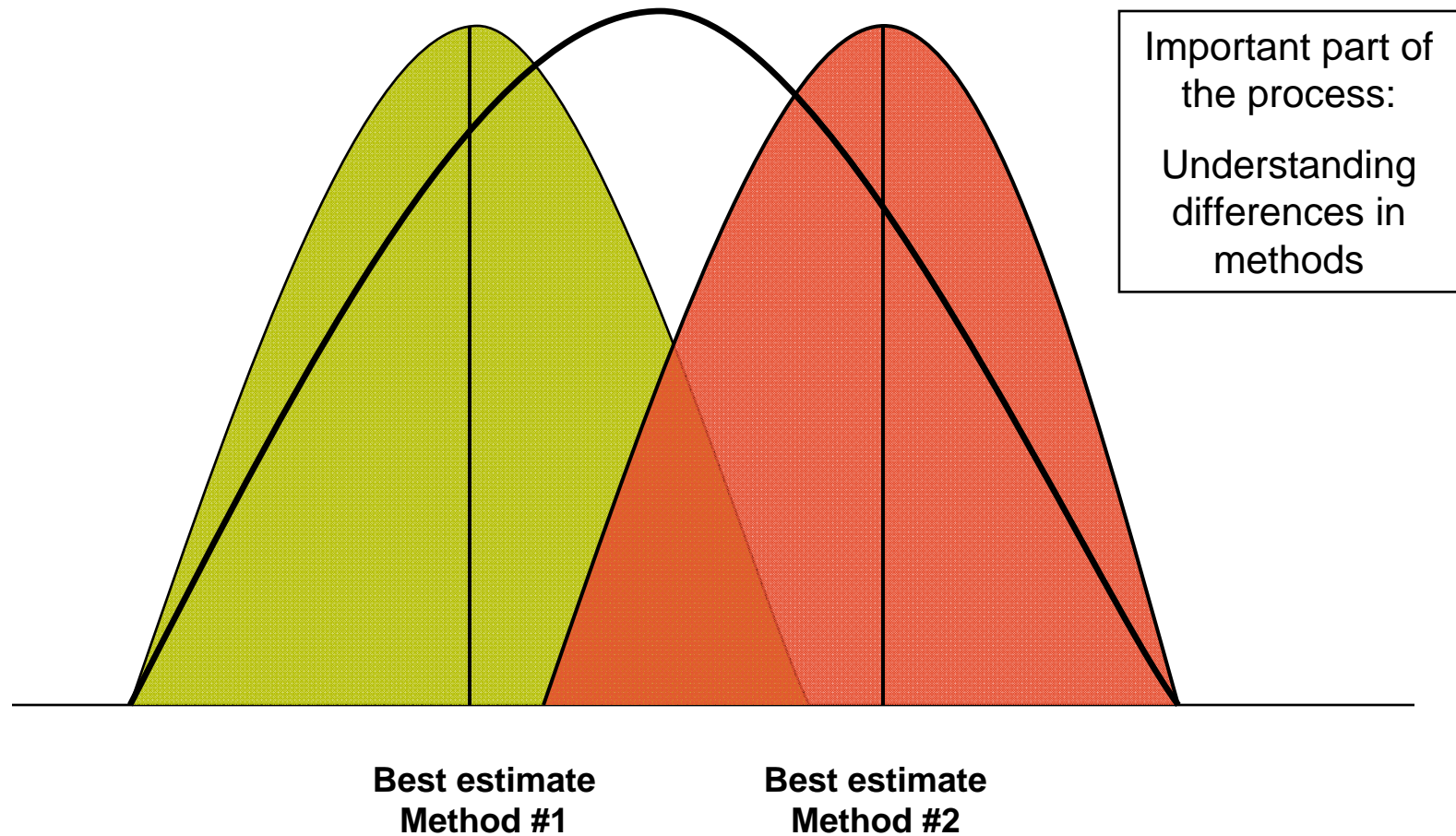
Stochastic techniques quantify the claim liability uncertainty

- Loss development is a stochastic process; the historical data is a specific realization

- Deterministic methods provide a point estimate of claim liabilities
  - Multiple methods can give a range of estimates
  - Best estimate usually chosen judgmentally

- Stochastic methods are more informative than deterministic methods
  - Produce a full distribution of possible outcomes
    — Confidence levels of held reserves

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Terminology

Method vs. Model

Method	Model
• Mathematical algorithm for estimating unpaid amounts • Parameters are selected • Selections assumed appropriate based on judgment • Chain Ladder algorithm	• Mathematical description of the world • "Best-Fitted" Parameters • Selections can be tested • Mack, Bootstrapping models

Presentation1

Several distinct types of risks are inherent in the measurement of claim liabilities — the actuary and the audience need to be clear about which are relevant to a particular application

Total Risk

Process Risk	Parameter Risk	Model Risk
Roll of fair die, equal chance of one to six	Roll of loaded die, no longer sure of probabilities	Roll of trick die not numbered one to six, not sure what is on each side

Decreases with volume

Constant with volume

Actual Outcome

True Expected Outcome

Model Estimate of Expected Outcome
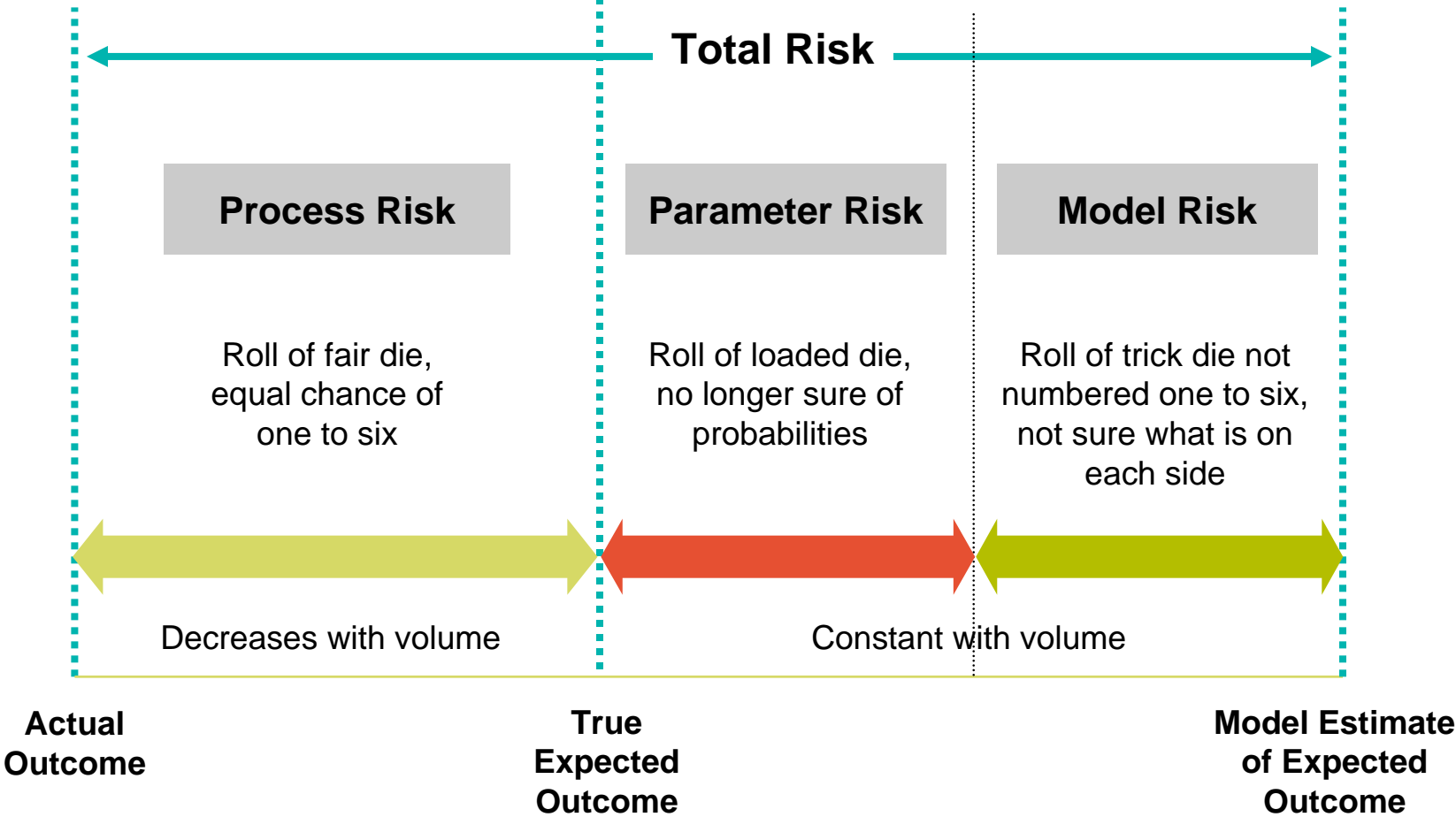
Types of stochastic reserving assignments

- Relevant sources of variability depend on the exercise on hand

- Financial Solvency/Capital adequacy context
  - "Stress testing" the balance sheet
  - Variation of actual outcome around the true expected outcome
  - All types of risk are relevant here

- Reserve variability context
  - Comparing two actuarial estimates
    — Variation around the true expected outcome
    — Parameter and model risk are relevant here

What "risk" do stochastic methods measure?

- Risk could mean different things to different audiences

- Actuaries usually think of risk in terms of "variance" and "standard deviation"

  - "coefficient of variation" (CV) is "scaled" by the mean and measures "relative" risk

- Other definitions

  - (VAR) - Value at Risk: a percentile (i.e., losses at the 75th)

  - (TVar) – Tail value at Risk: expected losses in excess of a given percentile

Presentation1

Deterministic: What range of estimates is implied by the actuarial techniques used?

Indicated Liabilities

High Estimate
Central Estimate
Low Estimate

Inc'd LDF — Paid LDF — Inc'd "BF" — "fxs" — Industry

Actuarial Technique
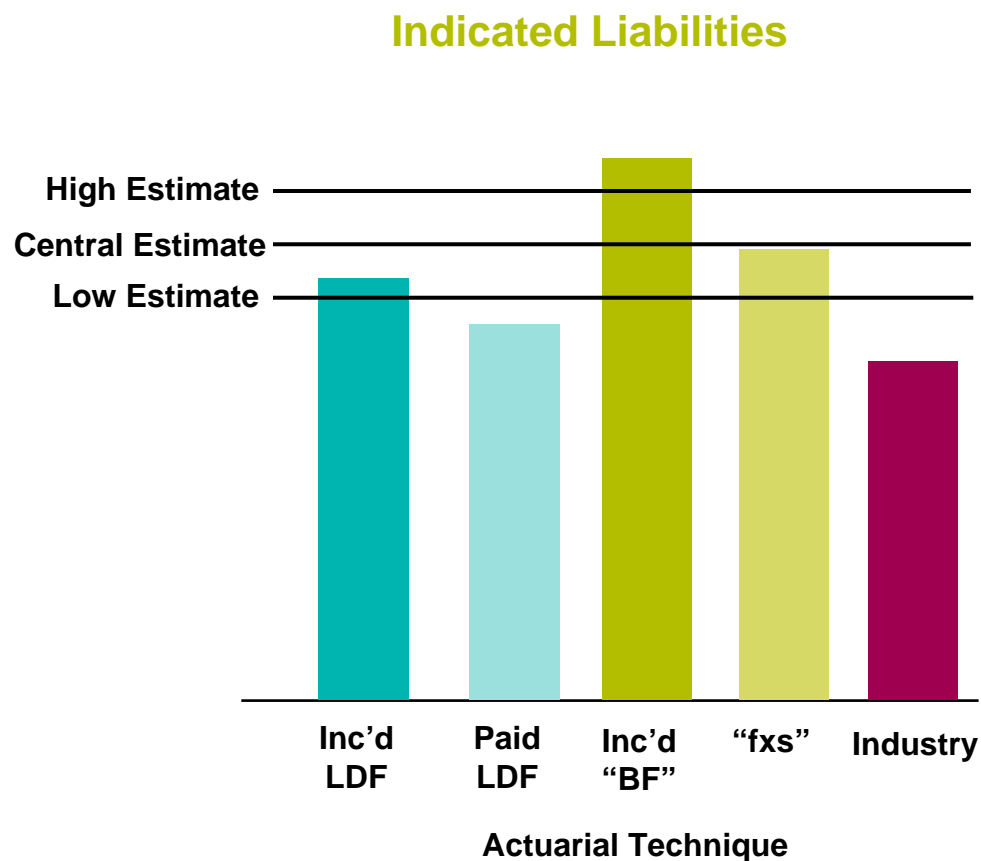
General Approach — Deterministic

- Estimate range of claim liabilities based on the results of several projections
- Applied to current data evaluation

Advantages	Disadvantages
• Easy to understand and apply	• Does not include process risk
• Based on liability estimates of traditional actuarial methods	• Does not separate model and parameter risk
• No extra work needed	• Does not produce confidence interval estimates
	• Highly judgmental
	• Simplistic

Performance Test: How accurate have the past estimates proven to be?

Actuarial Scorecard for Method X

Current view of % deficiency/redundancy

At year-end:

12/95
12/96
12/97
12/98
12/99
12/00
12/01
12/02
12/03
12/04
12/05
12/06
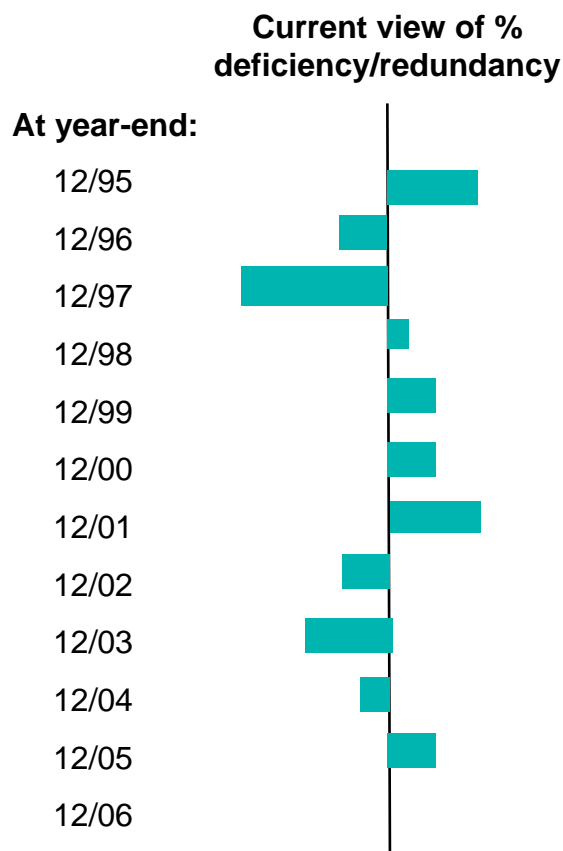
General Approach — Hindcast Test

- Retrospective test of a consistently applied methodology

- Uses current view of claim liabilities versus historical estimates

- Quantifies the degree of departure that has occurred around the results that would have been indicated by that methodology
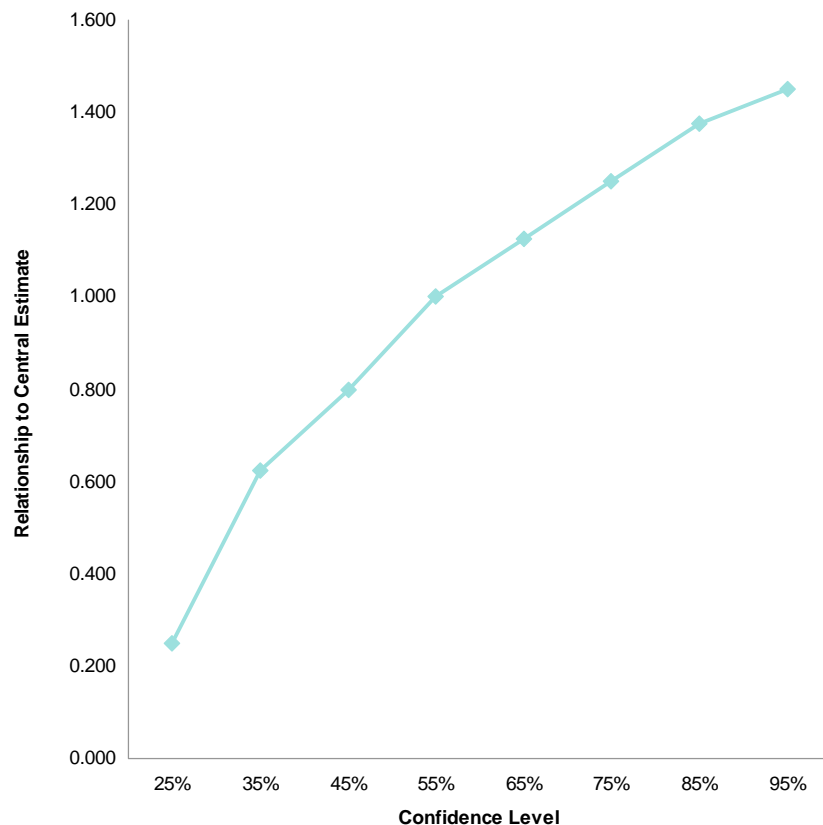
Advantages

- Easy to understand and apply

- Few assumptions needed for each model being tested

- Should do this test anyway in arriving at central estimate

Disadvantages

- Does not separate model, parameter and process risk

- Does not produce confidence interval estimates

- The actual "model" used is likely a combination of methods

Stochastic: What claim liability outcomes are reasonably likely?

Indicated Unpaid Claim Liabilities as of December 31, 2008

General Approach — Stochastic Methods
• Estimate probability distribution
• Based on statistical methods
• Applied to historical development data

Advantages	Disadvantages
• Produces estimates of confidence intervals	• Involves relatively complex statistical analysis
• Can approximately separate parameter and process risk	• An emerging practice within P/C actuarial field
• More complete description of loss generating process	• Lack of general agreement among actuaries on the best approach
• Feeds other analyses (ERM)	• Some exposures not amenable to this approach (A&E)

Popular Stochastic Methods

Analytical methods — Mack

- The Mack model measures the standard error of the chain ladder unpaid claim estimate
  - Based on the following simple regression model:

$$C_{ik+1} = f_k C_{i,k} + \sigma_k \varepsilon_{i,k} C_{i,k}^{1/2}$$

- This model is consistent with selected volume weighted RTRs

- Given the mean and standard error of claim liabilities percentiles are calculated
  - Recommended distributional formats are normal and log-normal

- Analytical calculation is based on
  - A "closed form" solution formula
  - A "recursive" calculation

Coefficients of Variation (CVs) by Accident Year

CVs by Accident Year
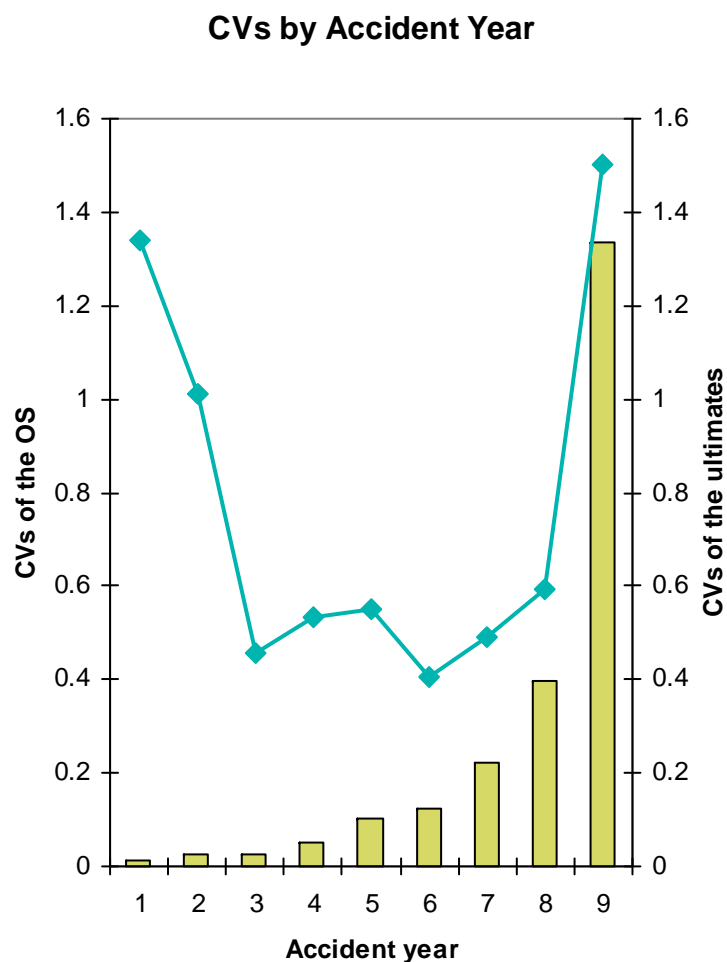
- The CVs of OS are higher for:
  - Older years where the remaining OS amounts are very low
  - Recent years where the uncertainty of the liabilities increases
- The CVs of Ultimate amounts increases in recent years
  - The uncertainty of the liabilities increases

Mack method: Pros and Cons

- Pros
  - Intuitive, based on chain-ladder assumptions
  - Widely accepted among actuaries
- Usually provides stable results
  - Very fast
  - Measures parameter, process and total risk

- Cons
  - Model provides, only, the mean and standard error of the claim distribution
  - Does not explicitly measure tail variability
  - Does not model well the situation when actuary selects factors other than weighted or simple average

Simulation approach: Monte-Carlo

- Simulation techniques help model the complex loss generating process

- Simulation methods assume that the simulated data has the same statistical characteristics as the actual data

- Simulation works as follows:
  - Start with a deterministic method that generates ultimate loss outcomes (i.e., chain ladder)
  - Makes assumptions about the method parameters
    — i.e. the mean and variance of the link ratios
      – Parameter risk needs to handled separately
    — Randomly generate input values
    — Calculate and save ultimate outcomes
    — Repeat many times

Presentation1

Output simulated distribution
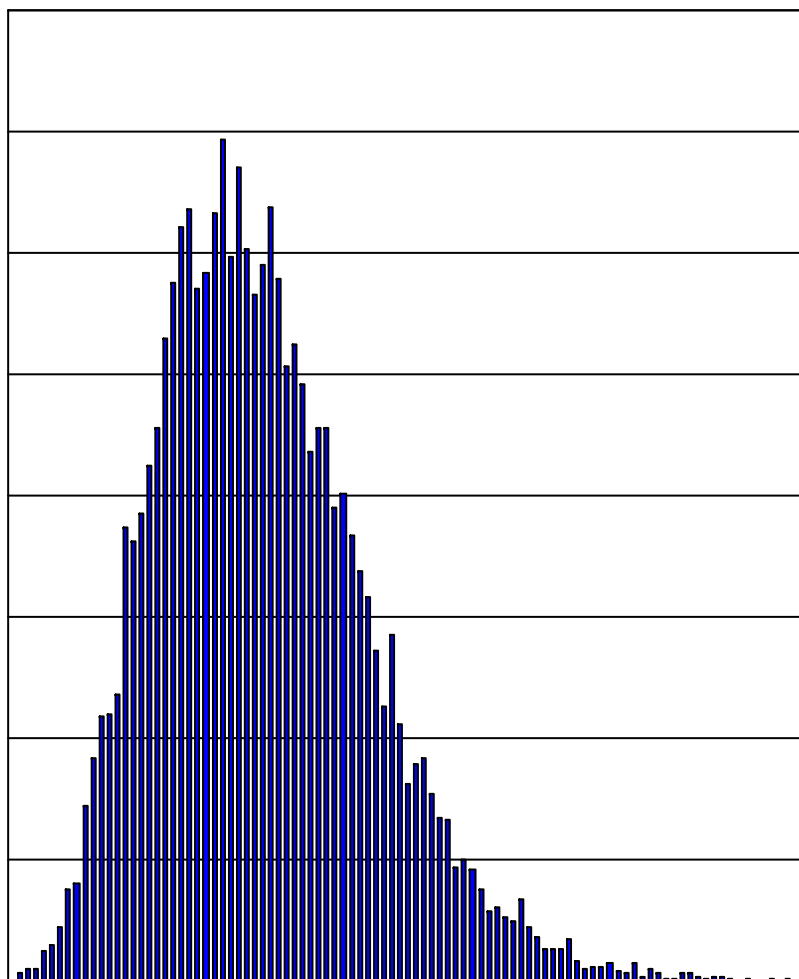
- Simulated "empirical" distribution estimates "theoretical" claim liabilities distribution

- A "wealth" of statistical information is produced (i.e. mean, variance, skewness, etc.)

- Simulated distribution "smooths" with a larger number of simulations

Presentation1

Monte Carlo simulations: Pros and Cons

- Pros

  - Popular method in many sciences

  - Produces an empirical distribution of the reserves

  - Method can be applied to incomplete data triangles (i.e. trapezoids)

  - It explicitly calculates tail volatility

- Cons

  - Data outliers can have a leveraged effect on the results

  - Slow to run

  - Needs additional complexity to measure parameter risk

Presentation1

Bootstrapping is a "second generation" simulation technique

- Monte Carlo techniques simulate the parameter inputs of a method

- Bootstrapping simulates the actual data employed by these methods
  - If the distribution of the data is known then we sample from that distribution
    — Parameters are estimated
    — This is called Parametric Bootstrapping
  - If we do not know the distribution of the data then we simulate from the actual data
    — This is called Nonparametric Bootstrapping
    — The process "resample" the residuals with "replacement"

Presentation1

Step-by-step description of the Bootstrapping algorithm

Actual Cumulative Historical Losses:

Accident	Development Age				
Year	12	24	36	48	60
1	1,000	1,500	1,800	2,000	2,100
2	1,200	1,900	2,150	2,300	
3	1,700	2,400	3,000		
4	2,000	2,900			
5	2,100				

	12-24	24-36	36-48	48-60	60-Ult
Selected RTRs	1.475	1.198	1.089	1.050	1.000

- Keep current diagonal intact
- Employ selected RTR factors to calculate expected cumulative payments

Expected Cumulative Historical Losses:

Accident	Development Age				
Year	12	24	36	48	60
1	1,040	1,533	1,837	2,000	2,100
2	1,196	1,763	2,113	2,300	
3	1,698	2,504	3,000		
4	1,967	2,900			
5	2,100				

For Example:
2,504 = 3,000 / 1.198

The Bootstrapping technique calculates residuals based on incremental losses

Actual Incremental Historical Losses:

Accident Year	Development Age				
	12	24	36	48	60
1	1,000	500	300	200	100
2	1,200	700	250	150	
3	1,700	700	600		
4	2,000	900			
5	2,100				

Expected Incremental Historical Losses:

Accident Year	Development Age				
	12	24	36	48	60
1	1,040	493	304	163	100
2	1,196	567	350	187	
3	1,698	806	496		
4	1,967	933			
5	2,100				

"Unscaled" Pearson Residuals: $\boxed{r_{P_{ij}}}$

Accident Year	Development Age				
	12	24	36	48	60
1	-1.233	0.295	-0.229	2.916	0.000
2	0.124	5.564	-5.327	-2.719	
3	0.052	-3.726	4.650		
4	0.752	-1.091			
5	0.000				

For Example:

$$-1.091 = \frac{900 - 933}{\sqrt{933}}$$

- The "unscaled" Pearson residuals are defined as:

$$r_{P_{ij}} = \frac{C_{ij} - m_{ij}}{\sqrt{m_{ij}}}$$

- The denominator represents the standard error of the incremental losses

- The Pearson residuals are "unscaled" in the sense they exclude ϕ which is needed only when considering the process error

- The (5,12) and (1,60) residuals will be zero. They could be excluded from the remainder of the analysis

The "unscaled" Pearson residuals need to be adjusted for the "degrees of freedom"

"Unscaled" Pearson Residuals: $r_{P_{ij}}$

Accident	Development Age				
Year	12	24	36	48	60
1	-1.233	0.295	-0.229	2.916	0.000
2	0.124	5.564	-5.327	-2.719	
3	0.052	-3.726	4.650		
4	0.752	-1.091			
5	0.000				

Degrees of Freedom adjustment factor: **1.581** $= \sqrt{\dfrac{15}{15 - 9}}$

"Adjusted" Pearson Residuals:

Accident	Development Age				
Year	12	24	36	48	60
1	-1.950	0.466	-0.363	4.611	0.000
2	0.195	8.797	-8.422	-4.300	
3	0.083	-5.891	7.352		
4	1.188	-1.725			
5	0.000				

For Example:
-1.725 = -1.091 x 1.581

- The "Unscaled" Pearson residuals need to be adjusted for the difference in the degrees of freedom between the analytical model and the bootstrapping technique

- The adjustment is equal to:

$$\sqrt{\frac{n}{n - p}}$$

Where n = 15 is the number of data points and

p = 9 are the parameters that need to be estimated

- In general, n-p represent the degrees of freedom of a model

Presentation1

Simulation of "pseudo" incremental loss data

"Adjusted" Pearson Residuals:

Accident Year	Development Age 12	24	36	48	60
1	-1.950	0.466	-0.363	4.611	0.000
2	0.195	8.797	-8.422	-4.300	
3	0.083	-5.891	7.352		
4	1.188	-1.725			
5	0.000				

Sampling with replacement of the Pearson Residuals: $r^*_{P_{ij}}$

Accident Year	Development Age 12	24	36	48	60
1	-5.891	7.352	-0.363	-0.363	4.611
2	8.797	8.797	-5.891	7.352	
3	4.611	-1.950	-4.300		
4	-8.422	1.188			
5	-8.422				

"Pseudo" incremental loss data: C^*_{ij}

Accident Year	Development Age 12	24	36	48	60
1	850	657	298	158	146
2	1,500	777	239	288	
3	1,888	750	401		
4	1,593	970			
5	1,714				

For Example:

$$970 = 1.188 * \sqrt{933} + 933$$

- The resampling of the "Adjusted" Pearson residuals is based on the assumption that the residuals are independent and identically distributed

- The sampling with replacement could cause the sampled residuals to appear more than once

- The "pseudo" incremental loss data is created by solving the Pearson residual equation

$$C^*_{ij} = r^*_{P_{ij}} \sqrt{m_{ij}} + m_{ij}$$

Incorporation of process risk

Cumulative "pseudo" loss data and "squaring" of the triangle

Accident Year	Development Age				
	12	24	36	48	60
1	850	1,507	1,804	1,962	2,109
2	1,500	2,277	2,516	2,804	3,013
3	1,888	2,638	3,039	3,353	3,602
4	1,593	2,563	2,937	3,240	3,481
5	1,714	2,641	3,027	3,339	3,588

	12-24	24-36	36-48	48-60	60-Ult
Simulated RTRs	1.541	1.146	1.103	1.074	1.000

Incremental future loss data:

Accident Year	Development Age				
	12	24	36	48	60
1					
2					209
3				314	250
4			374	303	241
5		927	386	312	249

Simulate Incremental payments from a Gamma distribution with parameters α = mean / ϕ, and β = ϕ

Accident Year	Development Age					Estimated Reserves:
	12	24	36	48	60	
1						0
2					299	299
3				229	335	564
4			349	464	225	1,038
5		822	300	214	129	1,466
					Total:	**3,367**

- The bootstrapping technique, up to now, has considered parameter risk only

- The scale parameter can be estimated as the Chi-square statistic divided by the degrees of freedom

$$\phi = \frac{\sum r_{ij}^2}{n - p} = 19.027$$

- We simulate from a Gamma distribution with the appropriate parameter's transformation. Advantages:

  - a) Simulate from the continuous Gamma distribution, and

  - b) avoid simulating values that are a multiple of ϕ from the overdispersed Poisson distribution

Bootstrapping: Pros and Cons

- Pros
  - Actual data "guides" the simulation
  - No assumption needed for simulation of parameters
  - It is a "modern" simulation technique

- Cons
  - Data outliers can have a leveraged effect on the results
  - Needs additional complexity to measure process risk
  - Residuals might needed to be divided into similar resampling groups

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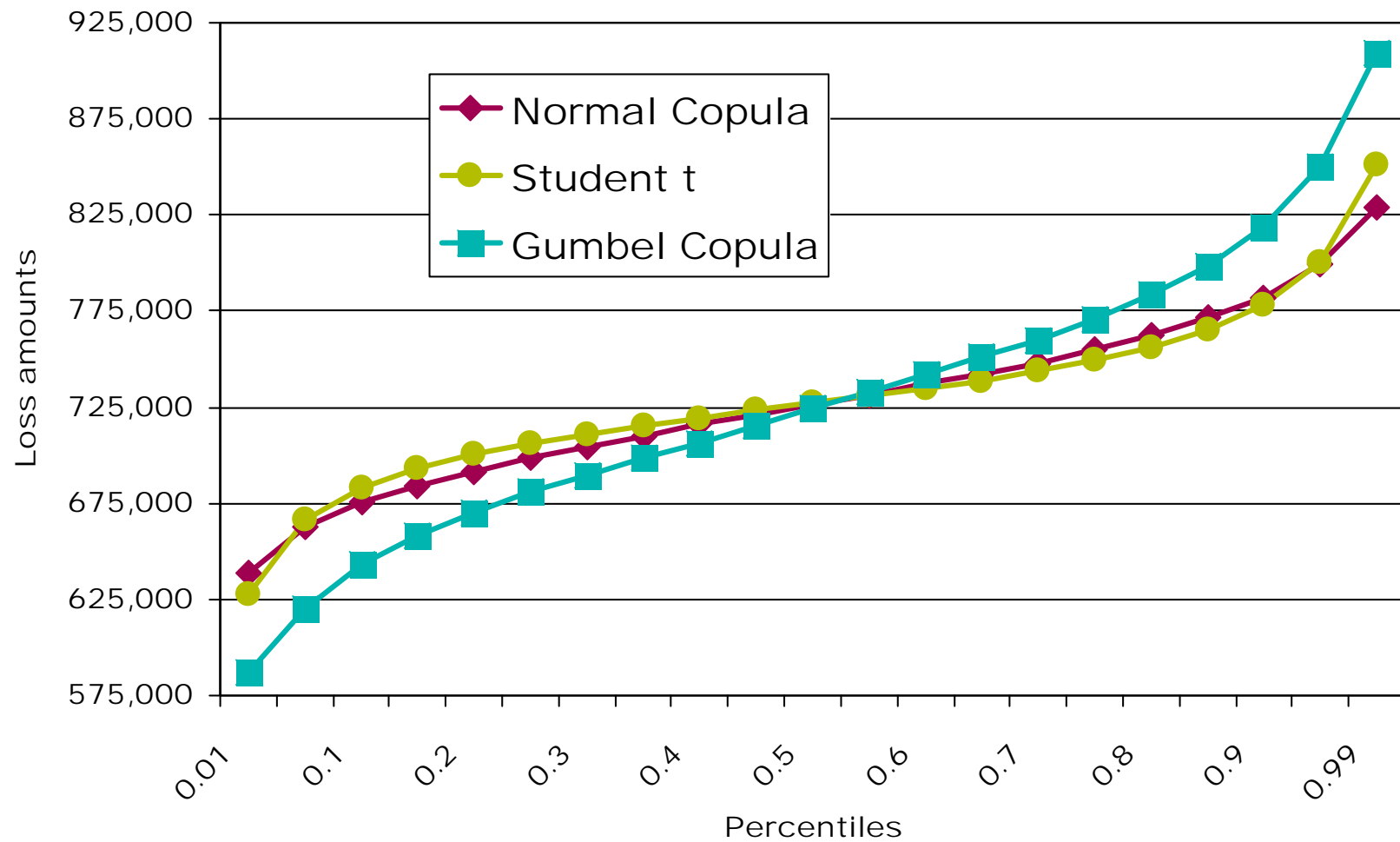Presentation1

Aggregation of Liabilities

Aggregation: Correlation between Lines of Business

- Strength of the correlation is irrelevant if we only care about the mean reserve indication for two lines A and B:
  - mean(A + B) = mean(A) + mean(B)

- Strength of correlation matters when we look towards the ends of the aggregate distribution of (A+B)!

- Generally, the aggregate distribution is less risky than the distribution of the individual lines:
  - 75thpercentile(A + B) < 75thpercentile(A) + 75thpercentile(B)
  - Equality only occurs in the case of perfect correlation across lines (this is very unlikely!)

- The volatility of the aggregate distribution increases:
  - By the volatility of the individual lines
  - By the correlation between the lines

Theory of Copulas

- Copulas provide a convenient way to express the aggregate distributions of several random variables

- Copula components:
  - The distributions of individual random variables
  - Correlations of these variables

- Correlation coefficients measure the overall strength of association across various distributions

- Copulas can vary that degree of association over the various parts of the aggregate distribution
  - Example: for workers comp and property losses the correlation is higher in the tail of the distribution

Comparison of Copulas

Questions?