# Loss Simulation Model

## Testing and Enhancement

*Casualty Loss Reserve Seminar*

*By*

*Kailan Shang*

*Sept. 2011*

# Agenda

- Research Overview
- Model Testing
- Real Data
- Model Enhancement
- Further Development

# I. Research Overview

# Background – Why use the LSM

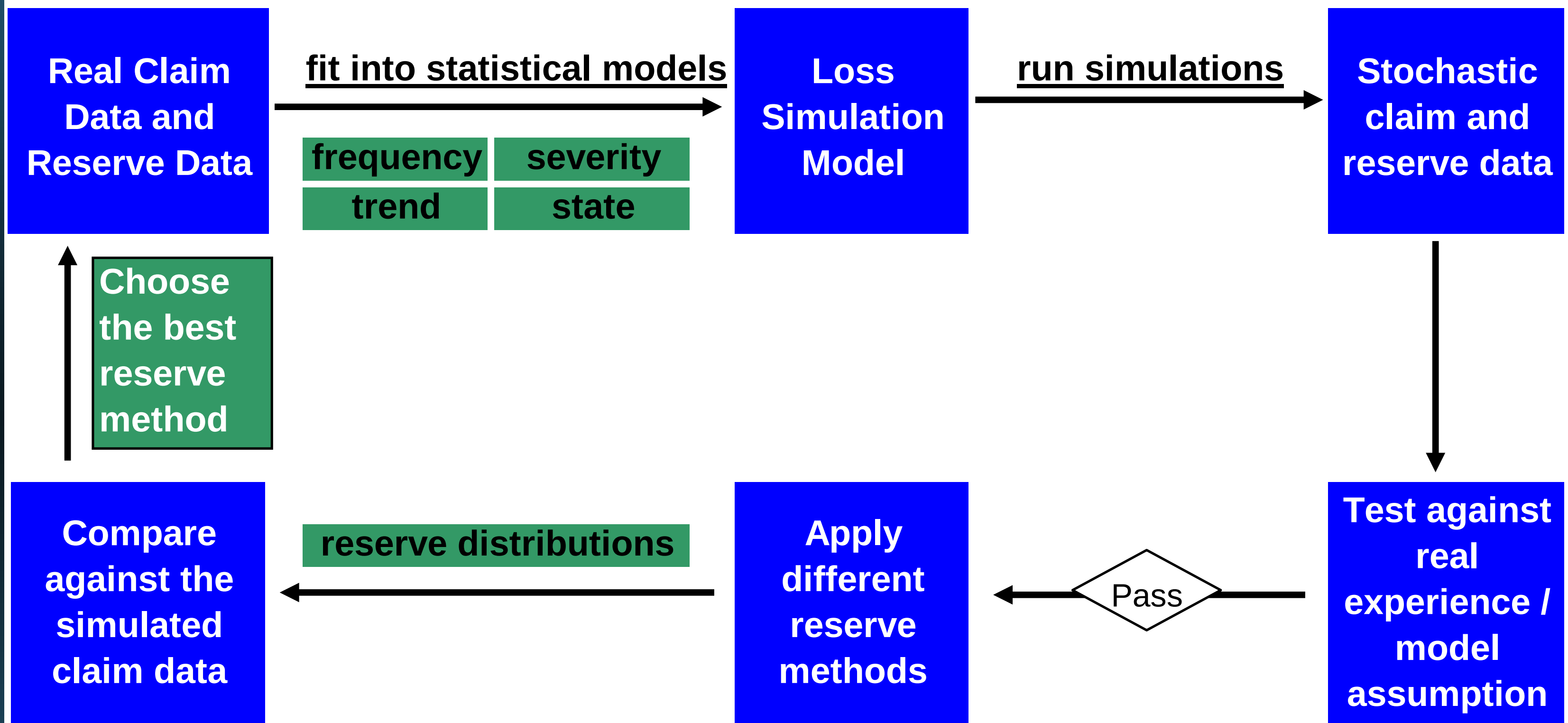
Reserving is a challenging task which requires a lot of judgements on assumption setting

The loss simulation model (LSM) is a tool created by the CAS Loss Simulation Model Working Party (LSMWP) to generate claims that can be used to test loss reserving methods and models

It helps us understand the impact of assumptions on reserving from a different perspective – distribution based on simulations that resemble the real experience

In addition, stochastic reserving is also a popular trend.

# Background – How to use the LSM

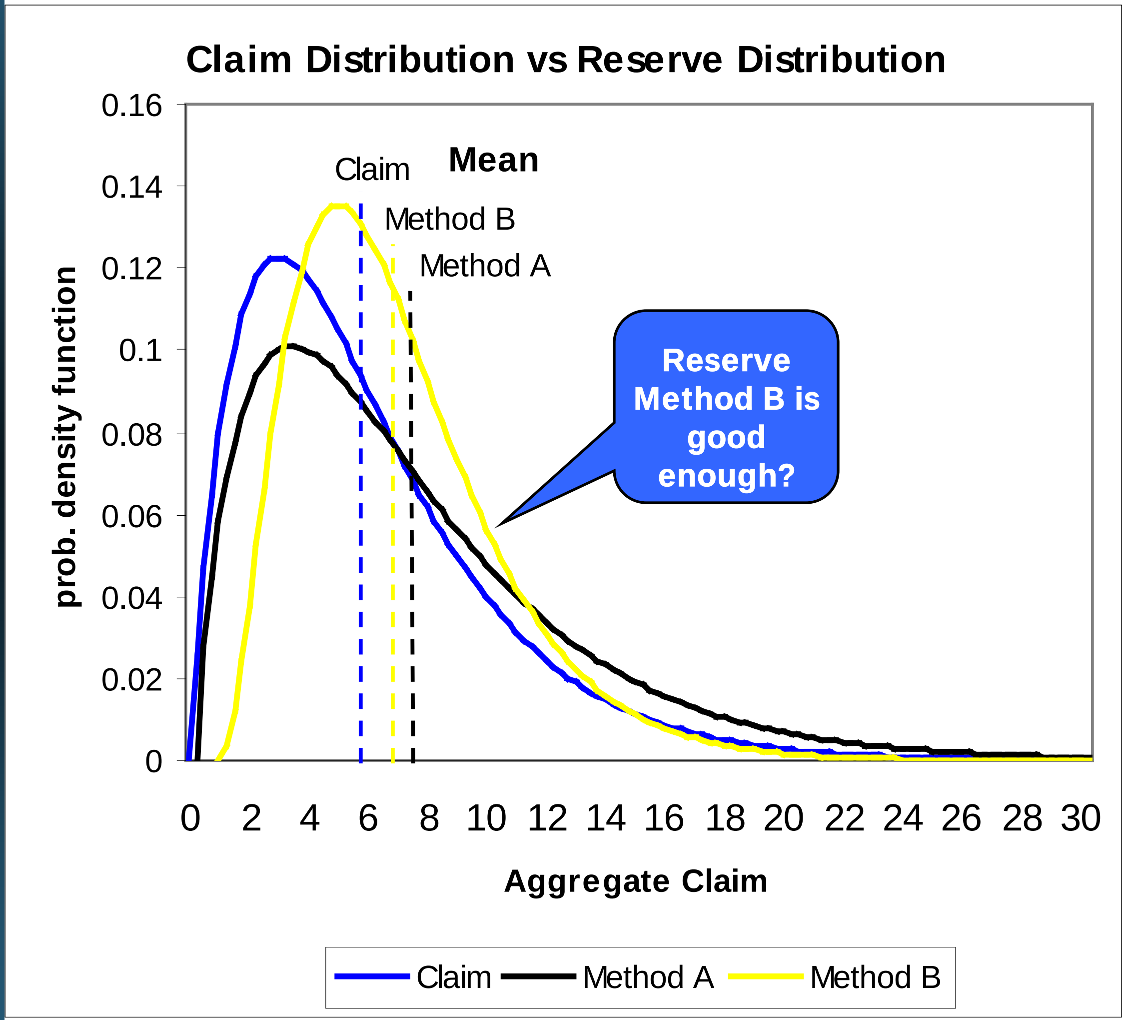



**We do not expect an accurate estimation of the claim amount.**

**We are more concerned about the adequacy of our reserve.**

**At what probability that the reserve is expected to be below the final payment?**

# Background – How to use the LSM




| Amount | Claim | Method A | Method B |
|---|---|---|---|
| 10 | 83.5% | 73.7% | 81.2% |
| 15 | 95.7% | 90.3% | 96.7% |
| 20 | 99.0% | 96.6% | 99.5% |
| 25 | 99.8% | 98.9% | **99.9%** |
| 30 | **99.9%** | 99.6% | 100.0% |

99.9% percentile of method B

**<**

99.9% percentile of claim

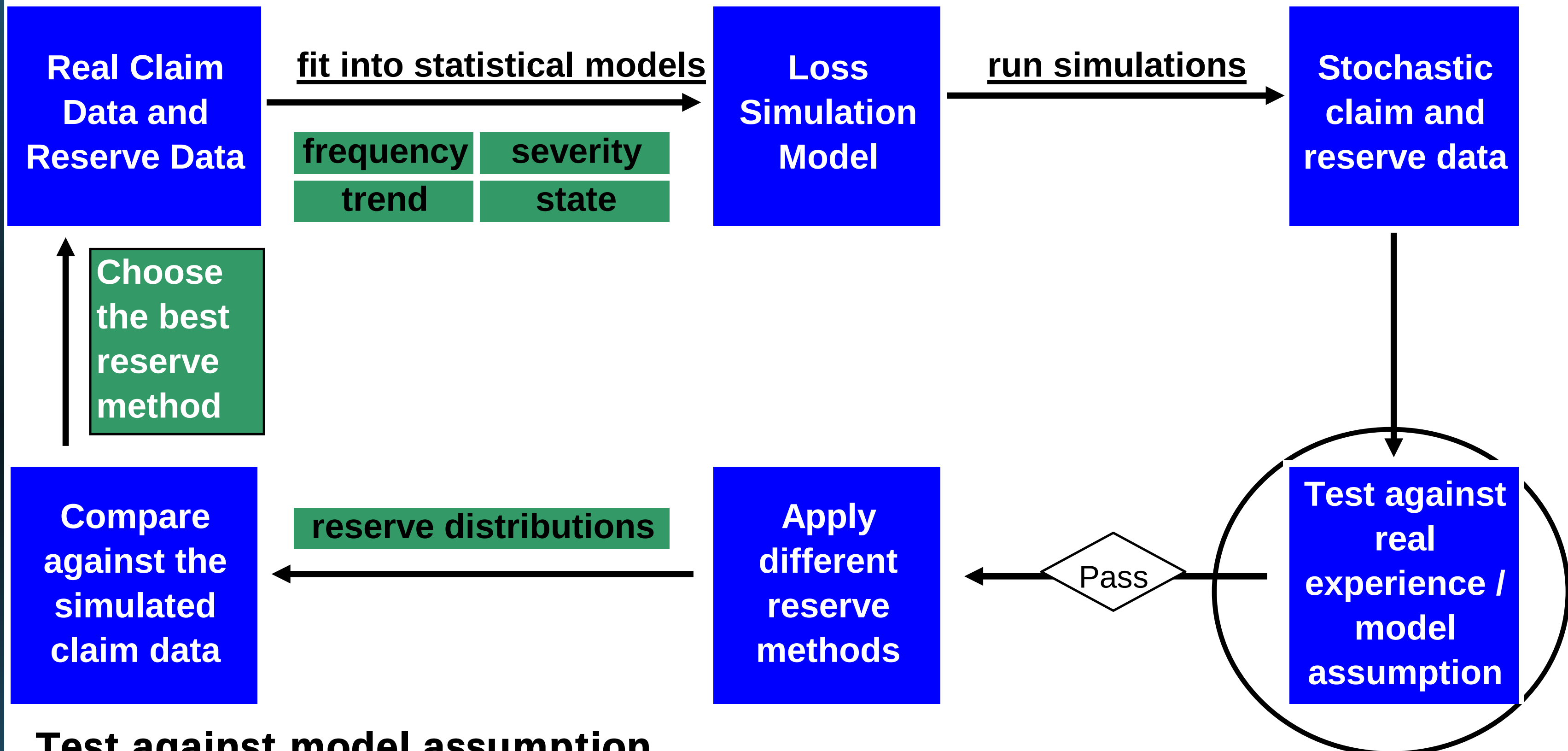
Without stochastic analysis, method B might be chosen. **The LSM can help you on it!**

**One out of hundreds of examples**

# Overview

- Test some items suggested but not fully addressed in the CAS LSMWP summary report "Modeling Loss Emergence and Settlement Processes"
- Fit real claim data to models.
- Build two-state regime-switching feature in the LSM to add an extra layer of flexibility to describe claim data.
- Software: LSM and R. The source code of model testing and model fitting using R is provided.

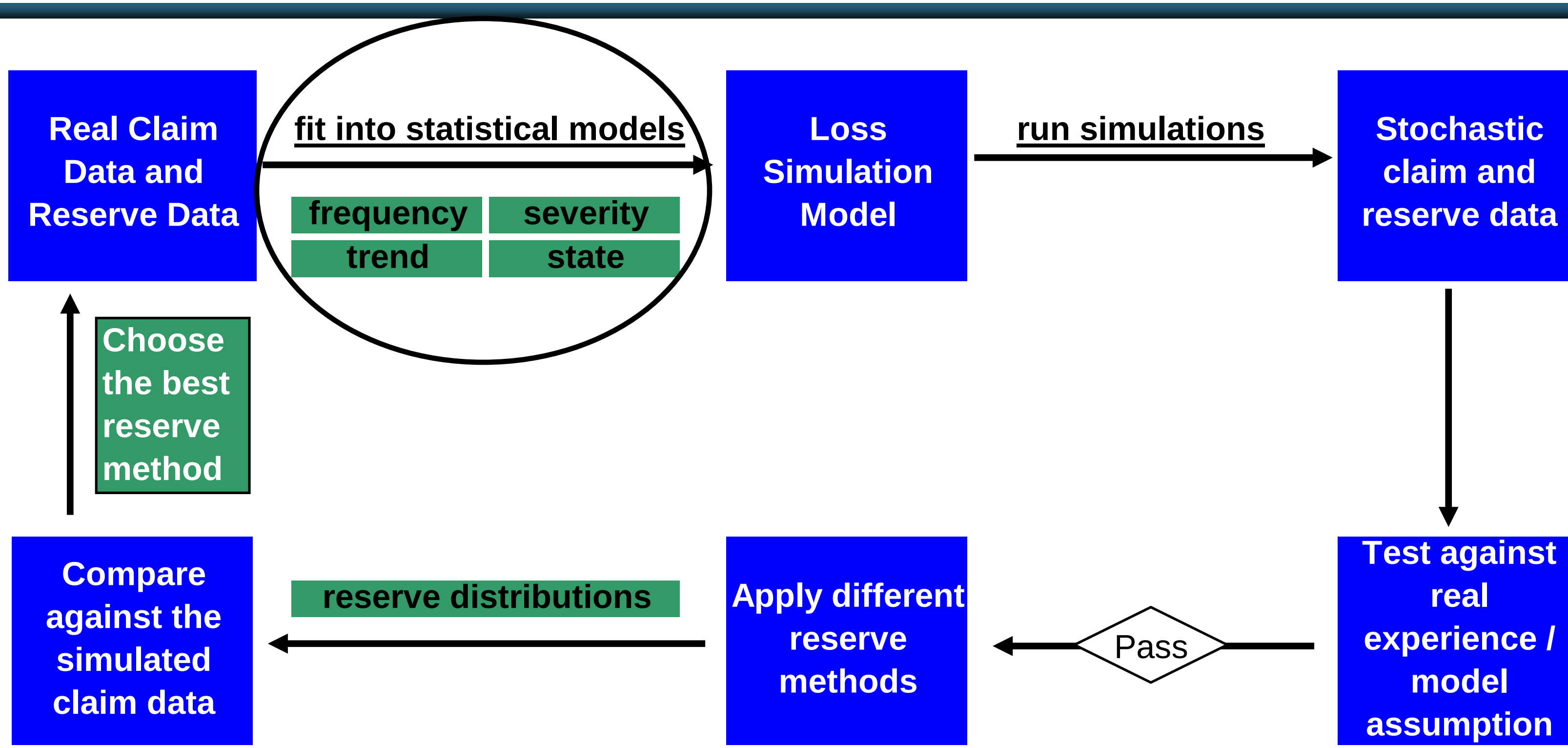
# Model Testing




**Test against model assumption**

- ✓ Negative binomial frequency distribution
- ✓ Correlation
- ✓ Severity trend
- ✓ Case reserve adequacy distribution

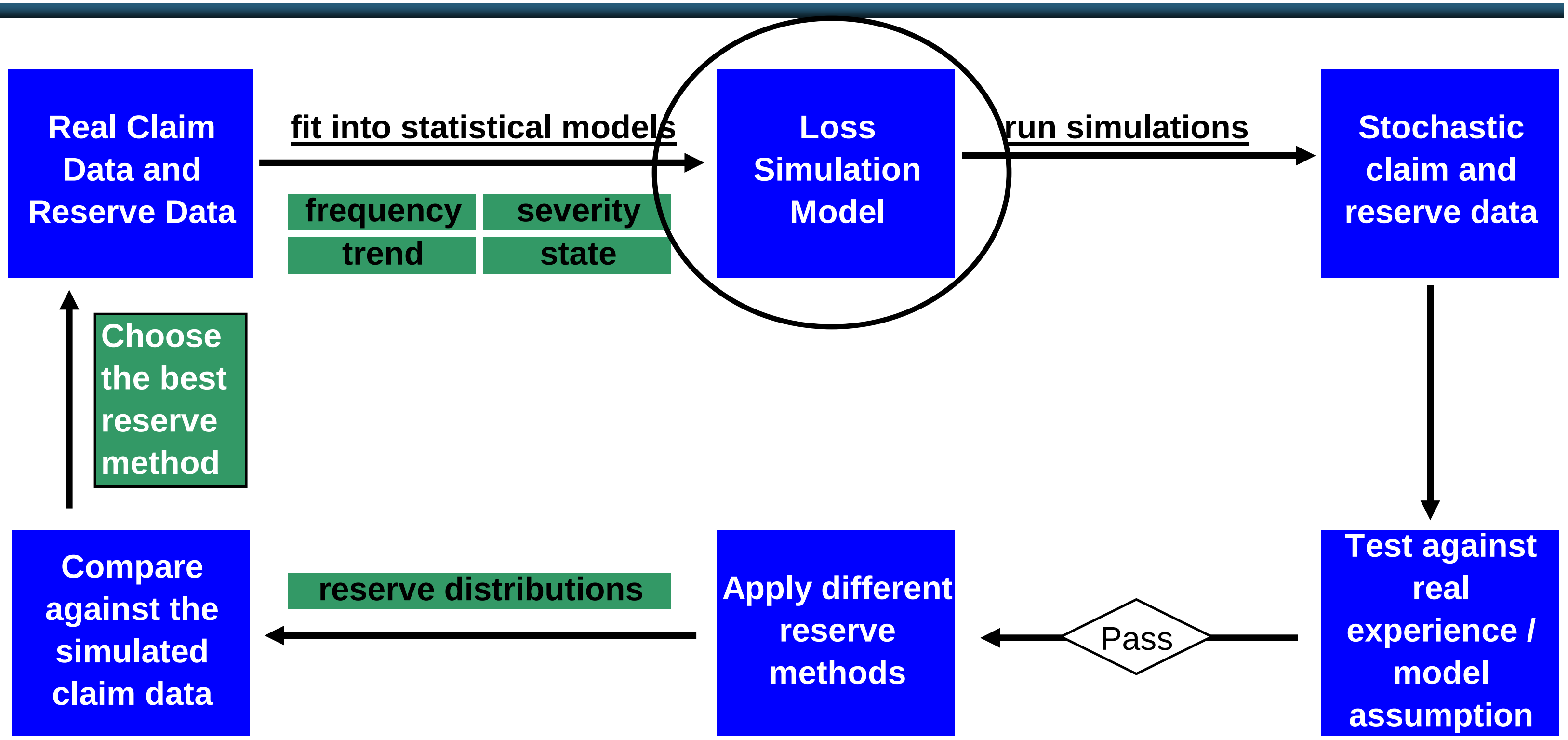
# Real Data Model Fitting




**Fit real claim data to statistical models**

- ✓ **frequency**
- ✓ **Severity**
- ✓ **Trend**
- ✓ **Correlation**

# Model Enhancement




**Two-state regime-switching distribution**

- ✓ **Switch between states at specified probability**
- ✓ **Each state represents a distinct distribution**

# II. Model Testing

# DAY ONE

## 9 AM

Tom, our company plans to use the loss simulation model to help our reserving works. Let's do some tests first to get a better understanding of the model.

Start from the frequency model.

# Negative Binomial Frequency Testing

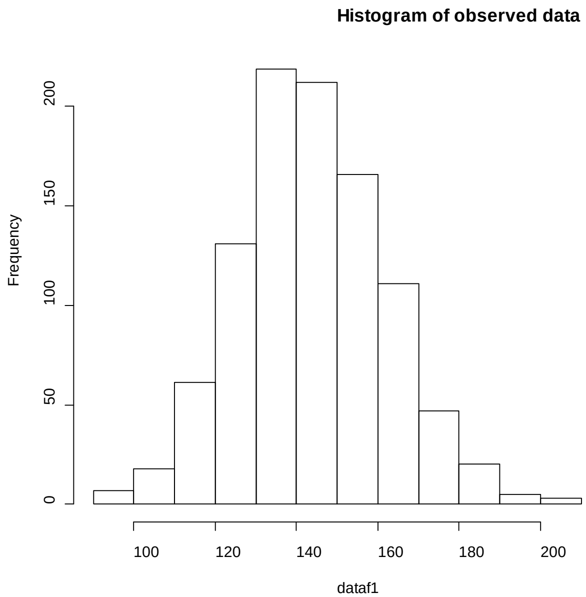
- **Frequency simulation**
  - ✓One Line with annual frequency Negative Binomial (size=100, prob.=0.4)
  - ✓Monthly exposure: 1
  - ✓Frequency Trend: 1
  - ✓Seasonality: 1
  - ✓Accident Year: 2000
  - ✓Random Seed: 16807
  - ✓No. of Simulations: 1000

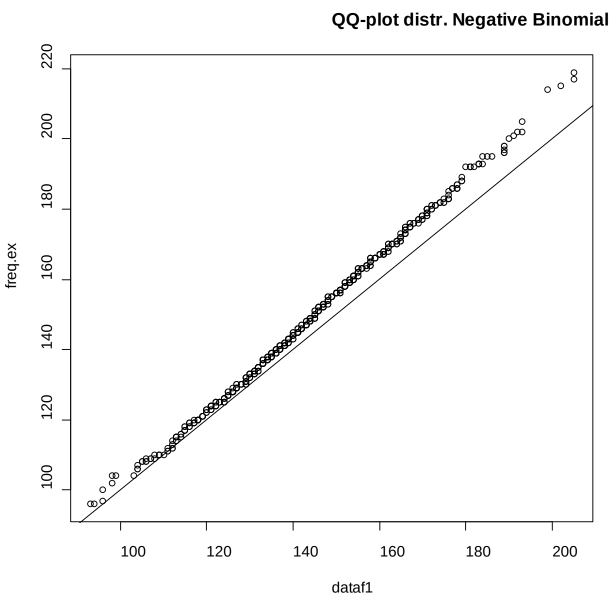
**R code extract**

```
# draw histogram
hist(dataf1,main="Histogram of observed data")

# QQPlot
freq.ex<-(rnbinom(n=1000,size=100,prob=0.4))
qqplot(dataf1,freq.ex,main="QQ-plot distr. Negative Binomial")
abline(0,1) ## a 45-degree reference line is plotted
```

- **Histogram and QQ plot**



# Negative Binomial Frequency Testing

- **Goodness of fit test - Pearson's $\chi^2$**

| | $\chi^2$ | *p value* |
|---|---|---|
| Pearson | 197.4 | 0.64 |

- **Maximum likelihood (ML) estimation**

| | *size* | $\mu$ |
|---|---|---|
| Estimation | 117.2 | 144.2 |
| S.D. | 9.5 | 0.57 |

| | Model Assumption | ML estimation |
|---|---|---|
| Size | 100 | 117 |
| Prob. | 0.4 | 0.448 |
| Mean $(\mu)$ | 150 | 144.2 |
| Variance | 375 | 321.5 |

**R code extract**

```
# Goodness of fit test
library(vcd) #load package vcd
gf<-goodfit(dataf1,type="nbinom",par=list(size=100,prob=0.4))

# Maximum likelihood estimation
gf<-goodfit(dataf1,type= "nbinom",method= "ML")
fitdistr(dataf1, "Negative Binomial")
```

# DAY ONE

## 5 PM

Good job Tom!
Let’s get the correlation test done tomorrow.

# Correlation

- **Correlation among frequencies of different lines**
  - Gaussian Copula
  - **Clayton Copula**
  - **Frank Copula**
  - **Gumbel Copula**
  - ***t* Copula**
- **Correlation between claim size and report lag**
  - **Gaussian Copula**
  - Clayton Copula
  - Frank Copula
  - Gumbel Copula
  - *t* Copula

**Use R package “copula”**

# Frequencies – Frank Copula

**Gumbel Copula:** $C_\theta^n(u) = -\frac{1}{\theta}\ln(1+\frac{(e^{-\theta u_1}-1)(e^{-\theta u_2}-1)\cdots(e^{-\theta u_n}-1)}{(e^{-\theta}-1)^{n-1}}$ $\quad \theta > 0$

- **$U_i$: marginal cumulative distribution function (CDF)**
- **C(u): joint CDF**

- **Frequencies simulation**
  - Two Lines with annual frequency Poisson ($\lambda$ = 96)
  - Monthly exposure: 1
  - Frequency Trend: 1
  - Seasonality: 1
  - Accident Year: 2000
  - Random Seed: 16807
  - Frequency correlation: $\Theta$ = 8, n = 2
  - # of Simulations: 1000

- **Test Method**
  - Scatter plot
  - Goodness-of-fit test

**1. Parameter estimation based on maximum likelihood and inverse of Kendall's tau**

**2. Cramer-von Mises (CvM) statistic**

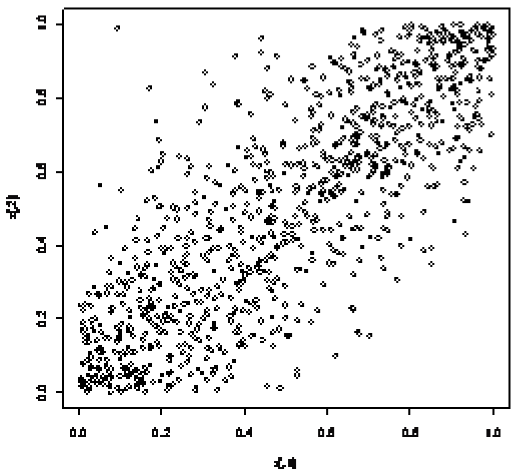
$$S_n^{(k)} = \sum_{i=1}^{n}\{C_n^{(k)}(\widehat{U}_i^{(k)}) - C_{\theta_n}^{(k)}(\widehat{U}_i^{(k)})\}^2$$

**3. *p* value by parametric bootstrapping**

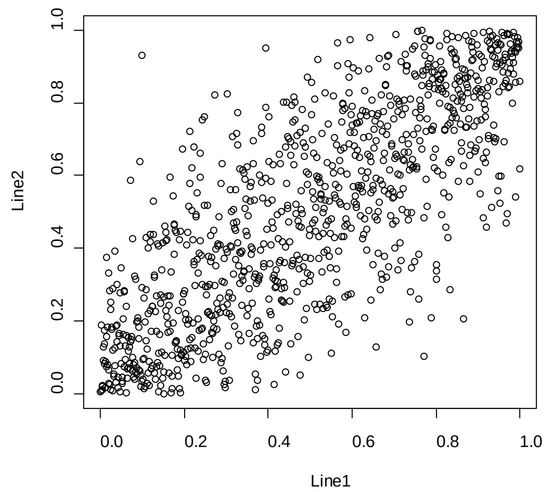
# Frequencies – Frank Copula

- **Scatter plot**

**Frank Copula (Θ=8)**



**Simulated Frequencies**




- **Goodness-of-fit test**
  - **Maximum Likelihood method**

    Parameter estimate(s): 7.51

    Std. error: 0.28
    CvM statistic: 0.016 with *p*-value 0.31
  - **Inversion of Kendall's tau method**
    Parameter estimate(s): 7.54
    Std. error: 0.31
    CvM statistic: 0.017 with *p*-value 0.20

**R code extract**

```
# construct a Gumbel copula object
gumbel.cop <- gumbelCopula(3, dim=2)

# parameter estimation
fit.gumbel<-fitCopula(gumbel.cop,x,method="ml")
fit.gumbel<-fitCopula(gumbel.cop,x,method="itau")

#Copula Goodness-of-fit test
gofCopula(gumbel.cop, x, N=100, method = "mpl")
gofCopula(gumbel.cop, x, N=100, method = "itau")
```

# Claim Size and Report Lag – Normal Copula

**Normal Copula a.k.a. Gaussian Copula:** $C_{\Sigma}^{n}(u) = \Phi_{\Sigma}(\Phi^{-1}(u_1), \cdots, \Phi^{-1}(u_n))$

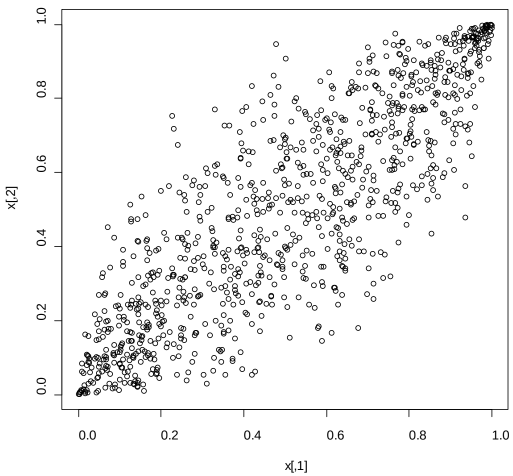
- **$\Sigma$: correlation matrix**
- **$\Phi$: normal cumulative distribution function**

- **Claim simulation**
  - One Line with annual frequency Poisson ($\lambda$ = 120)
  - Monthly exposure: 1
  - Frequency Trend: 1.05
  - Seasonality: 1
  - Accident Year: 2000
  - Random Seed: 16807
  - Payment Lag: Exponential with rate = 0.00274, which implies a mean of 365 days.
  - Size of entire loss: Lognormal with $\mu$ = 11.17 and $\sigma$ = 0.83
  - Correlation between payment lag and size of loss: normal copula with correlation = 0.85, dimension 2
  - # of Simulations: 10

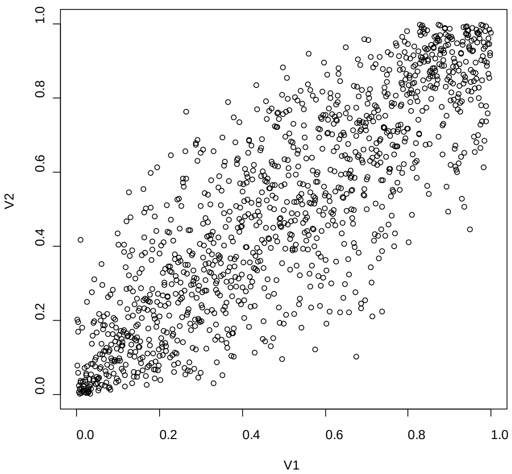
# Claim Size and Report Lag – Normal Copula

- **Scatter plot**

**Normal Copula (0.85)**




**Simulated claim size vs. report lag**




- **Goodness-of-fit test**
  - **Maximum Likelihood method**

    Parameter estimate(s): 0.83

    Std. error: 0.01

    CvM statistic: 0.062 with *p*-value 0.05
  - **Inversion of Kendall's tau method**

    Parameter estimate(s): 0.85

    Std. error: 0.01

    CvM statistic: 0.029 with *p*-value 0.015

# DAY THREE

## 9 AM

We often see trends in our claim data. How is it handled in the simulation model?

# Severity Trend

**The LSM has two ways to model it**

- **Trend factor (cum)**
- **$\alpha$ (Persistency of the force of the trend)**

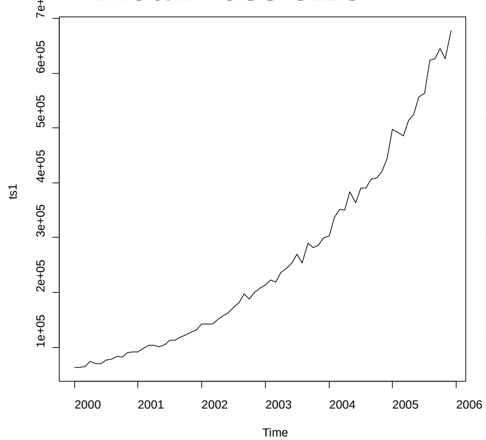
$$trend = (cum_{acc\_date})\left(\frac{cum_{pmt\_date}}{cum_{acc\_date}}\right)^{\alpha} = (cum_{acc\_date})^{1-\alpha}(cum_{pmt\_date})^{\alpha}$$

- **Trend factor Test Parameters**
  - One Line with annual frequency Poisson ($\lambda$ = 96)
  - Monthly exposure: 1
  - Frequency Trend: 1
  - Seasonality: 1
  - Accident Year: 2000 to 2005
  - Random Seed: 16807
  - Size of entire loss: Lognormal with $\mu$ = 11.17 and $\sigma$ = 0.83
  - Severity trend: 1.5
  - \# of Simulations: 300

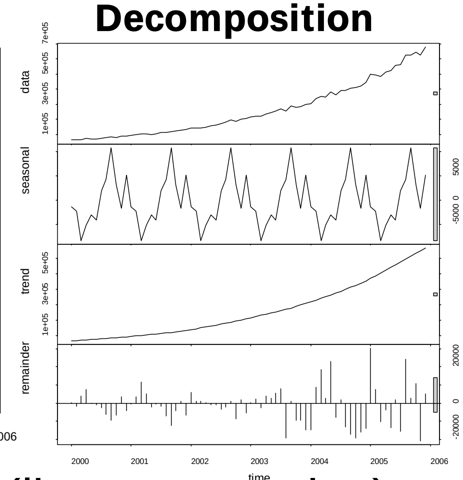
# Severity Trend

- **Trend factor Test**
  - Decomposition of Time Series by Loess (Locally weighted regression) into trend, seasonality, and remainder

**Mean loss size**




**Decomposition**




**R code extract**

```
#set up time series
ts1<-ts(data,start=2000,frequency=12)
plot(ts1)
#decomposition
plot(stl(ts1,s.window="periodic"))
#linear trend fitting
trend = time(ts1)-2000
reg = lm(log(ts1)~trend, na.action=NULL)
```

  - Time series analysis (linear regression)

**Log(Mean Loss Size) = Intercept + trend * (time – 2000) + error term**

**Coefficients:**

| | Estimate | Std. Error | *t* value | Pr(>\|*t*\|) |
|---|---|---|---|---|
| (Intercept) | 11.034162 | 0.007526 | 1466.1 | <2*e*-16 |
| trend | 0.405552 | 0.002196 | 184.7 | <2*e*-16 |

**Residual standard error: 0.03226 on 70 degrees of freedom**
**Multiple R-squared: 0.998, Adjusted R-squared: 0.9979**
***F*-statistic: 3.412e+04 on 1 and 70 DF, *p*-value: < 2.2*e*-16**

**exp(0.405552) = 1.50013 vs. model input 1.5**

# Severity Trend

- **Trend persistency $\alpha$ Test Parameters**
  - One Line with annual frequency Poisson ($\lambda$ = 96)
  - Monthly exposure: 1
  - Frequency Trend: 1
  - Seasonality: 1
  - Accident Year: 2000 to 2001
  - Random Seed: 16807
  - Size of entire loss: Lognormal with m = 11.17 and s = 0.83
  - Severity trend: 1.5
  - Alpha = 0.4
  - # of Simulations: 1000

**But how do we test it?**

Choose the loss payments with report date during the 1st month and payment date during the 7th month.

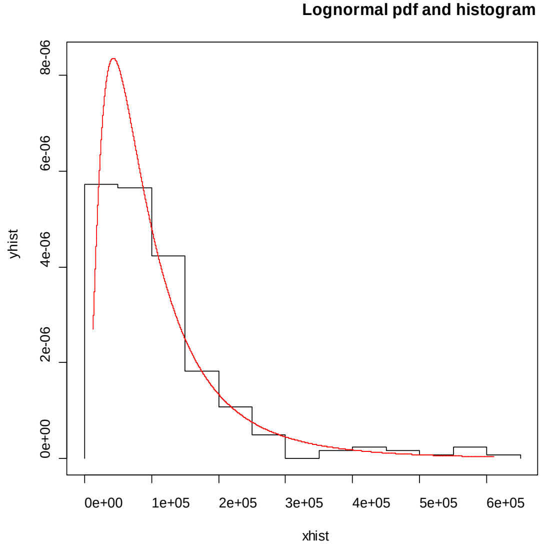
The severity trend is $(1.5^{1/12})^{(1-0.4)} \cdot (1.5^{7/12})^{0.4} \approx 1.122$

The expected loss size is $1.122 \cdot e^{11.17+0.83^2/2} \approx 112{,}175$

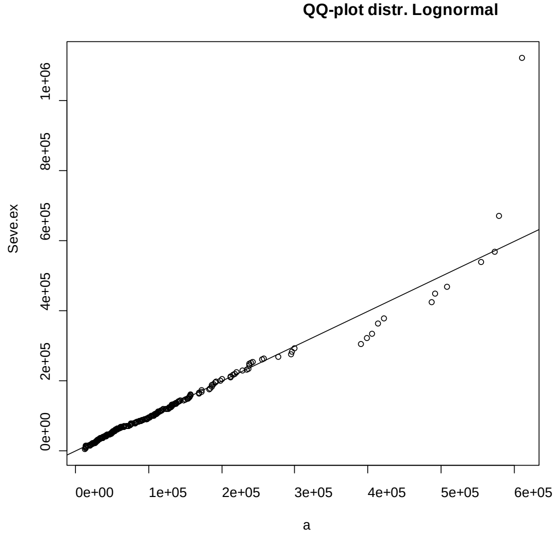
# Severity Trend

- **Trend persistency $\alpha$ Test**

**Histogram and fitted *pdf***




**QQ plot of severity**




- Maximum likelihood estimation (mean of severity=113,346**)**

| | **meanlog** | **sdlog** |
|---|---|---|
| **Estimation** | **11.32** | **0.80** |
| **Standard Deviation** | **0.052** | **0.037** |

- Normality test of log (severity)

  **Kolmogorov-Smirnov test**: *p*-value = 0.82

  **Anderson-Darling normality test**: *p*-value = 0.34

**R code extract**

```
#Kolmogorov-Smirnov Tests
ks.test(a,"plnorm", meanlog=11.32,
      sdlog=0.8)
#Anderson-Darling Test
library(nortest) ## package loading
ad.test(datas1.norm)
```

# DAY FOUR

## 9 AM

I heard you guys plan to use the loss simulation model. Is it capable of modeling case reserve adequacy?

# Case Reserve Adequacy

**In the LSM, the case reserve adequacy (CRA) distribution attempts to model the reserve process by generating case reserve adequacy ratio at each valuation date**

- **Case reserve = generated final claim amount × case reserve adequacy ratio**

- ## Case Reserve Simulation
  - One Line with annual frequency Poisson ($\lambda$ = 96)
  - Monthly exposure: 1
  - Frequency Trend: 1
  - Seasonality: 1
  - Accident Year: 2000 to 2001
  - Random Seed: 16807
  - Size of entire loss: Lognormal with $\mu$ = 11.17 and $\sigma$ = 0.83
  - Severity trend: 1
  - $P(0)$ = 0.4
  - Est $P(0)$ = 0.4
  - # of Simulations: 8

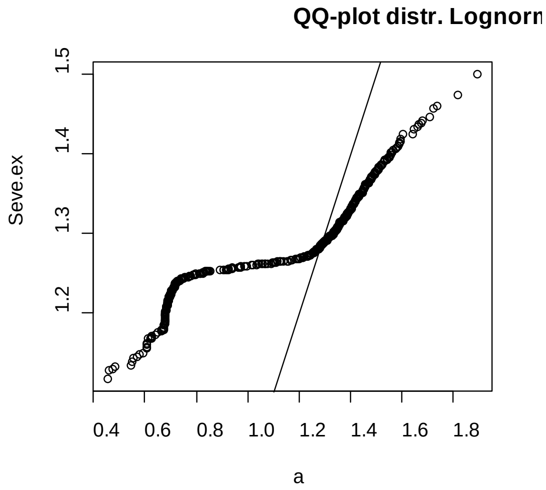
**Test 40% time point (60×report date + 40%×final payment date) case reserve adequacy ratio**

**Mean:** $e^{0.25+0.05^2/2} \approx 1.2856$

# Case Reserve Adequacy

- ## Case Reserve Adequacy Test

**QQ plot of CRA ratio**




- Maximum likelihood estimation

| | meanlog | sdlog |
|---|---|---|
| **Estimation** | **0.08** | **0.32** |
| **Standard Deviation** | **0.014** | **0.010** |

- Normality test of log (CRA ratio)

**Kolmogorov-Smirnov test: p-value = 0.00**

**Anderson-Darling normality test: p-value = 0.00**

**Where went wrong?**

**case reserve is generated on the simulated valuation dates.**

**Linear interpolation method is used to get case reserve ratio at 40% time point.**

**On the report date, a case reserve of 2,000 is allocated for each claim.**

**If the second valuation date > 40% time point, linear interpolation method is not appropriate.**

# III. Real Data

# DAY FIVE

## 5 PM

Wait a minute Tom! I want you to think about how to use real claim data for model calibration during the weekend!

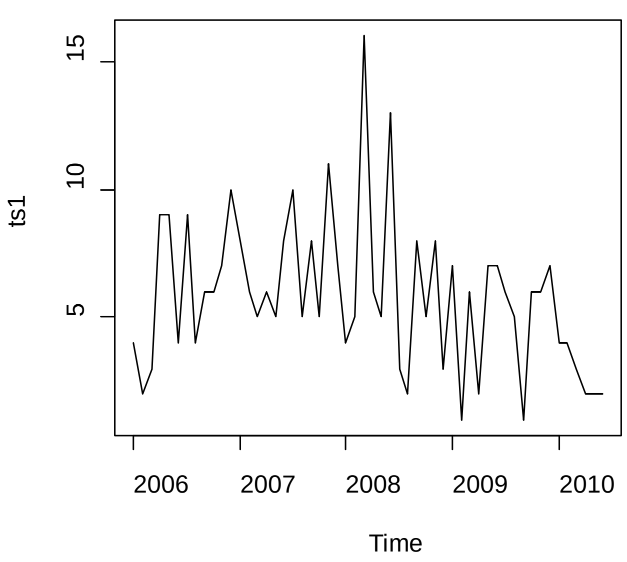

# Real Data

**Marine claim data for distribution fitting, trend analysis, and correlation analysis**

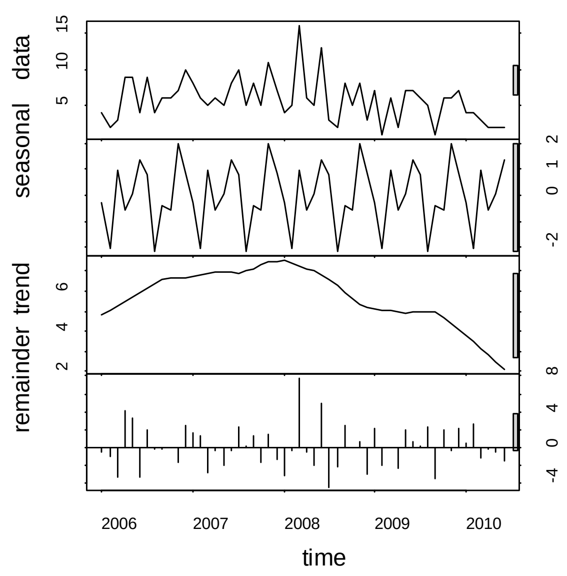
- **two product lines: Property and Liability**
- **data period: 2006 – 2010**
- **accident date, payment date, and final payment amount**

- **Fit the frequency**
  - Draw time series and decomposition

**Historical Frequency**




**Decomposition**



# Real Data

- **Fit the frequency (continued)**
  - Linear regression for trend analysis

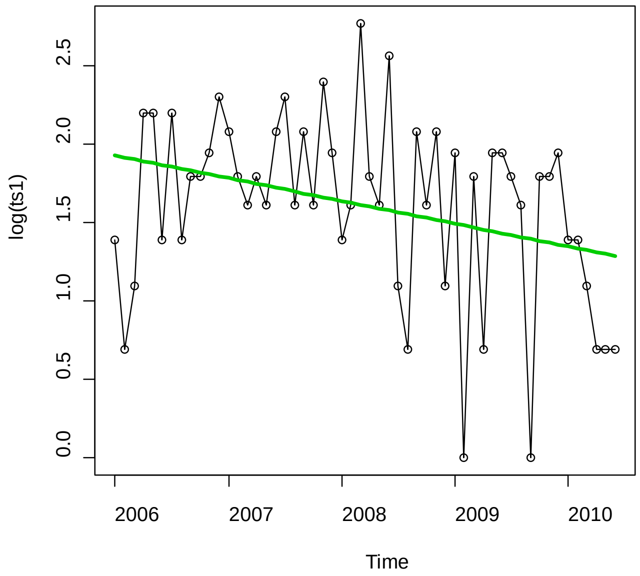
**Log(Monthly Frequency) = Intercept + trend * (time – 2006) + error term**

**Coefficients:**

| | Estimate | Std. Error | $t$ value | Pr(>\|$t$\|) |
|---|---|---|---|---|
| (Intercept) | 1.93060 | 0.15164 | 12.732 | <2$e$-16 |
| trend | -0.14570 | 0.05919 | -2.462 | 0.0172 |

**Residual standard error: 0.5649 on 52 degrees of freedom.**
**Multiple R-squared: 0.1044, Adjusted R-squared: 0.08715.**
**$F$-statistic: 6.06 on 1 and 52 DF, $p$-value: 0.01718.**

**Trend Fitting**



# Real Data

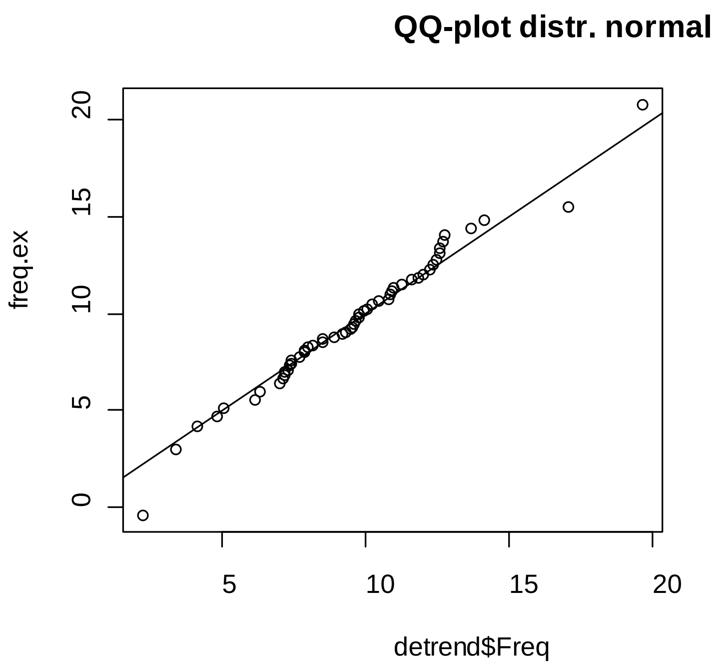
- **Fit the frequency (continued)**
  - Detrend the frequency and fit to the lognormal distribution

| | meanlog | sdlog |
|---|---|---|
| Estimation | 9.5539259 | 3.1311762 |
| Standard Deviation | 0.4260991 | 0.3012976 |

  - Normality test of log (detrended freq.)

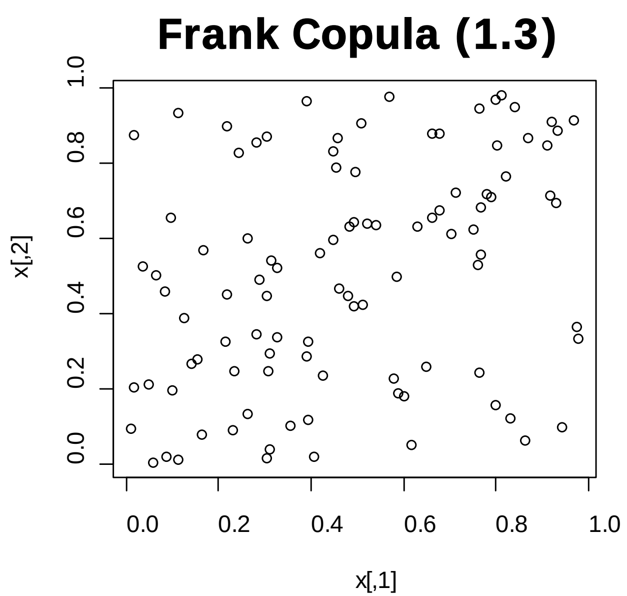
    **Kolmogorov-Smirnov test: p-value = 0.84**

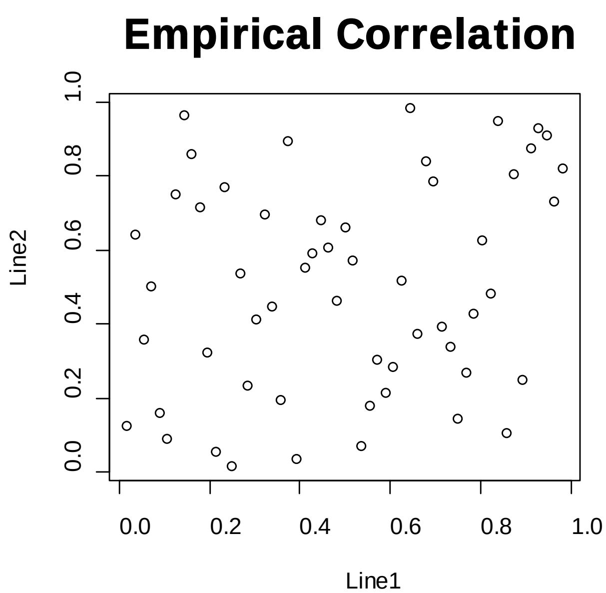
**QQ plot of detrended freq.**



# Real Data

- **Fit the Severity**
- **Correlation calibration**




- **Maximum Likelihood method**

  Parameter estimate(s): 1.51

  CvM statistic: 0.027 with *p*-value 0.35

- **Inversion of Kendall's tau method**

  Parameter estimate(s): 1.34

  CvM statistic: 0.028 with *p*-value 0.40

**What is missing?**

**Historical reserve data which are essential for case reserve adequacy modeling.**

# IV. Model Enhancement

# Two-state regime-switching model

**Sometimes the frequency and severity distribution are not stable over time**

- **Structural change**
- **Cyclical pattern**
- **Idiosyncratic character**

## •The model

- Two distinct distributions represent different states
- Transition rules from one state to another

**$P_{11}$: state 1 persistency, the probability that the state will be 1 next month given that it is 1 this month.**
**$P_{12}$: the probability that the state will be 2 next month given that it is 1 this month.**
**$P_{21}$: the probability that the state will be 1 next month given that it is 2 this month.**
**$P_{22}$: state 2 persistency, the probability that the state will be 2 next month given that it is 2 this month.**
**$\Pi_1$: steady probability of state 1.**
**$\Pi_2$: steady probability of state 2.**

$$\begin{pmatrix}\Pi_1 & \Pi_2\end{pmatrix}\begin{pmatrix}P_{11} & P_{12} \\ P_{21} & P_{22}\end{pmatrix} = \begin{pmatrix}\Pi_1 & \Pi_2\end{pmatrix}$$

$$P_{11} = 1 - P_{12}$$

$$P_{21} = 1 - P_{22}$$

$$\Pi_1 + \Pi_2 = 1$$

# Two-state regime-switching model

- **The Simulation**
  - Steps
  1. **Generate uniform random number randf$_0$ on range [0,1].**
  2. **If randf$_0$< $\Pi_1$, state of first month state is 1, else, it is 2.**
  3. **Generate uniform random number randf$_i$ on range [0,1].**
  4. **For previous month state I, if randf$_i$<P$_{i1}$, then state is 1, else it is 2.**
  5. **Repeat step 3 and 4 until the end of the simulation is reached.**
  - Test Parameters
  - ✓ **State 1: Poisson Distribution ($\lambda$ = 120)**
  - ✓ **State 2: Negative Binomial Distribution (size = 36, prob = 0.5)**
  - ✓ **Assume the trend, monthly exposure, and seasonality are all 1**
  - ✓ **State 1 persistency: 0.5**
  - ✓ **State 2 persistency: 0.7**
  - ✓ **Seed: 16807**

$$\Pi_1 = \frac{1-P_{22}}{2-P_{11}-P_{22}} = \frac{1-0.7}{2-0.5-0.7} = 0.375$$

$$\Pi_2 = \frac{1-P_{11}}{2-P_{11}-P_{22}} = \frac{1-0.7}{2-0.5-0.7} = 0.625$$

| Random Number (RN) | State | Criteria |
|---|---|---|
| 0.634633548790589 | 2 | RN>0.375 |
| 0.801362191326916 | 1 | RN>0.7 |
| 0.529508789768443 | 2 | RN>0.5 |
| 0.0441845036111772 | 2 | RN<0.7 |
| 0.994539848994464 | 1 | RN>0.7 |
| 0.21886122901924 | 1 | RN<0.5 |
| 0.0928565948270261 | 1 | RN<0.5 |
| 0.797880138037726 | 2 | RN>0.5 |
| 0.129500501556322 | 2 | RN<0.7 |
| 0.24027365935035 | 2 | RN<0.7 |
| 0.797712686471641 | 1 | RN>0.7 |
| 0.0569291599094868 | 1 | RN<0.5 |

# Two-state regime-switching model

- ## The Test – Transition Matrix
  - Frequency

    **State 1: Poisson ($\lambda$ = 120); State 1 persistency: 0.2**

    **State 2: Negative Binomial (size = 36, prob = 0.5); State 2 persistency: 0.9**

**Line 1 Frequency**

$$\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{pmatrix} 0.15 & 0.85 \\ 0.1 & 0.9 \end{pmatrix}$$

$$\begin{pmatrix} \Pi_1 & \Pi_2 \end{pmatrix} = \begin{pmatrix} 10.53\% & 89.47\% \end{pmatrix}$$

**Line 2 Frequency**

$$\begin{pmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{pmatrix} = \begin{pmatrix} 0.2 & 0.8 \\ 0.1 & 0.9 \end{pmatrix}$$

$$\begin{pmatrix} \Pi_1 & \Pi_2 \end{pmatrix} = \begin{pmatrix} 11.11\% & 88.89\% \end{pmatrix}$$

**<u>Non Zero Cases:</u>**

| | |
|---|---|
| **State 1: 391** | **State 1: 410** |
| **State 2: 2797** | **State 2: 2733** |

**<u>Probability of Zero Cases:</u>**

| | |
|---|---|
| **State 1: 0.005% ($e^{-10}$)** | **State 1: 0.005% ($e^{-10}$)** |
| **State 2: 0.125 (prob$^3$)** | **State 2: 0.135 ($e^{-2}$)** |

**<u>Estimated all Cases:</u> Non Zero Cases/ (1 – Probability of Zero Cases)**

| | |
|---|---|
| **State 1: 391** | **State 1: 410** |
| **State 2: 3188 (2797/(1-0.125))** | **State 2: 3161 (2733/(1-0.135))** |

**Total Cases: # of simulations * 12 months = 3600**

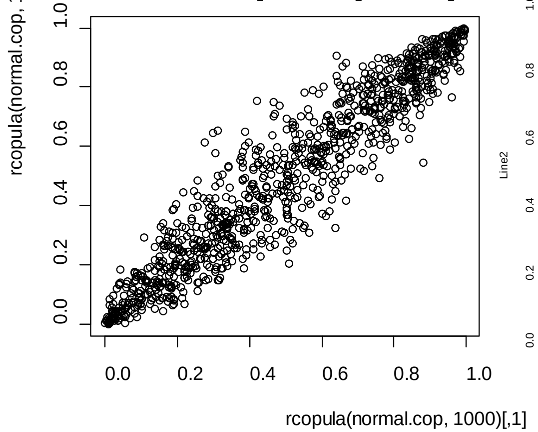
**Steady-state probability (compared with $P_1$ & $P_2$)**

| | |
|---|---|
| **State 1: 391/3600 = 10.86%** | **State 1: 410/3600 = 11.4%** |
| **State 2: 1-10.86% = 89.14%** | **State 2: 1-11.4% = 88.6%** |

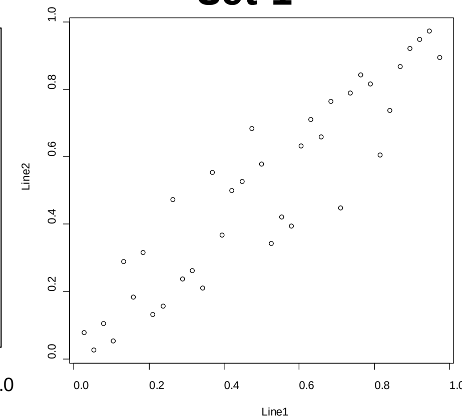
# Two-state regime-switching model

- **The Test – Correlation**

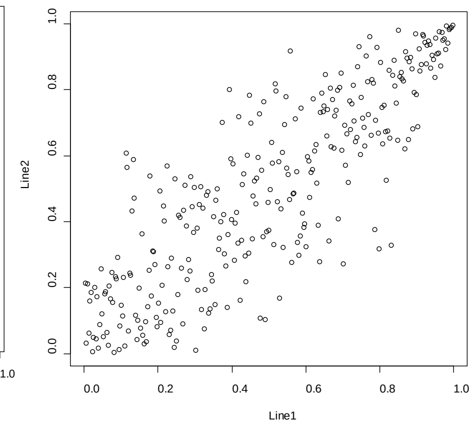
**Normal Copula (0.95)**



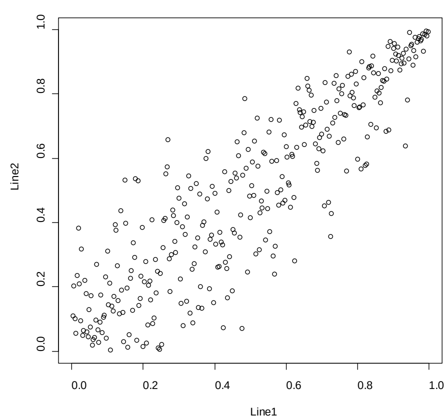

**Set 1**



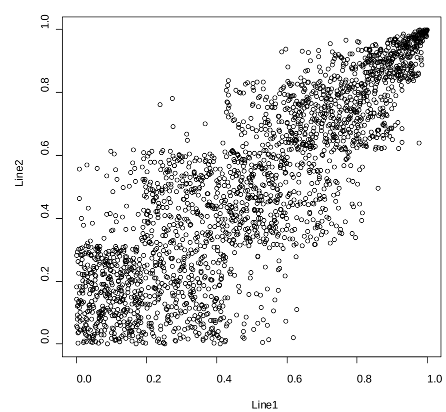

**Set 2**




**Set 3**




**Set 4**




**Set 1: State 1 for line 1 and state 1 for line 1**

**Set 2: State 1 for line 1 and state 2 for line 2**

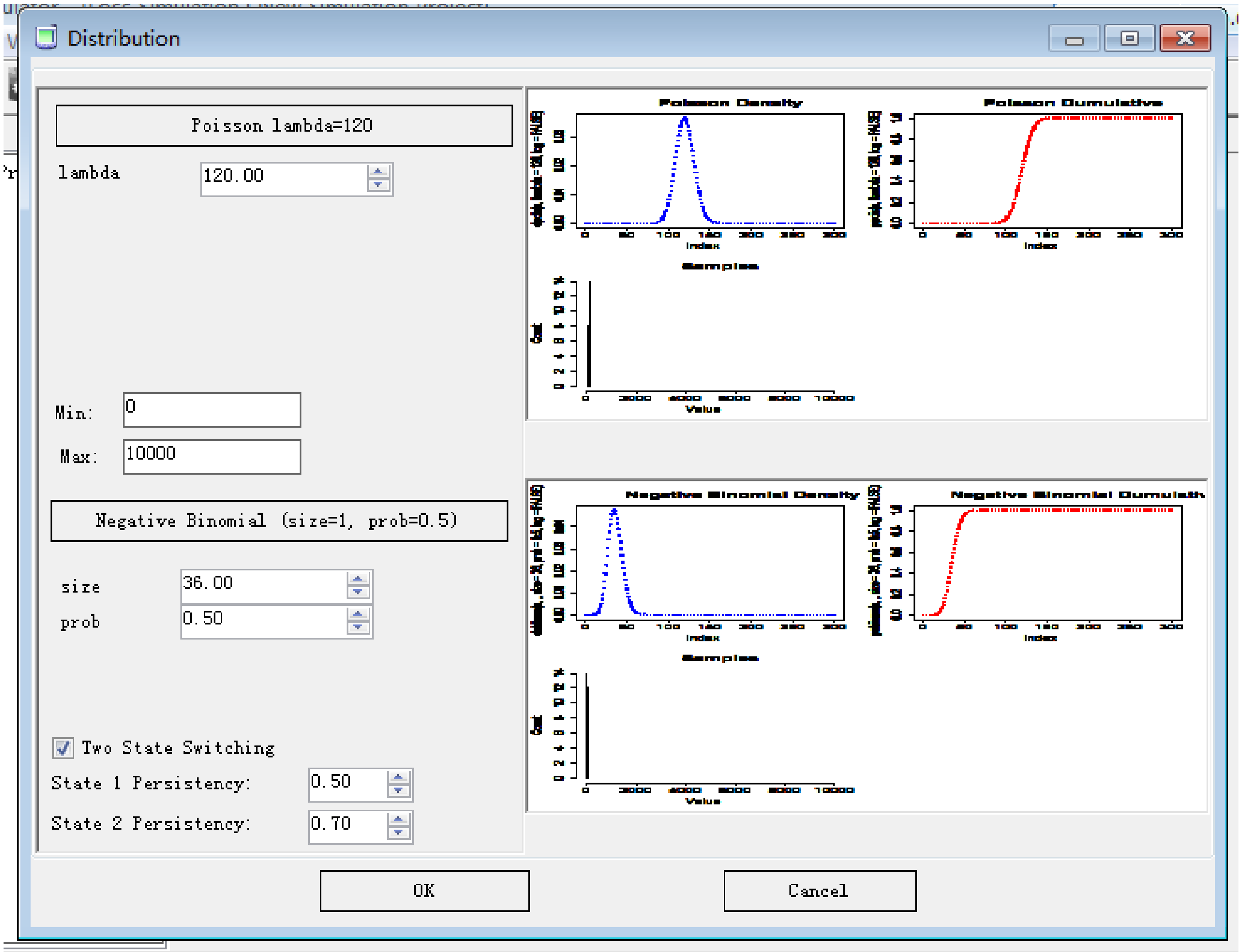
**Set 3: State 2 for line 1 and state 1 for line 1**

**Set 4: State 2 for line 2 and state 2 for line 2**

**Goodness-of-fit test is also conducted.**

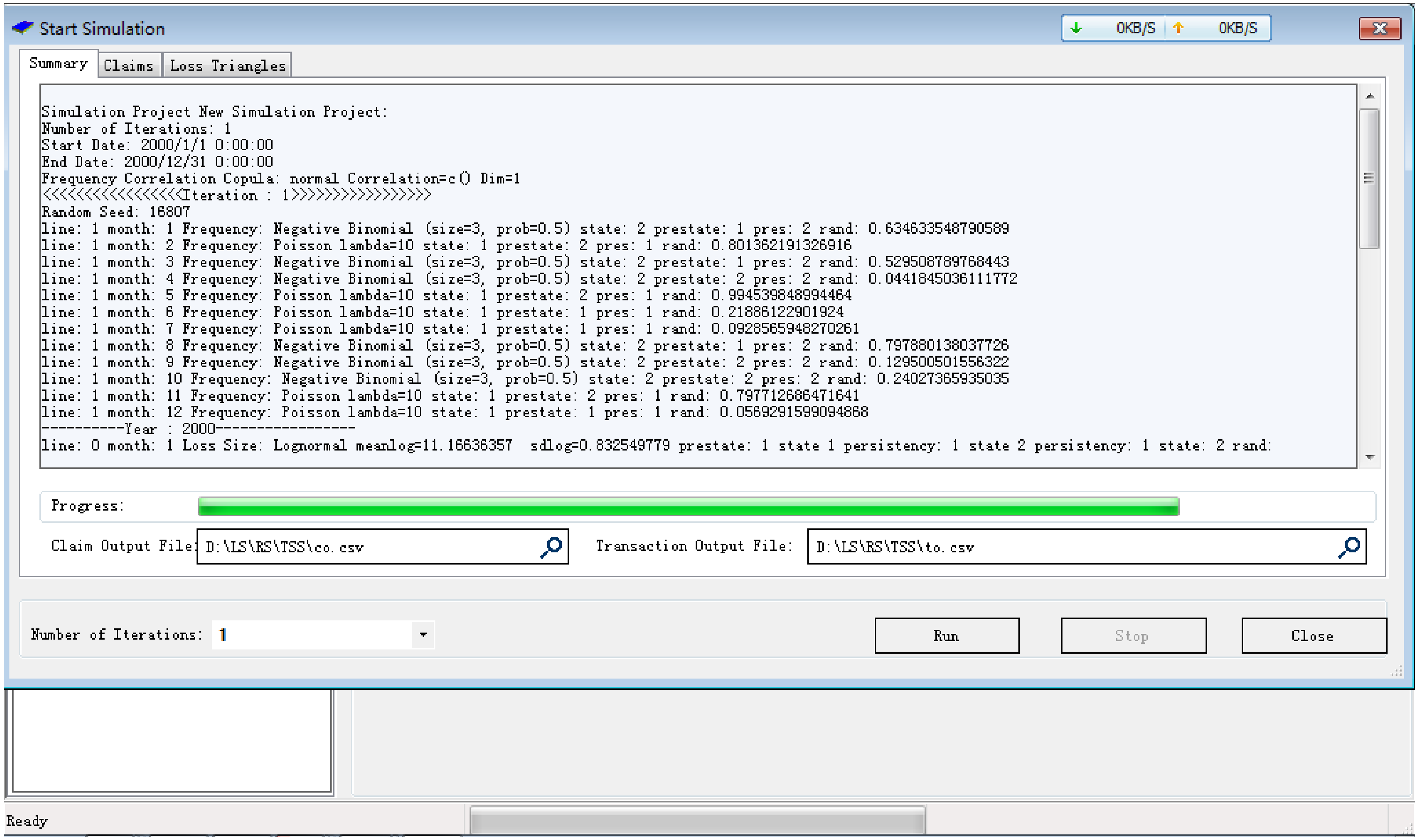
# Interface

- **Input**



# Interface

- **Output**
  - Additional column in claim and transaction output files to record the state
  - Showing state and random number while simulating



# THREE MONTHS LATER

Well done! It improved our reserve adequacy a lot and reduced our earnings volatility. We created a new manager position for you.

**Congratulations!**


# V. Further Development

# Further Development

Case reserve adequacy test shows that the assumption is not consistent with simulation data.

This may be caused by the linear interpolation method used to derive 40% time point case reserve.

It is suggested revising the way in which valuation date is determined in the LSM. In addition to the simulated valuation dates based on the waiting-period distribution assumption as in the LSM, some deterministic time points can be added as valuation dates.

In the LSM, 0%, 40%, 70%, and 90% time-points, case reserve adequacy distribution can be input into the model. Therefore, 0%, 40%, 70% and 90% time points may be added as deterministic valuation dates.

# Thank you!