A link between the one-year and ultimate perspective on insurance risk

Casualty Loss Reserve Seminar 2011

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Agenda

1. Introduction
   - One-year vs ultimate: Two perspectives of the same risk
   - Issues in one-year loss parameterisation
2. Models of loss ratio development
   - Approaches to link the one-year and ultimate perspective
3. Conclusions / Discussion

1. Introduction

One-Year vs Ultimate: Two perspectives of the same risk

- Ultimate view: “The risk that the current estimate of the claims reserve is insufficient to cover the full run-off of the liabilities”
- Another perspective is the *one-year* view, which considers the claims development over a single annual time period

- Regulatory regimes have converged on the one-year view
  - Complete run-off of liabilities under the Solvency II regime is satisfied by additionally holding the present value of the cost of future one-year capital requirements to run-off the liabilities, otherwise known as a *market value margin*
- For many existing stochastic reserving models, generating one-year reserve distributions is more complex than it is for the ultimate perspective:
  - A one-year method needs to re-estimate the claims reserve at the end of the time period, using the new information gained

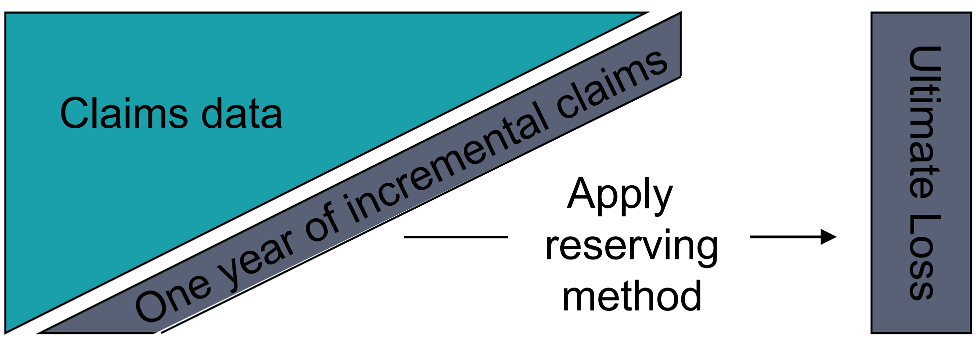
Key Issue: Timing of loss recognition is important in one-year models

Typical differences in one-year versus ultimate reserve risk models

Ultimate Risk Model

1. Simulate the completion of paid/incurred triangle
2. Sum these claims to get to an ultimate loss estimate
3. Compare with existing held reserve

One-Year Risk Model



1. Simulate one new year of new claims
2. Apply reserving method to get an estimate of ultimate losses one-year out
3. Compare with existing held reserve

Key Issue: Timing of loss recognition is important in one-year models

Issues in One-Year Loss Parameterisation
The "Actuary in a Box"

- The reserving step in the one-year model is complex!
  - For many existing stochastic reserving models, generating one-year reserve distributions is more difficult than it is for the ultimate perspective
  - A one-year method needs to re-estimate the claims reserve at the end of the time period, using the new information gained

  - The "Actuary in a Box"

Issues in One-Year Loss Parameterisation

- Wacek*: suggests two ways in which the estimate of the ultimate may vary as a result of the extra one year of claims information
  - The year end claims payments will generally have been different from those expected, and reapplying the same development factors will give rise to a new indication for the claims reserve
  - Secondly, the extra claims experience may also result in a different selection of development factors

- There is also a third: mechanically applied reserving methods do not reflect the reality. Actuaries will take into account information not contained in the triangle – this may result in bigger changes to ultimate loss estimates than the claims data would suggest.

* *Wacek, M.G., 2007, The Path of the Ultimate Loss Ratio Estimate. Casualty Actuarial Society Forum, Winter 2007, 339-370*

Issues in One-Year Loss Parameterisation

- A literal view of one-year risk will rely on loss emergence patterns
  - With long tail lines of business in particular this presents problems.
  - Usually little extra claim specific information is gained over a single year resulting in small changes to reserves using typical stochastic methods and consequently very low measures of one-year reserve risk
  - Is this view realistic?

- Consider the following example:
  - Period of high inflation begins during year that will impact casualty claims
  - At end of year, uncertain as to how long the inflationary environment will continue and what impact it will ultimately have on the liabilities so may only recognize <20% of the ultimate impact
  - However, the view of the liabilities and the associated uncertainty have changed => change in capital requirements

Summary of “Actuary in a Box” problems

Issue	Summary
Mechanical Reserving Methods	Do not necessarily give a good approximation to actual approaches
Non Claims Information	Changes in external environment are likely to give rise to the largest changes in claims estimates
Claims Information not in Triangles	Eg Ground Up Loss information for a claim not yet in layer
Long Tail Lines	Often unrealistically small results
Inflation	Recognition of the impact of inflationary changes over a one year period is difficult
Mean Reinsurance	Recognition of the XoL reinsurance protection over a one year period is difficult
Complexity	Actuary in a Box is a large, complex model that is hard to parameterise

Summary of “Actuary in a Box” problems

Issue
Large Model Error
Large Parameter Error
Often does not give reasonable results
Difficult to programme
Simulation time large

- This session will explore alternatives to the “Actuary in a Box”, that are based upon the more reliable “to ultimate” simulated results.
- We will look at proxies that we can use to estimate one year distributions

2. Models of Loss Ratio Development

Models of Loss Ratio Development

- Rather than relying on an Actuary in a Box setup, we consider some simple theoretical models of loss ratio development
- These imply a relationship between the one year and ultimate distributions and provide proxies to estimate one year capital requirements given an ultimate distribution

Some Notation

$$P_t = \text{Incremental payment in time period } t$$

$$P_t^U = \text{Ultimate future payments from time period } t = \sum_{s=t} P_s$$

$$C_t = \text{One year capital requirement in time period } t$$

$$C_t^U = \text{To ultimate capital requirement from time period } t$$

$$\%\{X, y\} = \text{The } (100y) \text{th percentile of the distribution X}$$

Some Notation

The Capital Signature

$$C_t = \%\left\{P_t + E\left(\sum_{t+1} P_s \middle| P_t\right), 0.995\right\} - E\left(P_t + E\left(\sum_{t+1} P_s \middle| P_t\right)\right)$$

$$C_t^U = \%\left\{P_t^U, 0.995\right\} - E\left(P_t^U\right)$$

$$\lambda_t = \frac{E(C_t)}{C_1}$$

$$\text{Capital Signature} = \lambda_1, \lambda_2, \ldots, \lambda_N$$

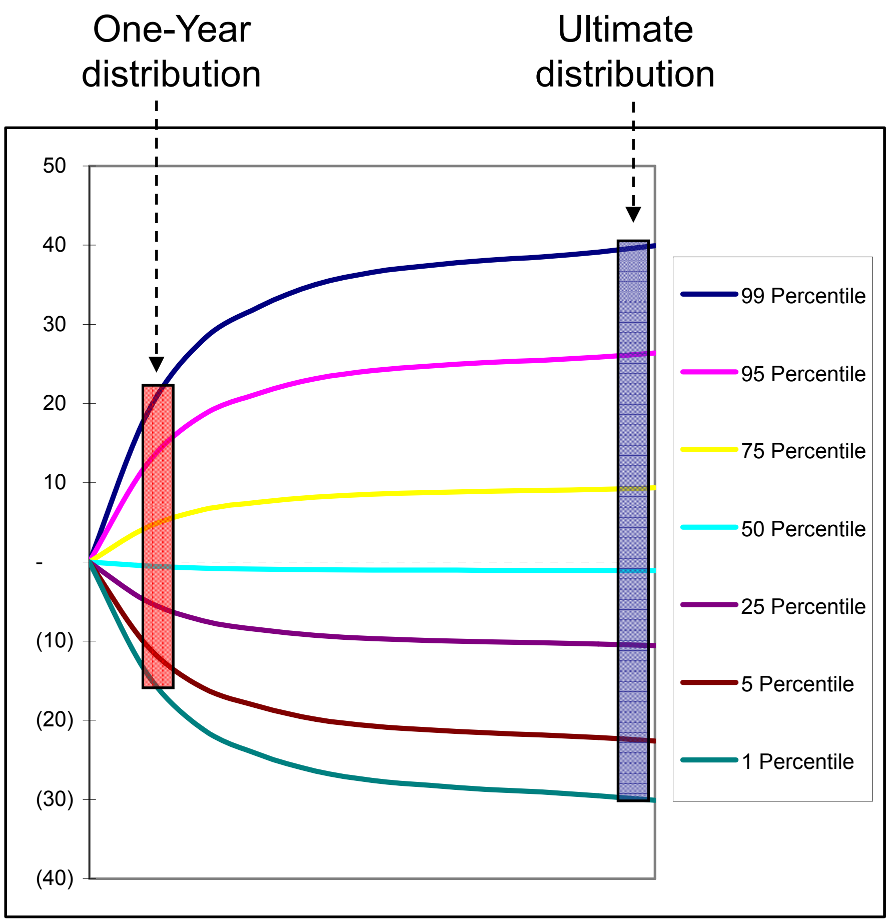
Model 1
Run off reserve risk using loss pattern

– Given a pattern of $\frac{E(P_1)}{E(P_1^U)}, \frac{E(P_2)}{E(P_1^U)}, \ldots, \frac{E(P_{U-1})}{E(P_1^U)}$ then one year capital requirement is estimated as:

$$C_1 = \frac{E(P_1)}{E(P_1^U)} \%\{P_1^U, 0.995\} - E(P_1^U)$$

– Runs risk off linearly with loss development
  – The result of a 'strict' Bornhuetter-Ferguson reserving method
– No variability in timing of loss recognition causes understatement of one-year risk (similar to many existing one-year models)
– Ultimate loss increments are 100% correlated using this approach. This produces a smooth path for claims development in line with the selected pattern

Model 1
Run off reserve risk using loss pattern



- Implied development paths are smooth under this model
  - x percentile for the 1 year loss distribution is the x percentile for the ultimate
- Not a suitable model for development paths in most situations, and is likely to significantly understate one-year risk
- Similar issues are apparent as seen with mechanical reserving methods – there is additional uncertainty in loss recognition not captured by this approach

Model 2
Use stochastic incremental % to recognise the ultimate loss

- Assume increments P_i are independent and normally distributed:

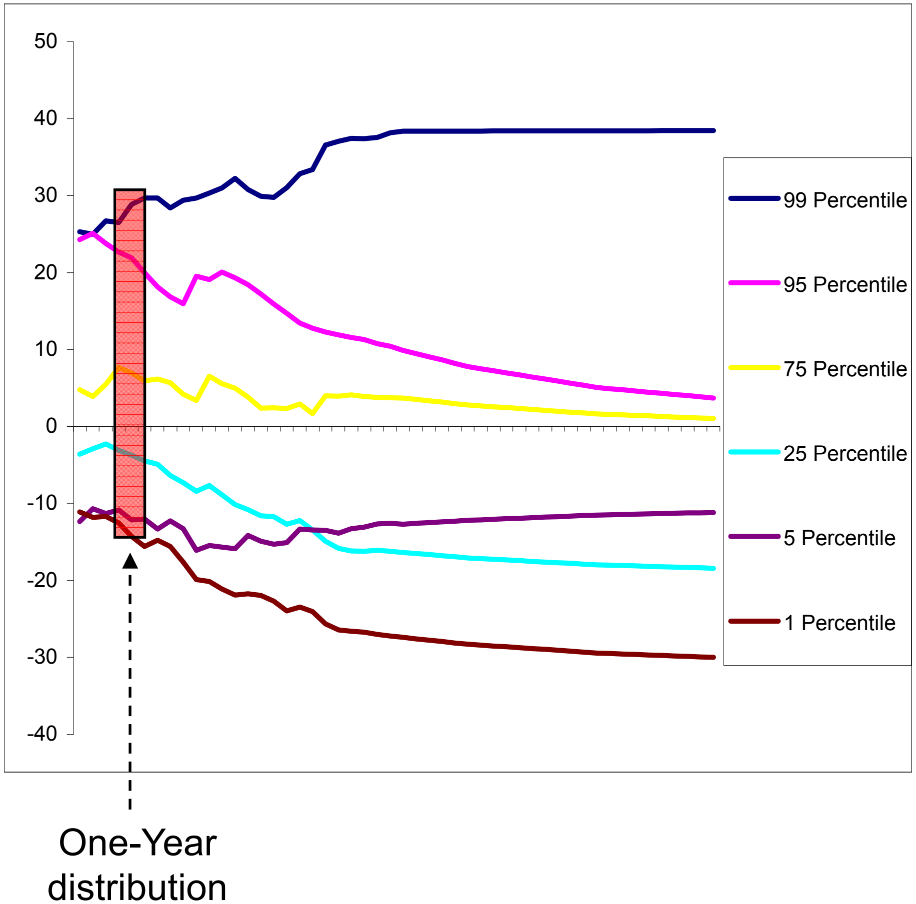
$$P_i \sim N(0, \sigma_i^2)$$

- Parameterisation allows for a variety of loss recognition patterns
  - Extreme cases of low frequency, high severity losses which lead to spikes in the recognition patterns
  - This is a generalised version of Model 1 which assumes 'average' loss recognition

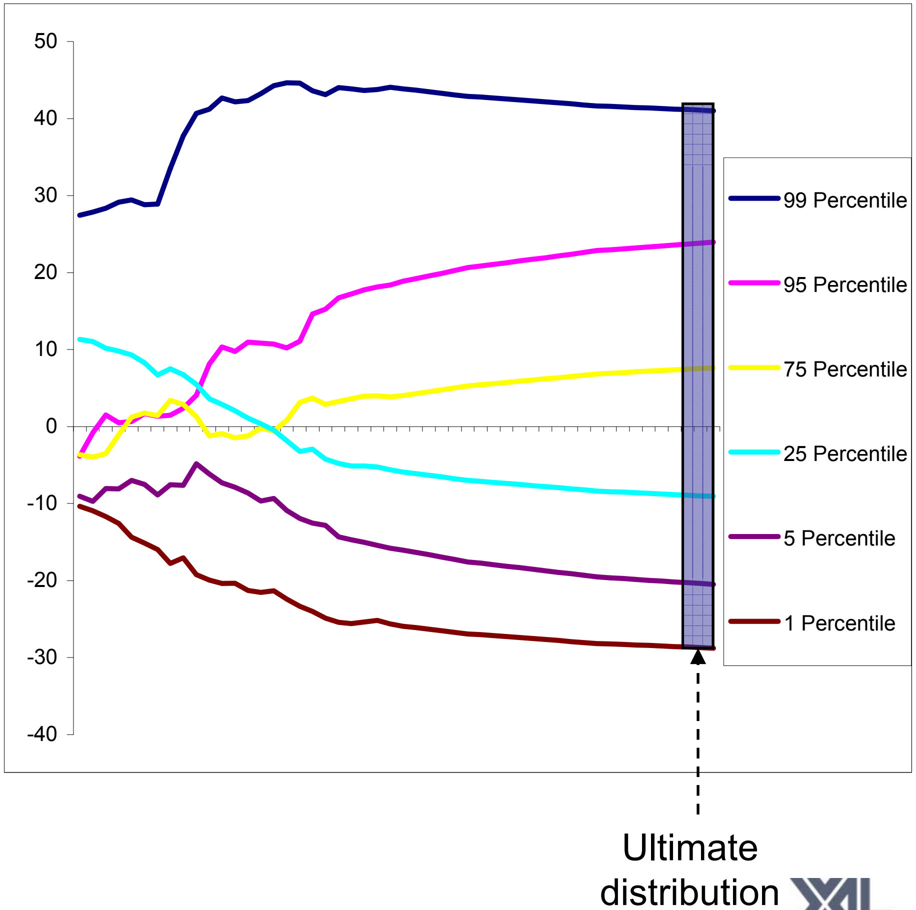
Model 2

Use stochastic incremental % to recognise the ultimate loss

Percentiles for CDR_1



Percentiles for CDR_{Ult}



Model 2
Comments

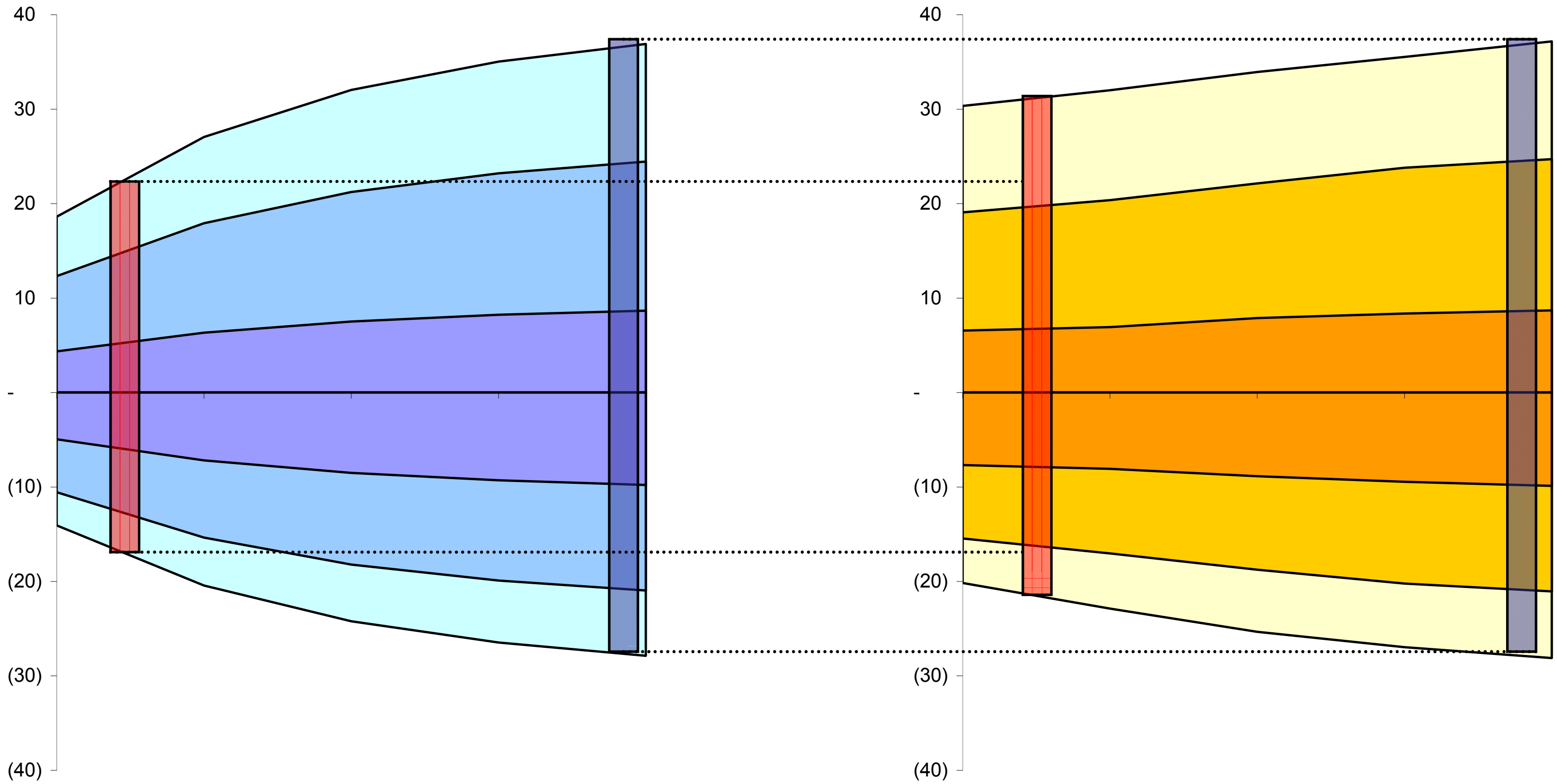
- Assuming that incremental claim amounts are independent is a ***conservative*** assumption, since:
  - In a model with positively correlated increments, an adverse result in the first year will tend to get worse over time
  - In an independent model the “to ultimate” capital requirement will have all the diversification benefit of diversification between consecutive time periods
  - All this diversification credit has to be unwound to give the resultant one year capital requirement
  - Negative correlation is the most conservative approach

- Most models assume there does exist positive correlation between consecutive time periods

Comparison of Models 1 and 2
"Cone of Uncertainty"

Model 1
Fixed Recognition Pattern

Model 2
Stochastic Recognition Pattern



Using a fixed recognition pattern results in a significantly lower estimate for one-year distributions

Model 2 - Independent Normal Incrementals
One-year vs Ultimate theoretical results

- In order to compare the one-year and ultimate confidence levels we need to solve the following equation for the probability p:

$$\%\left\{P_1^U - E\left(P_1^U\right), p\right\} = C_1$$

- The capital signature becomes:

$$\lambda_t = \frac{\sigma_t}{\sigma_1}$$

Model 2
Theoretical Results

– The exact solution for p is as follows:

$$p = \Phi\left(\frac{\Phi^{-1}(0.995)}{\sqrt{1 + \frac{1}{\sigma_1^2} \sum_{s=2} \sigma_s^2}} \right) = \Phi\left(\frac{\Phi^{-1}(0.995)}{\sqrt{\sum \lambda_t^2}} \right)$$

– This gives an estimate for the link between one-year and ultimate confidence levels:

$$C_1 = \%\left\{P_1^U, p\right\} - E\left(P_1^U\right)$$

Model 3
"Time-scaling"

- The concept of *Time-scaling* is to use the duration of projected risk capital to adjust the confidence level employed to calculate economic capital
  - Market value margins require the projection of risk capital

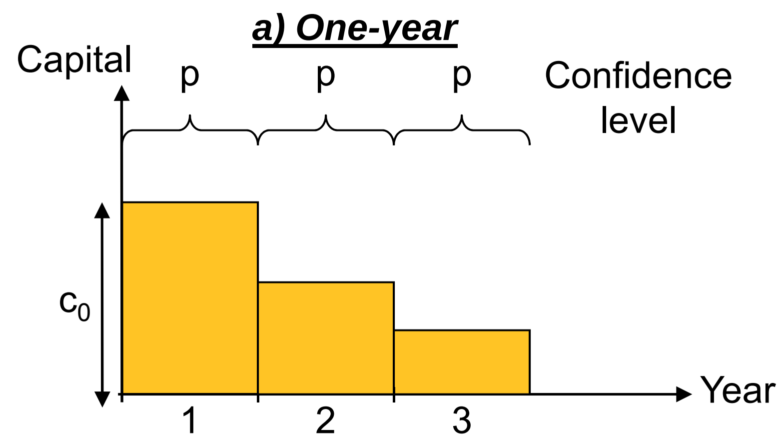
- Estimate for one year capital is given by the ultimate confidence level

$$p = 0.995^{d}; \quad where \quad d = \sum \lambda_t$$

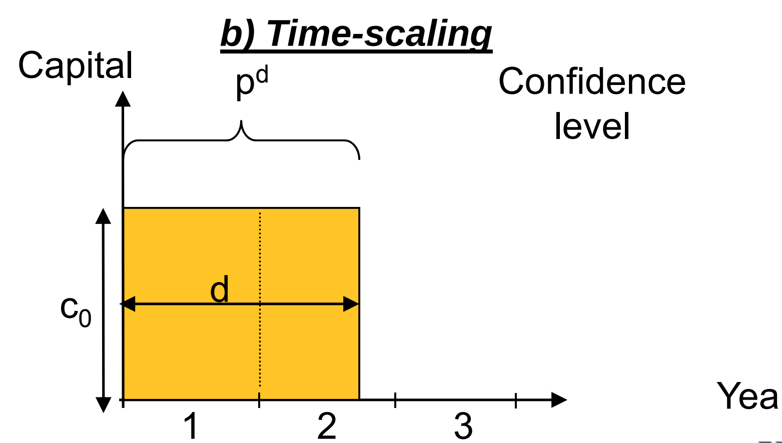
- For example:
  - Confidence level of 99.5% over a one year time horizon
  - Duration of economic capital = 3 years
  - Confidence level for ultimate distribution = 0.995^3 = 98.51%

Model 3
Time-scaling Example

- Capital requirements for run-off of liabilities, viewed as a series of one-year capital requirements, or one-year survival probabilities
- Duration of capital is 3 in this example



- Approximated by a single level capital requirement for the 3 year run-off of liabilities
- Ultimate confidence level is set as equivalent to a series of one year probabilities



The time-scaling duration, d, is calculated so that the overall capital requirement (i.e. size of the shaded areas) are identical in each diagram

Model 3
Loss ratio model underlying time-scaling (Cauchy distribution)

- As with Model 2 assume increments P_i are independent
- Increments are Cauchy distributed (t-distribution with 1 degree of freedom)

$$P_i \sim \text{Cauchy}(0, \gamma_i)$$

- The Cauchy distribution has the transformation property that if X has a Cauchy(0, γ_1) distribution, Y has a Cauchy(0, γ_2) distribution and k_1, k_2 are constants then $k_1X + k_2Y$ has a Cauchy(0, $k_1\gamma_1 + k_2\gamma_2$) distribution. The result follows from this key property
- The mean and variance of the Cauchy distribution do not exist as the distribution has very heavy tails, making it a conservative choice for loss distributions and inferring one-year confidence levels

Model 3
Time-scaling

- Uses the concept of ‘duration at risk’ to estimate the relationship between one-year and ultimate risk. This is something that needs to be estimated for Market Value Margins
- Removes the need for detailing a process by which loss emergence is recognised – “Actuary in a Box”
  - Instead it relies on duration of risk to estimate the ‘average’ one-year default probability for the run-off of a portfolio
  - Offers a way to cope with the problem of external information impacting loss recognition
- Allows the actuary to focus on parameterising the ultimate loss distribution, which can better model issues such as claims inflation or reinsurance

- A similar approach is frequently used within Life Insurance
  - GN46 Section 6.6: “*There is no scientific method of determining exactly the equivalent confidence level over a longer term to a 99.5% level over one year. Hence it will be necessary to justify any confidence level assumed for such a term and in particular one that is less than a (100-0.5N)% confidence level for an assessment of the capital necessary using an N-year projection*”
  - (100-0.5N)% is very close to $99.5\%^N$, and this is used as a baseline for converting ultimate confidence levels to one-year in Life ICAs

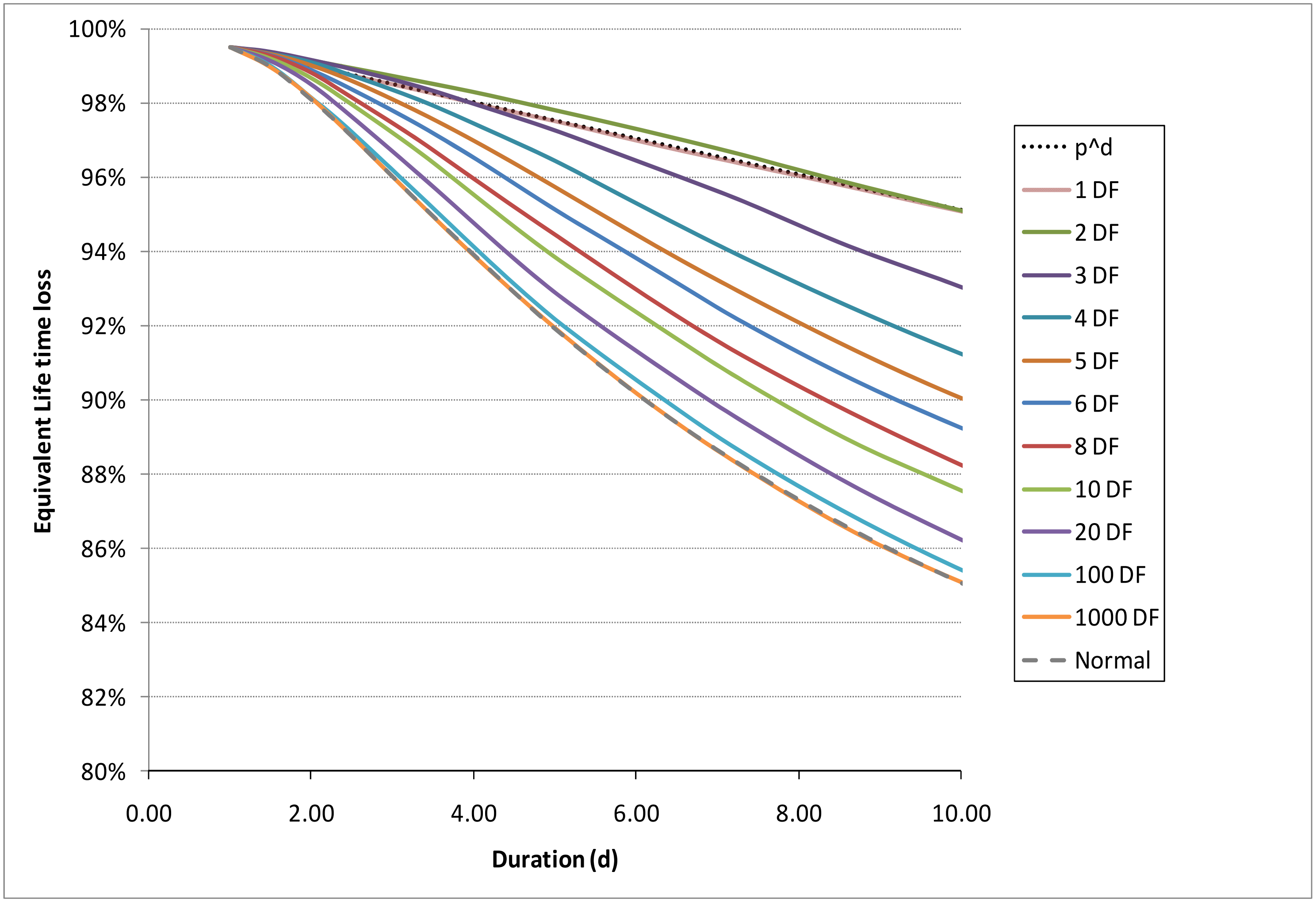
Model 4
Stochastic incrementals are t-distributed

- All models described so far are part of a broader family of distributions where incrementals are assumed to be from a t-distribution
  - Model 2: Normal distribution has infinite DF
  - Model 3: Cauchy distribution has DF = 1
- For these two models we can compare analytically:

$$p = \Phi\left(\frac{\Phi^{-1}(0.995)}{\sqrt{\sum \lambda_t^2}}\right) \; \textit{and} \;\; p = 0.995^{\lambda}$$

- For other degrees of freedom comparisons can be made using a simulation model – the next chart shows this for a particular shape of capital signature (simple decay patterns of differing durations)
- In the case when the liabilities have one year duration then all models give the same answer that p=0.995

Comparison of confidence levels for t-distributions with various degrees of freedom



Model 5
Stochastic development factor model

- Assumption of independent increments may not be realistic. Another approach is model stochastic development factors
  - $LDF_i \sim N(\mu_i,\sigma_i^2)$

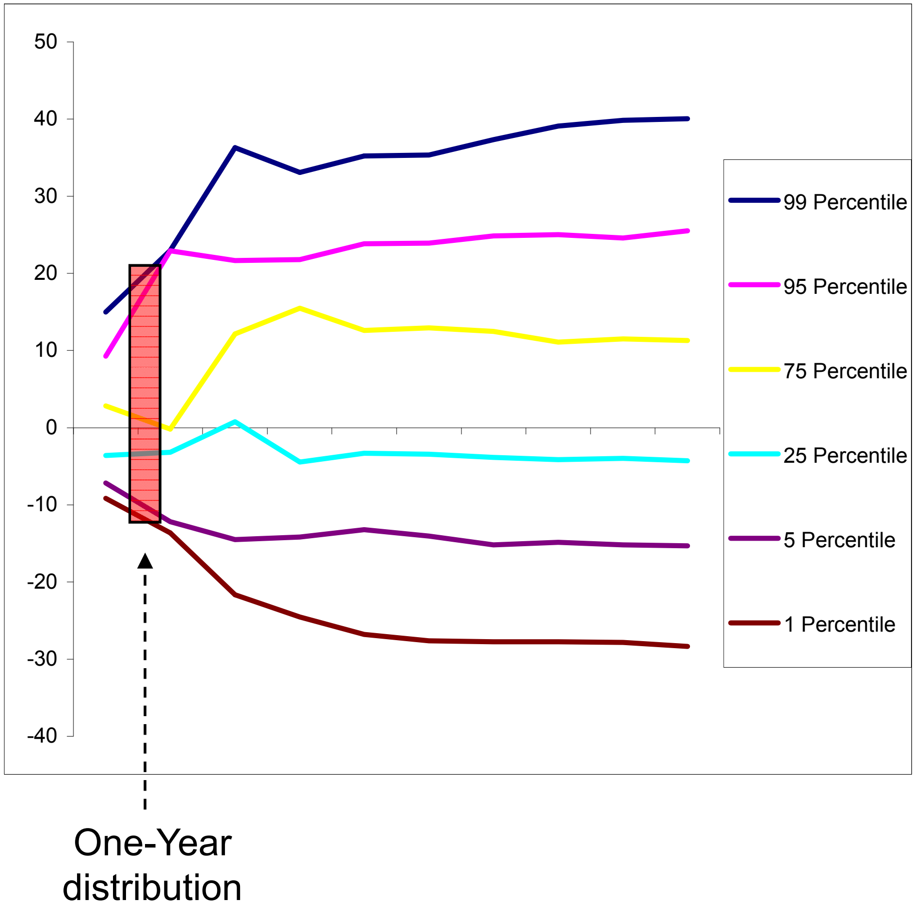
- In each trial of a simulation
  1. Generate P_1^U
  2. Use each random LDF_i to calculate

- Introduces some dependence in incremental ultimate loss recognition

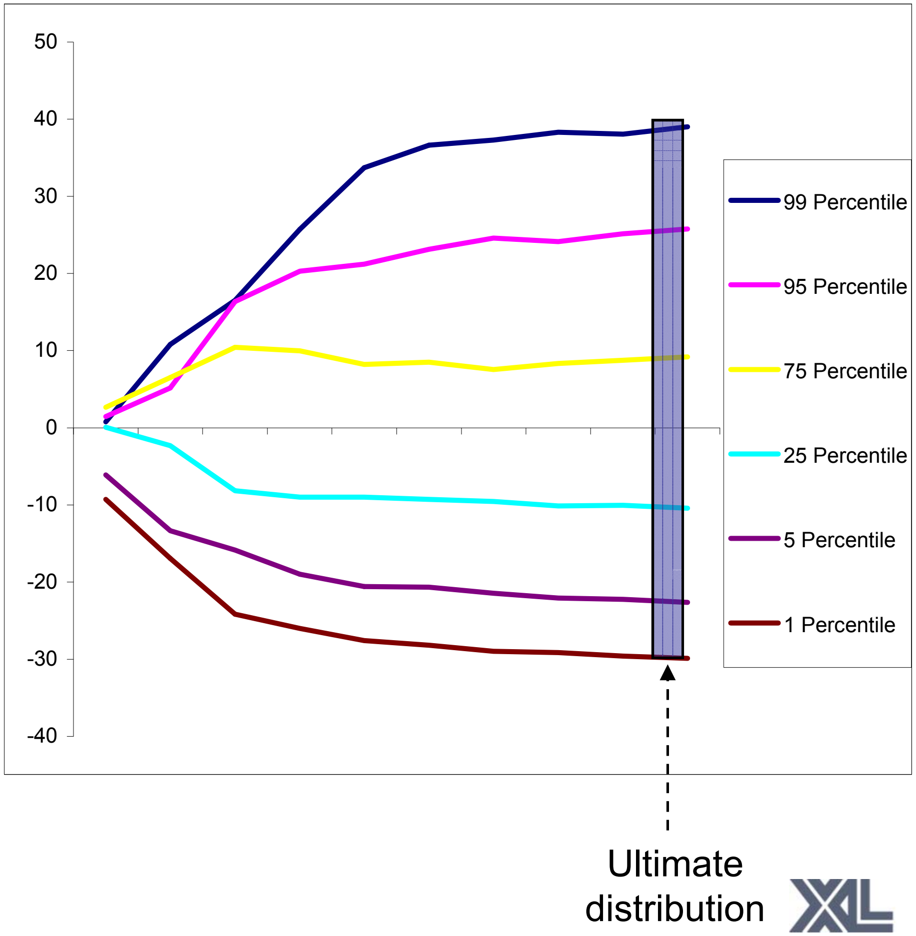
$$P_1 = P_1^U \Big/ \prod_i LDF_i$$

Model 5
Stochastic development factor model

Percentiles for CDR_1



Percentiles for CDR_{Ult}



Model 5
Conclusions

- Using development factors in this model introduces positive correlation in the claims development process
  - This produces a much narrower estimate for the one year capital requirement. I.e. it is much more optimistic in its one year capital estimate than all of the models discussed so far

3. Conclusions / Discussion

Conclusion

- We have discussed the reasons why a simple approach to moving from a "to ultimate" basis to a "1 year basis" may be desirable and possibly preferable to an "actuary in a box" approach
- We have given a couple of examples of such an approach that are simple to implement
  - Normal increments: $p = \Phi\left(\dfrac{\Phi^{-1}(0.995)}{\sqrt{\sum \lambda_t^2}}\right)$
  - Time-scaling: $p = 0.995^{\lambda}$

Positive/Negative Dependence vs Independence

- Which is the more realistic assumption?
  - Lines where the timing of loss recognition is uncertain will tend to exhibit negative correlation – a large movement in one development period would be expected to be followed by small increments. E.g. excess claims
  - Other lines where exposure to risk is a key driver will tend to see bad experience continue to develop (i.e. exhibit positive correlation). E.g. clash policies

Other Considerations

- Care must be taken with any proxies used in the construction of the capital signature
- We have not discussed methods for developing full one-year distributions consistent with time-scaling
  - Can resize the ultimate distribution to generate a one-year version –keep a consistent mean and adjust to a new desired one-year percentile
- We have not discussed the complicated issue of dependency between “1 year” distributions.
- The rationale in the previous slides takes the conservative assumption that future increments are independent

Discussion