ST-2:Using Multivariate Models to Test Traditional Reserving Methods

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Today's Discussion

- Overview of why using multivariate regression analysis may be useful for testing traditional methods
- Outline the method for projecting losses and associated prediction ranges
- Consider three real life examples

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Why test?

- Informed judgment is an important component at many points in the loss reserving process
  - Data organization
  - Method selection
  - Parameter selection
  - Reserve selection

Why test?

- Personal biases can potentially exist in each of these steps.
- It is useful to have an objective way to test our estimates
  - Too aggressive?
  - Too conservative?
  - Appropriate methods?

The Reserving Problem

- The reserving actuary considers many predictors when trying to predict future loss payments
  - Paid Losses
  - Incurred Losses
  - Case Reserves
  - Premium
  - Counts
- The implicit goal in choosing among these is predictive accuracy

Enter Regression

- Multivariate regression is a useful, objective method for **testing** our predictions for bias.
- By simultaneously analyzing the predictive variables available to us, we can also gain important **insight** into model selection

Data for Example – Industry CMP

Paid	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	6,012,756	9,312,546	11,257,739	12,812,561	13,586,182	14,297,514	14,718,682	15,011,236	15,217,920	15,353,100
2002	4,812,214	7,802,211	9,415,277	10,693,032	11,638,785	12,223,959	12,595,145	12,814,035	12,991,656	
2003	5,059,317	7,895,919	9,257,518	10,554,878	11,474,131	12,087,424	12,420,888	12,619,639		
2004	5,824,983	9,325,949	10,883,543	12,181,421	13,152,332	13,644,354	13,958,093			
2005	5,650,710	10,478,308	11,975,097	13,448,030	14,378,528	14,940,670				
2006	5,477,628	8,823,355	10,395,561	11,703,269	12,591,344					
2007	6,209,345	9,803,947	11,482,120	12,843,154						
2008	8,520,931	13,564,525	15,392,295							
2009	6,873,732	10,739,686								
2010	7,477,552									

Incurred	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10	Premium
2001	9,962,679	12,476,803	13,866,817	14,566,272	14,746,643	15,032,490	15,198,904	15,312,149	15,430,321	15,503,366	20,580,181
2002	8,267,681	10,258,482	11,627,151	12,358,156	12,691,520	12,871,972	12,997,907	13,108,227	13,215,388		23,191,400
2003	8,618,499	10,431,567	11,493,158	12,105,431	12,498,330	12,674,146	12,798,931	12,881,279			25,979,567
2004	9,998,162	11,955,935	13,055,555	13,733,794	13,977,058	14,159,753	14,273,213				28,243,474
2005	10,776,107	13,404,860	14,305,501	15,002,870	15,307,201	15,501,390					28,853,202
2006	9,444,040	11,544,522	12,577,134	13,228,889	13,594,179						30,701,289
2007	10,532,592	12,652,145	13,850,910	14,510,933							31,377,086
2008	13,930,479	16,831,090	17,940,460								30,635,085
2009	11,258,589	13,780,811									29,002,596
2010	12,225,649										28,533,933

Target a Particular Loss Projection

Pd	Increm	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
	2001	6,012,756	3,299,790	1,945,193	1,554,822	773,621	711,332	421,168	292,554	206,684	135,180
	2002	4,812,214	2,989,997	1,613,066	1,277,755	945,753	585,174	371,186	218,890	177,621	
	2003	5,059,317	2,836,602	1,361,599	1,297,360	919,253	613,293	333,464	198,751		
	2004	5,824,983	3,500,966	1,557,594	1,297,878	970,911	492,022	313,739			
	2005	5,650,710	4,827,598	1,496,789	1,472,933	930,498	562,142				
	2006	5,477,628	3,345,727	1,572,206	1,307,708	888,075					
	2007	6,209,345	3,594,602	1,678,173	1,361,034						
	2008	8,520,931	5,043,594	1,827,770							
	2009	6,873,732	3,865,954								
	2010	7,477,552									

We will begin our example with estimating the incremental loss in Age 4 for Accident Year 2009

Available Predictors

Paid	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	6,012,756	9,312,546	11,257,739	12,812,561	13,586,182	14,297,514	14,718,682	15,011,236	15,217,920	15,353,100
2002	4,812,214	7,802,211	9,415,277	10,693,032	11,638,785	12,223,959	12,595,145	12,814,035	12,991,656	
2003	5,059,317	7,895,919	9,257,518	10,554,878	11,474,131	12,087,424	12,420,888	12,619,639		
2004	5,824,983	9,325,949	10,883,543	12,181,421	13,152,332	13,644,354	13,958,093			
2005	5,650,710	10,478,308	11,975,097	13,448,030	14,378,528	14,940,670				
2006	5,477,628	8,823,355	10,395,561	11,703,269	12,591,344					
2007	6,209,345	9,803,947	11,482,120	12,843,154						
2008	8,520,931	13,564,525	15,392,295							
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Incurred	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10	Premium
2001	9,962,679	12,476,803	13,866,817	14,566,272	14,746,643	15,032,490	15,198,904	15,312,149	15,430,321	15,503,366	20,580,181
2002	8,267,681	10,258,482	11,627,151	12,358,156	12,691,520	12,871,972	12,997,907	13,108,227	13,215,388		23,191,400
2003	8,618,499	10,431,567	11,493,158	12,105,431	12,498,330	12,674,146	12,798,931	12,881,279			25,979,567
2004	9,998,162	11,955,935	13,055,555	13,733,794	13,977,058	14,159,753	14,273,213				28,243,474
2005	10,776,107	13,404,860	14,305,501	15,002,870	15,307,201	15,501,390					28,853,202
2006	9,444,040	11,544,522	12,577,134	13,228,889	13,594,179						30,701,289
2007	10,532,592	12,652,145	13,850,910	14,510,933							31,377,086
2008	13,930,479	16,831,090	17,940,460								30,635,085
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Historical Observations

Pd Increm	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	6,012,756	3,299,790	1,945,193	1,554,822	773,621	711,332	421,168	292,554	206,684	135,180
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Regression Model

Future Incremental Paid Loss =

C_0

x $e^{(Year * C_1)}$

x Earned Premium^{C_2}

x Current Cumulative Paid Loss^{C_3}

x Current Case Reserve^{C_4}

x Most Recent Incremental Payment^{C_5}

Some observations

- Paid LDF method is simple case with $c_3 = 1$, other exponents = 0, and $c_0 = LDF_2 * (LDF_3 - 1)$
- Paid BF method is simple case with $c_2 = 1$, other exponents = 0, and c_0= a priori loss ratio * $(1/(LDF_4 * LDF_5 * \ldots) - 1/(LDF_3 * LDF_4 * \ldots))$
- Incurred LDF and incurred BF methods not simple cases, but use of case reserve and paid loss includes the information.

X Matrix

Unity	Year	ln(Prem)	ln(Cumulative Paid)	ln(Case Reserve)	ln(Most Recent Incremental Payment)
1	1	16.83983908	16.04687308	14.96742883	15.00936939
1	2	16.95929208	15.86991771	14.7141549	14.91078294
1	3	17.0728209	15.8818566	14.74595978	14.85811741
1	4	17.15637298	16.04831129	14.78248908	15.06854949
1	5	17.17773153	16.16481777	14.8893355	15.38985959
1	6	17.2398152	15.99291274	14.81657139	15.02319457
1	7	17.26158844	16.09829562	14.86219707	15.09494383

Y vector

ln(Observed Payment)
14.2568716
14.0606152
14.075842
14.0762412
14.2027662
14.0837865
14.1237553

Regression - Matrix Formula

- Vector of Regression Factors:

$\beta = ((X^TX)^{-1})(X^TY)$

- Std Error of each factor:

$se(\beta_i) = s * [((X^TX)^{-1})_{i,i}]^{.5}$

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	Factor	Std Error
Constant	4.493960265	7.601179296
Year	-0.004894893	0.023045288
Premium	-0.082407021	0.360912549
Cumulative Paid	-0.234246537	0.368566088
Case	0.794372336	0.317167603
Incremental Paid	0.201653841	0.144537839
d.f.	1	
s	0.02233145	

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14.1237553

	Factor	Std Error
Constant	4.493960265	7.601179296
Year	-0.004894893	0.023045288
Premium	-0.082407021	0.360912549
Cumulative Paid	-0.234246537	0.368566088
Case	0.794372336	0.317167603
Incremental Paid	0.201653841	0.144537839
d.f.	1	
s	0.02233145	

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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X Matrix

Unity	Year	ln(Prem)	ln(Cumulative Paid)	ln(Case Reserve)	ln(Most Recent Incremental Payment)
1	1	16.83983908	16.04687308	14.96742883	15.00936939
1	2	16.95929208	15.86991771	14.7141549	14.91078294
1	3	17.0728209	15.8818566	14.74595978	14.85811741
1	4	17.15637298	16.04831129	14.78248908	15.06854949
1	5	17.17773153	16.16481777	14.8893355	15.38985959
1	6	17.2398152	15.99291274	14.81657139	15.02319457
1	7	17.26158844	16.09829562	14.86219707	15.09494383

Y vector

ln(Observed Payment)
14.2568716
14.0606152
14.075842
14.0762412
14.2027662
14.0837865
14.1237553

	Factor	Std Error
Constant	4.493960265	7.601179296
Year	-0.004894893	0.023045288
Premium	-0.082407021	0.360912549
Cumulative Paid	-0.234246537	0.368566088
Case	**0.794372336**	**0.317167603**
Incremental Paid	**0.201653841**	**0.144537839**
d.f.	**1**	
s	0.02233145	

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Prediction

- Mean of the prediction (logarithm) is $X_0\beta$ where X_0 is the row vector of predictive variables corresponding to our prediction and β is the column vector of regression factors.
- Standard Deviation of the Prediction is given by: $s(1+X_0(X^TX)^{-1}X_0^T)^{.5}$

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Prediction

- Standard Deviation of Prediction
  - Improves with general fit of the regression (value of 's')
  - Increases when degrees of freedom decrease
  - Increases with greater extrapolation

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Prediction

- In this example:

	ln	Transformed
Prediction	14.15840183	1,424,486
SD of Prediction	0.147782929	211,669

- Transformed Mean = $\exp(\mu + .5\sigma^2)$
- Transformed SD = Transformed Mean * $(\exp(\sigma^2)-1)^{.5}$

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Fewer Parameters

X Matrix

Unity	ln(Case Reserve)	ln(Most Recent Incremental Payment)
1	14.96742883	15.00936939
1	14.7141549	14.91078294
1	14.74595978	14.85811741
1	14.78248908	15.06854949
1	14.8893355	15.38985959
1	14.81657139	15.02319457
1	14.86219707	15.09494383

Y vector

ln(Observed Payment)
14.2568716
14.0606152
14.075842
14.0762412
14.2027662
14.0837865
14.1237553

	Factor	Std Error
Constant	2.202530022	2.222877406
Case	**0.79487443**	**0.176686582**
Incremental Paid	0.009221987	0.090432319
d.f.	**4**	
s	0.032231917	

	ln	Transformed
Prediction	14.20808381	1,481,841
SD of Prediction	0.037729804	55,929

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Fewer Parameters Still

X Matrix

Unity	ln(Case Reserve)
1	14.96742883
1	14.7141549
1	14.74595978
1	14.78248908
1	14.8893355
1	14.81657139
1	14.86219707

Y vector

ln(Observed Payment)
14.2568716
14.0606152
14.075842
14.0762412
14.2027662
14.0837865
14.1237553

	Factor	Std Error
Constant	**2.199982026**	**1.990659021**
Case	**0.804408391**	**0.134271093**
d.f.	**5**	
s	0.028866554	

	ln	Transformed
Prediction	14.20797978	1,481,477
SD of Prediction	0.033778048	50,056

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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The Set of Possible Parameters

- With each parameter being in or out (except the constant) the total number of parameter sets is $2^5 = 32$
- This is for a single Accident Year-Age projection
- Luckily computers don't mind repetitive tasks!

Helpful Hints for Automating in Microsoft Excel

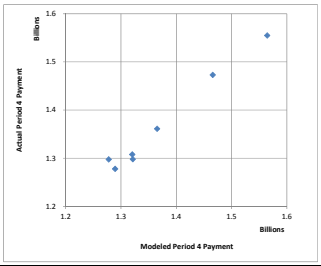
- With variable size of X matrix, it is useful to use the following features together
  - Array Functions – MMULT, MINVERSE, TRANSPOSE
  - INDIRECT Function

Constant	Year	Premium	Cuml. Pd	Case	Increm Pd	df	s	ln mean	ln std dev	mean	std dev	
6.2991	0	-0.19432	0	0.6206	0.12949	3	0.01533	14.188432	0.0186647	1,452,223	27,108	Smallest Prediction SD
2.14754	-0.0128	0	0	0.70187	0.10788	3	0.01646	14.14615	0.023786	1,392,251	33,121	
2.80593	-0.01	0	-0.2632	0.84489	0.20254	2	0.0162	14.140018	0.0248983	1,383,778	34,459	
6.03728	0	-0.15625	-0.2202	0.7572	0.20339	2	0.01614	14.175688	0.024826	1,434,026	35,607	
4.74307	0	-0.12162	0	0.77316	0	4	0.02289	14.194831	0.027584	1,461,848	40,331	
5.84357	0	-0.19776	0.21075	0.5591	0	3	0.02245	14.203532	0.0282407	1,474,650	41,653	
2.14202	-0.009	0	0	0.81075	0	4	0.02175	14.16355	0.0294553	1,416,902	41,744	
1.75739	-0.0126	0	0.15488	0.67037	0	3	0.0224	14.158173	0.0304401	1,409,346	42,911	
2.19998	0	0	0	0.80441	0	5	0.02887	14.20798	0.033778	1,481,477	50,056	
3.67228	0	0	-0.5985	1.07335	0.27414	3	0.02462	14.162981	0.0368511	1,416,444	52,215	
2.20253	0	0	0	0.79487	0.00922	4	0.03223	14.208084	0.0377298	1,481,841	55,929	
2.45154	0	0	-0.1078	0.9039	0	4	0.03083	14.199317	0.038685	1,468,960	56,848	
10.0813	0	-0.36317	0.64035	0	0	4	0.03397	14.207876	0.0426562	1,481,827	63,238	
3.43689	-0.0245	0	0.67356	0	0	4	0.04388	14.120849	0.0573633	1,359,319	78,039	
14.7639	0	-0.3511	0	0	0.35652	4	0.05152	14.138691	0.0572024	1,383,776	79,220	
9.94695	0	-0.36182	0.66244	0	-0.0161	3	0.03919	14.20996	0.0564928	1,485,937	84,012	
9.06117	-0.0204	0	0	0	0.34193	4	0.06306	14.063471	0.0768445	1,285,198	98,906	
2.76429	-0.0243	0	0.80168	0	-0.0917	3	0.04972	14.133467	0.0764278	1,378,336	105,497	
10.7469	0	0	0	0	0.22449	5	0.07097	14.15197	0.0784016	1,404,291	110,268	
14.1257	0	0	0	0	0	6	0.07536	14.125697	0.080562	1,368,112	110,397	Constant Incremental
17.3029	0	-0.18579	0	0	0	5	0.07633	14.110494	0.0832588	1,347,767	112,408	Test of Paid BF
7.12809	0	0	0.43695	0	0	5	0.06413	14.202051	0.0804618	1,476,652	119,007	Test of Paid Link Ratio
14.158	-0.0081	0	0	0	0	5	0.08031	14.085325	0.0978235	1,316,003	129,045	Trended Incremental
5.59195	-0.0023	-0.16124	0	0.63379	0.1265	2	0.01871	14.180801	0.1023674	1,448,502	148,669	
6.18305	0	0	0.61205	0	-0.1235	4	0.07078	14.218192	0.1019161	1,503,620	153,642	
0.39903	-0.0147	0.081615	0	0.8357	0	3	0.02487	14.14449	0.1153719	1,398,828	161,924	
3.7279	-0.0067	-0.09594	0.18642	0.61245	0	2	0.02711	14.179482	0.1271145	1,450,707	185,153	
4.49396	-0.0049	-0.08241	-0.2342	0.79437	0.20165	1	0.02233	14.158402	0.1477829	1,424,486	211,669	
16.2168	0.02408	-0.67166	0.58063	0	0	3	0.03515	14.292615	0.1542683	1,630,690	253,068	
26.2226	0.04927	-0.9976	0	0	0.31666	3	0.04528	14.327482	0.1843964	1,697,187	315,634	
17.3893	0.02696	-0.71292	0.50111	0	0.05282	2	0.04259	14.295896	0.2814086	1,681,995	482,855	
32.8493	0.06855	-1.11091	0	0	0	4	0.07078	14.377526	0.2879475	1,828,457	537,604	

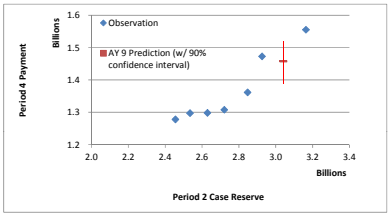
Factors in red have t statistics less than 1.0

Selected Model

$$Loss\ Projection = \frac{6.299 \times CaseRes^{0.621} \times IncrmPd^{0.129}}{Premium^{0.194}}$$

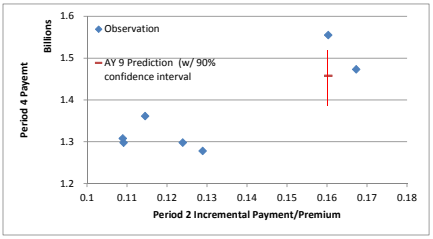


Selected Model



Prediction interval was set here as:
$[exp(\mu + t(0.05,3)\sigma), exp(\mu + t(0.95,3)\sigma)]$ using Student's T distribution with 3 degrees of freedom

Selected Model



Prediction of Outcomes vs. Confidence in the Mean

- Although it is interesting to consider the distribution of potential outcomes for the loss projection, we are specifically interested here in hypothesis testing on the mean projection from a standard method (LDF, BF, etc.).
- We should not be comfortable with the standard method projecting an amount within X% of future outcomes, but rather within X% confidence of the mean modeled outcome
- In the example we have been using, dividing the standard deviation by the square root of the number of observations gives: $27,108/7^{0.5} = 10,246$
- Using the Log Student's T distribution (still with 3 degrees of freedom) gives a 90% confidence interval of [$1.428 Billion, $1.476 Billion] for the mean loss estimate for Accident Year 2009 (AY 9) in Calendar year 2012 (Age 4)

Paid LDF Estimate- Long Term Averages

Paid	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	6,012,756	9,312,546	11,257,739	12,812,561	13,586,182	14,297,514	14,718,682	15,011,236	15,217,920	15,353,100
2002	4,812,214	7,802,211	9,415,277	10,693,032	11,638,785	12,223,959	12,595,145	12,814,035	12,991,656	
2003	5,059,317	7,895,919	9,257,518	10,554,878	11,474,131	12,087,424	12,420,888	12,619,639		
2004	5,824,983	9,325,949	10,883,543	12,181,421	13,152,332	13,644,354	13,958,093			
2005	5,650,710	10,478,308	11,975,097	13,448,030	14,378,528	14,940,670				
2006	5,477,628	8,823,355	10,395,561	11,703,269	12,591,344					
2007	6,209,345	9,803,947	11,482,120	12,843,154						
2008	8,520,931	13,564,525	15,392,295							
2009	6,873,732	10,739,686								
2010	7,477,552									
	Wtd Avg LDF (All Periods Included)	1.612	1.169	1.128	1.076	1.046	1.028	1.018	1.014	1.009

- 2009 Age 4 Incremental Projection = $10.74B * 1.169 * (1.128 -1) =
- $1.61B – considerably outside our confidence interval for the mean

Paid LDF – Last 3

- Using only the last 3 development factors in our averages gives a 2-3 factor of 1.158 and a 3-4 factor of 1.122
- This projects an incremental paid loss in 2009 – age 4 of $1.52 Billion
- While closer to our modeled range than the LDF(all) estimate, still outside the range
- Not terribly surprising since our best model suggested strong reliance on case reserves and the Paid LDF method does not use it.

Paid BF

- Test the method of setting the a priori loss ratios based on cumulative average of previous accident year's estimated loss ratios

Year	Premium	Current Loss	% of Ultimate	A Priori Loss Ratio	BF Estimate	Est Ultimate Loss Ratio
2001	20,580,181	15,353,100	100.0%	N/A	15,353,100	74.6%
2002	23,191,400	12,991,656	99.1%	74.6%	13,143,988	56.7%
2003	25,979,567	12,619,639	97.8%	65.1%	12,996,951	50.0%
2004	28,243,474	13,958,093	96.1%	59.5%	14,621,353	51.8%
2005	28,853,202	14,940,670	93.5%	57.3%	16,007,593	55.5%
2006	30,701,289	12,591,344	89.7%	56.9%	14,387,997	46.9%
2007	31,377,086	12,843,154	83.5%	54.9%	15,691,038	50.0%
2008	30,635,085	15,392,295	74.4%	54.1%	19,639,516	64.1%
2009	29,002,596	10,739,686	64.2%	55.5%	16,495,324	56.9%
2010	28,533,933	7,477,552	40.7%	55.7%	16,896,471	59.2%

- The estimated loss payment in Age 4 for 2009 is $16.495B x (83.5% - 74.4%) = $1.67B, also outside of the modeled range

Incurred LDF

Incurred	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	9,962,679	12,476,803	13,866,817	14,566,272	14,746,643	15,032,490	15,198,904	15,312,149	15,430,321	15,503,366
2002	8,267,681	10,258,482	11,627,151	12,358,156	12,691,520	12,871,972	12,997,907	13,108,227	13,215,388	
2003	8,618,499	10,431,567	11,493,158	12,105,431	12,498,330	12,674,146	12,798,931	12,881,279		
2004	9,998,162	11,955,935	13,055,555	13,733,794	13,977,058	14,159,753	14,273,213			
2005	10,776,107	13,404,860	14,305,501	15,002,870	15,307,201	15,501,390				
2006	9,444,040	11,544,522	12,577,134	13,228,889	13,594,179					
2007	10,532,592	12,652,145	13,850,910	14,510,933						
2008	13,930,479	16,831,090	17,940,460							
2009	11,258,589	13,780,811								
2010	12,225,649									
Wtd Avg (Last 3)	1.2111432	1.0814265	1.0493241	1.0217532	1.013228	1.0091719	1.0074621	1.0079286	1.0047339	

- The estimated ultimate loss for 2009 is $16.669B. The total of all future payments is $5.929B. Allocating this reserve to year, using the paid development pattern and an assumption of case reserve at Age 10 of 1% of incurred suggests an incremental payment in Age 4 of $1.47B – which is within the model range

Some Observations

- Consider the high level of correlation between the methods. For example, the incurred loss as of age two (the age from which we are projecting) is about 80% paid loss.
- We may want to pay extra attention to changes over time in development patterns and loss ratios in this case, as well as considering alternative methods of development.
- Also we may want to consider a different segmentation of data (property split out from liability in this example)
- Should look at the other cells and see if we see a similar result

Expansion to the Other Cells

- 45 lower triangular projections
- The nine Age 10 projections do not have enough data to suggest a range even under the most simple model, so therefore there are only 36 to model.

Other Constraints

- Only selected from among those models where all parameters had t statistics > 1.0
- Only selected from within those models that were in the best 50% of models from a logarithm perspective.
- Constrained the parameters based on premium or losses to be in the range [-1,2]. While this dramatically increases the models to be reviewed (need to analyze the endpoints), it is a reasonable constraint.
- From among the models considered acceptable pick the one for each cell with smallest prediction SD.

AY	Age	μ	σ	Mean	St Dev	5th %ile	95th %ile	Constant	Age	Prem	Pd	Case Increm Pd	dF
10	2	15.26036	0.012639	4,241,565	53,611	4,140,875	4,344,009	(4.672)	0.000	0.000	0.000	1.297	0.000	7
10	3	14.41588	0.030744	1,823,649	56,080	1,713,291	1,939,282	13.515	0.000	(0.441)	0.000	0.000	0.557	5
10	4	14.18528	0.018122	1,447,642	26,237	1,392,551	1,504,418	12.989	0.000	(0.411)	0.000	0.536	0.000	4
10	5	13.74573	0.03266	933,098	30,483	869,877	999,848	8.092	0.000	0.329	0.000	0.000	0.000	4
10	6	13.19047	0.045215	535,787	24,238	481,212	595,333	26.738	0.000	(0.749)	0.000	0.000	0.000	3
10	7	12.65982	0.00165	314,839	519	311,576	318,135	27.361	0.000	(0.951)	0.000	0.106	0.000	1
10	8	10.82696	0.113875	50,682	5,799	24,693	102,706	12.747	(0.193)	0.000	0.000	0.000	0.000	1
10	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
9	3	14.35447	0.030409	1,715,014	52,165	1,612,333	1,822,547	6.072	0.000	(0.252)	0.000	0.845	0.000	5
9	4	14.18858	0.007055	1,452,223	10,246	1,428,275	1,476,499	6.299	0.000	(0.194)	0.000	0.621	0.129	3
9	5	13.64504	0.010249	843,309	8,643	823,169	863,851	23.596	0.000	0.000	0.000	(0.977)	0.306	3
9	6	13.26549	0.03676	577,328	21,230	518,218	642,312	6.685	0.000	0.000	(1.000)	1.525	0.000	2
9	7	12.64414	0.001661	309,941	515	306,707	313,208	27.388	0.000	(0.933)	0.092	0.000	0.000	1
9	8	11.0191	0.107416	61,382	6,612	30,974	120,246	12.747	(0.193)	0.000	0.000	0.000	0.000	1
9	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
8	4	14.12848	0.030492	1,368,112	41,726	1,288,805	1,450,949	14.126	0.000	0.000	0.000	0.000	0.000	6
8	5	13.60515	0.022908	810,504	18,569	771,671	850,844	28.355	0.000	0.000	(1.000)	0.000	0.000	4
8	6	13.16422	0.011205	521,405	5,843	485,762	559,594	16.055	0.000	(0.820)	0.000	1.531	(0.737)	1
8	7	12.61415	0.001127	300,786	339	298,654	302,933	26.924	0.000	(0.921)	0.095	0.000	0.000	1
8	8	11.21119	0.101427	74,335	7,559	38,980	140,305	12.747	(0.193)	0.000	0.000	0.000	0.000	1
8	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
7	5	13.63701	0.027392	836,816	22,927	784,299	892,223	(14.901)	(0.113)	1.699	0.000	0.000	0.000	3
7	6	13.08904	0.023432	483,746	11,336	417,107	560,722	4.357	(0.025)	0.000	(1.000)	0.000	1.790	1
7	7	12.56484	0.001269	286,313	363	284,027	288,616	26.748	0.000	(0.916)	0.100	0.000	0.000	1
7	8	11.40322	0.094807	90,014	8,553	49,249	163,050	12.747	(0.193)	0.000	0.000	0.000	0.000	1
7	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
6	6	13.30432	0.014333	599,842	8,598	575,196	625,414	11.547	0.000	0.000	0.000	0.686	(0.563)	2
6	7	12.5745	0.00064	289,092	185	287,927	290,262	26.623	0.000	(0.920)	0.111	0.000	0.000	1
6	8	12.42568	0.033772	249,259	8,420	201,279	308,323	(20.273)	0.000	0.000	2.000	0.000	0.000	1
6	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
5	7	12.64099	0.001329	310,826	413	308,229	313,445	26.442	0.000	(0.915)	0.117	0.000	0.000	1
5	8	12.09614	0.052361	179,425	9,401	128,740	249,381	(10.962)	0.000	0.000	0.000	1.742	0.000	1
5	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
4	8	11.90169	0.005266	147,518	777	142,692	152,503	(8.647)	0.000	0.000	0.000	1.623	0.000	1
4	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.163	0.000	0.000	0.000	0.000	0.000	1
3	9	12.16746	0.092998	193,259	18,012	106,969	346,151	12.563	0.000	0.000	0.000	0.000	0.000	1

Factor Comparisons

AY Factor (trend)

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	-	-	-
7				(0.11)	(0.03)	-	(0.19)	-
8			-	-	-	-	(0.19)	-
9		-	-	-	-	-	(0.19)	-
10	-	-	-	-	-	-	(0.19)	-

Premium Factor

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						(0.92)	-	-
6					-	(0.92)	-	-
7				1.70	-	(0.92)	-	-
8			-	-	(0.82)	(0.92)	-	-
9		(0.25)	(0.19)	-	-	(0.93)	-	-
10	-	(0.46)	(0.41)	0.33	(0.79)	(0.95)	-	-

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Factor Comparisons

Cumulative Paid

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						0.12	-	-
6					-	0.11	2.00	-
7				-	(1.00)	0.10	-	-
8			-	-	-	0.09	-	-
9		-	-	-	(1.00)	0.09	-	-
10	-	-	-	-	-	-	-	-

Case Reserve

	2	3	4	5	6	7	8	9
3								-
4							1.62	-
5						-	1.74	-
6					0.69	-	-	-
7				-	-	-	-	-
8			-	(1.00)	1.53	-	-	-
9		0.85	0.62	(0.98)	1.53	-	-	-
10	1.30	-	0.54	-	-	0.11	-	-

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Factor Comparisons

Incremental Paid

	2	3	4	5	6	7	8	9
3								0.00
4							0.00	0.00
5						0.00	0.00	0.00
6					(0.56)	0.00	0.00	0.00
7				0.00	1.79	0.00	0.00	0.00
8			0.00	0.00	(0.79)	0.00	0.00	0.00
9		0.00	0.13	0.31	0.00	0.00	0.00	0.00
10	0.00	0.56	0.00	0.00	0.00	0.00	0.00	0.00

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Ranges

5th %tile

	2	3	4	5	6	7	8	9
3								106,969
4							142,692	106,969
5						308,229	128,740	106,969
6					575,196	287,927	201,279	106,969
7				784,299	417,107	284,027	49,249	106,969
8			1,288,805	771,671	485,762	298,654	38,980	106,969
9		1,612,333	1,428,275	823,169	518,218	306,707	30,974	106,969
10	4,140,875	1,713,291	1,392,551	869,877	481,212	311,576	24,693	106,969

95th percentile

	2	3	4	5	6	7	8	9
3								346,151
4							152,503	346,151
5						313,445	249,381	346,151
6					625,414	290,262	308,323	346,151
7				892,223	560,722	288,616	163,050	346,151
8			1,450,949	850,844	559,594	302,933	140,305	346,151
9		1,822,547	1,476,499	863,851	642,312	313,208	120,246	346,151
10	4,344,009	1,939,282	1,504,418	999,848	595,333	318,135	102,706	346,151

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Interval Testing

Paid LDF Wtd Avg - All Periods

	2	3	4	5	6	7	8	9
3								174,294
4							249,479	196,226
5						411,610	274,398	215,826
6					581,042	362,894	241,922	190,282
7				976,481	637,722	398,295	265,521	208,844
8			1,972,715	1,320,283	862,254	538,527	359,007	282,374
9		1,820,341	1,609,724	1,077,343	703,594	439,435	292,947	230,416
10	4,574,416	2,042,769	1,806,417	1,208,984	789,566	493,130	328,743	258,571

>95%tile
<5%tile

- On average about 10% of projections should be outside the range, in this case 3.6, if the projections are consistent with the regression models.
- In this case 25 were outside the range, all on the high side.

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Interval Testing

Paid LDF Wtd Avg - Last 3

	2	3	4	5	6	7	8	9
3								174,294
4							249,479	196,226
5						400,873	274,206	215,675
6					538,278	352,280	240,968	189,532
7				959,635	590,067	386,174	264,152	207,767
8			1,883,151	1,290,814	793,705	519,446	355,313	279,469
9		1,694,148	1,521,202	1,042,714	641,151	419,607	287,020	225,754
10	4,327,921	1,862,272	1,672,163	1,146,191	704,778	461,248	315,504	248,157

- No dramatic improvement by going to the last 3 weighted average…

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Interval Testing

Paid BF - Set A Priori from previous AY's

	2	3	4	5	6	7	8	9
3								175,501
4							251,017	197,436
5						401,765	274,816	216,155
6					551,778	361,115	247,011	194,285
7				978,633	601,749	393,820	269,381	211,880
8			1,786,984	1,224,896	753,172	492,920	337,168	265,198
9		1,671,535	1,500,896	1,028,796	632,593	414,006	283,189	222,741
10	3,979,116	1,712,184	1,537,397	1,053,815	647,977	424,074	290,076	228,157

- …or in the Bornhuetter-Ferguson…

Interval Testing

Incurred LDF Wtd Avg - Last 3

	2	3	4	5	6	7	8	9
3								180,864
4							201,402	158,411
5						336,239	229,995	180,901
6					541,434	354,346	242,381	190,643
7				933,700	574,120	375,738	257,013	202,152
8			1,590,446	1,090,178	670,336	438,707	300,085	236,030
9		1,634,343	1,467,502	1,005,905	618,518	404,794	276,888	217,785
10	4,073,437	1,752,770	1,573,839	1,078,794	663,337	434,126	296,952	233,566

- …or from Incurred LDF.

Example 2 – Industry WC

Paid	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	4,911,312	11,447,685	15,497,376	17,819,233	18,891,225	19,596,939	20,244,282	20,960,131	21,349,825	21,658,369
2002	4,781,495	11,058,159	14,895,451	17,064,796	18,528,035	19,386,562	20,255,245	20,812,015	21,268,653	
2003	4,753,103	10,795,467	14,396,535	16,570,896	18,034,474	19,153,900	19,857,006	20,488,214		
2004	4,989,812	10,657,111	13,723,128	15,693,871	17,100,909	17,980,283	18,695,839			
2005	5,108,280	10,499,569	13,630,744	15,667,236	16,993,881	17,955,132				
2006	5,303,253	11,169,253	14,861,279	16,981,098	18,417,003					
2007	5,355,908	11,760,363	15,390,396	17,702,859						
2008	5,458,348	11,812,302	15,594,168							
2009	4,978,929	10,848,558								
2010	5,112,693									

Incurred	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10	Premium
2001	12,175,274	17,747,731	20,533,604	21,793,420	22,250,838	22,727,347	23,163,185	23,610,287	23,948,973	24,269,330	28,974,288
2002	11,995,861	17,889,636	20,397,284	21,340,492	22,058,364	22,592,920	23,130,637	23,530,834	23,916,752		34,098,234
2003	12,510,030	17,828,650	19,828,441	20,872,924	21,686,393	22,455,295	22,898,518	23,407,115			39,406,594
2004	12,412,357	16,940,426	18,728,358	19,775,010	20,582,538	21,195,976	21,739,955				43,839,345
2005	12,710,198	16,662,570	18,508,990	19,702,869	20,411,529	21,117,353					44,975,942
2006	12,994,647	17,677,476	20,112,140	21,405,232	22,406,243						45,140,227
2007	13,314,139	18,604,657	21,000,304	22,522,495							42,181,287
2008	13,405,971	18,766,138	21,374,385								39,194,976
2009	12,084,057	17,043,773									34,673,046
2010	12,302,291										32,652,882

Factor Comparisons – Ex 2

AY Factor (trend)

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	-	-	-
7				-	-	-	-	-
8			-	-	-	-	-	-
9		-	(0.01)	-	-	-	-	-
10	-	-	-	-	-	-	-	-

Premium Factor

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	(0.60)	-	-
7				0.43	-	-	-	-
8			-	0.35	1.91	-	-	-
9		-	-	0.51	0.75	-	-	-
10	(0.61)	(0.74)	(0.49)	0.43	0.42	-	-	-

Factor Comparisons – Ex 2

Cumulative Paid

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					(1.00)	-	-	-
7				-	-	-	-	-
8			1.10	-	-	-	-	-
9		-	1.38	-	-	-	-	-
10	-	-	0.42	-	-	-	-	-

Case Reserve

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	-	-	-
7				-	-	-	-	-
8			-	2.00	-	-	-	-
9		-	-	1.55	1.88	-	-	-
10	2.00	2.00	1.37	-	2.00	-	-	-

Factor Comparisons – Ex 2

Incremental Paid

	2	3	4	5	6	7	8	9
3								-
4							(0.80)	-
5						-	-	-
6					0.85	1.10	(0.61)	-
7				-	-	-	2.00	-
8			-	(0.43)	2.00	-	-	-
9		1.28	-	-	-	-	-	-
10	-	-	-	-	-	-	-	-

Ranges – Ex 2

5th %tile

	2	3	4	5	6	7	8	9
3								229,280
4							535,907	229,280
5						623,266	497,339	229,280
6					739,538	442,098	407,595	229,280
7				1,277,838	759,625	623,266	424,160	229,280
8			2,262,436	1,555,518	946,660	623,266	497,339	229,280
9		3,323,915	1,961,323	1,107,431	678,977	623,266	497,339	229,280
10	5,893,638	3,486,204	2,180,398	1,137,937	642,699	623,266	497,339	229,280

95th percentile

	2	3	4	5	6	7	8	9
3								783,437
4							775,914	783,437
5						865,869	812,583	783,437
6					1,181,621	1,088,722	889,027	783,437
7				1,554,250	1,082,836	865,869	1,117,610	783,437
8			2,367,855	1,741,717	1,322,754	865,869	812,583	783,437
9		3,578,098	2,080,155	1,265,490	865,392	865,869	812,583	783,437
10	6,294,680	3,907,229	2,268,157	1,400,506	975,138	865,869	812,583	783,437

Interval Testing – Ex 2

Paid LDF Wtd Avg - All Periods

	2	3	4	5	6	7	8	9	307,371
3								415,105	302,091
4							589,723	390,739	284,359
5						692,253	588,195	389,726	283,622
6					930,489	745,935	633,807	419,948	305,615
7				1,448,979	967,615	775,698	659,096	436,704	317,809
8			2,300,419	1,464,672	978,095	784,099	666,234	441,433	321,251
9		3,501,360	2,116,869	1,347,807	900,053	721,536	613,076	406,212	295,619
10	6,094,848	3,617,221	2,186,917	1,392,406	929,836	745,412	633,363	419,653	305,401

>95%tile
<5%tile

- The number of projections that are outside of the modeled range is about as expected here.

Interval Testing – Ex 2

Paid LDF Wtd Avg - Last 3

	2	3	4	5	6	7	8	9	307,371
3								415,105	302,091
4							589,723	390,739	284,359
5						726,628	589,279	390,444	284,144
6					1,045,771	787,641	638,759	423,229	308,003
7				1,526,898	1,091,921	822,401	666,948	441,906	321,596
8			2,298,760	1,543,292	1,103,645	831,230	674,109	446,651	325,048
9		3,467,327	2,110,326	1,416,785	1,013,177	763,093	618,851	410,038	298,404
10	6,030,414	3,561,468	2,167,624	1,455,253	1,040,686	783,812	635,653	421,171	306,506

- Even though this is not as troublesome as the CMP example, the probability of having eight or more outside the range is only 2.3% under the null hypothesis

Interval Testing – Ex 2

Paid BF - Set A Priori from previous AY's

	2	3	4	5	6	7	8	9
3								418,973
4							604,165	400,307
5						749,263	607,635	402,607
6					1,069,594	805,584	653,310	432,870
7				1,530,251	1,094,319	824,207	668,413	442,877
8			2,246,060	1,507,911	1,078,343	812,174	658,654	436,411
9		3,299,694	2,008,300	1,348,289	964,194	726,200	588,932	390,214
10	5,246,930	3,098,755	1,886,002	1,266,183	905,478	681,977	553,068	366,452

- In case you thought the discrepancies could only be on the high side

Interval Testing – Ex 2

Incurred LDF Wtd Avg - Last 3

	2	3	4	5	6	7	8	9
3								581,972
4							773,132	512,261
5						829,066	672,354	445,488
6					1,227,048	924,174	749,484	496,593
7				1,726,809	1,234,882	930,074	754,269	499,763
8			2,519,476	1,691,472	1,209,612	911,041	738,834	489,536
9		3,581,908	2,180,064	1,463,605	1,046,659	788,310	639,301	423,588
10	6,039,289	3,566,709	2,170,814	1,457,394	1,042,217	784,965	636,589	421,791

Example 3 – Specific Company

Paid	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10
2001	8,146	43,443	68,797	85,256	111,928	121,597	147,139	157,403	159,147	164,660
2002	8,106	34,738	45,862	59,862	71,269	78,248	86,295	87,808	88,907	
2003	14,244	46,678	78,507	107,679	128,103	142,043	145,848	149,888		
2004	12,546	43,044	94,212	120,133	141,586	150,916	158,117			
2005	8,039	45,177	90,802	125,888	141,262	161,063				
2006	6,076	34,912	71,860	101,927	119,775					
2007	5,073	37,032	68,574	108,590						
2008	9,001	30,431	59,901							
2009	3,674	16,752								
2010	5,043									

Incurred	Age1	Age2	Age3	Age4	Age5	Age6	Age7	Age8	Age9	Age10	Premium
2001	26,210	68,365	90,570	109,589	125,121	142,201	156,228	161,268	166,576	165,861	190,607
2002	19,199	54,651	67,229	84,791	85,788	89,864	89,592	90,630	91,798		254,088
2003	39,147	91,446	111,628	138,641	145,990	154,717	155,760	158,650			361,513
2004	35,689	89,687	129,207	145,424	159,072	166,418	167,445				440,653
2005	27,432	96,569	139,121	154,288	161,432	172,300					452,639
2006	36,607	84,216	109,041	126,924	138,422						418,092
2007	29,225	81,560	118,652	126,990							374,405
2008	22,337	63,356	89,985								349,931
2009	17,063	45,172									325,691
2010	21,373										303,557

Factor Comparison – Ex 3

AY Factor (trend)

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						(0.30)	-	-
6					-	(0.45)	-	-
7				-	-	(0.45)	-	-
8			-	-	(0.25)	(0.75)	-	-
9		-	0.18	(0.39)	-	(0.45)	-	-
10	(0.08)	-	-	(0.27)	-	(0.45)	-	-

Premium Factor

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	-	-	-
7				(0.97)	-	-	-	-
8			-	(0.50)	-	-	-	-
9		(1.00)	-	-	0.52	-	-	-
10	-	0.73	-	-	-	-	-	-

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Factor Comparison – Ex 3

Cumulative Paid

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	-	-	-
7				1.41	-	-	-	-
8			-	-	-	2.00	2.00	-
9		-	1.52	-	-	-	-	-
10	-	-	-	-	-	-	-	-

Case Reserve

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						2.00	2.00	-
6					1.84	-	-	-
7				-	2.00	-	-	-
8			1.11	(0.70)	2.00	-	-	-
9		2.00	-	2.00	-	-	-	-
10	0.48	0.64	2.00	2.00	-	-	-	-

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Factor Comparison – Ex 3

Incremental Paid

	2	3	4	5	6	7	8	9
3								-
4							-	-
5						-	-	-
6					-	-	2.00	-
7				-	-	-	-	-
8			-	0.83	-	-	-	-
9		-	-	-	2.00	-	-	-
10	-	-	-	-	-	-	-	-

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CREATIVE SOLUTIONS TO COMPLEX PROBLEMS

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Ranges – Ex 3

5th %tile

	2	3	4	5	6	7	8	9
3								232
4							583	232
5						288	99	232
6					10,228	455	1,156	232
7				16,119	2,849	257	583	232
8			21,100	15,318	1,931	0	42	232
9		15,436	14,442	632	728	83	583	232
10	15,505	20,183	11,657	1,240	7,779	47	583	232

95th percentile

	2	3	4	5	6	7	8	9
3								8,907
4							51,116	8,907
5						26,704	86,555	8,907
6					21,125	12,476	13,005	8,907
7				23,607	14,325	9,533	51,116	8,907
8			24,638	21,564	6,146	512,504	578,789	8,907
9		23,743	20,275	4,377	7,506	5,433	51,116	8,907
10	21,050	33,408	27,524	4,531	18,688	4,049	51,116	8,907

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Interval Testing – Ex 3

Paid LDF Wtd Avg - All Periods

	2	3	4	5	6	7	8	9
3								1,738
4							6,594	1,910
5						14,575	7,324	2,121
6					12,039	11,928	5,994	1,736
7				20,458	12,971	12,852	6,458	1,870
8			22,028	15,435	9,786	9,696	4,873	1,411
9		13,970	11,298	7,916	5,019	4,973	2,499	724
10	17,322	18,650	15,083	10,569	6,701	6,639	3,336	966

>95%tile
<5%tile

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Interval Testing – Ex 3

Paid LDF Wtd Avg - Last 3

	2	3	4	5	6	7	8	9
3								1,738
4							6,594	1,910
5						8,267	7,061	2,045
6					12,554	6,792	5,802	1,680
7				17,063	13,170	7,126	6,086	1,763
8			27,244	13,694	10,569	5,718	4,884	1,415
9		16,030	14,910	7,494	5,784	3,130	2,673	774
10	18,886	22,896	21,296	10,704	8,262	4,470	3,818	1,106

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Interval Testing – Ex 3

Paid BF - Set A Priori from previous AY's

	2	3	4	5	6	7	8	9
3								1,765
4							6,775	1,962
5						8,452	7,219	2,091
6					13,280	7,185	6,137	1,777
7				17,344	13,386	7,243	6,186	1,792
8			30,407	15,284	11,796	6,383	5,452	1,579
9		28,841	26,826	13,483	10,407	5,631	4,809	1,393
10	24,651	29,887	27,798	13,972	10,784	5,835	4,984	1,443

Interval Testing – Ex 3

Incurred LDF Wtd Avg - Last 3

	2	3	4	5	6	7	8	9
3								2,727
4							7,136	2,066
5						6,429	5,491	1,590
6					12,849	6,952	5,938	1,720
7				14,126	10,903	5,899	5,039	1,459
8			23,595	11,860	9,153	4,953	4,230	1,225
9		19,852	18,465	9,281	7,163	3,876	3,310	959
10	20,555	24,920	23,179	11,650	8,992	4,865	4,156	1,203

Other Candidate Predictive Variables

- Open Claim Count
- Current Rate Level Earned Premium
- Inflation Index (Cal Year or Accident Year)

Ways to Increase Degrees of Freedom

- Include older data
- Consider quarterly or monthly data instead of annual
- Subdivide claims (but still fitting common parameters)
- Consider estimating parameters that cover multiple development periods (this starts to approach building new models of loss development)

Summary

- Multivariate regression is a useful tool for testing our loss projections
- It is objective, which balances the significant amounts of judgment that is used in reserving
- It may indicate bias in our estimates
- It may suggest alternative methods of reserving for particular data