Extreme development techniques

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Overview

- Background and motivation
- Walkthrough of specific methods
  - Incremental paid/incurred loss development method
  - Case reserve run-off method
  - Recursive method
  - Munich chain ladder method

What are extreme development techniques?

Extreme development techniques are methods that may be necessary in the following situations:

- Claims and exposure data are limited to nearly non-existent
- Traditional development patterns are not available
- Data are so mature that ultimate loss estimates are "extremely" volatile

Some of these methods are extensions of traditional development methods, others are novel approaches to viewing loss development and projecting future claims.

When are extreme development techniques useful?

This session will discuss a number of examples of such extreme development methods and models that may be useful to actuaries who are modeling the following:

- Long-tailed lines of business
- Run-off portfolios
- Reinsurance liabilities

Techniques to be discussed today

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich chain ladder method

Incremental loss development method

- When is this method appropriate?
  - When reliable data are only available from a certain point in time onward (e.g., after a systems conversion)
  - When the liabilities are very mature and paid-to-date or incurred-to-date measures are of limited value
- What data are needed?
  - Paid losses from a fixed point in time forward
  - Case reserve at date
  - Incurred losses from a fixed point in time forward

Step 1: calculation of change in paid losses

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

- Step 1: Calculate the change in paid loss based on the incremental paid triangle
  - Assumption: evaluated as of 31 December 2010
  - The following triangle is the incremental paid/loss triangle; we are going to calculate the incremental paid/loss development factors based on this triangle

Few more ages are not shown here due to limited room

	Age (yrs)																		
U/W Year	12	13	14	15	16	17	18	19	20	21	22	27	28	29	30	31	32	33	34
1977								2,811,530	2,482,581	1,551,050	24,397	(10,000)	73,910	0	29,900	30,528	928	221	2
1978							5,302,785	2,773,356	3,971,550	1,327,150	355,550	65,604	38,706	16,950	0	106,000	21,220	438	
1979						7,286,341	1,020,570	1,018,529	682,414	1,312,383	419,963	0	36,550	27,932	1,922	823	2,201		
1980					13,738,448	11,320,482	2,662,400	5,516,100	1,695,950	(50,091)	(39,171)	42,192	2,102	1,821	3,105	920			
1981				7,241,050	6,012,428	1,785,059	525,718	401,611	261,705	758,351	722,135	4,550	10,291	0	3,910				
1982			3,825,050	1,710,305	1,361,162	3,656,080	4,814,300	533,656	338,776	216,700	216,691	523	1,190	949					
1983		6,709,700	3,808,744	2,609,950	2,602,120	1,386,939	5,233,688	4,960,051	170,624	26,350	73,799	120,192	201						
1984	5,161,750	5,784,645	4,606,044	4,573,758	836,374	128,119	239,651	430,221	220,731	81,321	101,293	2,120							

Incremental paid/loss development factors

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

	Age																	
U/W Year	13	14	15	16	17	18	19	20	21	22	27	28	29	30	31	32	33	34
1977								0.883	0.625	0.016	2.323	(7.391)	0.000	0.000	1.021	0.030	0.238	0.009
1978							0.523	1.432	0.334	0.268	1.866	0.590	0.438	0.000	0.000	0.200	0.021	
1979						0.140	0.998	0.670	1.923	0.320	1.923	0.000	0.764	0.069	0.428	2.674		
1980					0.824	0.235	2.072	0.307	(0.030)	0.782	(6.510)	0.050	0.866	1.705	0.296			
1981				0.830	0.297	0.295	0.764	0.652	2.898	0.952	0.317	2.262	0.000					
1982			0.447	0.796	2.686	1.317	0.111	0.635	0.640	1.000	0.559	2.275	0.797					
1983		0.568	0.685	0.997	0.533	3.774	0.948	0.034	0.154	2.801	0.119	0.002						
1984	1.121	0.796	0.993	0.183	0.153	1.871	1.795	0.513	0.368	1.246	0.051							
Wtd Average	1.121	0.673	0.727	0.670	0.744	0.567	0.790	0.533	0.532	0.359	1.145	0.567	0.293	0.108	0.924	0.177	0.030	0.009
Straight Avg	1.121	0.682	0.708	0.702	0.899	1.272	1.030	0.641	0.864	0.923	0.081	(0.369)	0.478	0.591	0.582	0.968	0.129	0.009
Straight Avg Ex H/L	1.121	0.682	0.685	0.813	0.551	0.929	1.006	0.610	0.674	0.761	0.806	0.726	0.500	0.069	0.428	0.200	0.129	0.009
Select		0.682	0.708	0.813	0.712	0.751	1.006	0.641	0.864	0.761	0.806	0.567	0.500	0.591	0.582	0.200	0.129	0.000
	144	156	168	180	192	204	216	228	240	252	264	324	336	348	360	372	384	396
Incremental Pattern	1.000	0.682	0.483	0.393	0.280	0.210	0.211	0.135	0.117	0.089	0.072	0.016	0.008	0.005	0.003	0.001	0.000	0.000
Accumulated Values	**1.000**	**1.682**	**2.165**	**2.558**	**2.838**	**3.048**	**3.259**	**3.394**	**3.511**	**3.600**	**3.672**	**3.847**	**3.855**	**3.859**	**3.862**	**3.863**	**3.863**	**3.863**

Calculation of change in paid loss

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

(1) U/W year	(2) Start age	(3) End age	(4) Total paid *At start age*	(5) Total paid *At end age*	(6) Total change *From start age to end age*
1977	19	34	2,811,530	7,131,041	4,319,511
1978	18	33	5,302,785	15,012,037	9,709,252
1979	17	32	7,286,341	12,634,556	5,348,215
1980	16	31	13,738,448	36,226,919	22,488,471
1981	15	30	7,241,050	18,501,792	11,260,742
1982	14	29	3,825,050	19,294,363	15,469,313
1983	13	28	6,709,700	27,847,579	21,137,879
1984	12	27	5,161,750	22,455,375	17,293,625
Total			52,076,654	159,103,662	107,027,008

Calculation details (use U/W yr **1984** as an example and refer to **triangle on page 7**):

1. Paid during age 12 = **5,161,750**
2. Total paid through age 27 = 5,161,750+5,784,645+…+2,120 = **22,455,375** (sum up all the incremental paid loss for U/W yr 1984)
3. Total change = 22,455,375 – 5,161,750 = **17,293,625**

Step 2: Curve fitting

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

We fitted x and y values into different distributions (e.g., Weibull, Gompertz and Richards model) to get the coefficients.

Age (in months)	X = Age (in years)	Actual Y = Accumulated incremental selections
144	12	1.000
156	13	1.682
168	14	2.165
180	15	2.558
192	16	2.838
204	17	3.048
216	18	3.259
228	19	3.394
240	20	3.511
252	21	3.600
264	22	3.672
276	23	3.726
288	24	3.766
300	25	3.802
312	26	3.831
324	27	3.847
336	28	3.855
348	29	3.859
360	30	3.862
372	31	3.863
384	32	3.863
396	33	3.863
408	34	3.863

From curve fitting software

Weibull model: y=a-b*exp(-c*x^d)	
Coefficient Data:	
a =	3.870
b =	20.470
c =	0.058
d =	1.423
Standard error:	0.0213885
Correlation coefficient:	0.999683

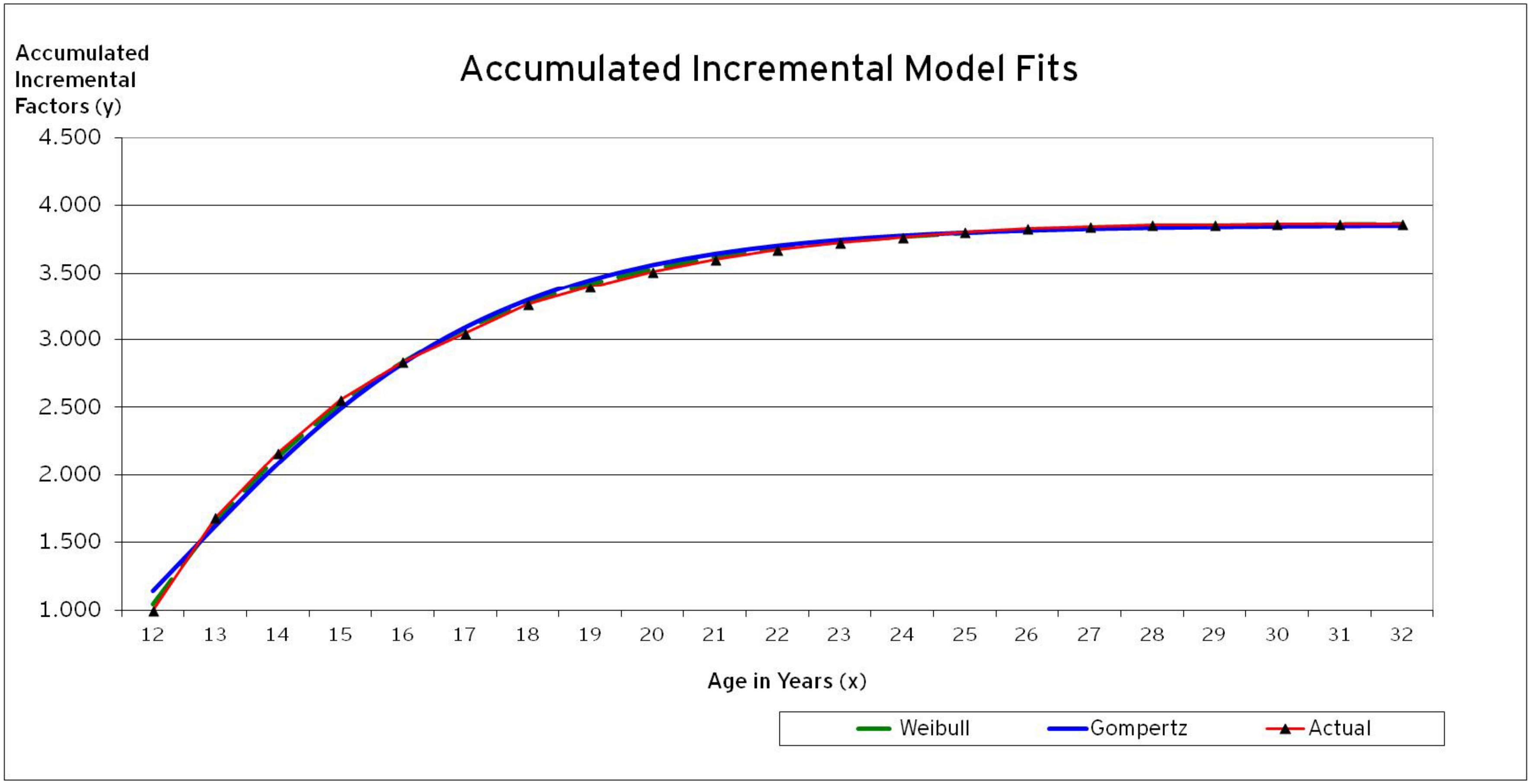
Gompertz relation: y=a*exp(-exp(b-cx))	
Coefficient data:	
a =	3.854
b =	4.284
c =	0.341
Standard error:	0.0494986
Correlation coefficient:	0.9982117

This column is from the triangle on page 8

Weibull Y^ = a - b*exp (-c*X^d)	Gompertz Y^ = a*exp (-exp(b-c*X))
1.046	1.141
1.646	1.621
2.133	2.081
2.523	2.486
2.834	2.822
3.078	3.087
3.269	3.292
3.416	3.445
3.530	3.558
3.617	3.641
3.682	3.701
3.732	3.745
3.769	3.776
3.796	3.798
3.817	3.814
3.832	3.826
3.842	3.834
3.850	3.839
3.856	3.844
3.860	3.847
3.863	3.849
3.865	3.850
3.866	3.851

Accumulated incremental paid ratio model selection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method



Step 3: Accumulated incremental ratios calculation

- Step 3: calculate accumulated incremental ratios implied after fitting and comparing different distributions that behave like (transformable to) cumulative distribution functions
  - Assumption: we use Weibull model as an example; in practice, other models can also be used

(1) U/W year	(2) Start age	(3) End age	Weibull (7) Accumulated incremental (at start)	Weibull (8) Accumulated incremental (at end)
1977	19	34	3.416403	3.866466
1978	18	33	3.268574	3.865007
1979	17	32	3.077762	3.862942
1980	16	31	2.833444	3.860034
1981	15	30	2.523254	3.855958
1982	14	29	2.132930	3.850278
1983	13	28	1.646396	3.842404
1984	12	27	1.046024	3.831549

From page 10

Weibull model: y=a-b*exp(-c*x^d)
coefficient data:

a =	3.870
b =	20.470
c =	0.058
d =	1.423

Weibull model: $y = a - b * \exp(-c * \mathbf{x} \,\hat{}\, d)$

$3.870 - 20.470 * \exp(-0.058 * \mathbf{27}^{1.423}) =$ **3.831549**

Step 4: Incremental ratios calculation and reserve projection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

- Step 4: calculate the incremental loss development ratio to ultimate development based on curve fit and estimate the total reserves.

(1)	(2)	(3)	(4)	(5)	(6)	Weibull		Ratio to total period change	Estimated total reserves
U/W year	Start age	End age	Total paid	Total paid	Total change	(7)	(8)	(9)	(10)
			At start age	*At end age*	*From start age to end age*	Accumulated incremental (at start)	Accumulated incremental (at end)	[(Ult)-(8)] / [(8)-(7)]	(6) * (9)
1977	8	34	2,811,530	7,131,041	4,319,511	3.416403	3.866466	0.007409	32,004
1978	7	33	5,302,785	15,012,037	9,709,252	3.268574	3.865007	0.008037	78,029
1979	6	32	7,286,341	12,634,556	5,348,215	3.077762	3.862942	0.008735	46,714
1980	5	31	13,738,448	36,226,919	22,488,471	2.833444	3.860034	0.009514	213,947
1981	4	30	7,241,050	18,501,792	11,260,742	2.523254	3.855958	0.010386	116,957
1982	3	29	3,825,050	19,294,363	15,469,313	2.132930	3.850278	0.011367	175,847
1983	2	28	6,709,700	27,847,579	21,137,879	1.646396	3.842404	0.012475	263,702
1984	1	27	5,161,750	22,455,375	17,293,625	1.046024	3.831549	0.013732	237,477
Total			52,076,654	159,103,662	107,027,008	Ultimate:	3.869800		1,164,676

Ultimate value = 3.869800
According to the Weibull model y = a – b * exp(-c* **x** ^d), when x → ∞, y → a=3.869800

Incremental ratio for U/W Yr 1984: (3.869800 – 3.831549) / (3.831549– 1.046024) = **0.013732**
Estimated unpaid reserve for U/W Yr 1984: 0.013732* $17,293,625= $237,477

Case reserve run-off method

- When is this method appropriate?
  - When there is a long history of incremental paid/incurred losses
  - When the incremental activity is more significant than in cases where incremental method may be more appropriate

- What data are needed?
  - Incremental paid/loss
  - Cumulative incurred loss

Step 1: data aggregation and preparation

- Step 1: construct case reserve run-off triangle
  - Given incremental paid triangle and case reserve triangle

Case reserve triangle

U/W Year	Age in years Prior	17	18	19	20	21	22	23	24	25
1986	…	56,300	67,280	51,888	44,987	25,461	26,830	24,093	19,015	17,699
1987	…	59,382	39,246	23,925	22,175	19,418	24,326	19,161	16,370	–
1988	…	52,489	58,013	71,744	66,143	33,791	21,906	17,383	–	–
1989	…	32,175	30,946	33,684	36,091	12,301	12,181	–	–	–
1990	…	49,900	64,371	75,530	80,570	69,592	–	–	–	–

Incremental paid loss triangle

U/W Year	Age in years Prior	17	18	19	20	21	22	23	24	25
1986	…	…	(7,208)	4,759	(268)	20,034	85	844	4,642	1,378
1987	…	…	16,063	5,411	(10,991)	4,033	(9,692)	6,314	2,279	–
1988	…	…	5,289	6,980	11,902	(3,304)	14,542	6,210	–	–
1989	…	…	1,004	(955)	(1,605)	24,381	–	–	–	–
1990	…	…	(238)	7,898	10,933	1,660	–	–	–	–

Case reserve run-off triangle from the start age 17

U/W year	17	18	19	20	21	22	23	24	25
1986	56,300	60,072	49,439	42,271	42,778	44,232	42,339	41,903	41,965
1987	59,382	55,309	45,399	32,658	33,934	29,150	30,299	29,787	–
1988	52,489	63,302	84,013	90,314	54,658	57,315	59,002	–	–
1989	32,175	31,950	33,733	34,536	35,627	35,007	–	–	–
1990	49,900	64,633	83,190	99,162	89,845	–	–	–	–

25,461 + [(7,208)+4,759+(268)+20,034] = **42,778**

58,013 + 5,289=**63,302**

Step 2: Run-off factor calculation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

► Step 2: calculate the run-off ATA and ATU factors

Case reserve run-off triangle from the start age 17

U/W year	17	18	19	20	21	22	23	24	25
1986	56,300	60,072	49,439	42,271	42,778	44,232	42,339	41,903	41,965
1987	59,382	55,309	45,399	32,658	33,934	29,150	30,299	29,787	–
1988	52,489	63,302	84,013	90,314	54,658	57,315	59,002	–	–
1989	32,175	31,950	33,733	34,536	35,627	35,007	–	–	–
1990	49,900	64,633	83,190	99,162	89,845	–	–	–	–

Case run-off ATA factor

U/W year	18/17	19/18	20/19	21/20	22/21	23/22	24/23	25/24	
1986	1.067	0.823	0.855	1.012	1.034	0.957	0.990	1.001	
1987	0.931	0.821	0.719	1.039	0.859	1.031	0.983	–	
1988	1.206	1.327	1.075	0.605	1.049	1.029	–	–	
1989	0.993	1.056	1.024	1.032	0.983	–	–	–	
1990	1.295	1.287	1.192	0.906	–	–	–	–	
Avg x hi/lo	1.089	1.055	0.985	0.983	1.008	1.029			
Wtd avg	1.100	1.075	1.011	0.859	0.992	1.007	0.987	1.001	
Selected	1.080	1.058	1.031	1.023	1.019	1.012	0.993	1.001	Tail
Implied ATU	1.496	1.386	1.310	1.270	1.242	1.218	1.204	1.211	1.210

Tail factor is usually selected based on industry factors

Step 3: case to case: run-off ratio calculation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Curve fitting method

Case reserve triangle

U/W year	17	18	19	20	21	22	23	24	25
1986	56,300	67,280	51,888	44,987	25,461	26,830	24,093	19,015	17,699
1987	59,382	39,246	23,925	22,175	19,418	24,326	19,161	16,370	–
1988	52,489	58,013	71,744	66,143	33,791	21,906	17,383	–	–
1989	32,175	30,946	33,684	36,091	12,801	12,181	–	–	–
1990	49,900	64,871	75,530	80,570	69,592	–	–	–	–

These ratios are derived as:

$$\frac{\text{Case reserve}}{\text{Case-reserve-run-off}}$$

Case to case-reserve-run-off ratio

U/W year	17	18	19	20	21	22	23	24	25
1986	1.000	1.120	1.050	1.064	0.595	0.607	0.569	0.454	0.422
1987	1.000	0.710	0.527	0.679	0.572	0.834	0.632	0.550	–
1988	1.000	0.916	0.854	0.732	0.618	0.382	0.295	–	–
1989	1.000	0.969	0.999	1.045	0.359	0.348	–	–	–
1990	1.000	1.004	0.908	0.813	0.775	–	–	–	–
Avg	1.000	0.944	0.867	0.867	0.584	0.543	0.499	0.502	0.422
Wtd Avg	1.000	0.946	0.868	0.836	0.627	0.514	0.461	0.494	0.422
Selection	1.000	0.944	0.867	0.836	0.584	0.543	0.499	0.494	0.422

Step 4: Case to Case: run-off ratio application and reserve projection

	Age in years	17	18	19	20	21	22	23	24	25
(1) (Slide 16)	Case-run-off factor	1.496	1.386	1.310	1.270	1.242	1.218	1.204	1.211	1.210
(2) (Slide 17)	Case to case-reserve-run-off ratio	1.000	0.944	0.867	0.836	0.584	0.543	0.499	0.494	0.422
((1)-1)/(2)	**Selected IBNR-to-case reserve ratio**	0.496	0.409	0.357	0.323	0.414	0.402	0.408	0.428	0.498

Age in years as of 31 December 2010	**U/W year**	**Case ($)**	**IBNR-to-Case ratio**	**Estimated IBNR ($)**
25	1986	17,699	0.553	9,785
24	1987	16,370	0.428	7,014
23	1988	17,383	0.408	7,095
22	1989	12,181	0.402	4,891
21	1990	69,592	0.414	28,804

Recursive method

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

- When is this method appropriate?
  - When only incremental loss data are available
  - When we assume the relationship of ΔP/ΔC is consistent as the exposure approaches ultimate
  - When only aggregate calendar year losses for all exposure years are available, particularly when all years are very mature
- What data are needed?
  - Incremental paid/loss
  - Change in case reserves

Theory and calculation steps

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

- Calculate (incremental) paid to prior case ratio: "p"
- Calculate case to prior case ratio: "c"
- Assumptions:
  - These consumption ratios are consistent over time
  - Initial case reserve is $1

Time	Paid losses	Case
0		1
1	p	c
2	pc	cc
3	pcc	ccc
4	pccc	c^4
5	pc^4	c^5
6	pc^5	c^6
7	pc^6	c^7
8	pc^7	c^8
9	pc^8	c^9
10	pc^9	c^10
11	pc^10	c^11
12	pc^11	c^12
13		
14		

- Required reserves= sum(pmts) = p * (1+c+c^2+c^3+c^4+c^5+ ….)
- Since c < 1, (a requirement), sum(pmts) = p/(1-c) (based on geometric theory)
- c = Case$(k) / Case$ (k-1);
- p = Paid$ movement (k) / Case$ (k-1)
  = (CumPaid$(k) – CumPaid$(k-1)) / Case$ (k-1)
- Since c and p share the same denominator,
  sum(pmts) = p/(1-c)
  = Paid$ movement (k) / (Case$(k-1) – Case$(k))
  = [CumPaid$(k) – CumPaid$(k-1)] / [Case$(k-1)–Case$(k)]

sum(pmts) = p/(1-c) = ΔP/ΔC
This is the ΔP/ΔC ratio we need to estimate

Few more things about this method

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

- **(ΔP/ΔC) x C = required reserves**
- If for every dollar of case reduction, there are Z (which is the selected ratio of ΔP/ΔC) dollars of paid losses, then the required reserves (case + IBNR) are (Z x C)
- ΔP/ΔC ratio: this ratio is a measurement of the interaction between paid and case movements. Paid losses almost always trigger case reserve changes
- We can interpret this as: future paid losses (to ultimate) will be related to case reserves in exactly the same ratio as ΔP/ΔC over the relevant period used
- This method does not require the availability of cumulative data. Thus if historical data are lost or missing, this method works. Since this is a calendar year method, it works well to combine exposure periods in order to stabilize the calculations

Numerical example

- Step 1: calculate and select the ratio of incremental payment relative to change in case reserves (ΔP/ΔC)

Calendar year	Company case reserves: Beginning	Company case reserves: Ending	Change case (-)	Company Incremental paid loss	ΔP/ΔC
	(1)	(2)	(3) = (1) - (2)	(4)	(5) = (4)/(3)
2000		3,235,000			
2001	3,235,000	2,910,000	325,000	488,000	1.50
2002	2,910,000	2,798,000	112,000	117,000	1.04
2003	2,798,000	3,038,000	(240,000)	33,000	(0.14)
2004	3,038,000	1,887,000	1,151,000	682,000	0.59
2005	1,887,000	1,826,000	61,000	19,000	0.31
2006	1,826,000	1,603,000	223,000	557,000	2.50
2007	1,603,000	1,344,000	259,000	388,000	1.50
2008	1,344,000	1,315,000	29,000	43,000	1.48
2009	1,315,000	1,145,000	170,000	359,000	2.11
				Avg 3 yrs	1.70
				Avg 5 yrs	1.58
				Selected ΔP/ΔC ratio	**1.70**

Numerical example

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. Munich Chain Ladder method

Step 2: calculate future payments and unpaid reserves

- Assumption: the ratio ΔP/ΔC would be stable for a mature set of exposure

Calendar year	Case reserves at 12/31/XX	Selected ΔP/ΔC factor	Company incremental paid loss	Paid Since date	Required reserves estimates
	(1)	(2)	(3)	(4) in 2000 = (3) total (4) = (4) prior - (3)	(5)=(1)*(2)-(4)
2000	3,235,000	1.70	–	2,686,000	2,805,513
2001	2,910,000	1.70	488,000	2,198,000	2,741,815
2002	2,798,000	1.70	117,000	2,081,000	2,668,692
2003	3,038,000	1.70	33,000	2,048,000	3,109,099
2004	1,887,000	1.70	682,000	1,366,000	1,837,241
2005	1,826,000	1.70	19,000	1,347,000	1,752,691
2006	1,603,000	1.70	557,000	790,000	1,931,142
2007	1,344,000	1.70	388,000	402,000	1,879,482
2008	1,315,000	1.70	43,000	359,000	1,873,253
2009	1,145,000	1.70	359,000	-	1,943,673
Total			2,686,000	Selected reserve	1,937,000

Selected the median value of estimated required reserves

Munich Chain Ladder Method

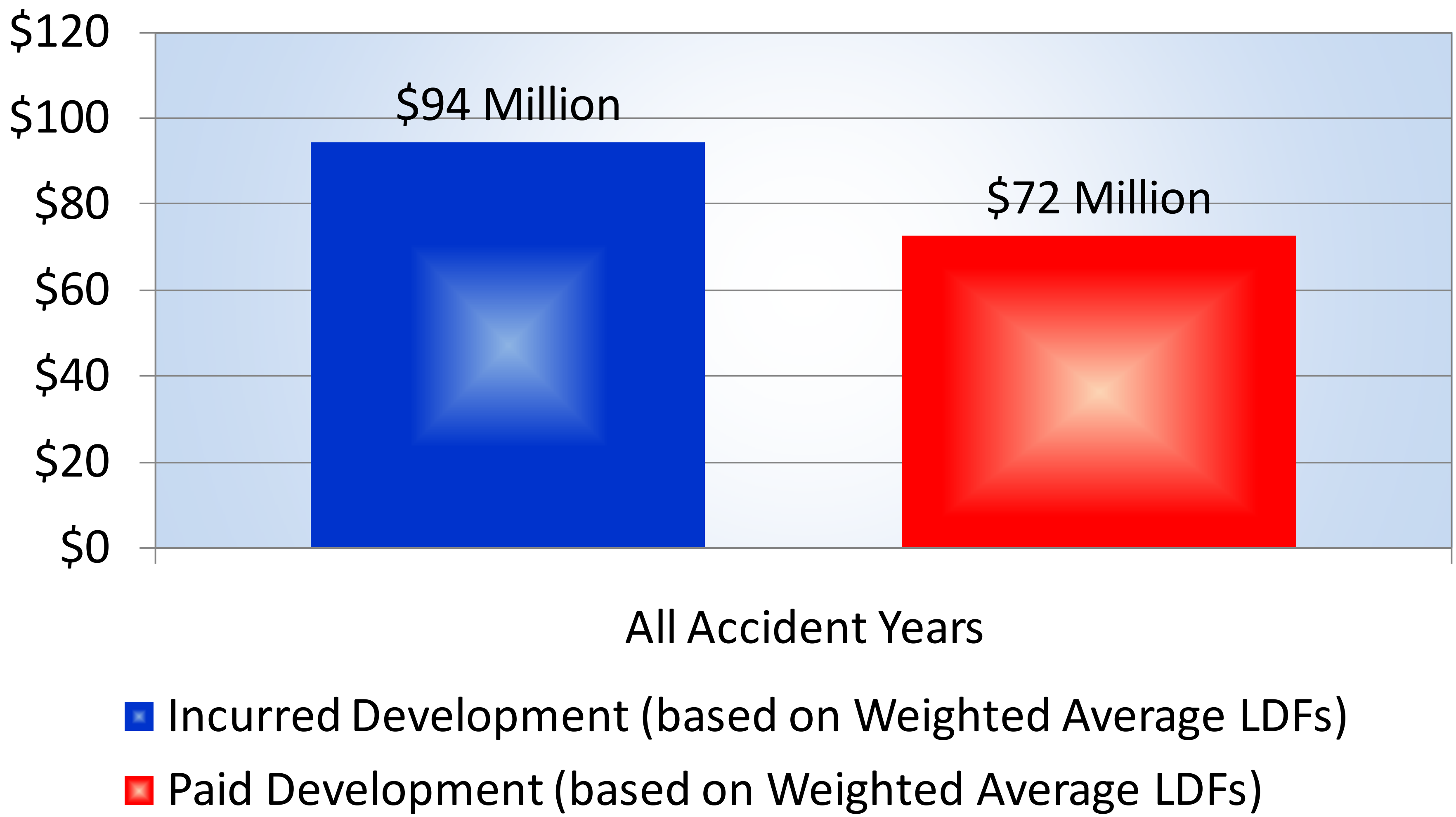
- Developed by Drs. Gerhard Quarg and Thomas Mack
- Originally published in a German journal in 2004
- Since reprinted in *Variance* (Fall 2008)
- Seeks to resolve the differences that arise between the standard paid and incurred chain ladder indications
  - MCL provides separate indications for paid and incurred, but they are much closer to one another
- Standard chain ladder methods ignore the correlation between paid losses and incurred losses

Munich Chain Ladder Example

- Drawn from actual insurance company data
  - Certain information altered to maintain confidentiality
- Commercial auto liability

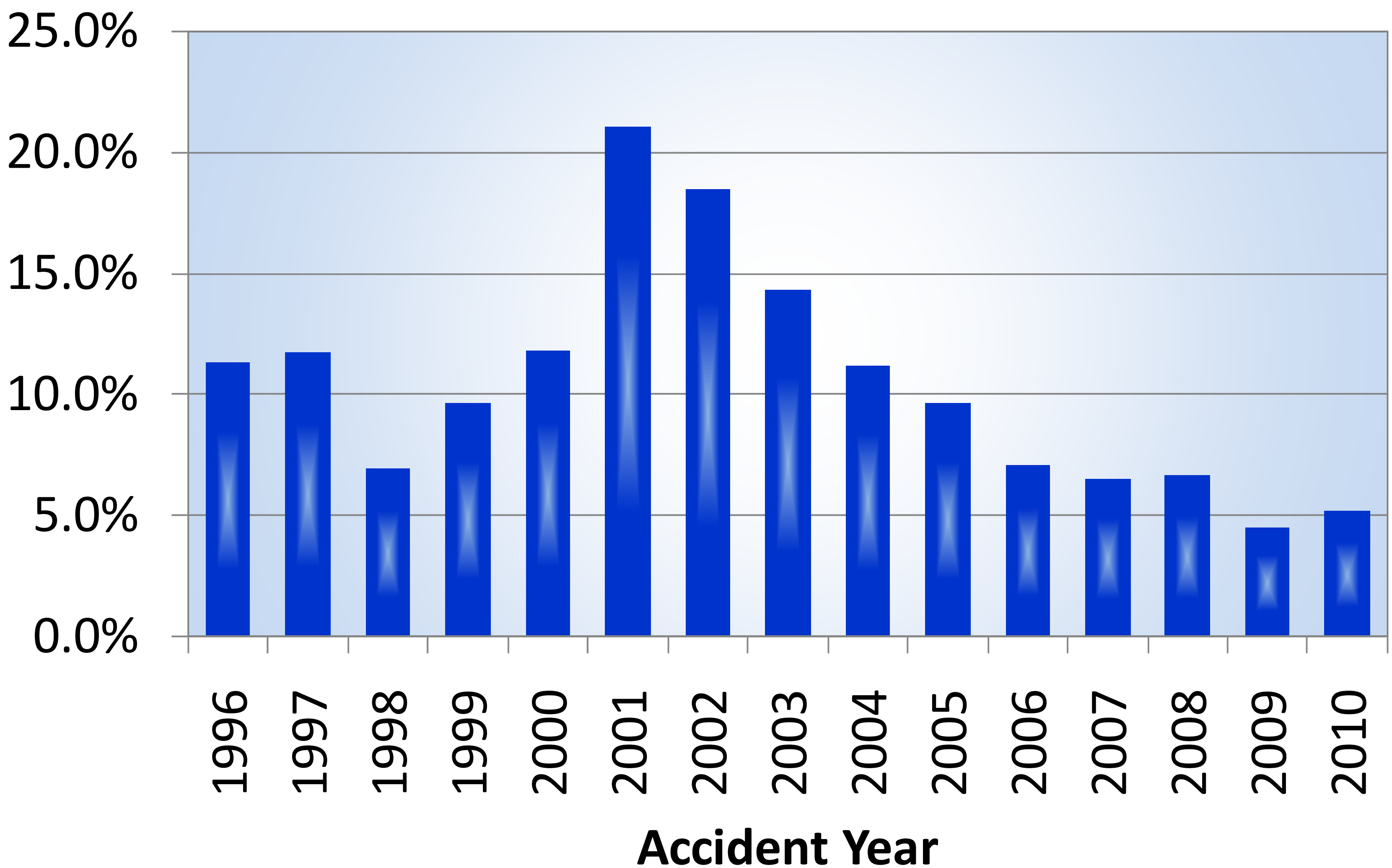
Indicated Unpaid Loss

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**



Paid-to-Incurred Ratios
at 6 Months of Development

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**

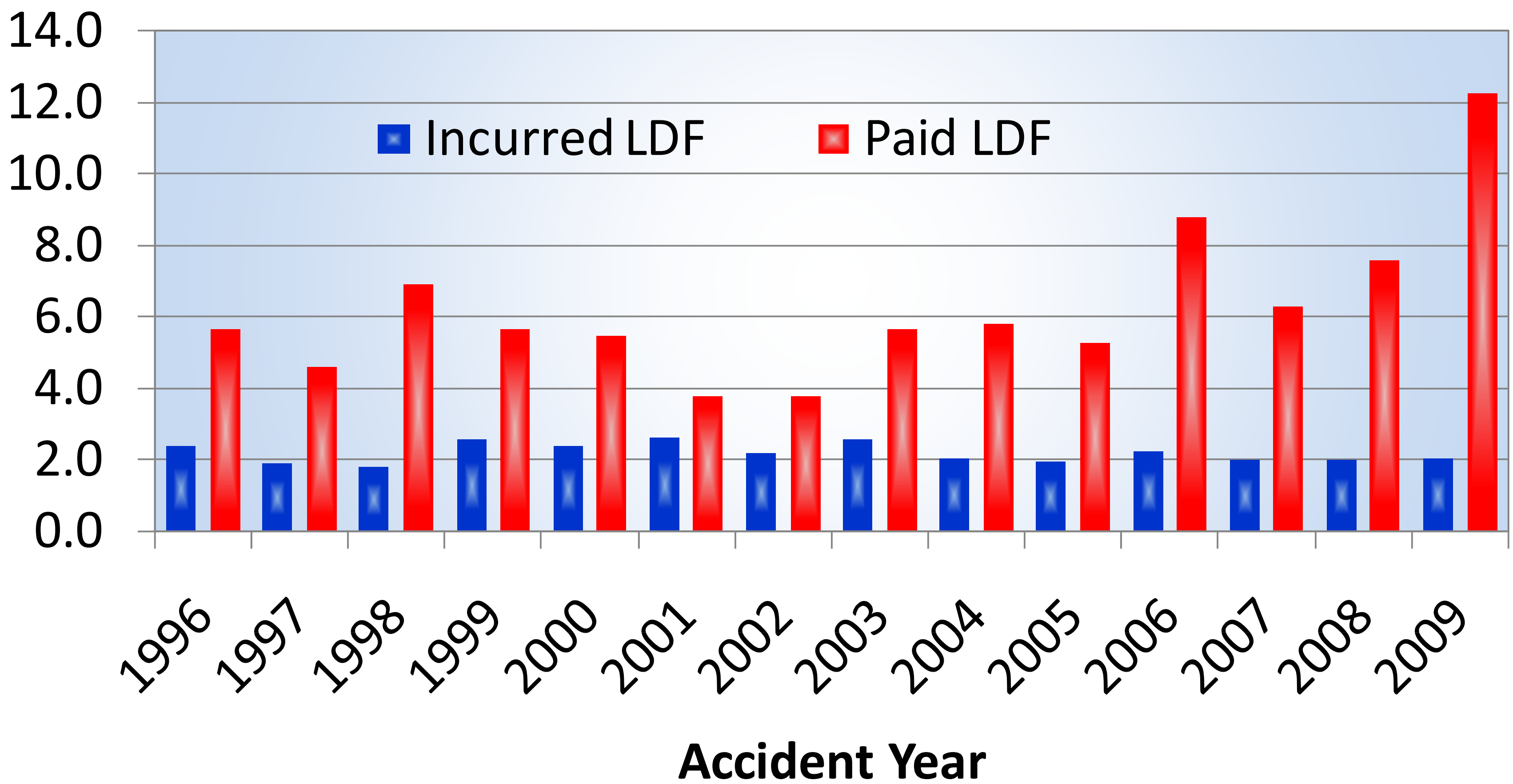


Possible Explanations

- Decrease in frequency
  - Recent claims on average more severe
  - May be causing slowdown in payment pattern
- Slowdown in payment pattern
  - Primarily driven by fewer small claims
  - Other claims may be closing more slowly too
- Case reserve strengthening
  - Not to be confused with change in average case reserve due to changing characteristics of open claims

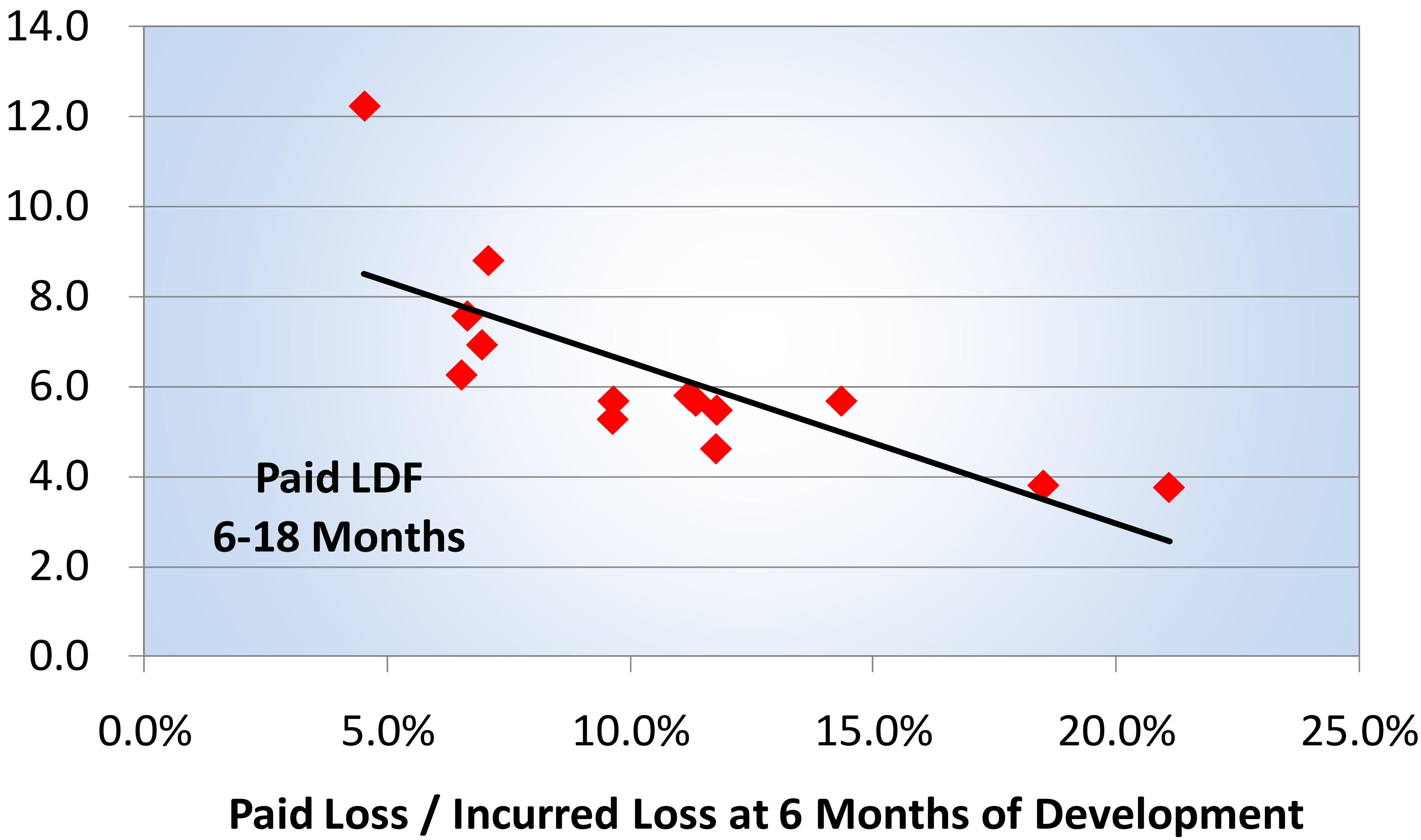
Incremental Loss Development Factors 6-18 Months of Development

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**

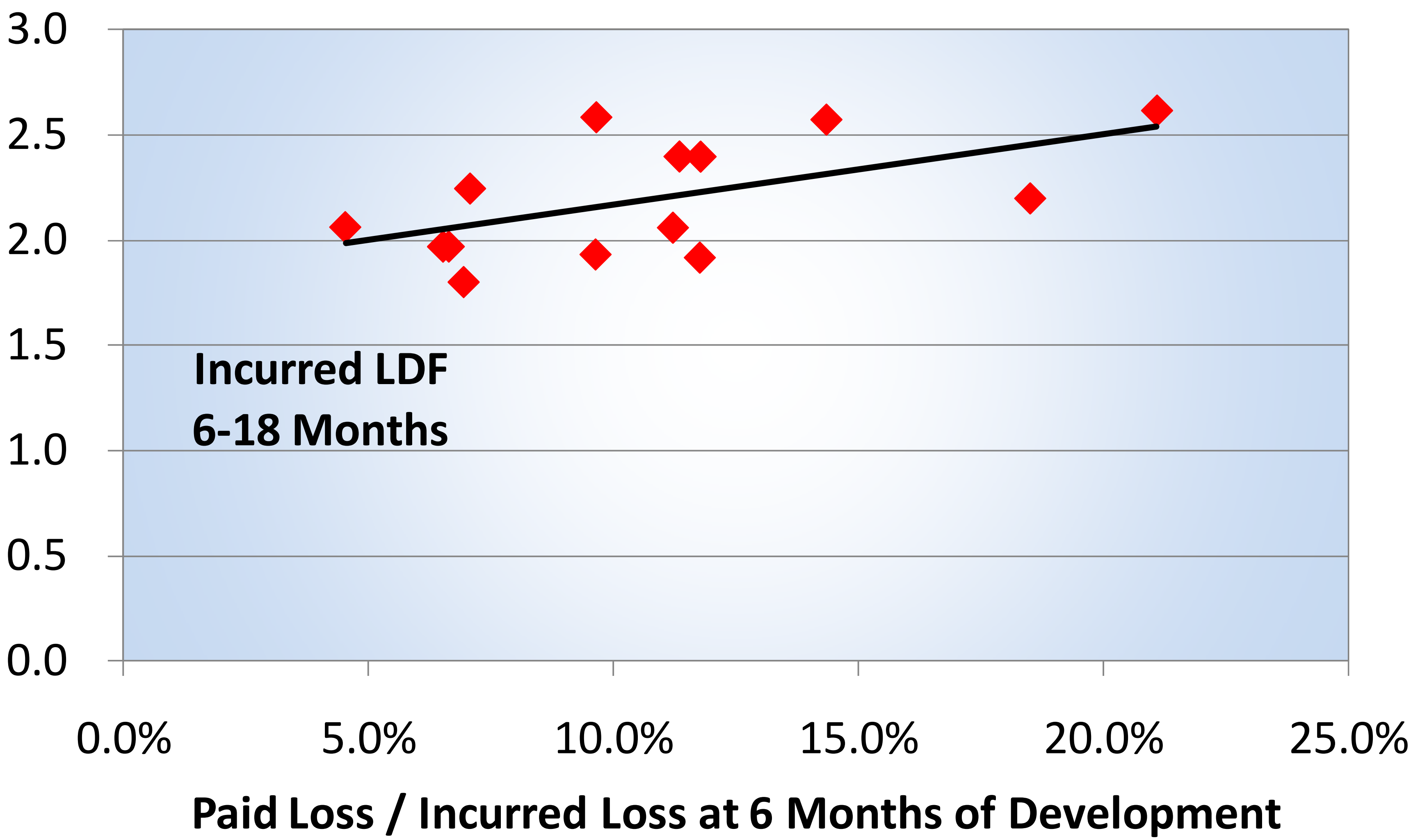


Paid LDFs vs. Paid-to-Incurred Ratio

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**



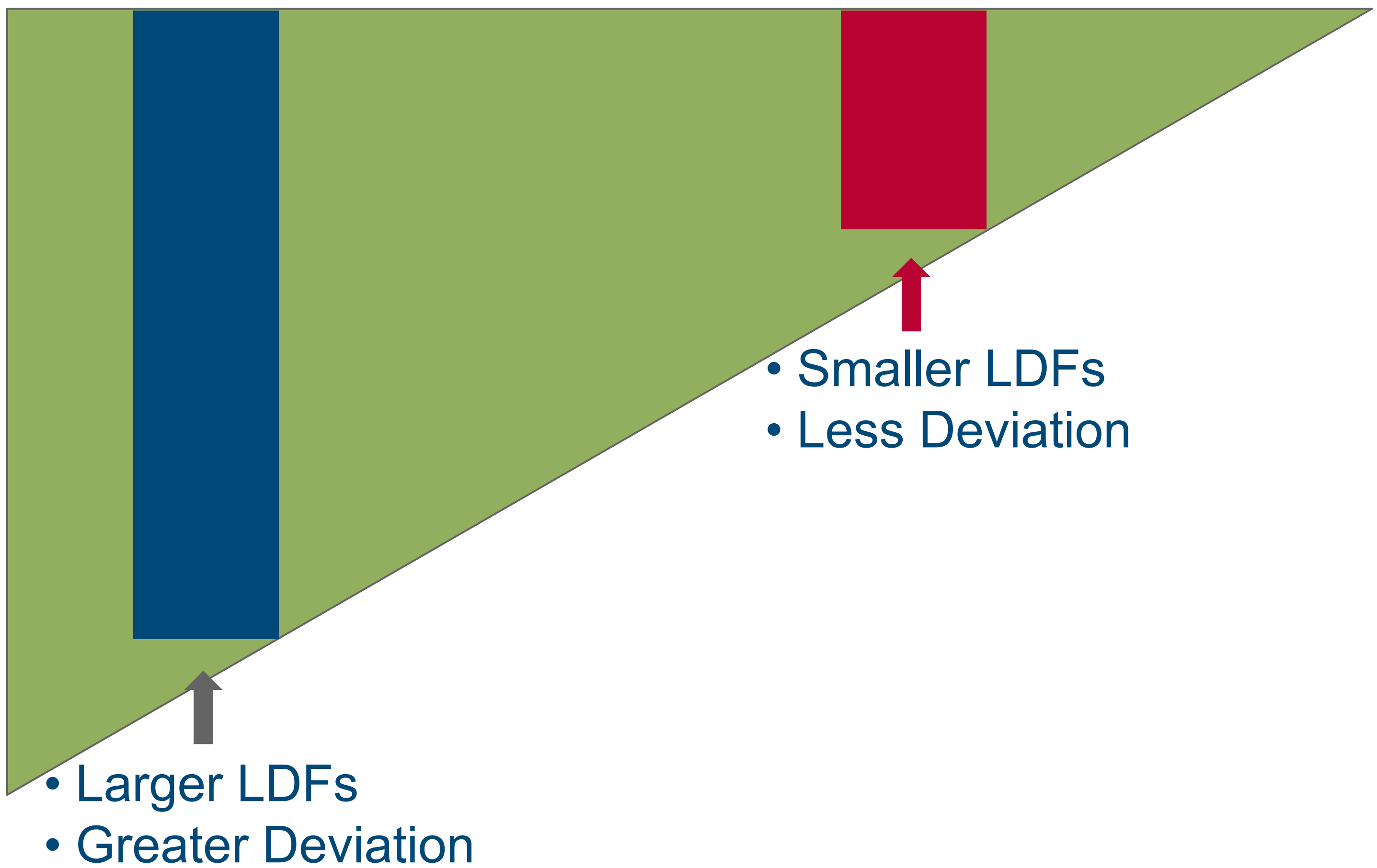
Incurred LDFs vs. Paid-to-Incurred Ratio



Munich Chain Ladder Method

- Reflects the relationship between paid-to-incurred ratios and subsequent development
  - Standard chain ladder methods magnify an unusual paid-to-incurred ratio in a given accident year (leverage effect)
  - Paid-to-incurred ratio should converge to 1.00 in each accident year if the chain ladder methods are to be consistent
- In doing so, considers all development periods as a whole

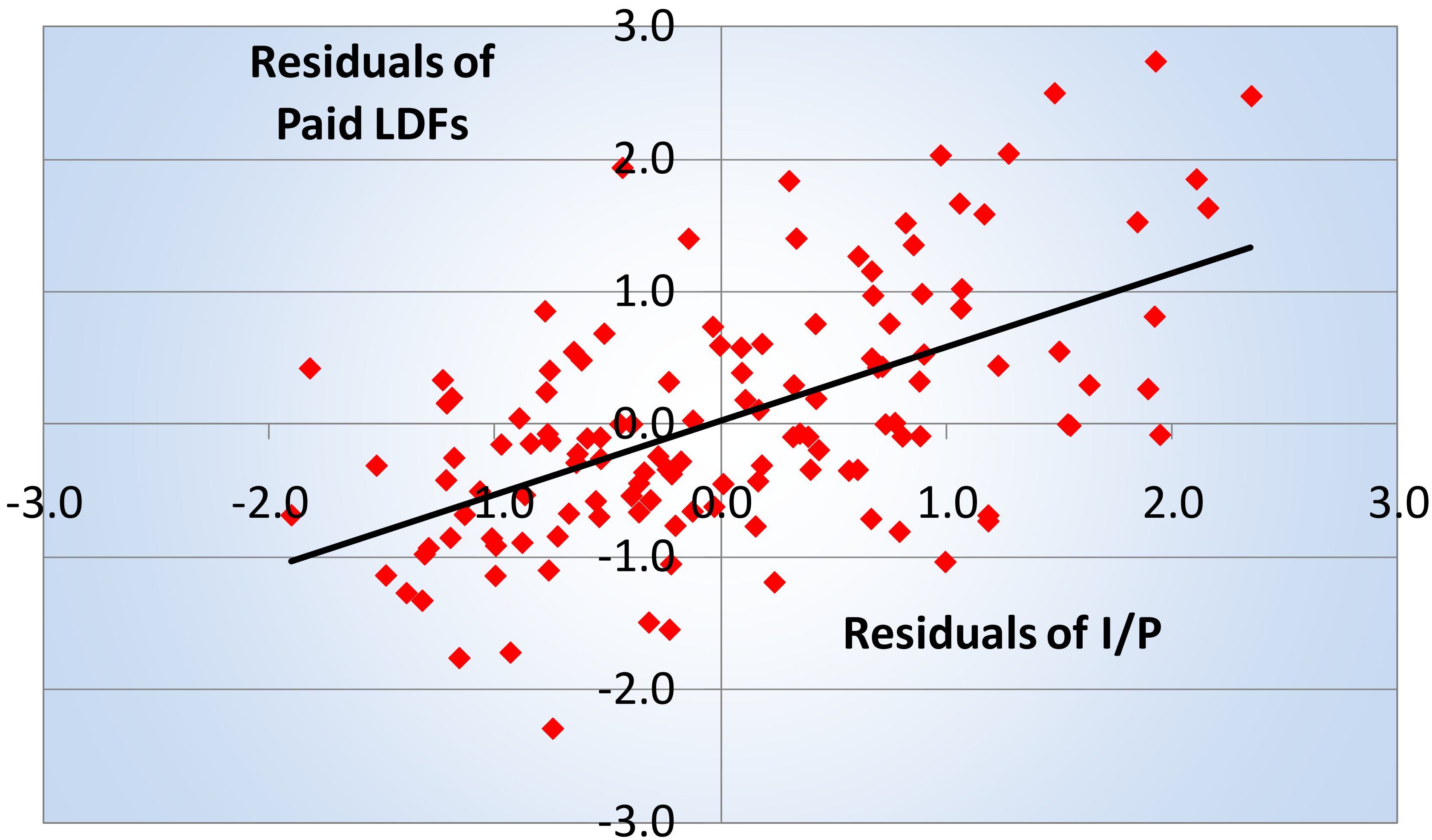
LDF Differences by Development Period



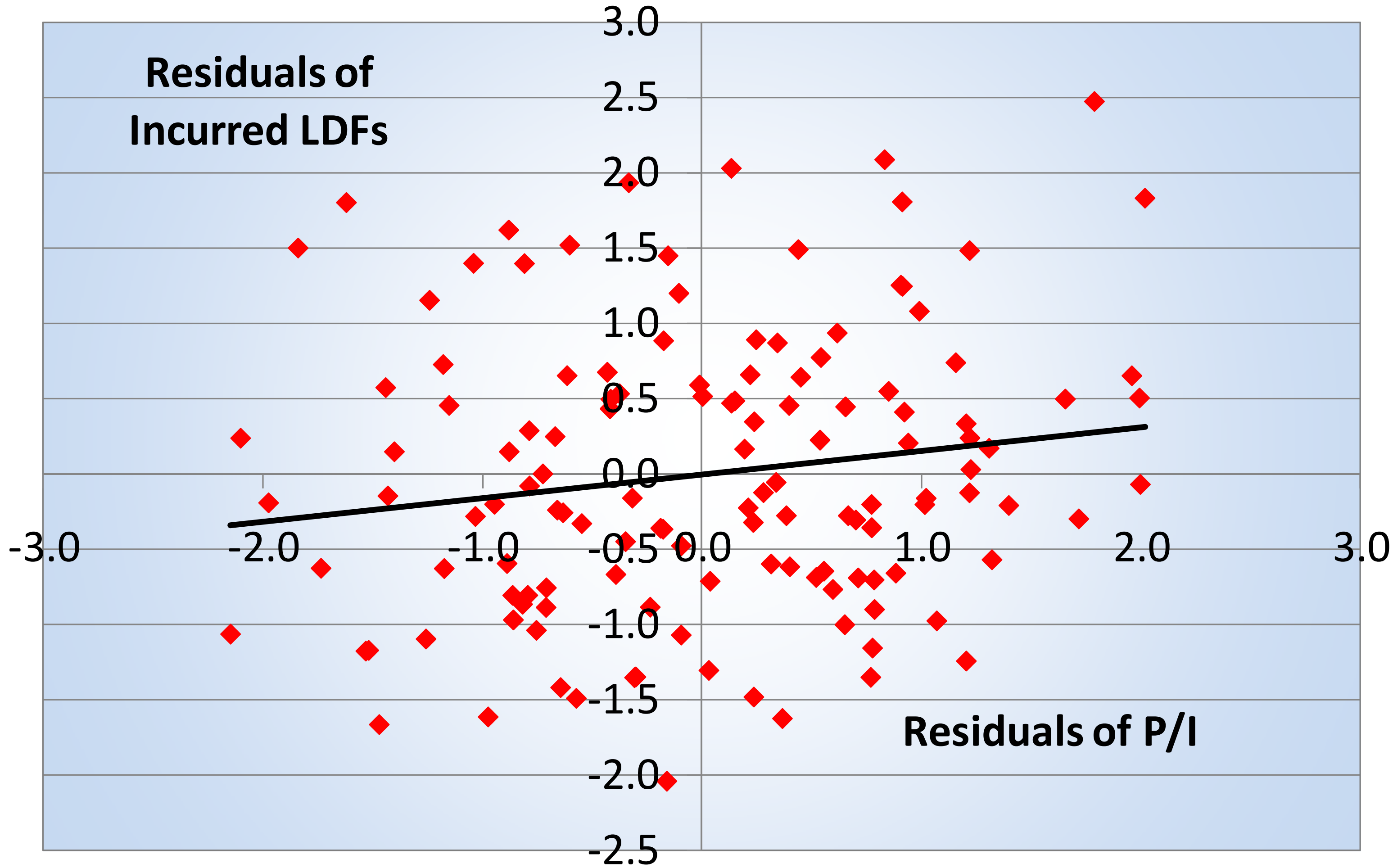
Adjustment for LDF Differences

- Residual = $\dfrac{\text{LDF - Wtd Avg LDF}}{\text{Std Deviation of LDFs}}$
- Assumption: other LDF differences due only to volatility
  - i.e., residuals are independent and identically distributed
- Allows use of all LDFs at once
- Method also considers residuals of paid-to-incurred and incurred-to-paid ratios

Paid Residual Plot

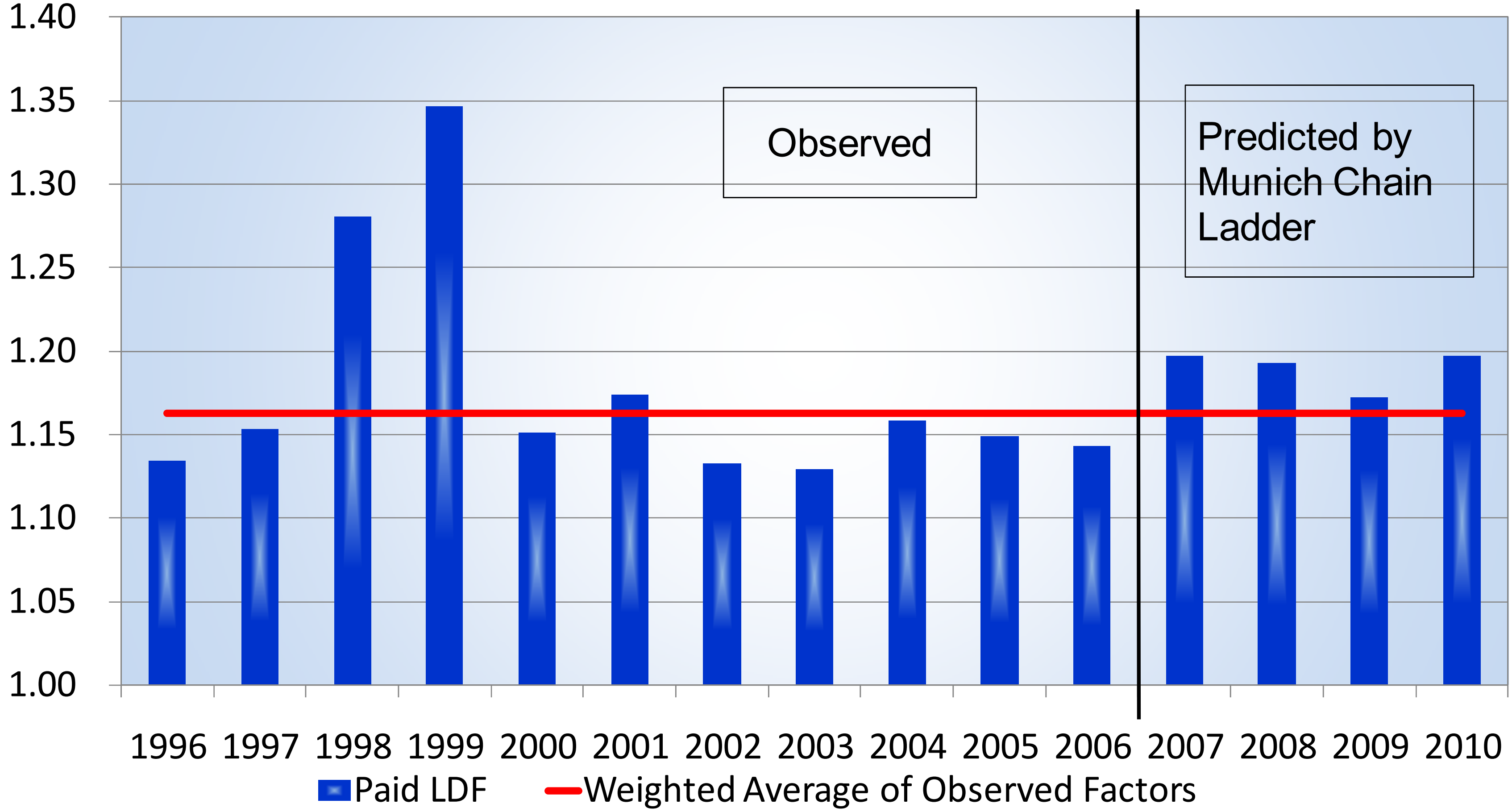


Incurred Residual Plot



Paid LDFs:
48-60 Months of Development

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**



Munich Chain Ladder – The Steps
Incurred Method

- Step 1: LDFs and Ratios
  - Incurred development factors and paid-to-incurred ratios
- Step 2: Weighted Averages and Standard Deviations
  - By development period, for each item in Step 1
- Step 3: Residuals
  - Now, data from different development periods has been standardized and can be grouped together
- Step 4: Conduct Linear Regression
  - Regress residuals of incurred LDFs against residuals of P/I ratios

Munich Chain Ladder – The Steps
Incurred Method (continued)

- Step 5: Calculate Indicated LDFs
  - Recursive process, based on regression parameters solved for in Step 4
  - LDFs will vary across accident years, in accordance with their paid-to-incurred ratios
- Step 6: Derive Ultimate Losses
  - Cumulate the indicated LDFs and multiply by the losses incurred-to-date

Munich Chain Ladder – Formulas
Incurred Method

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**

- Step 1: LDFs and Ratios
- Step 2: Weighted Averages and Standard Deviations
  - Weighted Squared Deviation of P/I Ratio (i,s) = Inc Loss (i,s) * (P/I Ratio (i,s) – Wgtd Avg P/I Ratio (s)) ^{2}
  - Weighted Standard Deviation of P/I Ratio (s) = Sqrt ((1/(n-s)) * $\sum_{i=1}^{n-s+1}$ Weighted Squared Deviation of P/I Ratio (i,s))
  - Weighted Squared Deviation of Incurred LDF (i,s) = Inc Loss (i,s) * (Inc LDF (i,s) – Wgtd Avg Inc LDF(s)) ^{2}
  - Weighted Standard Deviation of Incurred LDF (s) = Sqrt ((1/(n-s-1)) * $\sum_{i=1}^{n-s}$ Weighted Squared Deviation of Incurred LDF (i,s))
- Step 3: Residuals
  - Scaled Residual of P/I Ratio from Weighted Avg (i,s) = (P/I Ratio (i,s) – Wgtd Avg P/I Ratio (s)) * Sqrt (Inc Loss (i,s)) / Weighted Standard Deviation of P/I Ratio (s)
  - Scaled Residual of Incurred LDF from Weighted Avg (i,s) = (Inc LDF (i,s) – Wgtd Avg Inc LDF (s)) * Sqrt (Inc Loss (i,s)) / Weighted Standard Deviation of Incurred LDF (s)

Munich Chain Ladder – Formulas
Incurred Method (continued)

Step 4: Conduct Linear Regression

- Cross Product of Residuals of Incurred LDFs with Residuals of P/I Ratios =
  $\sum_{i=1}^{n} \sum_{s=1}^{m} \text{Scaled Residual of Incurred LDF from Weighted Avg } (i,s) * \text{Scaled Residual of } P/I \text{ Ratio from Weighted Avg } (i,s)$
- Cross Product of Residuals of P/I Ratios with Themselves =
  $\sum_{i=1}^{n} \sum_{s=1}^{m} (\text{Scaled Residual of } P/I \text{ Ratio from Weighted Avg } (i,s))^2$
- Slope of Regression Line of Incurred Development Against P/I Ratio (s) = (Cross Product of Residuals of Incurred LDFs with Residuals of P/I Ratios) / (Cross Product of Residuals of P/I Ratios with Themselves) * (Weighted Standard Deviation of Incurred LDFs (s)) / (Weighted Standard Deviation of P/I Ratios (s))

Step 5: Calculate Indicated LDFs

- Predicted Age-to-Age Incurred LDF (i,s) = Wgtd Avg Incd LDF (s) + Slope of Regression Line of Incurred Development Against P/I Ratio (s) * (Predicted P/I Ratio (i,s) – Wgtd Avg P/I Ratio (s))
- Predicted P/I Ratio (i,s) = Predicted P/I Ratio (i,s-1) * (Predicted Age-to-Age Paid LDF (i,s)) / (Predicted Age-to-Age Incurred LDF (i,s))
- Predicted Age-to-Age Paid LDF (i,s) – comes from Munich Chain Ladder Paid Method
- The above formulas are recursive

Step 6: Derive Ultimate Losses

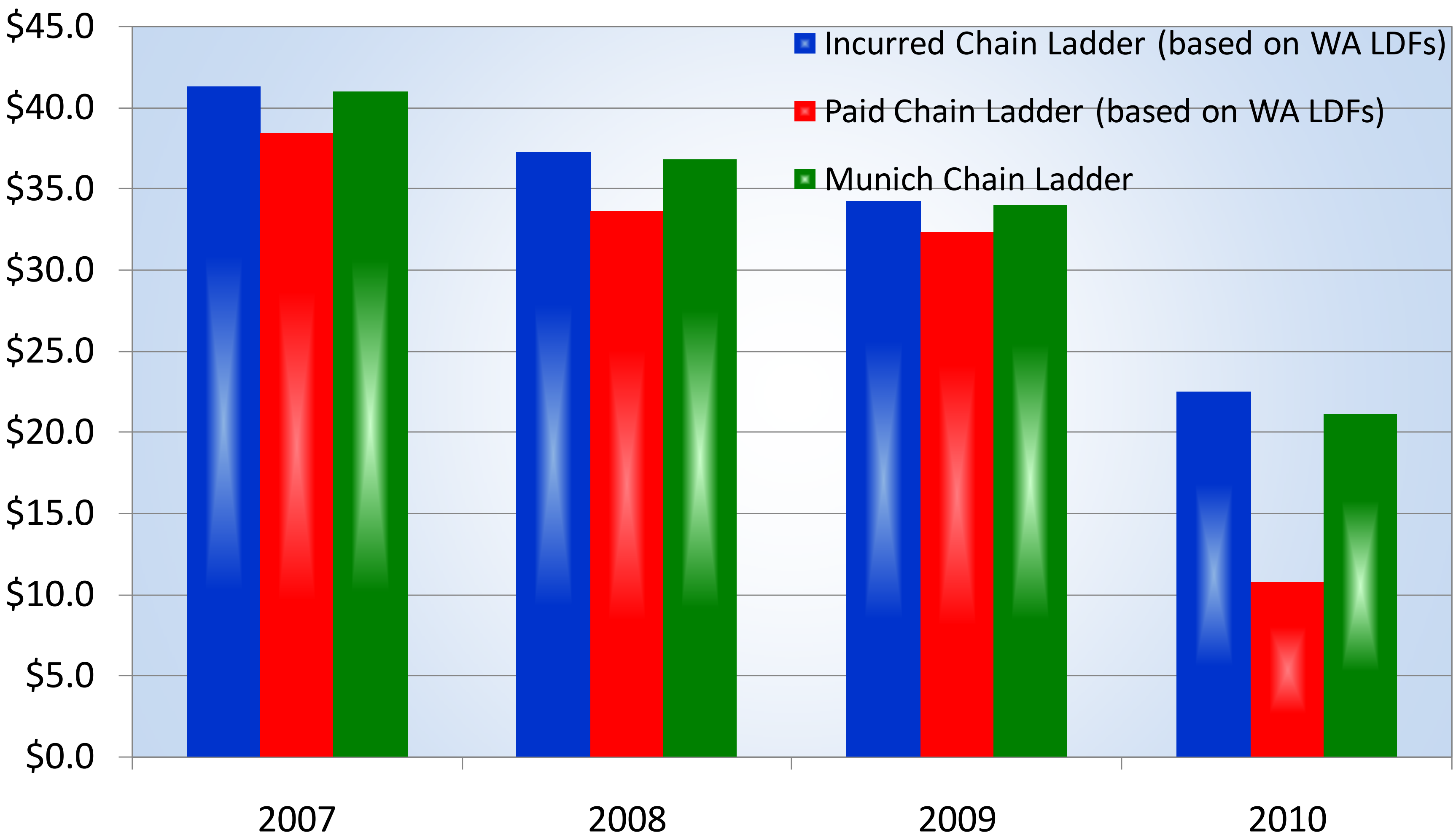
- Calculate cumulative incurred LDFs and multiply by incurred-to-date losses

Munich Chain Ladder – The Steps
Paid Method

- Step 1: LDFs and Ratios
  - *Paid* development factors and *incurred-to-paid* ratios
- Steps 2 - 6:
  - Same as Incurred Method, but using the data listed above

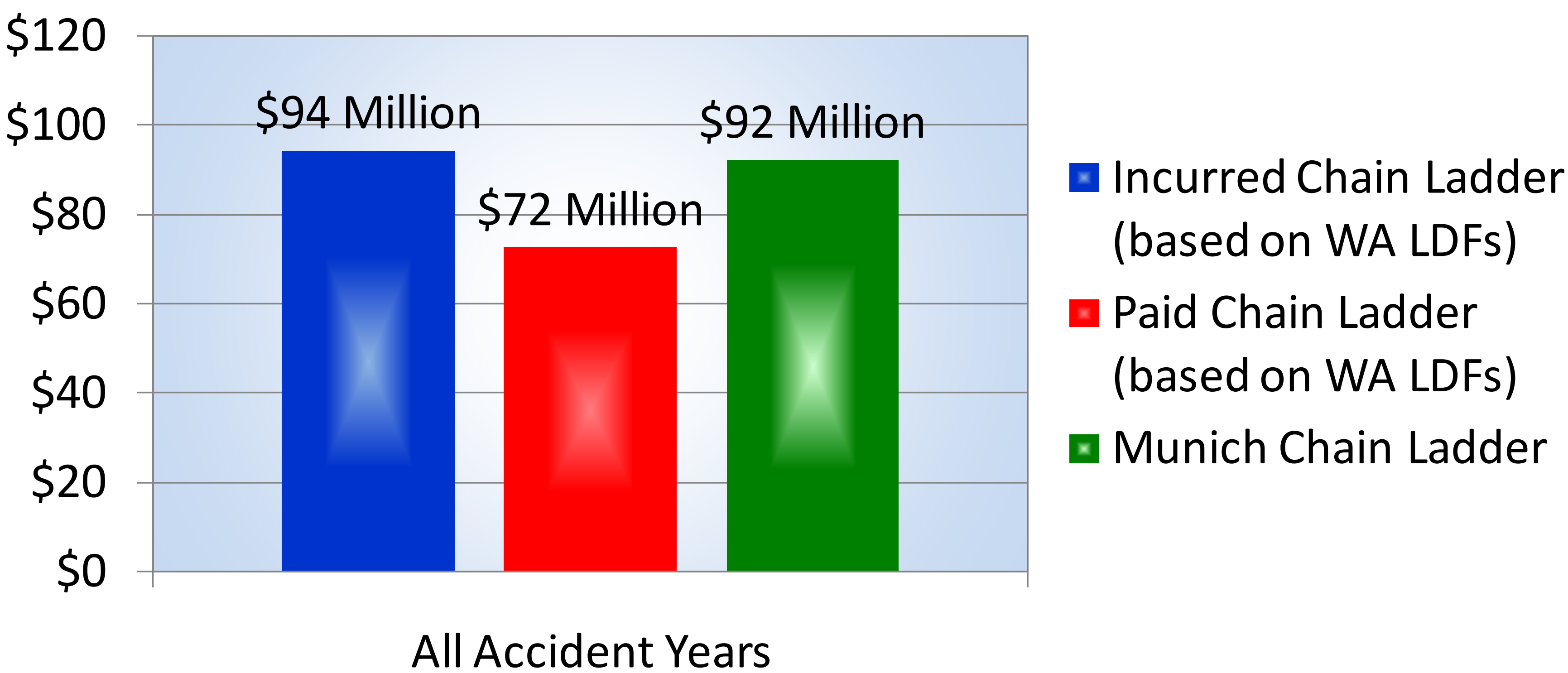
Indicated Ultimate Loss by Accident Year (in $Millions)

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**



Indicated Unpaid Loss
($ Millions)

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method
4. **Munich Chain Ladder method**



Advantages

- Uses paid and incurred information simultaneously
- Possibly more stable than other adjusted chain ladder methods (e.g., Berquist-Sherman, Brosius)
- Has a sound theoretical basis, yet is intuitive and understandable

Disadvantages

- More complex to implement than other reserving methods
- May not respond well to small data sets
- Parameters may need smoothing and extrapolation, especially when run-off extends beyond the most recent development period

Other Points

- Can also use for claim counts
  - e.g., closed with indemnity and incurred
- Two indications may still be derived
  - i.e., "paid" and "incurred" Munich Chain Ladder
- May not perform well when the paid-to-incurred ratio extends outside its of historical range

References

- Quarg, G., and T. Mack, "Munich Chain Ladder," *Variance* Vol. 2, 2008, pp. 266-299