Casualty Loss Reserve Seminar
Las Vegas, NV

The Mack/Murphy Model Framework: From Theory to Practice

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Agenda

- Background of the Mack/Murphy Chain Ladder Model framework
  - Common pitfalls as it is currently employed by practitioners today
- Introduction of the Chain Ladder Factor Model (CLFM) framework
  - CLFM point estimation theory
  - CLFM variance estimation theory
    - The Mack/Murphy Model is a special case of the CLFM
- Performance testing of Stochastic Methods
  - Scenario Generator results
- Questions and Discussion

Background of the Mack/Murphy Chain Ladder Model Framework

Chain Ladder: Method vs. Model

Method	Model
• Mathematical algorithm	• Mathematical description of the world
• Parameters are selected by the user	• “Best-Fitted” Parameters (i.e. MLE)
• Selections based on user judgment	• Selections can be tested based on statistical theory
• Chain Ladder algorithm	• Mack/Murphy models, CLFM

Chain Ladder "Method"

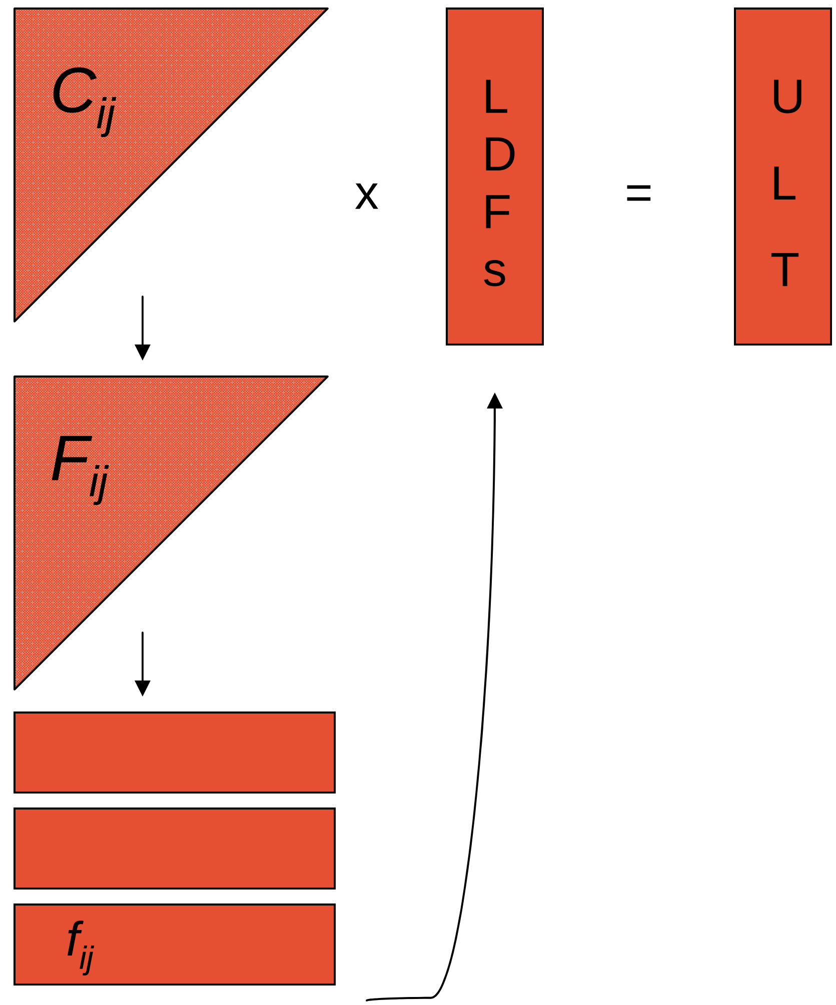
Loss Triangle

Triangle of Report-to-Report (RTR) Factors

Various Averages

Benchmark RTRs

***Selected* RTRs**



Chain Ladder "Model"

A chain ladder model superimposes a statistical framework on the traditional chain ladder algorithm with two requirements:

- Consistent with the standard chain ladder method
  - Must produce identical reserve estimates
- Chain Ladder parameter selections can be tested
  - Underlying statistical framework allows us to validate our judgmental actuarial assumptions (our link ratio "picks")

Sources of uncertainty in unpaid claim liability estimate from the model's point of view

C | $E(\hat{C})$ | $\hat{C}$

Total Risk

$E\{(C-E(C))^2\}$	$E\{(\hat{C}-E(\hat{C}))^2\}$	$(E(C)-E(\hat{C}))^2$
Process Risk	**Parameter Risk**	**Model Risk**
Even if selected development factor is correct, the actual outcome might be different from the expected one. Measured by Model	Are the selected development factors appropriate, given the data? Measured by Model	Is the Chain Ladder Method appropriate? Not measured by Model
Decreases with volume	Constant with volume	

Actual Outcome | **True Expected Outcome** | **Model Estimate of Expected Outcome**

Measure of risk is the mean square error (mse)

$$mse(\hat{C}) = E_{\hat{C}}(E((\hat{C}-C)^2 \mid \hat{C})) = E_C((C-E(C))^2 \mid \hat{C})^2 + E_{\hat{C}}(\hat{C}-E(\hat{C}) \mid \hat{C})^2 + (E(C)-E(\hat{C}))^2 =$$

$$Var(C) + Var(\hat{C}) + bias^2 = \Pr occesRisk + ParameterRisk + ModelRisk$$

- Mse measures the mean distance of the estimates from the unpaid claim liability amount
- Process risk captures the variability of the unpaid claim liabilities from their (unknown) mean value
- Parameter risk captures the variability of the estimates from the expected value of these estimates
  - Measures only dispersion, not bias
- Model risk captures the bias of the model when the expected value of the estimates of the unpaid claim liability is different from the actual (unknown) mean claim liability amount
  - Measures only bias, not dispersion

Mack/Murphy model is biased when the selected LDF are other than the straight or volume weighted averages

$$mse(\hat{C}) = Var(C) + Var(\hat{C}) + bias^2 = \Pr occes\ Risk + Parameter\ \ Risk + Model\ \ Risk$$

- If the expected value is based on selected link ratios other than the straight/volume weighted average one then the Mack/Murphy model is biased
- The resulting Mack/Murphy stochastic projections will be biased low since the underlying variance calculations do not include any correction for that bias (i.e. model risk)
  - Only Process and Parameter risk are relevant in the Mack/Murphy framework
  - The deterministic projections may be biased high, low or not at all (with low probability) depending on their expected value versus the actuary's belief

Risk can have different names

- Variance or standard error
  - Where standard error is the square root of the mse
- Value at Risk (VaR) represents a percentile
- Tail Value at Risk (TVar) represents the expected value of tail losses
- Coefficient of Variation measures relative risk $CV(X) = \sigma_X / \mu_X$
- "Scaling" principle, when CV(X) from one stochastic method is applied to the μ_Y of another method to calculate its standard deviation, i.e. $\sigma_Y = \sigma_X / \mu_X \; \mu_Y$

Mack/Murphy Model underlying structure

- The easiest way to think of this structure is to picture a linear regression across two consecutive columns in a loss triangle

$$C_{i,k+1} = f_k C_{i,k} + \sigma_k \varepsilon_{i,k} C_{i,k}^{a/2}, \quad where\, a = 0, 1 or\, 2$$

- i corresponds to accident year (i.e. row) and k corresponds to development year (i.e. column) of a triangle
- Alpha is assumed to be independent of k
- The random component of the error term is assumed to be independent and identically distributed (i.i.d.) around zero, with a variance of one
- The model is heteroscedastic since the variance of the error term is proportional to $\sigma_k^2 C_{i,k}^a$

Maximum likelihood solution for the Mack/Murphy model

$$\begin{cases} \hat{f}_k(\alpha) = \sum_{i=1}^{n-k} \dfrac{C_{i,k}^{1-\alpha}}{\sum_{j=1}^{n-k} C_{j,k}^{2-\alpha_k}} C_{i+1,k} = \sum_{i=1}^{n-k} w_{i,k}^{\alpha} \cdot F_{i,k}, \\ w_{i,k}^{\alpha} := \dfrac{C_{i,k}^{2-\alpha}}{\sum_{j=1}^{n-k} C_{j,k}^{2-\alpha}}, \quad F_{i,k} := \dfrac{C_{i,k+1}}{C_{i,k}} \end{cases}$$

Where $\hat{f}_k(a)$ represents the best linear unbiased estimator (BLUE) of the link ratio f_k from age k to age k+1

- What is the implied BLUE link ratio?
  - For alpha = 0 is the slope of a linear regression through the origin
  - For alpha = 1 is the all year volume weighted: $\hat{f}_k(a) = \sum_{i=1}^{n-k} C_{i,k+1} \Big/ \sum_{j=1}^{n-k} C_{j,k}$
  - For alpha = 2 is the all year simple average: $\hat{f}_k(a) = \sum_{i=1}^{n-k} \dfrac{C_{i,k+1}}{C_{i,k}} \Big/ (n-k)$

Resulting Mack/Murphy model Variance formulas

- There are three underlying assumptions of the model
  - (A1) $E(C_{i,k+1}|$ the Triangle$) = C_{i,k} f_k$
  - (A2) $Var(C_{i,k+1}|$ the Triangle$) = \sigma_k^2 C_{i,k}^a$
  - (A3) Accident years are independent

- The original Mack formulas were derived as a closed form solution
  - Mack formulas are impressive, yet daunting

- Both Murphy and subsequently Mack developed recursive variance formulas
  - Formulas are very close, differ by a cross-variance term in the Murphy model

What about if an actuary selects a link ratio other than the straight average or volume weighted one?

- Practitioners today are “scaling” the Mack/Murphy results when employing a model with different selected link ratios
  - The CV implied by the straight average and volume weighted Mack/Murphy models apply to reserves that have been calculated in different ways
    - The presenters are concerned that the “scaling” technique might significantly understate the mse estimate
- Two relevant questions easily come to mind:
  - Should actuaries employ link ratios other than the ones based on some type of averages of the empirical data?
  - Can we expand the Mack/Murphy model framework to consider other types of selected link ratios?

Actuaries can and should exercise judgment in the selection of the Chain Ladder link ratios

- According to ASOP No. 43 Section 3.6.2
  - "*The actuary should consider the reasonableness of the assumptions underlying each method or model used. Assumptions generally involve significant professional judgment as to the appropriateness of the methods and models used and the parameters underlying the application of such methods and models.*"

- Moreover, according to Jacqueline Friedland's "*Estimating Unpaid Claims using Basic Techniques*" in Chapter 7
  - "*When the credibility of the insurer's own historical experience is limited, there may be a need to supplement the insurer's own historical experience with certain benchmarks. One possible benchmark includes experience from similar lines with similar claims handling practices within the insurer.*"

Introduction of the Chain Ladder Factor Model (CLFM) Framework

CLFM Point Estimation Theory

CLFM Framework

- Simply an extension of the Mack/Murphy one

$$C_{i,k+1} = f_k C_{i,k} + \sigma_k \varepsilon_{i,k} C_{i,k}^{a_k/2}, \quad where\, a_k \in R$$

- Alpha is now dependent on k and is defined on the real line R
- We define the Link Ratio Function as the Maximum Likelihood solution

$$LR_k(\alpha) = \sum_{i=1}^{n-k} w_{i,k}^{\alpha} \cdot F_{i,k}$$

- where all factors are defined the same way as in slide 9
- In essence we can think of the CLFM as a family of models identified by an index alpha defined in the real line
  - The Mack/Murphy model is a special case with a in {0,1,2}

Asymptotic Properties of the Link Ratio Function

- When $a \to +\infty$ the BLUE of a link ratio approaches the link ratio experienced by the accident year with the smallest value of loss at the beginning of the development period

- When $a \to -\infty$ the BLUE of a link ratio approaches the link ratio experienced by the accident year with the largest value of loss at the beginning of the development period

- The alpha function is defined in the whole real line but not all possible link ratios correspond to an alpha value

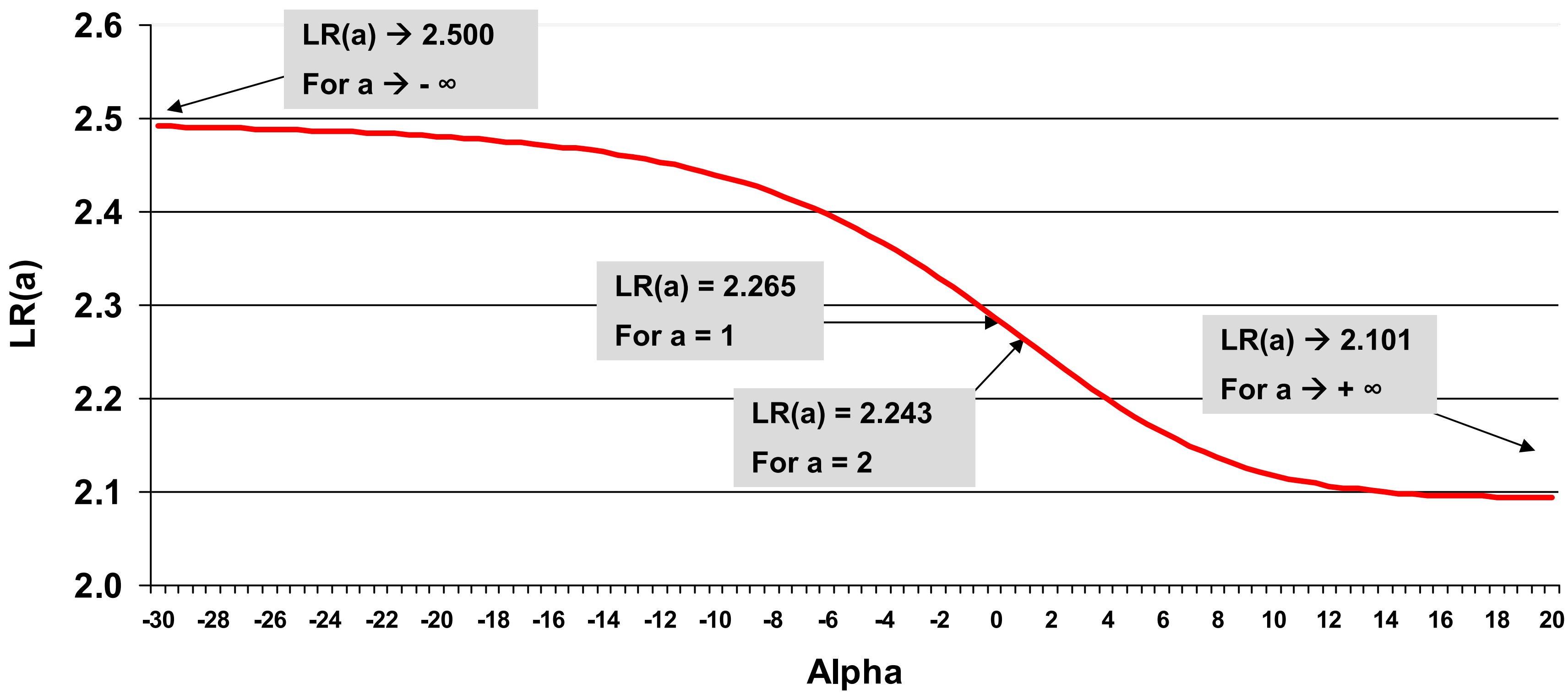
Illustrative Example #1

Table 1: Development Period Losses

$C_{i,j}$	j=1	j=2	$F_{i,1}$
j=1	280	680	2.429
j=2	250	550	2.200
j=3	300	750	2.500
j=4	235	466	1.983
j=5	207	435	2.101
Volume Weighted Avg.			2.265
Simple Avg.			2.243

- The Link Ratio Function is asymptotic to the y = 2.500 and y = 2.101 lines
- $LR_k(1.000) = 2.265$
- $LR_k(2.000) = 2.243$
- Given a link ratio, the alpha can be calculated through a numerical approximation technique (i.e. Newton-Rapshon)

Link Ratio Function for Table 1

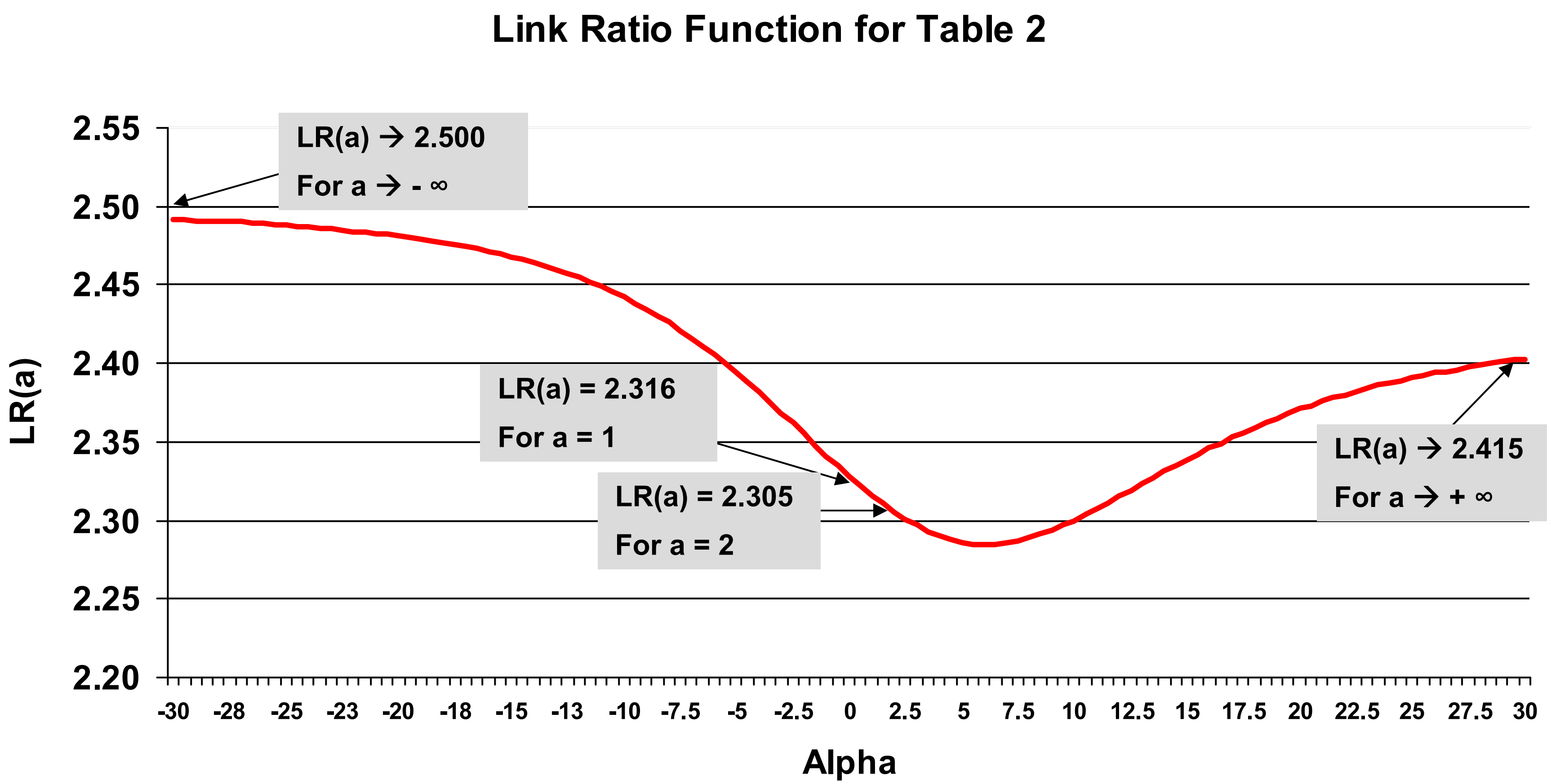


Illustrative Example #2

Table 2: Development Period Losses

$C_{i,j}$	j=1	j=2	$F_{i,1}$
j=1	280	680	2.429
j=2	250	550	2.200
j=3	300	750	2.500
j=4	235	466	1.983
j=5	207	500	2.415
	Volume Weighted Avg.		2.316
	Simple Avg.		2.305

- The Link Ratio Function is asymptotic to the y = 2.500 and y = 2.415 lines
- $LR_k(1.000) = 2.316$
- $LR_k(2.000) = 2.305$
- The Link Ratio Function is not necessarily a monotonic function
- The image of the link ratio function is not the entire real line, in fact the min link ratio is “off-the-chart” in this example


Link Ratio Function for Table 2
LR(a) → 2.500
For a → - ∞
LR(a) = 2.316
For a = 1
LR(a) = 2.305
For a = 2
LR(a) → 2.415
For a → + ∞
2.55
2.50
2.45
2.40
2.35
2.30
2.25
2.20
LR(a)
-30 -28 -25 -23 -20 -18 -15 -13 -10 -7.5 -5 -2.5 0 2.5 5 7.5 10 12.5 15 17.5 20 22.5 25 27.5 30
Alpha

Bridge to the Variance concept

- Variance estimation between the Mack/Murphy and CLFM follow similar logic
- One exception relates to the estimation of the process risk
- The process risk $\Gamma(C)^2$ is a function of the $E\{Var(C_{i,k+1})\}$ or $E(\sigma_k^2 C_{i,k}^a)$
  - Calculations are difficult for any number other than 0 and 1
- Take it over Dan!

Introduction of the Chain Ladder Factor Model (CLFM) Framework

CLFM Variance Estimation Theory

Performance Testing of Stochastic Methods

Various Stochastic Results

- 10.5%
- 9.8%
- 13.3%

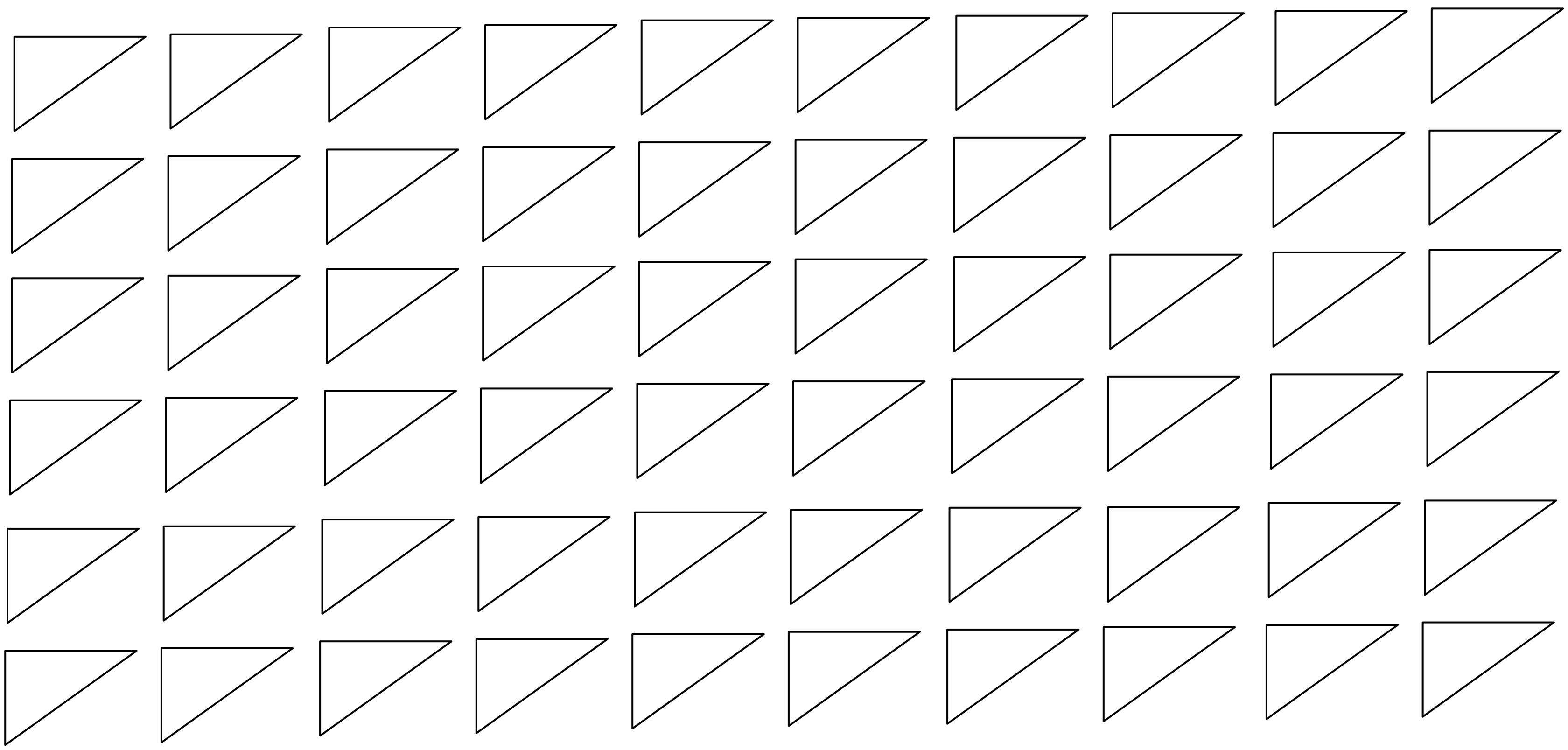
How do we differentiate which indication is the most appropriate?
How do we determine which model(s) are performing well?
When do models perform well and when do they perform poorly?

What We Measure

- Uncertainty in unpaid loss (& ALAE)
  - Uncertainty contains both process and parameter risk
  - Generally excludes model risk
    - — Uncertainty does <u>not</u> relate to uncertainty in estimates of ultimate loss (for this presentation at least)
- Uncertainty arises from a line of business' Risk Profile
  - What is risk profile?
    - — Concept generally describes underlying uncertainty of following:
      - – Frequency
      - – Severity
      - – Timing of paid, reported dollars and claim counts
      - – Various interactions of above components

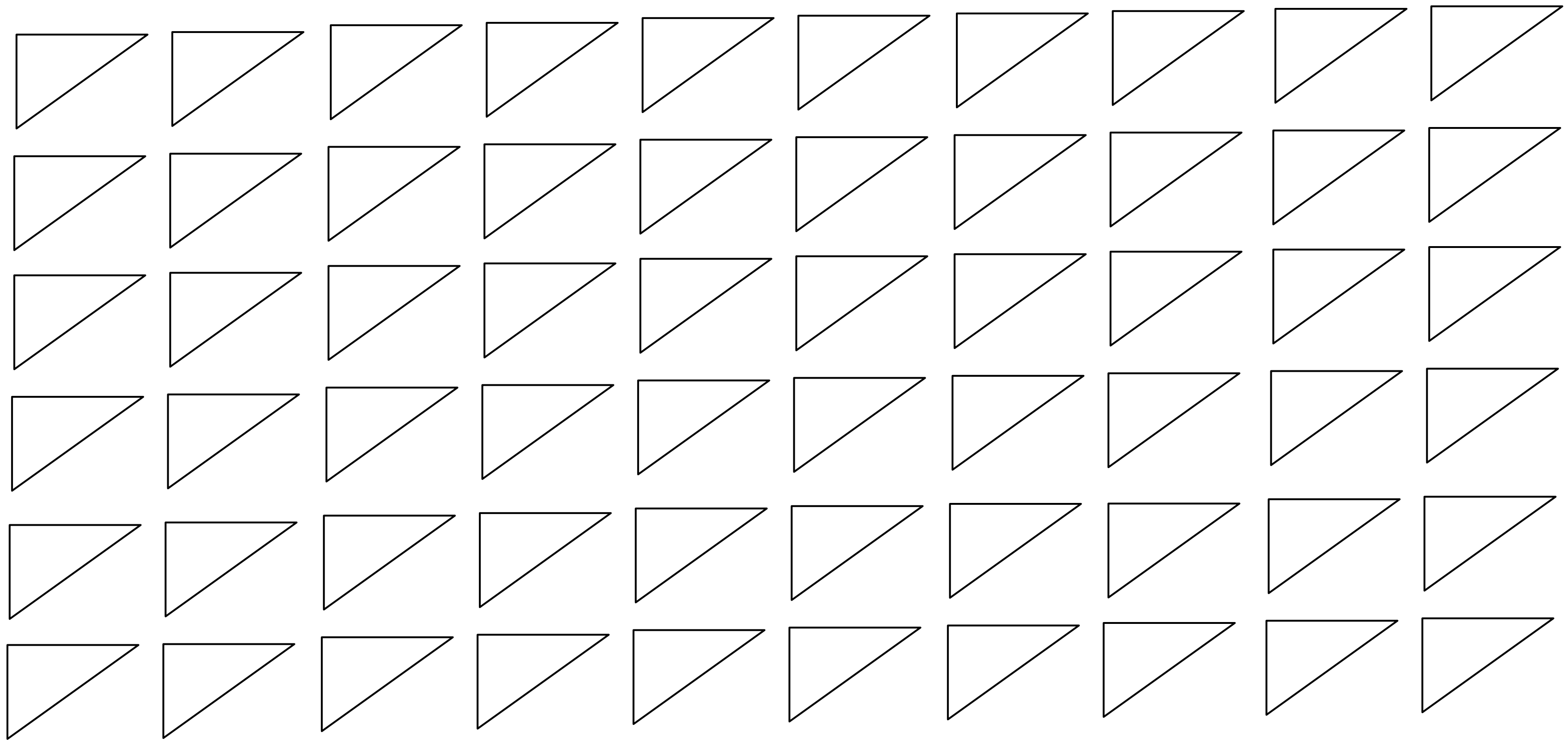
What We Measure

- The risk profile could manifest itself in a large number of ways
- Of these many potential manifestations, the unpaid losses are a measurable by-product



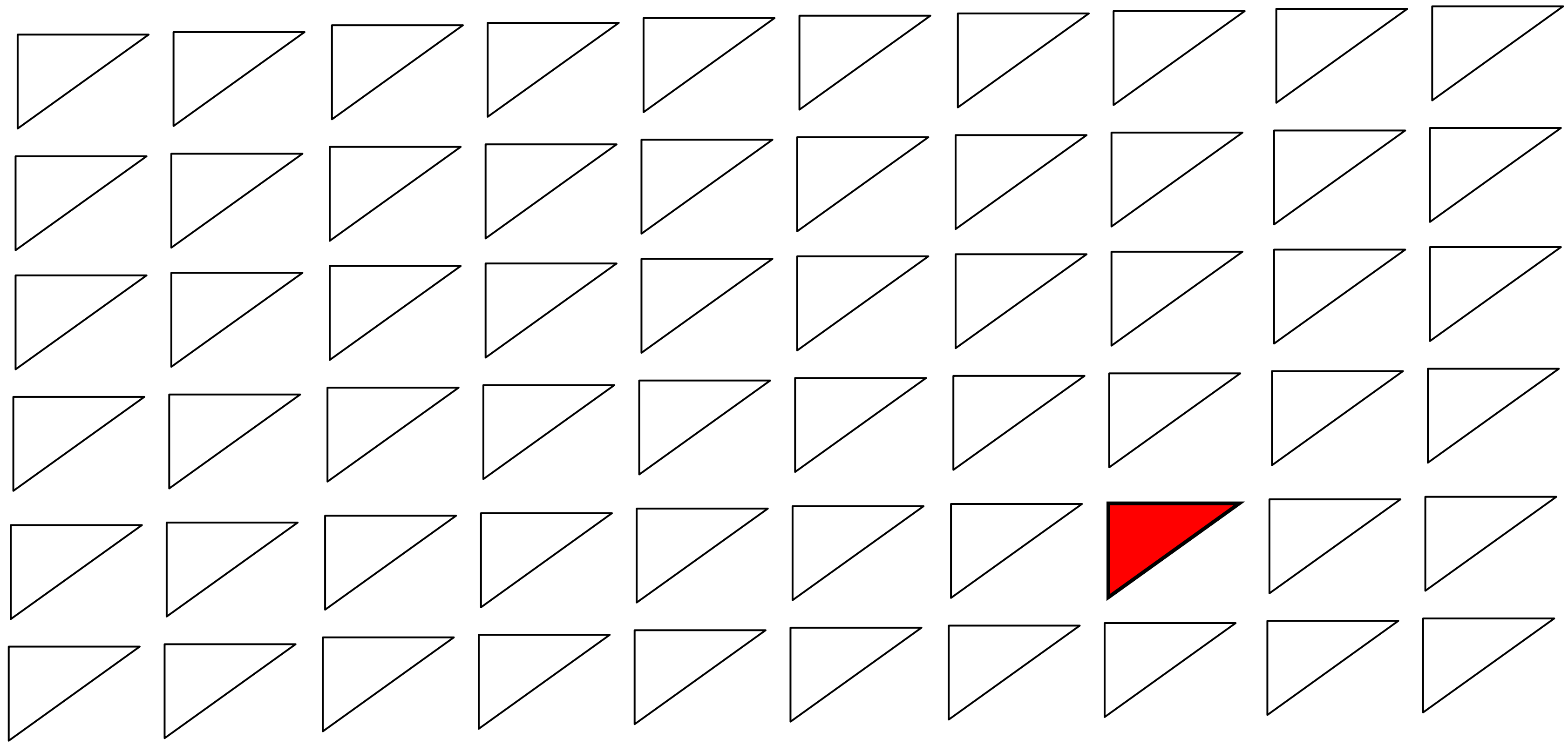
What We Measure

- However, out of all of the potential manifestations of reality…



What We Measure

- However, out of all of the potential manifestations of reality… we only get one single observation



How do we infer the probability distribution from a single observation? - Proposed Solutions

- Use whatever measurement approach desired
  - Bootstrap
  - Mack
  - CLFM
  - Practical
  - Other
- Brings us back to our original questions…
  - How do we differentiate which indication is the most appropriate?
  - How do we determine which model(s) are performing well?
  - When do models perform well and when do they perform poorly?
- Design test to answer questions
  - Compare to benchmark derived from
    - Empirical Data Sets (discussed by Dan)
    - User-Generated Data Sets
  - Our test should provide guidance as to which methods work well and when

Proposed Solutions

- Empirical Data Sets
  - Calculate reserve variation directly from tangible observations
    - Fundamental assumption of this approach: All observations come from distribution of similar risk profile
  - Main limitations of empirical data sets
    - Risk Profiles are different…
      - Between companies for same line

        e.g. Company A's "WC" comprised of large account financial services companies, Company B's "WC" comprised of construction and contractors
      - Across time within same company

        e.g. Company X's "Auto Liability" historically comprised 50/50 of rural Midwestern and urban Northeast, 3 years ago exited Northeastern market
    - History may reflect unique events that will not persist into future
      - One-off events
      - Forces underlying data that are not acknowledged or fully understood
    - History may not reflect events that will persist into future
      - Inflation

Proposed Solutions

- User-Generated Data Sets
  - Create many observations of “similar” risks
  - Calculate reserve variation directly from simulated universe of observations
    - Know that all observations come from distribution of similar risk profile
  - Main advantages of user-generated data
    - Have greater understanding of underlying forces and trends that created data
    - Have “unlimited” realizations from which to measure higher moments
    - Guarantees that all observed realizations come from identical underlying risk profile
  - For our work, utilized the Boles-Staudt Methodology of data creation
    - With minor tweaks to accommodate our focus on higher moments
  - Question: How “real” is the real data?
    - Subject data to Actuarial “Turing Test”
      - Subject data to various interrogative techniques
      - Confirm various qualities that would be expected in data exist in data
      - If professional can’t tell the difference, it is real for all intents and purposes

What We Found

- For simplicity only examining 4 scenarios
  1. Stationary
  2. Changes in claims settlement rates
  3. Accident year inflation
  4. Calendar year inflation
- Testing Four Methods
  1. Chain Ladder Factor Model (CLFM)*
  2. Bootstrap
  3. Mack Volume Weighted (Mack-VW)
  4. Mack Simple Average (Mack-SA)
- Hypothetical Data Archetype
  1. Workers Compensation

*Uses two RTR methodologies, “true hindsight” (HS) and hindsight RTRs bounded by max/min and VW/SA implied by data (HS[B])

Stationary

Workers Compensation

Reality	Bootstrap	Mack-VW	Mack-SA	CLFM-HS	CLFM-HS[B]
9.8%	10.1%	10.1%	9.7%	10.8%	9.9%

- Methods perform reasonably well when compared to reality

Changes in Claims Settlement Rates

Workers Compensation

Reality	Bootstrap	Mack-VW	Mack-SA	CLFM-HS	CLFM-HS[B]
9.7%	9.5%	10.6%	11.9%	20.5%	11.0%

- Methods perform moderately well though larger spread apparent
  - Some apparent overstatement
    - CLFM's high CV for HS red-flags that selected RTRs imply underlying changes in data

Accident Year Inflation

Workers Compensation

Reality	Bootstrap	Mack-VW	Mack-SA	CLFM-HS	CLFM-HS[B]
13.4%	9.2%	9.7%	10.3%	13.4%	9.7%

- Methods perform more poorly now that history is diverging from future
  - History doesn’t necessarily reflect future

Calendar Year Inflation

Workers Compensation

Reality	Bootstrap	Mack-VW	Mack-SA	CLFM-HS	CLFM-HS[B]
13.4%	8.8%	8.4%	9.4%	16.7%	9.0%

- Methods perform more poorly now that history is diverging from future
  - Relatively large understatement for all lines except for CLFM-HS which overstates

Conclusions

- When "future" is like past, history is generally a good predictor
  - Models generally work well
- When future diverges from past, history can be a poor predictor
  - Observation is intuitively consistent with conventional reserving
    - Actuaries don't generally rely on historical development when it is not predictive
  - Mixed results from various models
    - Note: Many conventional stochastic methods don't know how to disregard irrelevant history
- CLFM "tries" to adjust results for divergence of future development
  - Apparent recognition of "problem" of divergent future at least
    - Other models not-so-much
    - Nevertheless, sometimes over corrections apparent

Next Steps

- Explore use of adjusted methods in stochastic reserving
  - Which adjustments adjust history in manner that retains data integrity but is still reflective of future?
- Explore impact of sample error on models
  - This is the uncertainty in the uncertainty estimate

Questions and Discussion