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Calculation of Risk Adjusted Loss Reserves based on Cost of Capital

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Introduction

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Two leading starting points:

- ▶ **IFRS/IAS** (International Financial Reporting Standards/International Accounting Standards): all valuation standards are principle based, using market value.
- ▶ **Solvency II** rules by EU legislation: the European Insurance and Occupational Pension Authority (EIOPA).

IFRS/IAS:

- ▶ Market value loss reserve on a Cost of Capital approach: taking into account the remuneration of the resilient capital amount.

Solvency II:

- ▶ Amount of resilient capital is based on Value at Risk on a 99.5% probability. In formula:

$$1 - a = 99.5\%; a = 0.5\%$$

$$VaR_a(X) = \sup(x : P(X \leq x) \geq 1 - a)$$

$VaR_a(X)$ is de .5-percentile-amount

Solvency II:

- ▶ Until recently: prudence margin as a surcharge on undiscounted best estimate, based on a percentile amount (60-80%).
- ▶ In future: Risk Adjusted Loss also based on a minimum Cost of Capital (this will be discussed in the presentation).

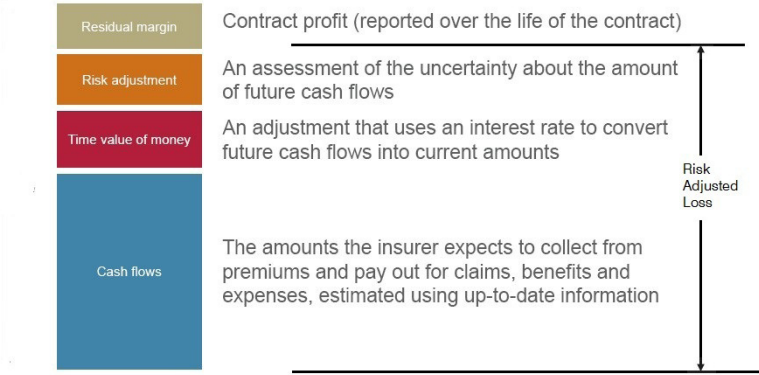
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Risk Adjusted Loss

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IAS Board: state of the art on Risk Adjustment

The proposals in the exposure draft



IAS Board: state of the art on Risk Adjustment

	ED proposal	Tentative decisions
Residual margin	Objective	
Risk adjustment	<ul style="list-style-type: none"> “The <i>maximum amount</i> the insurer would <i>rationaly pay</i> to be relieved of the risk that the ultimate cash flows exceed those expected.” 	<ul style="list-style-type: none"> “The <i>compensation</i> the insurer <i>requires to bear</i> the risk that the ultimate cash flows could exceed those expected”
Time value of money	Risk adjustment vs composite margin	
Cash flows	<ul style="list-style-type: none"> Include explicit estimate of the effects of uncertainty about future cash flows. 	<ul style="list-style-type: none"> ✓ measurement of liability should include explicit risk adjustment

Calculation of Risk Adjusted Loss Reserves based on a Cost of Capital

We integrated a calculation method of Risk Adjusted Loss Reserves based on Cost of Capital in our stochastic loss reserving application IFM (Integral Financial Modelling, CLRS Chicago 2009, Posthuma a.o.).

The idea:

- ▶ True liability is the amount a third party (e.g. a reinsurer) has to receive in order to be able to take over all insurance liabilities of a portfolio.
- ▶ Besides reserves for the regular loss payments an amount for resilient capital must be put aside in order to meet solvency regulations (e.g. Solvency II).
- ▶ Third party's equity involved in such a transaction has a target return on top of the risk free yield rate of $x\%$.

Under these conditions the question is at which (minimum) price such a third party is willing to take over the portfolio.

This can all be put into a simple differential equation to be calculated using a continuous stochastic reserving model (e.g. our IFM model).

Let $B(t_1, t_2)$ denote the future loss payments in $[t_1, t_2]$ and suppose we have

- ▶ a function $b(t)$ such that

$$\mathbb{E}B(t_1, t_2) = \int_{t_1}^{t_2} b(s) ds$$

- ▶ a function $V(t)$ such that

$$\mathbb{P}(B(t, \infty) \geq V(t)) = 0.005.$$

Note At time t the insurer must have a minimal resilient capital of $V(t)$. This part of his capital cannot be used to yield more than the risk-free rate.

Now suppose the liability is transferred to a third party. Denote

- ▶ R_f risk free rate
- ▶ R_t total rate = $R_{cOC} + R_f$

We will regard the third party's minimum price $E(t)$ as a fund which is (partly) available for investment, and from which the future losses are paid. This fund changes in a short time period Δt in the following way:

- ▶ The Resilient Capital $V(t)$ generates a risk-free interest $R_f \cdot \Delta t$.
- ▶ The rest $E(t) - V(t)$ generates the total interest $R_t \cdot \Delta t$.
- ▶ The loss payment $-b(t) \cdot \Delta t$.

This leads to the following differential equation for $E(t)$:

$$\frac{dE}{dt} = (E(t) - V(t))R_t(t) + V(t)R_f(t) - b(t).$$

In the end all liabilities have been winded down; no further loss payments are foreseen and no further resilient capital is needed. To determine a “fair price” we therefore take the boundary condition $E(\infty) = 0$.

Solving this differential equation, we find

$$E(t) = \int_t^\infty (V(s)(R_t(s) - R_f(s)) + b(s)) \exp\left(-\int_t^s R_t(u) du\right) ds.$$

and in particular, we find the **Risk Adjusted Loss** $E(0)$.

Interpretation

For the third party the following relation applies:

$$E(t) = DFL(t) + DMR(t)$$

The fund $E(t)$ consists of imbursements for:

- ▶ $DFL(t)$: future loss payments discounted at R_t (total rate)
- ▶ $DMR(t)$: missed remuneration on Resilient Capital also discounted at R_t (total rate).

Interpretation

$$E(t) = \int_t^\infty (V(s)R_{coc} + b(s)) \exp\left(-\int_t^s R_t(u) du\right) ds$$

Since the third party invests its capital at the total rate $R_t(t)$, a payment made at time $s \geq t$ (so in our setup this would be $b(s) ds$), should be discounted using the rate R_t . This leads to a Discounted Future Loss payment (DFL) at time t of

$$DFL(t) = \int_t^\infty b(s) \exp\left(-\int_t^s R_t(u) du\right) ds.$$

Interpretation

$$E(t) = \int_t^\infty (V(s)R_{coc} + b(s)) \exp\left(-\int_t^s R_t(u) du\right) ds$$

Reserving $V(s)$ means that the third party can invest this capital only at the risk-free rate. This corresponds to a *loss* in the time interval ds equal to $V(s)R_{coc} ds$. This loss is then discounted at the total rate, Discounted Margin on resilient capital (DMR) just like the payments:

$$DMR(t) = \int_t^\infty V(s)R_{coc} \exp\left(-\int_t^s R_t(u) du\right) ds.$$

Implementation

How to estimate the continuous functions $b(t)$ and $V(t)$?

Let

- ▶ $[t_{i-1}, t_i)$ be period i , with $t_n = T$.
- ▶ B_i expected payments in i
- ▶ V_i 99.5 % quantile of total payments after t_i with $V_n = 0$.

Implementation

$b(t)$ and $V(t)$ can be estimated as the following piecewise continuous functions whose accuracy increases when $[t_{i-1}, t_i)$ is shorter.

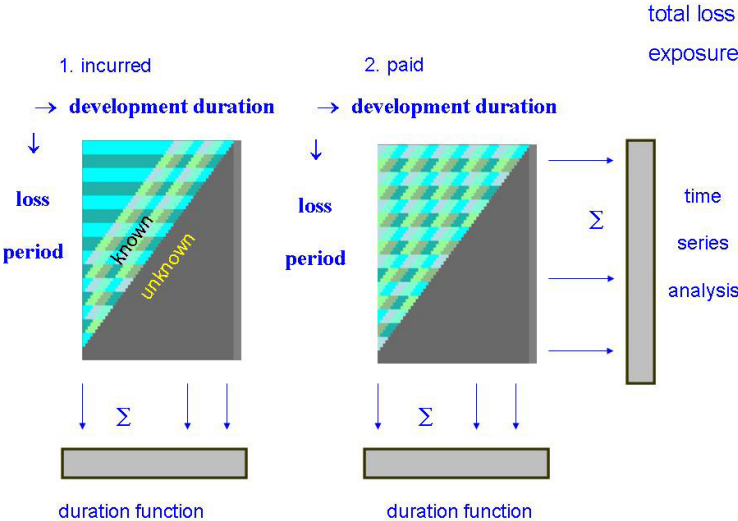
$$b(t) = \sum_{i=1}^n \frac{B_i}{t_i - t_{i-1}} \mathbf{1}_{[t_{i-1}, t_i)}(t), \text{ and}$$

$$V(t) = \sum_{i=1}^n V_{i-1} \mathbf{1}_{[t_{i-1}, t_i)}(t)$$

Risk Adjusted Loss in IFM

Before dealing with the integration of Risk Adjusted Loss Reserves based on Cost of Capital we will briefly recapitulate the IFM application.

Picture of the model



Initial definitions

We will use the following notation to define our model for the paid and the incurred run-off tables.

l indicates the loss period.

k indicates the development period.

$Y_{lk}^{(1)}$ indicates the incremental incurred.

$Y_{lk}^{(2)}$ indicates the incremental paid.

Our goal is to model the vector $(Y^{(1)}, Y^{(2)})$, including all future values.

Auxiliary variables

We start with auxiliary independent Gaussian random variables:

$$Z_{lk}^{(1)} \sim N\left(\mu_{lk}^{(1)}, V_{lk}^{(1)}\right)$$

$$Z_{lk}^{(2)} \sim N\left(\mu_{lk}^{(2)}, V_{lk}^{(2)}\right)$$

Now, define the event

$$R = \left\{ \sum_k Z_{lk}^{(1)} = \sum_k Z_{lk}^{(2)} \quad (\forall l) \right\}.$$

This says that for each loss period, the total amount incurred equals the total amount paid.

Final step

Finally we define the incremental losses by

$$Y^{(1)} \sim Z^{(1)} | R$$

$$Y^{(2)} \sim Z^{(2)} | R$$

This means that $(Y^{(1)}, Y^{(2)})$ is normally distributed, and that the row sums of the two tables are always equal.

Means and covariances of $(Y^{(1)}, Y^{(2)})$ are functions of $\mu_{lk}^{(i)}$ and $V_{lk}^{(i)}$.

The parameters in this model are given by $\mu_{lk}^{(i)}$ and $V_{lk}^{(i)}$ for $i = 1, 2$. We should reduce the number of parameters.

Product structure of parameters

We choose the following product structure for our parameters:

$$\mu_{lk}^{(i)} = W_l e^{(X\alpha)_l} \Pi_k^{(i)} \quad (i = 1, 2)$$

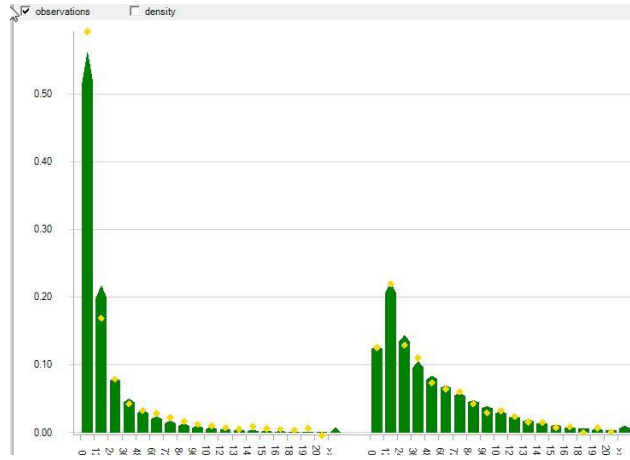
$$V_{lk}^{(i)} = \sigma^{(i)} W_l e^{(X\alpha)_l} \tilde{\Pi}_k^{(i)} \quad (i = 1, 2)$$

- ▶ W_l is the exposure measure for loss period l . (Note: l can be extended to future periods).
- ▶ $X\alpha$ is a linear model for the loss ratios, with design matrix X and parameter vector α .
- ▶ $\Pi^{(i)}$ and $\tilde{\Pi}^{(i)}$ distribute the total expectation, respectively variation, over all development periods:

$$\sum_k \Pi_k^{(i)} = \sum_k \tilde{\Pi}_k^{(i)} = 1 \quad (i = 1, 2).$$

- ▶ $\sigma^{(i)}$ ($i = 1, 2$) are parameters used to tune the total variation.

Example of parametric development curves



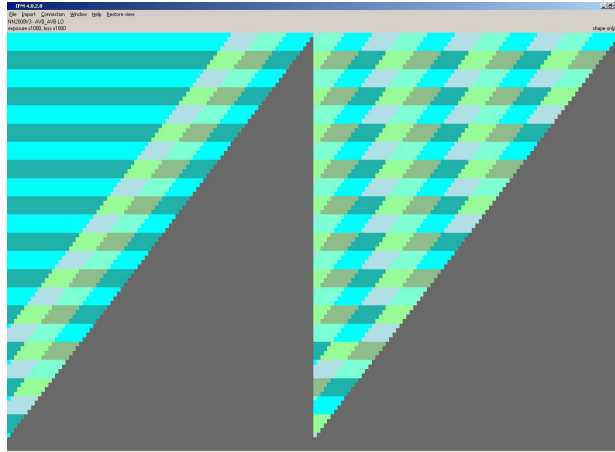
Data and estimation

The data usually consist of aggregated cells of the two run-off tables. We allow for any kind of aggregation given by some zero-one selection matrix S . So if $Y = (Y^{(1)}, Y^{(2)})$, our data is given by SY .

Note that SY is still normally distributed, with a known distribution, given the parameters. Therefore we can use Maximum Likelihood to estimate all the parameters.

Also prediction is relatively easy, since $Y | SY$ is again normally distributed. This structure makes our model very flexible. For a more detailed description, see “Combined analysis of Paid and Incurred Losses”, CAS e-forum, Fall 2008.

Example of aggregation



We have demonstrated that our joint model for paid and incurred losses performs very well on actual data.

Moreover, our model has several important advantages in managing “difficult” data:

1. It is flexible in aggregating various data sets and in handling aggregation levels of the input data.
2. It handles loss triangles with missing cells with ease.
3. It incorporates in a proper way negative data coming from negative adjustments to losses.

and

4. Future premium risk is added by extending the exposure measure, which in an integrated way produces future loss in force (Solvency II requires the loss risk on one-year future premium).

Examples

Two lines of business have been modeled in IFM to calculate the Risk Adjusted Loss Reserving based on Cost of Capital:

- 1. TPL Motor injuries (long tail)
- 2. Fire Real estate (short tail)

Excel sheet data available in:
www.posthuma-partners.nl (See downloads in Menu)
Examples loss triangles CAS Denver CLRS2012.xls

TPL Motor portfolio

Data:

- ▶ Incurred (paid + case reserves)
- ▶ Paid triangles

Loss periods:

- ▶ 1998Q1 - 2011Q3

One-year future premium:

- ▶ 2011Q4 - 2012Q3 \$ 10,000

TPL Motor portfolio

Integral Financial Modelling summarizes Management Information as follows:

- ▶ current nominal loss as a % of earned premium = 53.9%
- ▶ 50% of payments occur within 10 months
- ▶ 90% of payments occur within 89 months
- ▶ 99% of payments occur within 166 months

TPL Motor portfolio

		<i>loss on earned premium</i>	<i>including one-year future premium (\$ 10,000)</i>	<i>loss on one-year future premium (€ 10,000)</i>	
		<i>(a)</i>	<i>(b)</i>	<i>(b - a)</i>	<i>in %</i>
LR: Loss reserve best estimate	1	\$ 21,204	\$ 26,593	\$ 5,389	53.9
Time value of money	2	\$ (1,696)	\$ (2,061)	\$ (365)	-3.6
Risk margin based on CoC 7%	3	\$ 5,713	\$ 6,695	\$ 982	9.8
E(0): Risk adjusted loss 4= 1+2+3		\$ 25,221	\$ 31,227	\$ 6,006	60.1
DMR(0): Discounted Margin on resilient Capital	5	\$ 6,649	\$ 7,837	\$ 1,187	11.9
DFL(0): Discounted Future Loss payments	6	\$ 18,572	\$ 23,390	\$ 4,819	48.2
E(0) 7=5+6		\$ 25,221	\$ 31,227	\$ 6,006	60.1
V(0): VaR .005%	8	\$ 27,273	\$ 32,970	\$ 5,697	57.0
80% quantile Loss reserve	9	\$ 22,873	\$ 28,347	\$ 5,474	54.7

table: TLP Motor, Loss reserving outcomes including future premium (\$ 10,000)
time value of money based on zero-risk yield curve Dutch Supervisor

TPL Motor portfolio

In this scheme Risk Adjusted Loss is presented in two manners:

- The amounts in row 1-4 are in accordance to the statement in the IAS exposure draft as follows:
 - ▶ cash flow (best estimate)
 - ▶ time value of money
 - ▶ risk margin based on CoC 7%
 - ▶ (sums equals to) Risk Adjusted Loss on CoC
- The components of the fund $E(t)$ in row 5-7 as follows:
 - ▶ Discounted Margin on resilient capital
 - ▶ Discounted future loss payments on a total risk rate
 - ▶ (sums equals to) Risk Adjusted Loss on CoC

The resilient Capital based on future cash flow and the 80% quantile Loss Reserves are calculated in row 8 and 9.

TPL Motor portfolio

Risk adjusted loss on future premium as a % of future premium = 60.1%, and is the basis for adequate pricing in case of a targeted Cost of Capital of 7%.

This is much higher than the 80% quantile Loss Reserve amount (54.7%), which was the old rule for Adjusted Loss Reserve in Solvency Regulation and lead to false pricing in case of a targeted Cost of Capital of 7%.

Note: Risk Adjusted Loss is calculated on a 7% CoC surcharge (other surcharges are possible, but for Solvency II a minimum of 6% applies).

Fire (real estate) portfolio

Data:

- ▶ Incurred (paid + case reserves)
- ▶ Paid triangles

Loss periods:

- ▶ 2003Q1 - 2011Q3

One-year future premium:

- ▶ 2011Q4 - 2012Q3 \$ 10,000

Fire (real estate) portfolio

Integral Financial Modelling summarizes Management Information as follows:

- ▶ current nominal loss as a % of earned premium = 56%
- ▶ 50% of payments occur within 6 months
- ▶ 90% of payments occur within 16 months
- ▶ 99% of payments occur within 35 months

Fire (real estate) portfolio

		loss on earned premium	including one-year future premium (\$ 10,000)	loss on one-year future premium (€ 10,000)	
		(a)	(b)	(b - a)	in %
LR: Loss reserve best estimate	1	\$ 2,845	\$ 8,446	\$ 5,602	56.0
Time value of money	2	\$ (28)	\$ (133)	\$ (105)	-1.1
Risk margin based on CoC 7%	3	\$ 217	\$ 784	\$ 566	5.7
E(0): Risk adjusted loss 4= 1+2+3		\$ 3,034	\$ 9,097	\$ 6,063	60.6
DMR(0): Discounted Margin on resilient Capital	5	\$ 242	\$ 902	\$ 660	6.6
DFL(0): Discounted Future Loss payments	6	\$ 2,792	\$ 8,194	\$ 5,403	54.0
E(0) 7=5+6		\$ 3,034	\$ 9,097	\$ 6,063	60.6
V(0): VaR .005%	8	\$ 4,257	\$ 12,300	\$ 8,043	80.4
80% quantile Loss reserve	9	\$ 3,185	\$ 9,376	\$ 6,191	61.9

table: Fire insurance (real estate), Loss reserving outcomes including future premium (\$ 10,000)
time value of money based on zero-risk yield curve Dutch Supervisor

Fire (real estate) portfolio

Risk adjusted loss on future premium is 60.6% and differs slightly from the conventional 80% quantile Adjusted Loss Reserve amount (61.9%).

For this short tail insurance business the conventional Adjusted Loss Reserve would not give such a false pricing signal as was the case for the longer tailed TPL.

Conclusions

Given IFRS/IAS and European Solvency II developments Risk Adjusted Loss Reserves and Solvency will from now on be based on Cost of Capital. This will require a full stochastic model of future payments. More conventional modeling may not give the user enough information to determine the Risk Adjusted Loss based on Cost of Capital per line of business with satisfactory accuracy.

We have proposed a new method of Risk Adjusted Loss based on Cost of Capital. The method is in compliance with Solvency II and appears to produce very acceptable results for realistic examples. This new method is incorporated in our IFM model which is very well suited for these types of subtle predictions. The flexibility in structure of the data, and versatility in determining all kinds of predictions with margins makes IFM in our opinion the ideal model for dealing with loss triangles. The model also produces the necessary information for steering profitability bottom up, thanks to the future risk approach. Portfolio optimization therefore becomes feasible.