

More information will be available at

**CLRS – 2012**  
**Insureware’s Hospitality Suite**  
**Sheraton Denver Downtown Hotel, Denver, CO**  
**7:00pm-10:30pm**  
*Wednesday September 5<sup>th</sup>*  
*Thursday September 6<sup>th</sup>*

(Also learn about the Bootstrap [technique](#) for testing validity of a model or method)



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**Introduction and Summary / 1**

- We provide our solution to the Solvency II one-year risk horizon, SCR, Technical Provisions (TP) (Fair Value Liabilities), Market Value Margins (Risk Margins) for the aggregate of long tail LOBs
- The solution is non-recursive, non-circular, tractable and satisfies all the directives (requirements)
- IFRS4 requirements in respect of fungibility and ring-fencing is discussed
- Three types of correlations between LOBs
- How do we know if two LOBs have the same economic drivers?
- Is the economic inflation a principal driver of long tail liability calendar year trends?



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**Introduction and Summary / 2**

- Which probability distributions are required to compute the various risk measures for the aggregate of multiple LOBs?
- VaRs and T-VaRs
- Process Variance versus Parameter Uncertainty
- Reserve risk, underwriting risk and the combined risk
- Risk Capital allocation by LOB and calendar year
- The ultimate year risk horizon- conceptually much simpler
- Calendar year Payment stream probability distributions
  - what are the drivers?



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### Solvency II one-year risk horizon

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- Solvency Capital Requirement (SCR)
  - Market Value Margins (Risk Margins)
  - Technical Provisions (Fair Value of Liabilities)
- Under what (three) conditions is it necessary to compute these metrics?  
What is the cornerstone of Solvency II?  
Consistency of metrics from year to year on updating?  
What are the 'causes' of distress in the first year such that losses reach 99.5 percentile of the loss distribution?

Which are the LOBs that most contribute to distress?



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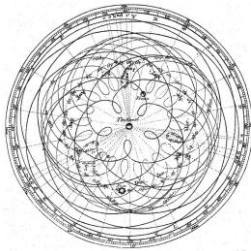
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### Most solutions to SII and IFRS4 metrics for long tail liabilities are like the Ptolemaic system

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If you look at things in the right way everything becomes simpler

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### Copernican Solution



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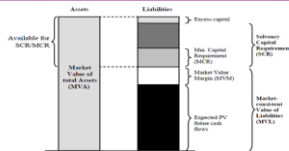
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### Example of risk diversification of SCR and Risk Margins

- SII metrics for the aggregate of real life six LOBs compared with SII metrics for the most volatile LOB to illustrate amongst other things risk diversification of SCR and (MVM (Risk Margin) component) of TP (Fair Value of Liabilities).
- Undiscounted reserves for the aggregate of six LOBs  
 = (approx) Technical Provisions + Solvency Capital Requirement (SCR)  
 = total in Economic Balance Sheet,  
 using a risk free rate of 4% and a spread of 6%.
- No need for additional capital in this example due to risk diversification SCR and MVM.
- Conditions for consistent estimates of prior accident year ultimates and SII risk measures on updating?
- We will explain how to avoid model error "distress".



### Solvency II – Economic Balance Sheet



Solvency II aims to establish a solvency regime that is better matched to the true risks of an insurance company

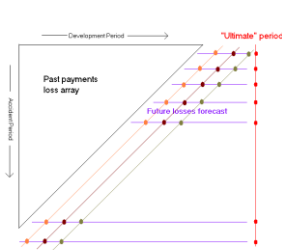
Ann Hagen in "Solvency II - Brave new world"

"Doing the job"

Under Solvency II, the way that work is carried out will change. For example, Solvency II is likely to require different actuarial techniques from the ones currently used. Technical provisions will be estimated as a probability-weighted average of expected future cash flows, taking into account the time-value of money and including a risk margin. Many of us are estimating claims reserves using traditional deterministic actuarial techniques, primarily relying on incurred claims data. Under Solvency II, not only will we need to discount these reserve estimates, requiring projected payment patterns, we will also need to demonstrate a deep understanding of the uncertainty of those reserves. We will additionally be required to apply the same approach to evaluating unexpired risk liabilities currently allowed for in the unearned premium reserves."



### Actual payments are made by calendar year

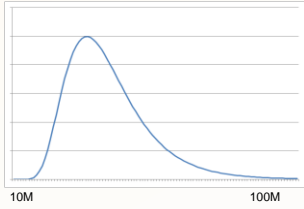


Summing future losses along the calendar year axis produces projections of the cash-flow, and the actual calls on the reserves. This is the dimension in which solvency issues arise.

Using cell distributions and correlations we can compute the distributions for each future year's cash flow.



Risk Capital: VaRs and T-VaRs (1)



Let's look at a typical forecast paid losses distribution. We expect to see a skew to the right, and more or less heavy tails. This could be the ultimate losses for a set of accident years, or the forecast losses for a single calendar year. We'll set the range as 10M-100M.




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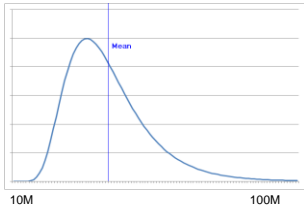
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Risk Capital: VaRs and T-VaRs (2)



The mean is, by definition, the "probability-weighted average of future outcomes". In this example Mean = 40.0M

Solvency II: "The Best Estimate shall correspond to the probability-weighted average of future cash-flows, taking into account the value of money (expected present value of future cash-flows, using the relevant risk-free interest rate term structure)."

To compute this we need to have the means by calendar period, as well as a projection of interest rates. Provision must be made for losses above the mean.




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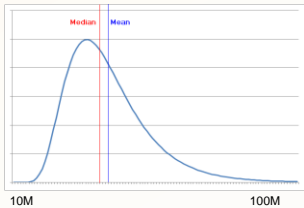
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Risk Capital: VaRs and T-VaRs (3)



Mean = 40M  
Median = 37M

In this example the mean is at the 59<sup>th</sup> percentile. That means that there is a 41% chance that the losses will come in above the mean. Since the right tail of the distribution is long, the loss may be much higher than the mean.




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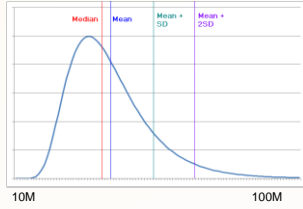
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Risk Capital: VaRs and T-VaRs (4)



SD = 13.16M  
 Mean+SD = 86.5<sup>th</sup> percentile  
 Mean + 2SD = 95.8<sup>th</sup> percentile

A risk measure is a measure of expected variability, and this is usually taken to be the Standard Deviation. However for a skewed, non-normal distribution it is not clear what we are measuring. In this example the Mean + 2SD is exceeded more than 4% of the time. (For the Normal distribution this percentage is 2.5%.)



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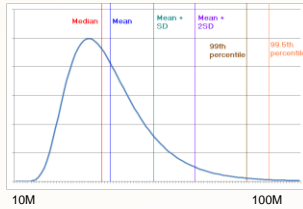
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Risk Capital: VaRs and T-VaRs (5)



99<sup>th</sup> percentile = 82.9M  
 99.5<sup>th</sup> percentile = 90.1M

The percentiles of the distribution provide a measure of risk that is uniform, in that it has the same practical meaning for any distribution. The Value at Risk (VaR) codifies this concept. The VaR at 99% is the amount by which the 99<sup>th</sup> percentile exceeds the reserve (reserve = best estimate is usually equal to the mean) and corresponds to the minimum amount lost if this quantile was reached.



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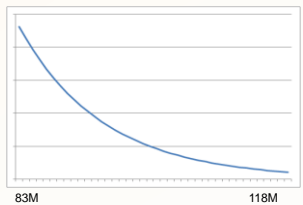
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Risk Capital: V@Rs and T-V@Rs (6)



99<sup>th</sup> p'tile = 82.9M  
 99.5<sup>th</sup> p'tile = 90.1M  
 TV@R<sub>99%</sub> = 93.1M  
 TV@R<sub>99.5%</sub> = 101.5M

If we are thinking about the possibility of losses exceeding a high threshold such as the 99<sup>th</sup> percentile then we need to look more closely at the tail of the distribution. Arguably we should consider the mean loss given that the loss exceeds the 99<sup>th</sup> percentile, this is the Tail-V@R (T-V@R).



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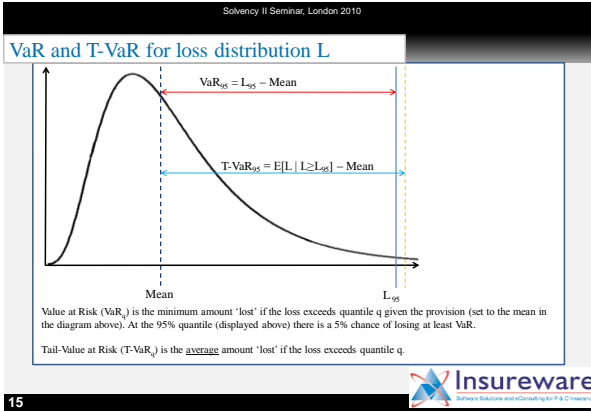
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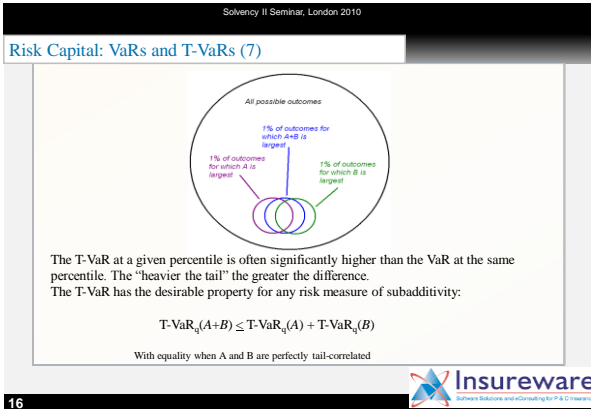
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**Solvency II one-year risk horizon: satisfies three conditions - Summary of decomposing the directives- What are the basic elements?**

- Risk Capital is raised at the beginning of each year and any unused capital is released at the end of the year;
- The analyses are conditional on the first (next) calendar year being in distress (99.5%);
- At the end of the first year in distress, the balance sheet can be "restored" in such away that the company has sufficient technical provisions (fair value of liabilities) to continue business or to transfer the liabilities to another risk bearing entity.

An important consideration is that **fungibility** by calendar year is only in the forward direction

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### One-Year Risk Horizon

- The cost of raising the risk capital, the **Market Value Margin (MVM)** or premium on the risk capital, also known as the **Risk Margin** is paid to the capital providers at the end of each year along with any unused risk capital.
- The Best Estimate of Liability (BEL) is the mean (suitably discounted)
- The **Technical Provision** or **Fair Value** = BEL +MVM.
- For calendar year k we have BEL(k) and MVM(k).

BELs and MVMs are additive.




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### The Concept of Risk - The SCR

As detailed in the **Insurance ERM analysis of Solvency II:**

#### **GREAT WEB SITE- decomposes the directives.**

"The fair value balance sheet is one of the cornerstones of Solvency II, and its impact is not restricted only to the calculation of fair value assets and liabilities. The concept of market value margin (MVM), and the related one-year risk approach in the calculation of the solvency capital requirement (SCR), find their origin in this fair value driven approach: re/insurance companies should have enough capital on their balance sheet to cover the risks that can emerge over a 12-month timeframe, and allow for a (theoretical) transfer of all (contractual) liabilities at the end of this balance-sheet period. This means that companies have to be able to calculate the impact of such shocks on their end-of-year balance sheets, and value these in such a way that they can be transferred to a third party."




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### The One-Year Risk Horizon

To satisfy Solvency II one-year risk horizon the Economic Balance Sheet at inception must have sufficient SCR and Fair Value to withstand a first (next) calendar year in distress at 99.5%, and be able to be restored to its Fair Value at the beginning of the second year.

$$\text{Fair Value} = \text{BEL} + \text{Market Value Margin (Risk Margin)}$$

It is also assumed that Risk Capital is raised from the capital providers at the beginning of each year and released if not fully used at the end of the year.




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## Definition of SCR

The above extracts lead to the following definition: the SCR for the one-year risk horizon is the Value-at-Risk for the first year plus the change (increase) in technical provisions (TP), equivalently, fair value, in the subsequent years (suitably discounted), **conditional** on the first year being in distress.

$$SCR = VaR_{99.5\%}(1) + \Delta TP(2) + \Delta TP(3) + \dots + \Delta TP(n),$$

where n is the limit of run-off.



## The Concept of Risk Horizon Perspective

### Quantification Requirements- What do we need to compute SII metrics?

- For the calculation of the **Technical Provisions, Market Value Margins** and **SCR** for both the **One-year Risk Horizon** (and **Ultimate Year Risk Horizon**) for the aggregate of all long-tail LOB's and each LOB separately the following critical information is required:
  - Probability distributions of paid losses (liability stream) by **calendar year** ( $k = 1, \dots, n$ ) and their correlations, for each LOB and the **aggregate** of all LOB's
  - Probability distributions of total reserves for each LOB and the aggregate of all LOB's.
  - Probability distributions of the aggregate paid losses from calendar year k to calendar year n for each LOB and the aggregate of all LOB's. This is required for each k ranging from 1 to n, where complete run-off is achieved at the ultimate calendar year n
  - Conditional Probability distributions, conditional on the first (next) calendar year being in "distress".
- Armed with these distributions **any risk measure can be computed**, including VaR(k) for the paid losses (total loss) in calendar year k; and **Market Value Margins, Technical Provisions** and VaRs conditional on the first year in distress, for each LOB and the aggregate of all LOB's.



## Risk Capital – One Year risk Horizon

### Simplest Case: Only One Year Runoff

$L_1$  = projected losses for the year. This is a random variable.

$BEL(1) = \frac{E(L_1)}{(1+d)^{0.5}}$  Where d = interest rate. Losses are paid uniformly through year, so we discount for half a year.

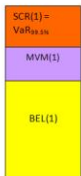
$$SCR(1) = VaR_{99.5\%}(L_1), \text{ i.e. } \Pr(L_1 \leq E(L_1) + SCR(1)) = 0.995$$

$MVM(1)$  is the cost incurred in having risk fund of  $SCR(1)$  available for the year. It is paid to capital provider at end of year and so is discounted by a full year.

$$MVM(1) = \frac{SCR(1) \cdot d}{(1+d)^1} \text{ if the interest on the risk fund is paid directly to capital provider, or } MVM(1) = \frac{SCR(1) \cdot d \cdot (1+d)}{(1+d)^2}, \text{ otherwise.}$$

$TP(1) = BEL(1) + MVM(1)$ . This is the Technical Provision and must be held in company own funds.

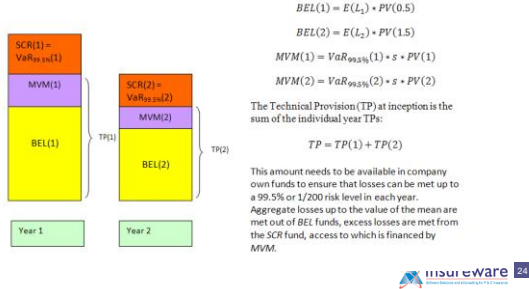
We will also let,  $PV(k;d)$ , or  $PV(k)$  be used to abbreviate the Present Value factor  $1/(1+d)^k$



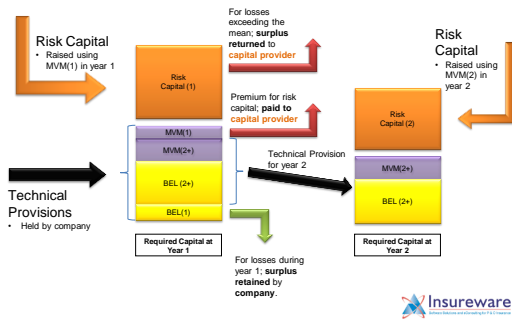


## Risk Capital – One Year risk Horizon

### Next Simplest Case: Two Year runoff, No correlation

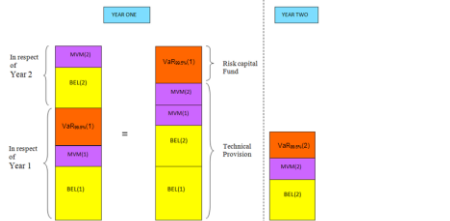


### Capital flow: Uncorrelated future calendar years



## Risk Capital – One Year risk Horizon

### Uncorrelated future calendar years



Two-year picture of accounts: In year 1 we require reserves to meet paid loss liabilities for years 1 and 2 and we also need to be able to fund the cost of access to the risk capital funds for years 1 and 2, however we only need access to the year 1 risk fund. When year 2 begins our accounts reset, since any cost over-runs from year 1 were paid out of the risk fund and do not degrade our prepared reserves for year 2. Provided the loss over-run is below  $RC(1) = VaR_{99.5}(L_1)$ .

**Risk Capital – One Year Horizon**

- This is fine, except for one thing:  
What if the distribution for the losses in year 2 has changed conditional on the losses in year one?
- Simply put, the previous picture assumes there is no correlation between the distributions for years 1 and 2. In other words, whatever the outcome observed after year 1 we are going to remain fixed on our previous course, full steam ahead  
**Typically calendar year distributions are positively correlated.**

The correlations are driven by parameter uncertainty.




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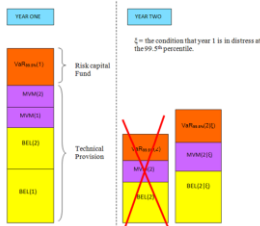
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**Risk Capital – One Year Horizon**



If year 1 is in distress at the 99.5th percentile, then our risk fund carries us over into year 2, but the conditional distributions are now different. Year 2 now must be re-evaluated in the light of conditional distributions and these increase the size of the BEL and the MVM, the cost of holding the risk fund. We need to include these adjustments in the year 1 risk fund.




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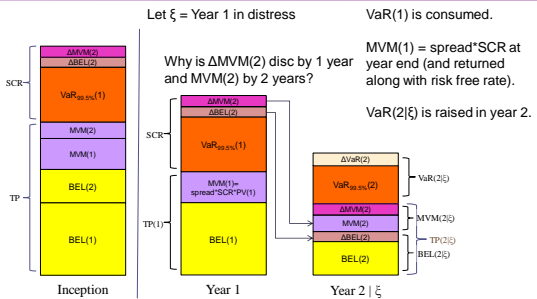
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**Two-year runoff with first year in distress**




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**What Causes Distress in the first year?**

1. "Inflation parameters" going forward. For example under the assumption 10%+ .3%, a 60% trend is distress.
2. Process volatility- large values from the tail of lognormal distributions.
3. Combinations of 1. and 2.
4. Which LOBs contribute more to distress than others?
  - Process volatility
  - Parameter uncertainty
  - "Size" of LOB




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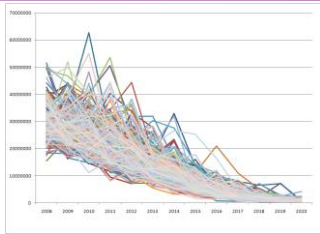
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**Conditional Statistics from Simulations**



Begin with a large number of simulations of the entire forecast table. This provides an equal number of sample paths through all future calendar years.




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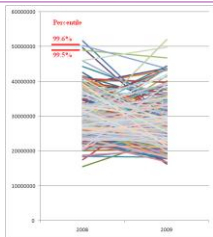
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**Conditional Statistics from Simulations**



Determine the sample paths corresponding to the distress scenario. If this is "next year at 99.5<sup>th</sup> percentile", then these paths belong in the [99.5, 99.6) order interval.




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### Conditional Statistics from Simulations



Restricting attention to only these sample paths we can then calculate any conditional statistic, such as  $BEL(k)|\xi$ ,  $MVM(k)|\xi$ ,  $Var(k)|\xi$  etc.



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### Multiple distress years

- In the previous solution, only the first year in distress is considered.
- What if receiver also subscribes to Solvency II?
- Fair value should include allowance for subsequent years also being in distress.
- One way to allow for this is to analyse subsequent years being in distress irrespective of the losses observed in previous years.



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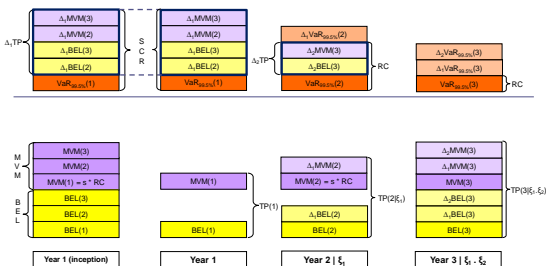
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### Three Year Run-off with One-Year Risk Horizon All-Inclusive, risk-free rate is 0.



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Three years in run-off with One-Year Risk Horizon				
R C	$\Delta_1 MVM(3) = s * \Delta_1 VaR_{loss}(3) * PV(2)$			
	$\Delta_1 MVM(2) = s * \Delta_1 VaR_{loss}(2) * PV(1)$	$\Delta_1 VaR_{loss}(2)$		
	$\Delta_1 BEL(3) = (E(L(3)   \xi_1) - E(L(3))) * PV(1.5)$	$\Delta_1 MVM(3) = s * \Delta_1 VaR_{loss}(3) * PV(1)$	$\Delta_1 VaR_{loss}(3)$	
	$\Delta_1 BEL(2) = (E(L(2)   \xi_1) - E(L(2))) * PV(0.5)$	$\Delta_1 BEL(3) = (E(L(3)   \xi_1) - E(L(3))) * PV(0.5)$	$\Delta_1 VaR_{loss}(3)$	
	<b>VaR</b>	$VaR_{loss}(1)$	$VaR_{loss}(2)$	$VaR_{loss}(3)$
T P	$MVM(3) = s * VaR_{loss}(3) * PV(3)$		$\Delta_1 MVM(3) = s * \Delta_1 VaR_{loss}(3) * PV(1)$	
	$MVM(2) = s * RC(2) * PV(2)$	$\Delta_1 MVM(2) = s * \Delta_1 VaR_{loss}(2) * PV(1)$	$\Delta_1 MVM(3) = s * \Delta_1 VaR_{loss}(3) * PV(1)$	
	$MVM(1) = s * RC(1) * PV(1)$	$MVM(2) = s * RC(2) * PV(1)$	$MVM(3) = s * VaR_{loss}(3) * PV(1)$	
	$BEL(3) = E(L(3)) * PV(2.5)$		$\Delta_1 BEL(3) = (E(L(3)   \xi_1) - E(L(3))) * PV(0.5)$	
	$BEL(2) = E(L(2)) * PV(1.5)$	$\Delta_1 BEL(2) = (E(L(2)   \xi_1) - E(L(2))) * PV(0.5)$	$\Delta_1 BEL(3) = (E(L(3)   \xi_1) - E(L(3))) * PV(0.5)$	
	$BEL(1) = E(L(1)) * PV(0.5)$	$BEL(2) = E(L(2)) * PV(0.5)$	$BEL(3) = E(L(3)) * PV(0.5)$	
	Inception	Year 2   $\xi_1$	Year 3   $\xi_2$	

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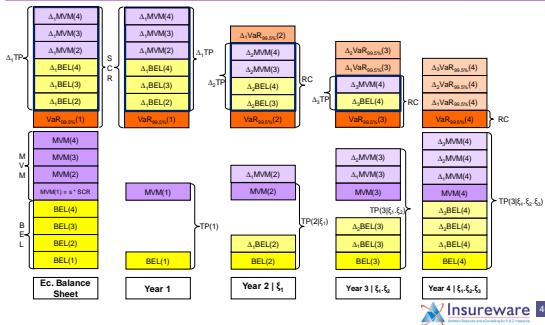
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**Four Year Run-off with One-Year Risk Horizon All-Inclusive, risk-free rate is 0.**




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**A single composite model for two long tail LOBs**

- Two LOBs written by the same company rarely have the same trend structure (including in the calendar year direction) and often process (volatility) correlation is either zero or very low. Reserve distribution correlation is often zero and if significant quite low.
- No two companies are the same in respect of trend structure, and process (volatility) correlation is often zero (for the 'same' LOB).
- No company is the same as the industry, unless it is a very large proportion of the industry.
- All the above are demonstrated with real life data.

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**A single composite model for all long tail LOBs**

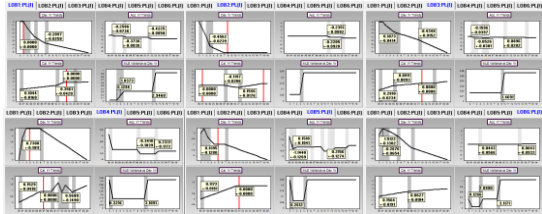
- For each LOB the corresponding model measures the volatility in the LOB with easily interpretable parameters.
- The model has descriptors that describe the risk characteristics of the data.
- Simulated triangles (including Bootstrap triangles) are indistinguishable from the real data in respect of statistical features.
- Forecast assumptions going forward are explicit and auditable.
- A single composite model also measures the correlations between the LOBs.
- There are three types of correlations between LOBs leading to reserve distribution correlation.



SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

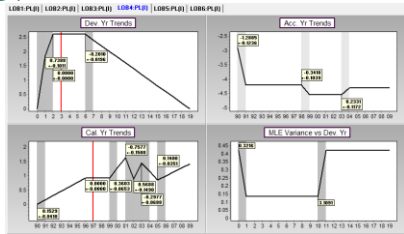
**Trends and volatility are unique to each LOB**

- LOB4 is the most volatile of the LOBs



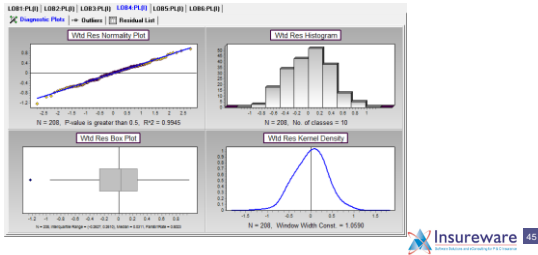
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- Model for LOB 4



SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- Diagnostics for LOB4 illustrating normality satisfied. But process variance (volatility) is high



SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- Forecast table for LOB4- Note there is no analytical closed form for the sum of lognormals

- Black:
  - Fitted mean
- Blue:
  - Observed
- Red:
  - Standard Deviation (log-normal)

Aggregate   LOB1 (PLB)   LOB2 (PLB)   LOB3 (PLB)   LOB4 (PLB)   LOB5 (PLB)   LOB6 (PLB)										
Accident Period vs Development Period										
Cal. Per. Total	0	1	2	3	4	5	6	7	8	
2004	3,848	16	68	162	186	214	247	284	268	254
2005	3,088	48	83	132	243	359	233	115	110	107
2006	2,318	16	102	246	283	328	378	435	411	389
2007	2,828	21	137	329	380	439	508	588	567	528
2008	2,884	21	137	329	380	439	508	588	567	528
2009	3,243	24	155	372	430	488	577	669	634	602
2010	3,900	49	64	154	183	220	266	323	319	317
Total Fitted Mean	29,918	2011	2012	2013	2014	2015	2016	2017	2018	2017
Cal. Per.	47,219	3,848	3,874	4,100	4,182	4,198	4,138	3,869	3,823	3,781
Total	47,449	104	732	851	995	1,139	1,245	1,334	1,429	1,429

SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- Forecast table for the aggregate of the six LOBs

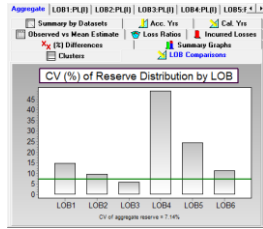
- Black:
  - Fitted mean
- Blue:
  - Observed
- Red:
  - Standard Deviation

Aggregate   LOB1 (PLB)   LOB2 (PLB)   LOB3 (PLB)   LOB4 (PLB)   LOB5 (PLB)   LOB6 (PLB)										
Accident Period vs Development Period										
Cal. Per. Total	0	1	2	3	4	5	6	7	8	
2004	287,031	95,735	109,358	36,115	17,279	9,935	7,361	8,040	4,984	4,194
2005	275,245	84,027	112,087	38,137	17,835	13,819	8,811	741	921	538
2006	288,482	97,481	116,668	43,660	19,640	13,268	8,438	6,548	5,768	4,268
2007	277,899	77,598	127,377	38,731	22,488	18,842	1,888	871	738	548
2008	306,715	104,752	124,282	44,480	21,295	12,361	9,192	7,828	6,619	4,228
2009	302,206	142,541	128,878	48,573	12,212	5,982	1,176	985	718	541
2010	319,815	100,488	118,963	42,746	20,666	12,618	8,023	6,862	4,955	3,788
2011	323,149	100,455	116,901	38,555	2,638	1,597	1,176	862	638	562
2012	329,853	107,790	127,871	45,668	22,083	12,798	6,556	6,269	4,675	3,598
2013	348,822	122,154	144,388	5,190	2,822	1,754	1,138	812	612	488
2014	344,119	109,831	130,819	47,800	22,729	11,857	7,884	5,912	4,428	3,428
2015	275,397	100,708	148,261	6,261	2,626	1,624	1,078	798	605	506
Total Fitted Mean	4,801,940	2010	2011	2012	2013	2014	2015	2016	2017	2017
Cal. Per.	4,801,940	243,927	118,931	78,350	51,163	37,784	28,882	22,533	18,015	18,015
Total	4,801,940	17,278	7,245	5,009	3,328	2,627	2,128	1,841	1,888	1,888



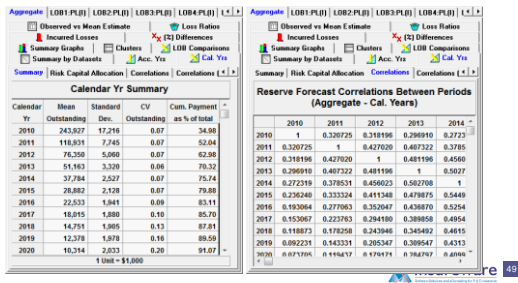
SII metrics for the aggregate of six LOBs compared with SII metrics for the most volatile LOB- risk diversification of SCR and TP

- LOB4 is the most volatile of the six LOBs (CV=49%)
- CV of Aggregate is 7.14%
- Substantially more Solvency II risk capital required if LOB4 was written on its own. It has a CV of 49%



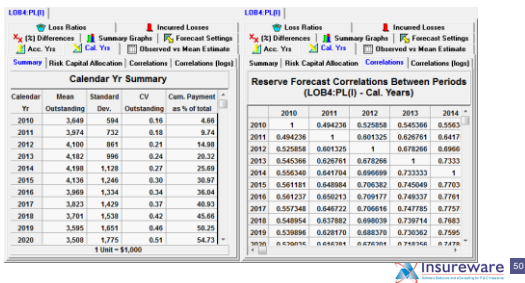
Liability stream by calendar year and calendar year correlations for the Aggregate of the six LOBs

52% of reserves paid within two years (2011).  
Calendar year correlations low and reduce quickly



Liability stream by calendar year and calendar year correlations for LOB4- long tail with high correlations

50% of reserves not paid before 2019 (10 years hence).  
Calendar year correlations high and do not dampen.



**One-year risk horizon – one year in distress**  
**Aggregate of six LOBs**

- Aggregate Solvency II capital required (**Technical Provisions + SCR**) is the same as undiscounted BEL

Metrics Summary		MVM, SCR and TP as % of BEL	
Value	%	% of BEL	% of Undisc. BEL
BEL	609,492	98.03	
MVM	12,223	1.97	10.98
Technical Provisions	621,715	100.00	
			9.68
			1.75
VaR(2010)	49,160	7.46	192.01
Delta TP	17,781	2.54	MVM + SCR
SCR	66,822	100.00	12.99
			TP + SCR
			112.09
			68.73
Technical Provisions	621,715	90.28	
SCR	66,822	8.72	
TP + SCR	688,537	100.00	
1 Unit = \$1,000			

- Bulk of SCR is the VaR for next year (2010)
- All calculations assume: risk free = 4% and spread = 6%



**One-year risk horizon – all years in distress**  
**Aggregate of six LOBs**

- Aggregate Solvency II capital required (**Technical Provisions + SCR**) is 6.6% higher than undiscounted BEL.

Metrics Summary		MVM, SCR and TP as % of BEL	
Value	%	% of BEL	% of Undisc. BEL
BEL	609,492	90.68	
MVM	62,618	9.32	11.99
Technical Provisions	672,108	100.00	10.27
			8.99
			30.28
VaR(2010)	49,571	89.68	MVM + SCR
Delta TP	21,658	20.42	21.96
SCR	71,239	100.00	121.86
			106.63
Technical Provisions	672,108	90.42	
SCR	71,239	9.58	
TP + SCR	743,347	100.00	
1 Unit = \$1,000			

- Main difference MVM
- Bulk of SCR is the VaR for next year (2010)
- All calculations assume: risk free = 4% and spread = 6%



**One-year risk horizon**  
**LOB4**

- Solvency II capital required (**Technical Provisions + SCR**) for LOB4 is substantially higher than for undiscounted BEL

Metrics Summary		MVM, SCR and TP as % of BEL	
Value	%	% of BEL	% of Undisc. BEL
BEL	52,713	85.28	
MVM	8,591	14.72	96.99
Technical Provisions	61,814	100.00	77.27
			11.63
			78.98
VaR(2010)	3,962	3.64	MVM + SCR
Delta TP	40,263	66.58	114.25
SCR	51,210	100.00	214.25
			144.71
Technical Provisions	61,814	54.73	
SCR	51,210	45.27	
TP + SCR	112,519	100.00	
1 Unit = \$1,000			

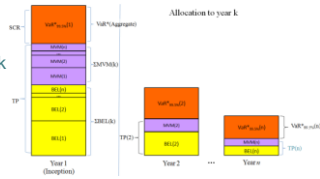
- Bulk of SCR is Delta TP – capital required to restore the balance sheet should the next year be in distress





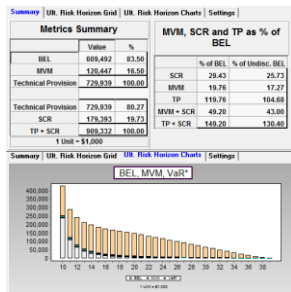
## Ultimate Year Risk Horizon

- Allocation of capital in the Ultimate Year Risk Horizon framework

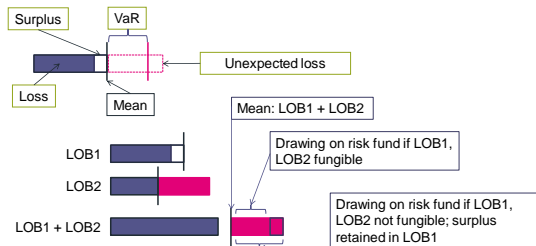


## Ultimate-year risk horizon Aggregate of six LOBs

- MVM is calculated based on the VaR 'to run-off' for each calendar year
- MVM is around 10x the one-year risk horizon's MVM



## Fungibility and Ring-fencing by example – drawing on the risk fund



- In the case of fungibility the risk fund is smaller since it is expected to be supplemented by surpluses from other LOBs/portfolios.



### Consistent estimates of prior year ultimates and Solvency II Risk Measures on updating

- Total reserve increases from year to year (with same accident (underwriting) exposure as previous year)
- What does a calendar year trend (inflation) of 5% imply in terms of estimates of prior year ultimates, loss reserves and premiums (per unit risk)?
- AXIOM  
Calendar year trends (inflation) project (impact) both the prior and future accident (underwriting) years  
Here is a simple example that illustrates the main ideas that reserve increases do not represent under-reserving. Indeed, they are necessary in order to maintain consistent estimates of prior year ultimates as the company writes new underwriting (accident) years.



### Consistent estimates of prior accident year ultimates and reserve increases on updating

- On a logarithmic scale the data were generated as follows
- $Y(w,d) = 10^{-0.3*d} + 0.05(w+d-1)$  where w is the accident year 1,...,7 and d is the development year 0,..., 5.
- The numbers down each column increase by 0.05 on a log scale (approximately 5% annual). The numbers along each row decrease by 0.25 (= -0.3+0.05) on a log scale We have assumed that the paid losses run-off after five years. Even if this is the case for 1999, this may not be the case for subsequent accident years especially if inflation is 'high'



### Consistent estimates of prior year ultimates and SII metrics updating

Reserves and ultimates as at year end 2004

Cal. Per. Total	Accident Period vs Development Period					Reserve	Ultimate
	0	1	2	3	4		
1999	22,028	22,028	17,154	13,369	10,000	0.183	0.311
2000	40,310	23,336	18,034	14,045	10,930	0.249	0.331
2001	55,740	24,443	18,960	14,765	11,608	0.295	0.342
2002	68,899	25,591	19,830	15,622	12,288	0.344	0.352
2003	80,639	26,803	20,642	16,519	12,980	0.397	0.362
2004	91,092	28,073	21,502	17,454	13,700	0.455	0.373
Total PrioYrPct		0.000	0.000	0.000	0.000		
Cal. Per. Total	358,736	66,857	48,251	37,987	30,787	0.103	0.277

Reserves and ultimates as at year end 2005

Cal. Per. Total	Accident Period vs Development Period					Reserve	Ultimate
	0	1	2	3	4		
1999	22,878	22,878	17,164	13,369	10,000	0.183	0.311
2000	40,310	23,336	18,034	14,045	10,930	0.249	0.331
2001	55,740	24,443	18,960	14,765	11,608	0.295	0.342
2002	68,899	25,591	19,830	15,622	12,288	0.344	0.352
2003	80,639	26,803	20,642	16,519	12,980	0.397	0.362
2004	91,881	28,073	21,502	17,454	13,700	0.455	0.373
2005	95,742	29,232	22,185	18,034	14,441	0.518	0.384
Total PrioYrPct		0.000	0.000	0.000	0.000		
Cal. Per. Total	454,646	89,487	64,833	49,127	39,841	0.149	0.378

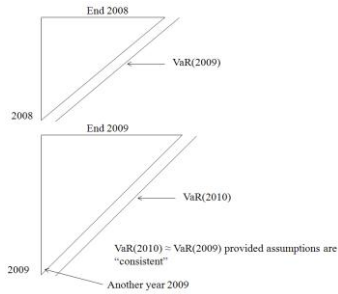




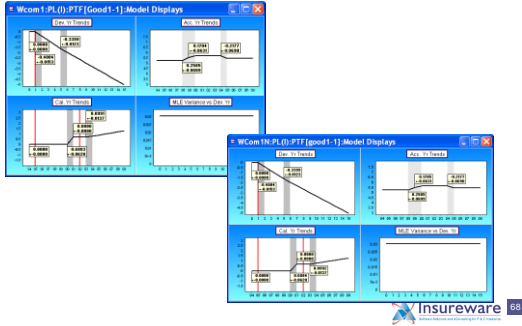
Conditional Statistics on next calendar period- volatility in ultimates on updating.

Acc. Yr	Mean		Standard Dev	CV		Cmt. on Next Cal. Per.	
	Outstandings	Ultimate		Outstandings	Ultimate	Std Dev (Std)	Ultimate
1994	64,595	4,307,621	12,000	0.19	0.00	0	12,000
1995	183,476	5,004,973	25,993	0.14	0.00	14,660	21,416
1996	348,961	6,811,802	42,843	0.12	0.01	27,242	30,918
1997	618,813	9,346,343	69,325	0.11	0.01	46,598	51,320
1998	941,181	10,645,342	102,898	0.11	0.01	70,502	74,950
1999	1,723,818	16,862,194	195,782	0.11	0.01	135,844	140,985
2000	3,276,295	24,725,342	308,618	0.10	0.01	231,508	233,135
2001	4,978,876	35,703,717	495,726	0.10	0.01	349,564	351,496
2002	7,758,325	43,405,885	778,323	0.10	0.02	541,788	547,597
2003	12,957,889	49,881,635	1,231,522	0.10	0.02	866,642	882,837
2004	17,798,981	58,626,983	1,855,874	0.10	0.03	1,284,855	1,329,654
2005	19,291,483	54,288,480	1,993,819	0.10	0.04	1,366,231	1,451,035
2006	29,237,716	62,418,911	3,042,497	0.10	0.05	2,066,529	2,232,990
2007	43,462,668	69,752,296	4,574,689	0.11	0.07	3,087,796	3,375,429
2008	54,378,314	64,985,786	5,829,819	0.11	0.09	3,912,252	4,322,161
Total	196,244,302	516,725,321	15,668,381	0.08	0.03	10,816,181	12,088,820

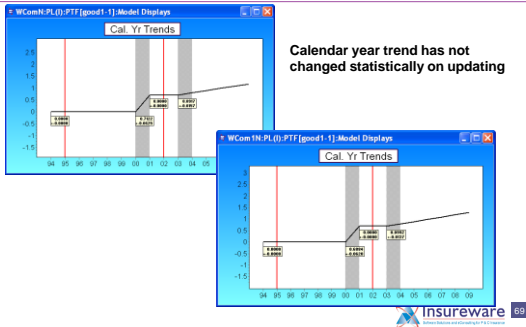
Updating and monitoring



Consistent estimates of prior year ultimates on and SII metrics updating



### Consistent estimates of prior year ultimates on and SII metrics updating

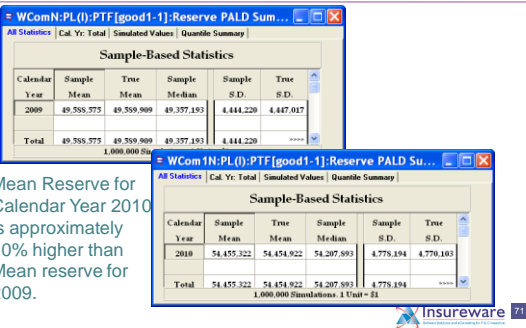


### Consistent Estimates of prior year ultimates on updating

Acc. Yr	Outstanding	Ultimate	Diff.	Outstanding	Ultimate
1994	49,105	4,367,021	52,621	0.19	0.00
1995	193,476	5,904,972	25,952	0.14	0.00
1996	248,961	6,811,162	42,843	0.12	0.01
1997	410,813	8,346,343	66,325	0.11	0.01
1998	941,181	10,645,342	102,899	0.11	0.01
1999	1,722,018	16,646,104	199,792	0.11	0.01
2000	3,225,295	24,725,342	326,618	0.10	0.01
2001	4,876,876	36,783,717	495,736	0.10	0.01
2002	7,758,325	43,485,885	778,323	0.10	0.02
2003	12,257,189	49,688,835	1,237,122	0.10	0.02
2004	17,768,981	56,820,983	1,865,911	0.10	0.03
2005	19,291,483	54,268,400	1,993,810	0.10	0.04
2006	29,273,276	62,418,911	3,084,497	0.10	0.06
2007	43,462,568	69,753,706	4,574,689	0.11	0.07
2008	54,378,314	64,995,706	5,829,819	0.11	0.09
Total	198,244,362	516,728,124	15,669,381	0.08	0.03

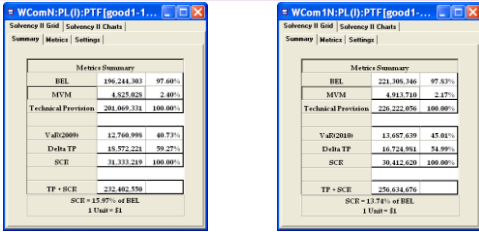
At end 2008, ultimate 2008=64.9+ 5.8, at end 2009 66.2+ 4.22

### Consistent estimates of prior year ultimates on and SII metrics updating





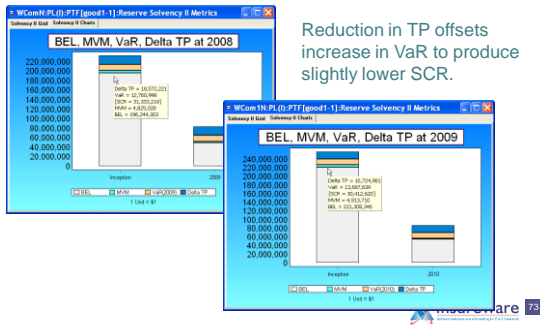
Consistent SII metrics on updating



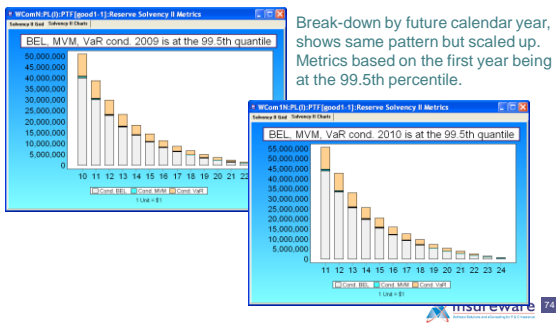
Solvency II calculations with no discounting: MVM for 2010 is almost the same as for 2009, and so is SCR.



Consistent SII metrics on updating



Consistent SII metrics on updating









**There are four types of correlations between LOBs**

1. Process (volatility) Correlation (that is, correlation between two sets of residuals)
2. Parameter Correlations
3. Same trend structure (especially along the calendar years)
4. Reserve distribution correlations

#1 induces #2. However, #3 is the 'worst' kind of relationship you can have between two LOBs as it results in very little, if any, risk diversification. It means that in terms of future calendar year trends the two LOBs move together, that is, a trend change in one LOB means a trend change in the other LOB, and is tantamount to the two LOBs having the same drivers. If two LOBs satisfy #3, then #1 and #2 are close to 1.

Fortunately, #3 we have only observed between layers of the same LOB, between segments of the same LOB, and between net of reinsurance and gross data (of the same LOB). #1, #2, #3 induce #4. #4 is typically much less than #1 in the absence of #3.

It is important to recognize that you cannot measure the relationship between two LOBs unless you first identify the trend structure and process variability in each LOB. It is only in the Probabilistic Trend Family (PTF) modelling framework that you can identify a parsimonious model that separates the trend structure in the three directions from the process variability.




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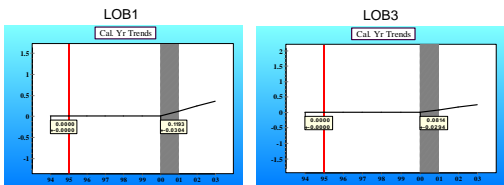
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**Two LOBs: LOB1 and LOB3**  
Actually same LOB different territories



Both LOBs have a calendar year trend change in 2000

That should be regarded as a concern!




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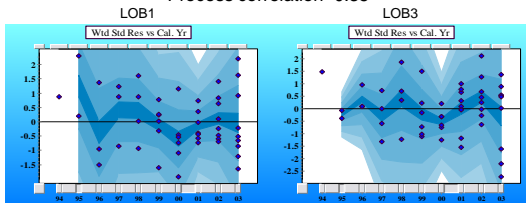
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**Two LOBs: LOB1 and LOB3**  
Actually same LOB different territories

Process correlation=0.35



Note 98-00 slight negative trend, 00-02 slight positive trend and 02-03 zero trend LOB1 and slight negative trend LOB3.




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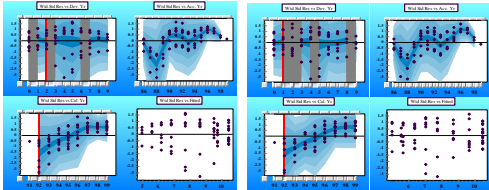
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## Two segments of WC

### Each segment only adjusted for development year trends

Accident year parameter correlations equal 1 after modelling accident years- major implications also for future underwriting years, where correlation in distributions of ultimates exceed 0.99




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## Reserve distribution correlations between two distinct LOBs- a very different story

- Highest process correlation observed between two different LOBs is about 0.6 (in our experience)
- But Reserve distribution correlation is typically lower.
- Trend structures for two LOBs typically different
- Parameter correlations low or zero
- See Private Passenger Automobile (PPA) versus Commercial Auto Liability (CAL) for Berkshire Hathaway below, for example




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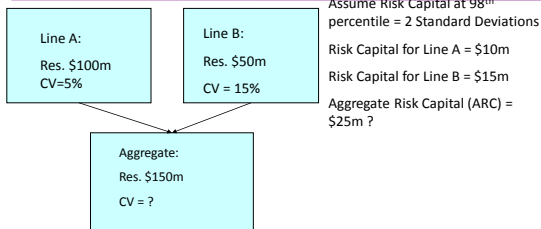
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## Risk Capital Allocation




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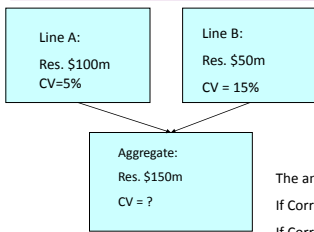
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### Risk Capital Allocation



Assume Risk Capital at 98<sup>th</sup> percentile = 2 Standard Deviations  
 Risk Capital for Line A = \$10m  
 Risk Capital for Line B = \$15m  
 Aggregate Risk Capital (ARC) = \$25m ?

The answer depends on the correlation.  
 If Corr = +1.0, ARC = \$25m  
 If Corr = 0.0 ARC = \$18m




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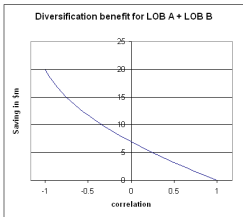
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### Risk Capital Allocation: Diversification benefit



Benefit =  
 Sum of individual risk capital assessments – aggregate risk capital assessment from *joint distribution* of the two (correlated) lines.




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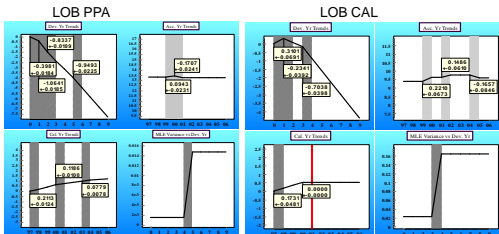
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### Berkshire Hathaway Schedule P 2006

No LOBs have the "same" trend structure and most LOBs have zero process correlation. Consider Private Passenger Automobile and Commercial Auto Liability



Note LOBs have very different trend structure and process variance




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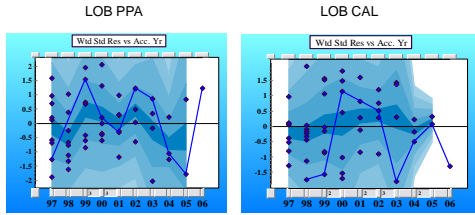
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Berkshire Hathaway Schedule P 2006



Note zero process correlation. Blue lines represent trace of calendar year 2006



Forecast lognormal distributions (and their correlations) for each future cell for each LOB based on an explicit forecast scenario

2003	3,202,436	1,525,975	1,107,844	413,244	194,092	104,193	66,317	2
	3,225,780	1,514,880	1,022,987	410,588	200,477	4,898	7,268	
2004	3,443,720	1,849,599	1,197,852	446,786	239,812	129,758	69,606	3
	3,380,493	1,663,027	1,130,227	426,291	11,342	7,137	9,202	
2005	3,714,166	1,783,247	1,294,820	552,022	296,359	159,145	86,048	4
	3,734,138	1,840,895	1,207,194	26,731	16,318	10,359	11,954	
2006	4,011,063	1,928,056	1,599,904	692,204	366,309	196,740	106,393	5
	4,008,908	2,020,277	79,001	39,777	23,877	14,966	15,494	
	Total Fitted/Paid		2007	2008	2009	2010	2011	
Cal. Per.	25,980,855		2,571,638	1,199,857	639,324	337,281	173,651	8
Total	25,985,528		97,713	65,891	37,506	25,895	19,419	1

1 Unit = \$1,000

Blue is observed, black is mean of lognormal and red is standard deviation of lognormal



Berkshire Hathaway Schedule P 2006  
One Year Time Horizon- "Change" in Ultimates conditional on next year's paid losses?

Related Question:

When do estimates of prior year ultimates stay consistent on updating (next valuation period) ?

No need for simulations! Why?

Acc. Yr	Mean		Standard Dev.	CV		Cond. on Next Cal. Per.	
	Outstanding	Ultimate		Outstanding	Ultimate	Std.Dev./Data	+Ult/Data
1999	404,928	7,341,745	96,842	0.24	0.01	54,108	80,437
2000	694,421	7,694,790	138,848	0.20	0.02	81,066	112,725
2001	1,169,064	8,854,591	187,454	0.16	0.02	113,677	149,051
2002	1,726,106	9,062,873	239,837	0.14	0.03	160,697	167,104
2003	2,512,700	7,324,908	245,950	0.12	0.03	190,312	165,797
2004	2,967,607	8,370,680	311,556	0.10	0.04	240,509	198,045
2005	4,895,219	10,343,879	608,057	0.10	0.05	336,132	382,302
2006	9,816,003	11,863,381	1,374,678	0.14	0.12	711,682	1,176,113
Total	23,898,022	80,529,585	2,040,345	0.09	0.03	1,172,221	1,670,001

1 Unit = \$1,000





Which risk characteristics of the data do calendar year payment stream distributions depend on?

1. Base development period trends, and
2. Calendar year trend assumptions for the future

Calendar Yr	Outstanding	Mean	Standard Dev.	CV	Cum. Payment as % of total
2007	7,622,060	1,089,829	0.14	31.89	
2008	5,917,632	819,307	0.12	62.89	
2009	3,489,890	389,994	0.11	67.48	
2010	2,486,601	292,826	0.12	77.81	
2011	1,733,841	236,258	0.14	85.06	
2012	1,202,066	192,478	0.16	90.09	
2013	797,860	128,916	0.16	93.27	
2014	595,861	107,367	0.21	95.38	
2015	351,672	65,873	0.24	96.86	
2016	250,232	67,360	0.27	97.80	
2017	182,100	56,547	0.31	98.68	
2018	127,728	43,278	0.34	99.20	




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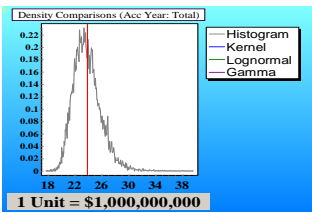
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Distribution of Aggregate Reserves=Sum of all lognormals for each future cell for each LOB



Note skewness (even) of aggregate distribution. Mean=23.9B, VaR at 95%=3.6B and T-VaR at 95%=5B




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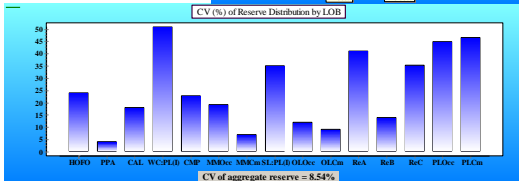
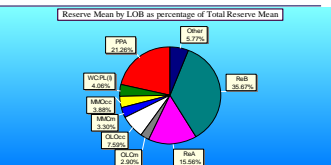
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Mean Reserve as a % of Total Reserve, and Reserve CV of aggregate versus CV for each LOB.

WC has largest CV but is not smallest LOB by Mean Reserve

ReA has large CV and is not so large proportion of total business.

Reserve distribution correlations essentially zero!



CV of aggregate less than most CVs




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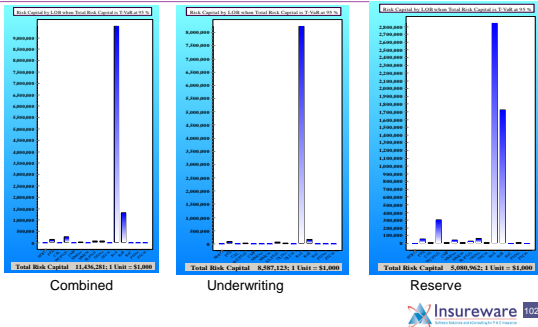
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Combined risk charge < underwriting risk charge + reserve risk charge,  
for any T-VaR . Why?




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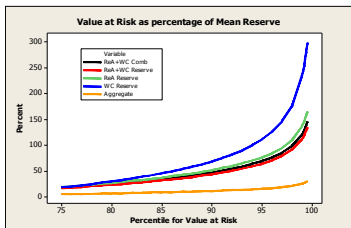
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### Berkshire Hathaway – Diversification Effects

This graph compares risk capital, calculated as Value at Risk and expressed as a percentage of the Forecast Mean Reserve. Only WC and ReA, two of the high CV lines are shown.



Blue = WC and Green = ReA Red = WC+ReA

Black = WC+ReA including next underwriting year.

Orange = Aggregate of all BH lines.

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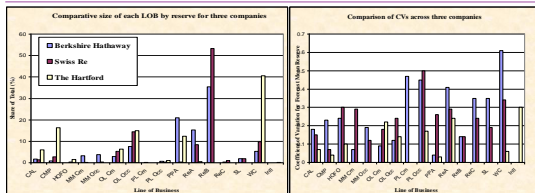
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### Berkshire Hathaway, Swiss Re and The Hartford, comparison by Line of Business



Higher Mean does not necessarily mean lower CV

For PPA, BH Mean > HF Mean, and CV BH > CV HF  
For OLOcc, Mean Hart > Mean Swiss Re > Mean BH & CV Hart > CV Swiss Re > CV BH

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WC for BH versus SR reserve distributions by calendar year

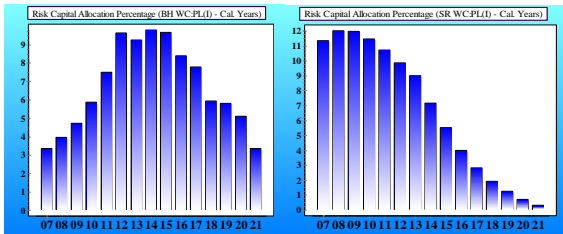
Calendar Yr Summary					Calendar Yr Summary				
Calendar Yr	Mean Outstanding	Standard Dev.	CV	Cum. Payment as % of total	Calendar Yr	Mean Outstanding	Standard Dev.	CV	Cum. Payment as % of total
2007	159,884	32,390	0.20	16.46	2007	276,468	77,826	0.28	21.62
2008	118,214	30,173	0.26	28.64	2008	221,282	77,816	0.35	39.47
2009	83,162	28,629	0.31	38.23	2009	177,917	76,607	0.42	63.68
2010	80,409	31,904	0.40	46.61	2010	143,055	71,663	0.50	64.82
2011	76,096	30,129	0.51	54.35	2011	115,136	67,018	0.58	74.05
2012	78,181	49,850	0.64	62.40	2012	92,834	62,235	0.67	81.41
2013	68,736	47,445	0.69	69.48	2013	74,967	67,661	0.77	87.35
2014	64,214	50,198	0.78	75.09	2014	55,040	45,899	0.83	91.71
2015	67,252	48,542	0.72	81.99	2015	39,168	35,277	0.90	94.81
2016	45,556	43,194	0.93	88.78	2016	28,129	25,878	0.99	96.88
2017	39,796	40,247	1.01	90.88	2017	17,101	18,737	1.10	98.24
2018	29,327	30,856	1.05	93.90	2018	10,828	13,282	1.23	99.10
2019	25,004	30,693	1.19	95.59	2019	6,479	9,065	1.40	99.61

Distributions of payment streams by calendar year are different

These depend on the base development period trends and the assumed forecast scenario calendar year trends.



Risk capital allocation (%) by calendar year for WC, BH versus SR



% allocation very different. It is based on a variance/covariance formula that is a function of three factors. 1. The future calendar year trend (mean and standard deviation thereof) assumption, 2. Development period trends and 3. process variance.



Summary

- All companies are different and no company is the same as the industry.
- Consistent estimates of prior year ultimates on updating can only be maintained within a sound modelling framework that incorporates calendar year parameters and assumptions about the future are explicit and auditable. The assumptions can easily be monitored on updating.
- Reserve distribution correlation is usually considerably less than process correlation.
- Segments of the same LOB such as 1. net of reinsurance and gross, 2. indemnity versus medical, 3. layers, for example, limited to 500K and limited to 1M; have common drivers and are highly correlated.
- Different LOBs very often do not have common drivers. That is, the trend structure (especially along calendar years) is not the same and process correlation is zero.
- A sound measurement of volatility and correlations (from the data) is essential to calculate risk capital allocation by LOB and calendar year, irrespective of capital risk measure.



Summary

- Combined reserve and underwriting risk charge < reserve risk charge + underwriting risk
- Mack and related methods can give answers that are wildly too low or too high and cannot capture the volatility in the data.
- The identified Probabilistic Trend Family (PTF) model measures the trends in the three directions and the distributions about the trend structure.
- In order to measure the process (volatility) correlation between two LOBs you need to first identify the PTF model for each LOB.



More information will be available at

**CLRS – 2012**  
**Insureware’s Hospitality Suite**  
 Sheraton Denver Downtown Hotel, Denver, CO  
 7:00pm-10:30pm  
 Wednesday September 5<sup>th</sup>  
 Thursday September 6<sup>th</sup>

(Also learn about the Bootstrap [technique](#) for testing validity of a model or method)

