Extreme development techniques

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Overview

- Background and motivation

- Walkthrough of specific methods
  - Incremental paid / incurred loss development method
  - Case reserve run-off method
  - Recursive method

What are extreme development techniques?

Extreme development techniques are methods that may be necessary in the following situations:

- Claims and exposure data are limited to nearly non-existent
- Traditional development patterns are not available
- Data are so mature that ultimate loss estimates are "extremely" volatile

Some of these methods are extensions of traditional development methods, while others are novel approaches to viewing loss development and projecting future claims.

When are extreme development techniques useful?

This session will discuss a number of examples of such extreme development methods and models that may be useful to actuaries who are modeling the following:

- Long-tailed lines of business
- Run-off portfolios
- Reinsurance liabilities

Incremental loss development method

- When is this method appropriate?
  - When reliable data are only available from a certain point in time onward (e.g., after a systems conversion)
  - When the liabilities are very mature, and paid-to-date or incurred-to-date measures are of limited value

- What data are needed?
  - Paid losses from a fixed point in time forward
  - Incurred losses from a fixed point in time forward
  - Case reserve at date

Data preparation

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method

- Calculate the change in paid loss based on the incremental paid triangle
  - Assumption: evaluated as of 31 December 2010
  - The following triangle is the incremental paid/loss triangle;
    we are going to calculate the incremental paid/loss development factors based on this triangle

A few more ages are not shown here due to limited room.

	Age (yrs)																		
U/W Year	12	13	14	15	16	17	18	19	20	21	22	27	28	29	30	31	32	33	34
1977								2,811,530	2,482,581	1,551,050	24,397	(10,000)	73,910	0	29,900	30,528	928	221	2
1978							5,302,785	2,773,356	3,971,550	1,327,150	355,550	65,604	38,706	16,950	0	106,000	21,220	438	
1979						7,286,341	1,020,570	1,018,529	682,414	1,312,383	419,963	0	36,550	27,932	1,922	823	2,201		
1980					13,738,448	11,320,482	2,662,400	5,516,100	1,695,950	(50,091)	(39,171)	42,192	2,102	1,821	3,105	920			
1981				7,241,050	6,012,428	1,785,059	525,718	401,611	261,705	758,351	722,135	4,550	10,291	0	3,910				
1982			3,825,050	1,710,305	1,361,162	3,656,080	4,814,300	533,656	338,776	216,700	216,691	523	1,190	949					
1983		6,709,700	3,808,744	2,609,950	2,602,120	1,386,939	5,233,688	4,960,051	170,624	26,350	73,799	120,192	201						
1984	5,161,750	5,784,645	4,606,044	4,573,758	836,374	128,119	239,651	430,221	220,731	81,321	101,293	2,120							

Select incremental development factors

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method

End Age																		
U/W Year	13	14	15	16	17	18	19	20	21	22	27	28	29	30	31	32	33	34
1977								0.883	0.625	0.016	2.323	(7.391)	0.000	0.000	1.021	0.030	0.238	0.009
1978							0.523	1.432	0.334	0.268	1.866	0.590	0.438	0.000	0.000	0.200	0.021	
1979						0.140	0.998	0.670	1.923	0.320	1.923	0.000	0.764	0.069	0.428	2.674		
1980					0.824	0.235	2.072	0.307	(0.030)	0.782	(6.510)	0.050	0.866	1.705	0.296			
1981				0.830	0.297	0.295	0.764	0.652	2.898	0.952	0.317	2.262	0.000					
1982			0.447	0.796	2.686	1.317	0.111	0.635	0.640	1.000	0.559	2.275	0.797					
1983		0.568	0.685	0.997	0.533	3.774	0.948	0.034	0.154	2.801	0.119	0.002						
1984	1.121	0.796	0.993	0.183	0.153	1.871	1.795	0.513	0.368	1.246	0.051							
Wtd Average	1.121	0.673	0.727	0.670	0.744	0.567	0.790	0.533	0.532	0.359	1.145	0.567	0.293	0.108	0.924	0.177	0.030	0.009
Straight Avg	1.121	0.682	0.708	0.702	0.899	1.272	1.030	0.641	0.864	0.923	0.081	(0.369)	0.478	0.591	0.582	0.968	0.129	0.009
Straight Avg Ex H/L	1.121	0.682	0.685	0.813	0.551	0.929	1.006	0.610	0.674	0.761	0.806	0.726	0.500	0.069	0.428	0.200	0.129	0.009

Select		0.682	0.708	0.813	0.712	0.751	1.006	0.641	0.864	0.761	0.806	0.567	0.500	0.591	0.582	0.200	0.129	0.000
	13	14	15	16	17	18	19	20	21	22	23	28	29	30	31	32	33	34
Incremental Pattern	1.000	0.682	0.483	0.393	0.280	0.210	0.211	0.135	0.117	0.089	0.072	0.016	0.008	0.005	0.003	0.001	0.000	0.000
Accumulated Values	**1.000**	**1.682**	**2.165**	**2.558**	**2.838**	**3.048**	**3.259**	**3.394**	**3.511**	**3.600**	**3.672**	**3.847**	**3.855**	**3.859**	**3.862**	**3.863**	**3.863**	**3.863**

Curve fitting

We fitted x and y values into different distributions (e.g., Weibull, Gompertz and Richards model) to get the coefficients.

Age (in months)	X = Age (in years)	Actual Y = Accumulated incremental selections
144	12	1.000
156	13	1.682
168	14	2.165
180	15	2.558
192	16	2.838
204	17	3.048
216	18	3.259
228	19	3.394
240	20	3.511
252	21	3.600
264	22	3.672
276	23	3.726
288	24	3.766
300	25	3.802
312	26	3.831
324	27	3.847
336	28	3.855
348	29	3.859
360	30	3.862
372	31	3.863
384	32	3.863
396	33	3.863
408	34	3.863

From curve fitting software

Weibull model: $y = a - b \times e^{-cx^d}$.	
Coefficient Data:	
a =	3.870
b =	20.470
c =	0.058
d =	1.423
Standard error:	0.0213885
Correlation coefficient:	0.999683

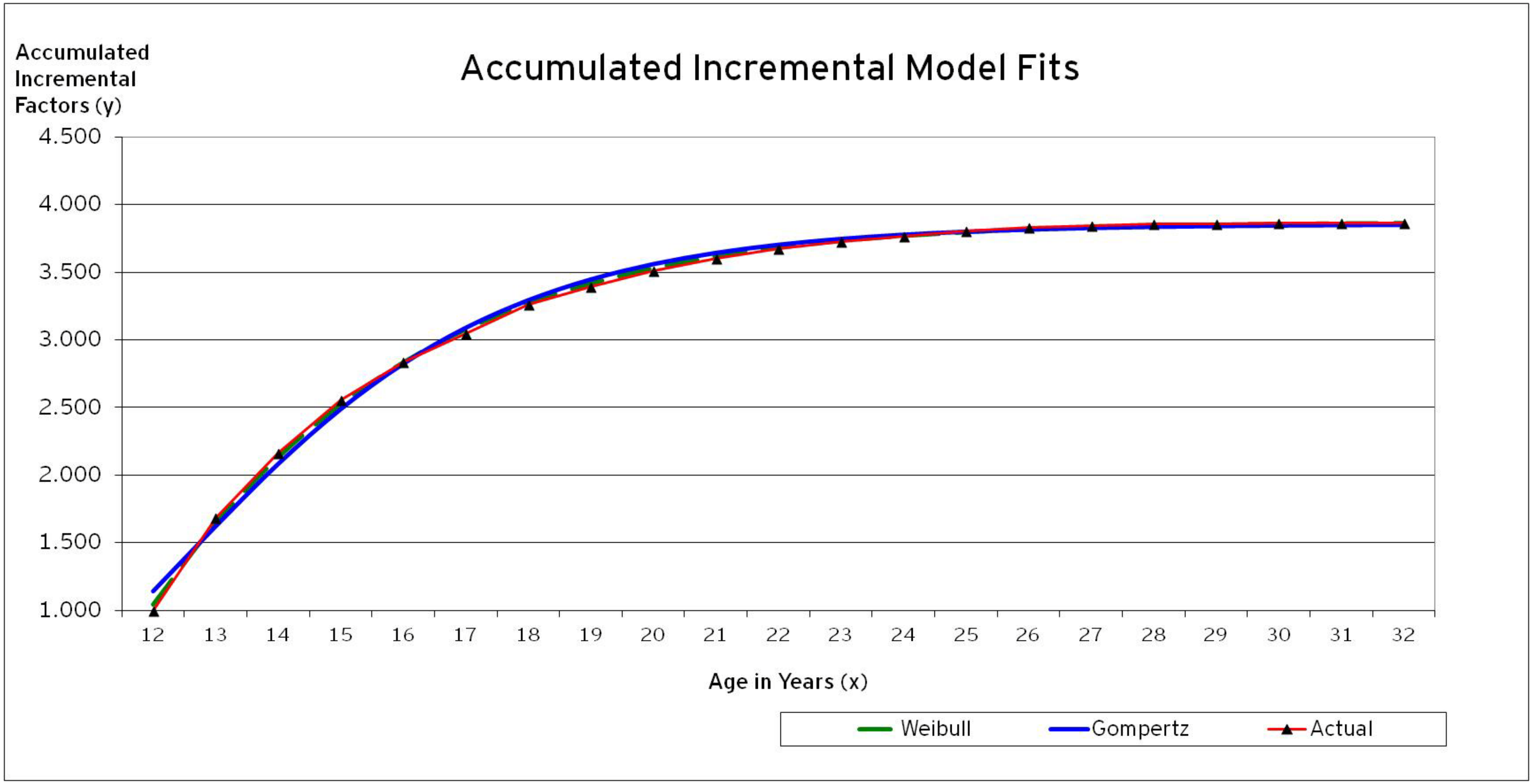
Gompertz relation: $y = a \times e^{-e^{b-cx}}$.	
Coefficient data:	
a =	3.854
b =	4.284
c =	0.341
Standard error:	0.0494986
Correlation coefficient:	0.9982117

This column is from the triangle on page 8.

Weibull $y = a - b \times e^{-cx^d}$	Gompertz $y = a \times e^{-e^{b-cx}}$
1.046	1.141
1.646	1.621
2.133	2.081
2.523	2.486
2.834	2.822
3.078	3.087
3.269	3.292
3.416	3.445
3.530	3.558
3.617	3.641
3.682	3.701
3.732	3.745
3.769	3.776
3.796	3.798
3.817	3.814
3.832	3.826
3.842	3.834
3.850	3.839
3.856	3.844
3.860	3.847
3.863	3.849
3.865	3.850
3.866	3.851

Accumulated incremental paid ratio model selection

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method



Incremental ratios calculation and reserve projection

- Calculate the incremental loss development ratio to ultimate development based on curve fit, and estimate the total reserves.

(1)	(2)	(3)	(4)	(5)	(6)	Weibull		Ratio to total period change	Estimated total reserves
U/W year	Start age	End age	Total paid	Total paid	Total change	(7) Accumulated incremental (at start)	(8) Accumulated incremental (at end)	(9) $\frac{Ult-(8)}{(8)-(7)}$	(10)
			At start age	*At end age*	*From start age to end age*				(6) * (9)
1977	19	34	2,811,530	7,131,041	4,319,511	3.416403	3.866466	0.007409	32,004
1978	18	33	5,302,785	15,012,037	9,709,252	3.268574	3.865007	0.008037	78,029
1979	17	32	7,286,341	12,634,556	5,348,215	3.077762	3.862942	0.008735	46,714
1980	16	31	13,738,448	36,226,919	22,488,471	2.833444	3.860034	0.009514	213,947
1981	15	30	7,241,050	18,501,792	11,260,742	2.523254	3.855958	0.010386	116,957
1982	14	29	3,825,050	19,294,363	15,469,313	2.132930	3.850278	0.011367	175,847
1983	13	28	6,709,700	27,847,579	21,137,879	1.646396	3.842404	0.012475	263,702
1984	12	27	5,161,750	22,455,375	17,293,625	1.046024	3.831549	0.013732	237,477
Total			52,076,654	159,103,662	107,027,008	Ultimate:	**3.869800**		1,164,676

Ultimate value = 3.869800

According to the Weibull model, $y = a - b \times e^{-cx^d}$; when $x \to \infty, y \to a = 3.869800$

Incremental ratio for U/W Yr 1984: $\frac{3.869800 - 3.831549}{3.831549 - 1.046024} = \mathbf{0.013732}$

Estimated unpaid reserve for U/W Yr 1984: $0.013732 \times \$17{,}293{,}625 = \mathbf{\$237{,}477}$

Case reserve run-off method

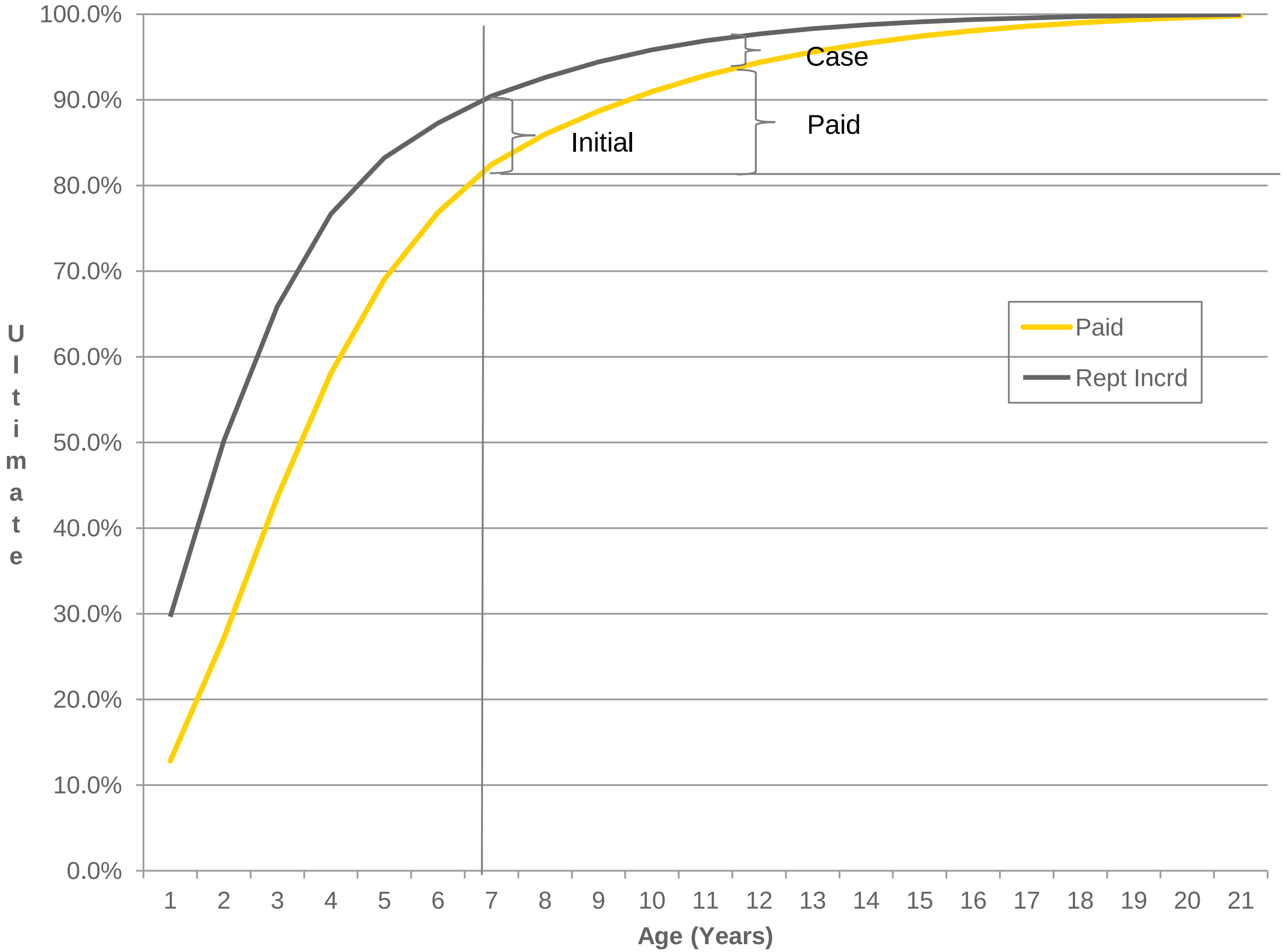
- When is this method appropriate?
  - When there is a long history of incremental paid / incurred losses
  - When the incremental activity is more significant than in cases where incremental method may be more appropriate

- What data are needed?
  - Incremental paid / incurred losses
  - Cumulative incurred loss from start age

Case reserve run-off method

Case reserve run-off method

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method



Case reserve run-off method

Case run-off ATA factor

Case to case-reserve-run-off ratio

1. Selected IBNR ratio to OLR
2. Estimated IBNR

Recursive method

- When is this method appropriate?
  - When only incremental loss data are available
  - When we assume the relationship of $\frac{\Delta P}{\Delta C}$ is consistent as the exposure approaches ultimate
  - When only aggregate calendar year losses for all exposure years are available, and particularly when all years are very mature
- What data are needed?
  - Incremental paid / incurred losses
  - Change in case reserves

Recursive method

Data

- Beginning Case Reserves
- Ending Case Reserves
- Cumulative Paid Loss

Differences

- Change in Case Reserves
- Incremental Paid Loss

Unpaid Reserves Estimation

- Calculate and select ΔP/ΔC
- Project from case outstanding
- Net paid since date

Questions?

1. Incremental paid/incurred loss development method
2. Case reserve run-off method
3. Recursive method