Applying Stochastic Reserving Models under a Common Flexible Framework

Presented by
Iva Yuan
FCAS, MAAA

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General Stochastic Framework

- Let A_{ij} = incremental averages in each cell
- By law of large numbers, average is asymptotically Normal
- So assume averages are independent and come from a Gaussian (Normal) distribution with:

$$\mathrm{E}\left(A_{ij}\right) = g_{ij}\left(\boldsymbol{\theta}\right)$$

$$\mathrm{Var}\left(A_{ij}\right) = e^{\kappa - e_i}\left(\mathrm{E}\left(A_{ij}\right)^2\right)^p$$

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Maximum Likelihood Estimation

- Maximum Likelihood Estimators
  - Asymptotically Normal
  - Asymptotically Unbiased
  - Consistent and Sufficient
  - Doesn't need to be linearized
  - Does require distribution assumption

- Minimize negative log likelihood function

$$\ell\left(A_{11}, A_{12}, \ldots, A_{n1}; \boldsymbol{\theta}, \kappa, p\right) =$$

$$\sum \frac{\kappa - e_i + \ln\left(2\pi\left(g_{ij}(\boldsymbol{\theta})^2\right)^p\right)}{2} + \frac{\left(A_{ij} - g_{ij}(\boldsymbol{\theta})\right)^2}{2e^{\kappa - e_i}\left(g_{ij}(\boldsymbol{\theta})^2\right)^p}$$

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Results Without Parameter Uncertainty

- Future amounts for each period *i:*

$$R_i \sim \mathrm{N}\left(E_i \sum_{j=n-i+2}^{n} g_{ij}\left(\hat{\boldsymbol{\theta}}\right), E_i^2 \sum_{j=n-i+2}^{n} e^{\hat{\kappa} - e_i}\left(g_{ij}\left(\hat{\boldsymbol{\theta}}\right)^2\right)^{\hat{p}}\right)$$

- Future total

$$R_T \sim \mathrm{N}\left(\sum_{i=1}^{m} E_i \sum_{j=n-i+2}^{n} g_{ij}\left(\hat{\boldsymbol{\theta}}\right), \sum_{i=1}^{m} E_i^2 \sum_{j=n-i+2}^{n} e^{\hat{\kappa} - e_i}\left(g_{ij}\left(\hat{\boldsymbol{\theta}}\right)^2\right)^{\hat{p}}\right)$$

- Note: No parameter uncertainty accounted for, only process uncertainty here

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Accounting for Parameter Uncertainty

- Assuming <u>parameters</u> have a multi-variate Gaussian distribution with:
  - Mean equal to maximum likelihood estimators
  - Variance-covariance matrix equal to inverse of Fisher Information matrix
- Posterior distribution through simulation
  - Randomly select parameters from multi-variate Gaussian distribution
  - Simulate R_i^* (future amounts) from appropriate Gaussian distribution

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Specific Model Parametrization

- Berquist Sherman

$$g_{ij}(\boldsymbol{\theta}) = \theta_j e^{i\theta_{n+1}}$$

  - Think: Average times Trend
  - θ_j s are average incrementals for 0^{th} exposure period at age j
  - θ_{n+1} is the natural log of the annual trend in the data

- Hoerl Curve

$$g_{ij}(\boldsymbol{\theta}) = \exp\left(\theta_1 + \theta_2 j + \theta_3 j^2 + \theta_4 \ln(j) + i\theta_5\right)$$

  - Curve fit with trend taken in account
  - Expected amounts must be positive in all cells

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Specific Model Parametrization

- Cape Cod

$$g_{ij}(\boldsymbol{\theta}) = \begin{cases} \theta_1 \text{ if } i = j = 1 \\ \theta_1\theta_i \text{ if } j = 1 \text{ and } i > 1 \\ \theta_1\theta_{m+j-1} \text{ if } i = 1 \text{ and } j > 1 \\ \theta_1\theta_i\theta_{m+j-1} \text{ if } i > 1 \text{ and } j > 1 \end{cases}$$

- Think: Exposure period parameters times lag parameters
- θ_1 is the scale parameter
- θ_i for i = 2, …, n is change from one accident year to the next
- θ_i for i = n+1, …, m+n-1 incremental change from one lag period (age of development) to the next

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Specific Model Parametrization

- Chain Ladder

$$g_{ij}(\boldsymbol{\theta}) = \begin{cases} P_1\theta_j \text{ if } j < n \text{ and } i = 1 \\ P_1\left(1 - \sum_{k=1}^{n-1}\theta_k\right) \text{ if } j = n \text{ and } i = 1 \\ \dfrac{P_i\theta_j}{\sum_{k=1}^{n-i}\theta_k} \text{ if } j < n \text{ and } i \neq 1 \\ \dfrac{P_i}{\sum_{k=1}^{n-i}\theta_k}\left(1 - \sum_{k=1}^{n-1}\theta_k\right) \text{ if } j = n \text{ and } i \neq 1 \end{cases}$$

  - Actual average paid to date for exposure period $i = P_i$
  - think θ_j as percent emerged at age j
  - Condition: expected amount to date = actual

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More

- For more, go to “Stochastic Reserving Today and (Mostly) Tomorrow”
- Hayne, Roger, “A Stochastic Framework for Incremental Average Reserve Models,” Casualty Actuarial Society Forum, Fall 2008, pp. 174-195
- Update recently submitted paper to Variance (review process)

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