Two Symmetric Families of Loss Reserving Methods

2012 CLRS – Denver

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TOWERS WATSON

Outline

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- Outline
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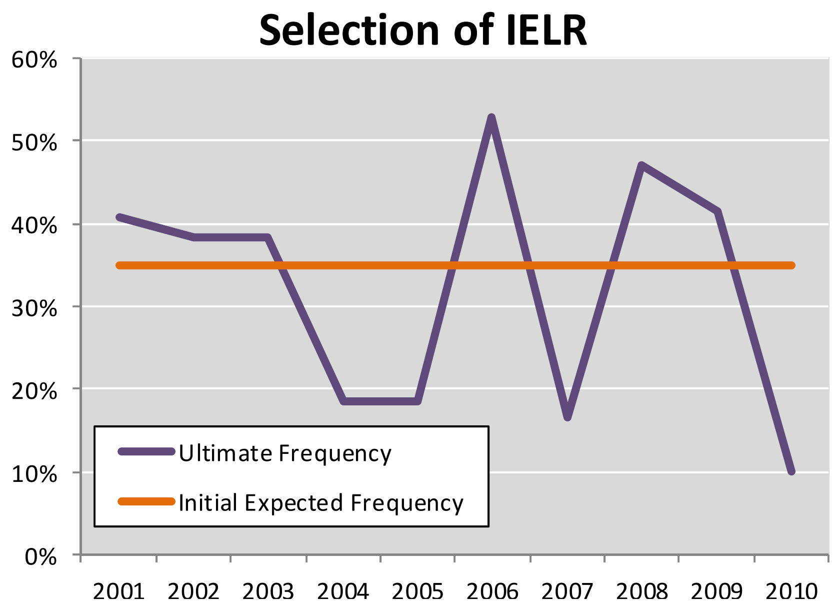
Data

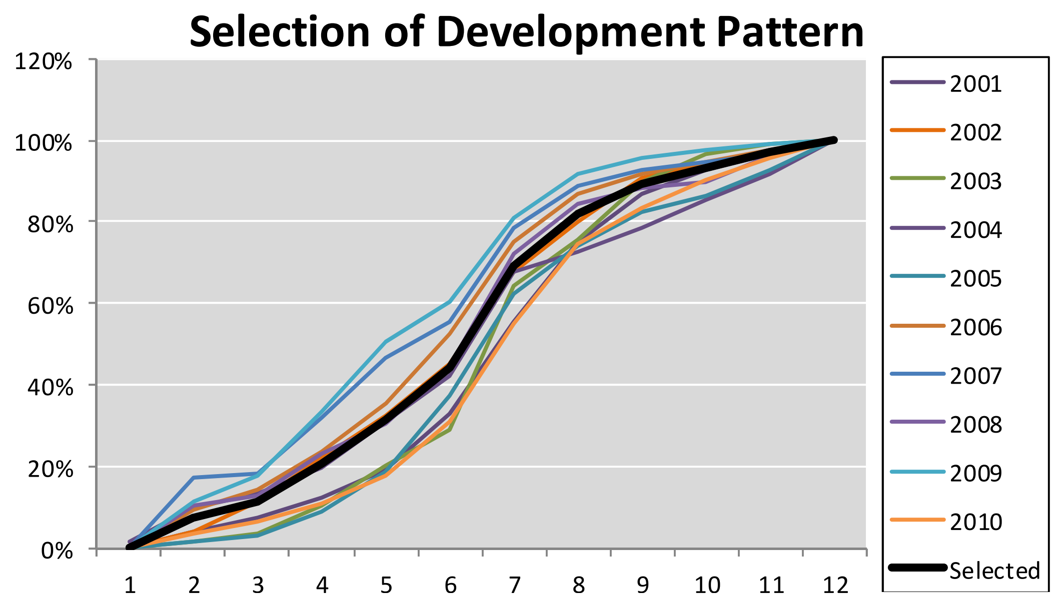
[redacted]

Federal Crop Insurance Program – Texas – Frequency

Year	Total Policies	Policies Indemnified	Ultimate Frequency	Init. Exp. Frequency	Policies Indemnified by Month: Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2001	232	95	40.7%	35.0%	0	4	7	12	18	31	53	71	82	88	91	95
2002	225	86	38.3%	35.0%	0	4	10	19	28	39	58	69	78	81	84	86
2003	228	87	38.4%	35.0%	0	1	3	9	17	25	56	66	78	84	86	87
2004	207	38	18.5%	35.0%	1	4	5	8	12	16	26	28	30	33	35	38
2005	194	36	18.4%	35.0%	0	1	1	3	7	13	22	26	29	31	33	36
2006	196	104	52.8%	35.0%	0	10	15	24	37	55	78	90	95	98	101	104
2007	226	37	16.5%	35.0%	0	6	7	12	17	21	29	33	35	35	36	37
2008	245	115	46.9%	35.0%	0	12	15	27	35	51	83	97	102	104	111	115
2009	237	98	41.5%	35.0%	0	11	17	33	50	60	80	90	94	96	97	98
2010	203	21	10.1%	35.0%	0	1	1	2	4	6	11	15	17	18	20	21
Total	2,194	718	32.7%	35.0%	1	53	81	149	225	318	497	586	641	669	696	718

Month	Pattern
1	0%
2	7%
3	11%
4	21%
5	31%
6	44%
7	69%
8	82%
9	89%
10	93%
11	97%
12	100%



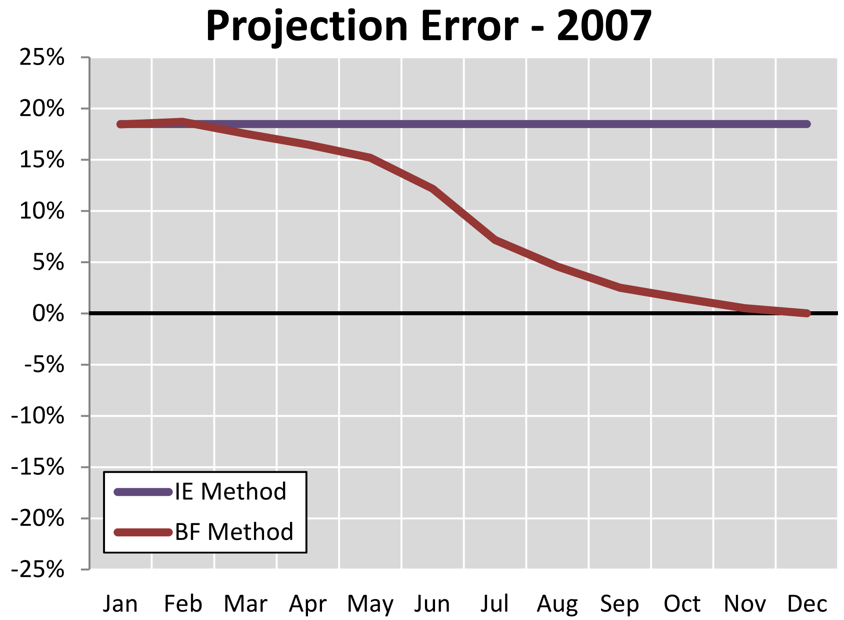


The Actual vs. Expected Family

(1) As an alternative to a fixed IELR

The trouble with a fixed IELR

- Consider the projection error over time for the…
  - Initial Expected (IE) method $U_{IE} = U_0$
  - Bornhuetter-Ferguson (BF) method $U_{BF} = C_k + (1 - p_k)U_0$



A (natural) way to adjust the fixed IELR

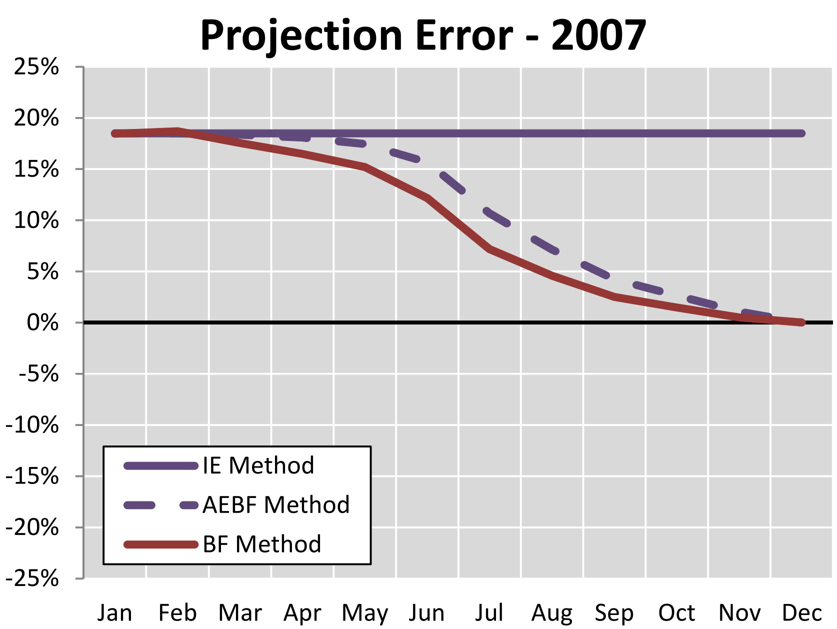
- The Actual vs. Expected (AE) Family
  - The general formulation

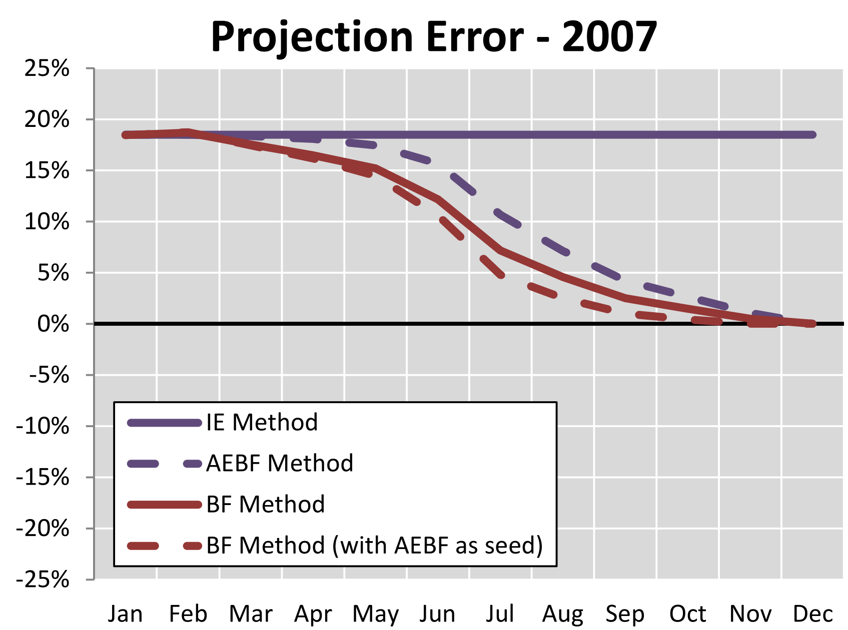
$$U_{AEi} = \underbrace{U_0}_{\text{Initial Expectation}} + \underbrace{w_i}_{\text{Weight}} \underbrace{(C_k - p_k U_0)}_{\text{Actual vs. Expected Adjustment}}$$

  - A specific member

$$U_{AEBF} = U_0 + p_k (C_k - p_k U_0)$$

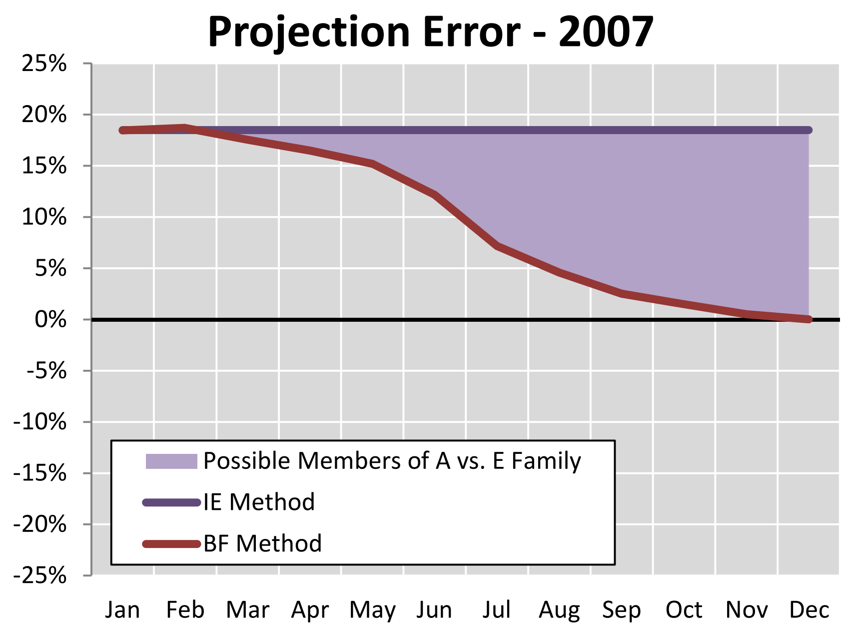
$$\hat{U}_{BF} = C_k + (1 - p_k) U_{AEBF}$$





But what is the best choice for the weight function

- Recall the general formulation $U_{AEi} = U_0 + w_i(C_k - p_k U_0)$
- Let's first establish some bounds $w_i \in [0,1]$
- Which leads us to the following possible members of the AE Family



- Note that the IE and BF methods are both members at the extremes

$$\text{If } w_i = 0 \text{ then } U_{AEi} = U_0 + w_i(C_k - p_k U_0) = U_0 + 0(C_k - p_k U_0) = U_0$$

$$\text{If } w_i = 1 \text{ then } U_{AEi} = U_0 + w_i(C_k - p_k U_0) = U_0 + 1(C_k - p_k U_0) = C_k + (1 - p_k)U_0$$

But that isn't really that useful (lets do some mathamagic)

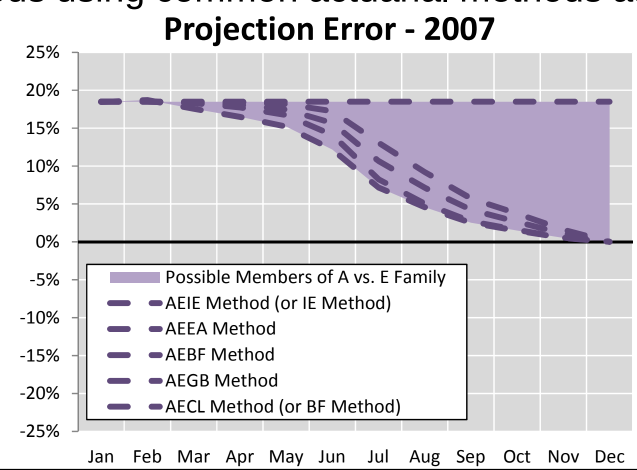
- Consider the alternative [credibility] formulation of the AE Family

$$U_{AEi} = p_k U_i + (1-p_k)U_0$$

- Plug-in an actuarial method and out pops an AE method

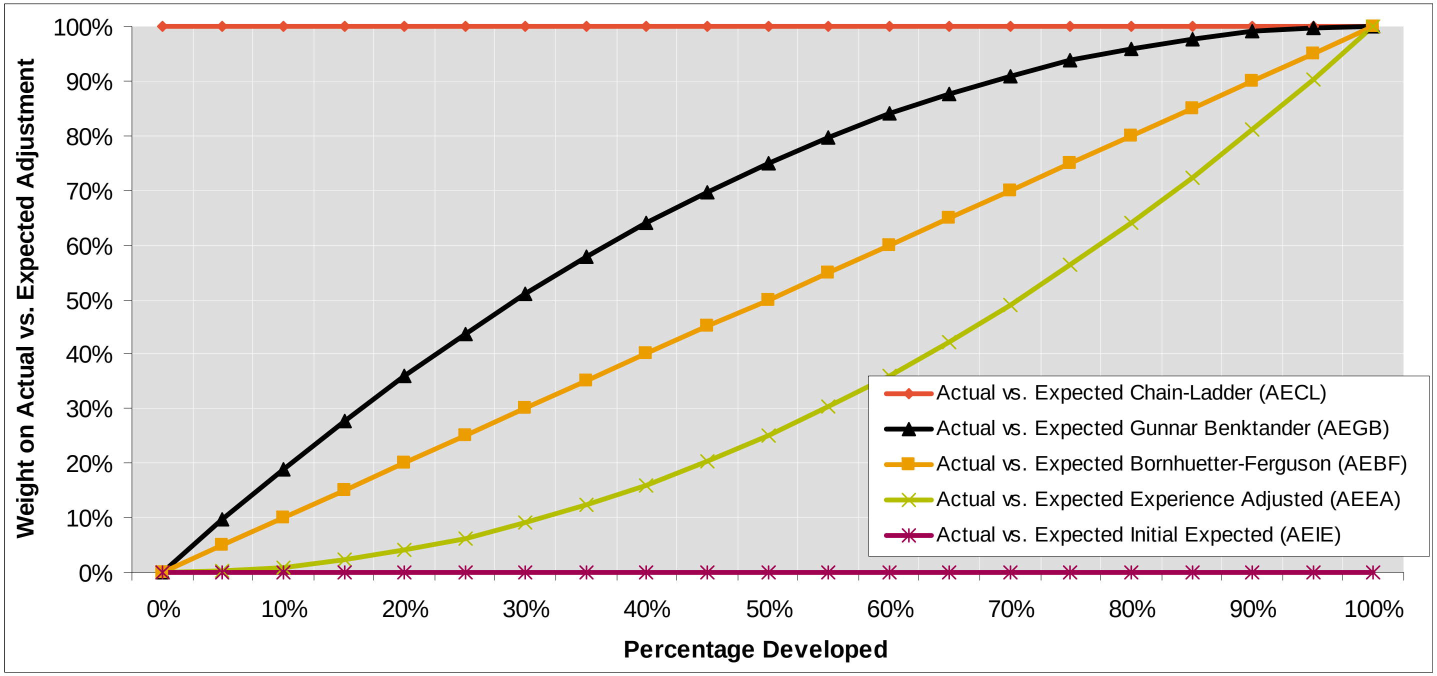
$$\begin{aligned}
U_{AEi} &= p_k U_i + (1-p_k)U_0 \\
U_{AEBF} &= p_k U_{BF} + (1-p_k)U_0 \\
&= p_k\left[C_k + (1-p_k)U_0\right] + (1-p_k)U_0 \\
&= p_k C_k + p_k U_0 - p_k^2 U_0 + U_0 - p_k U_0 \\
&= p_k C_k - p_k^2 U_0 + U_0 \\
&= U_0 + p_k\left(C_k - p_k U_0\right)
\end{aligned}$$

- And here are five AE methods using common actuarial methods as the plug-in



Method	Credibility Formulation	Actual vs. Expected Formulation	Weight Function
AEIE	$U_{AEIE} = p_k U_{IE} + (1-p_k)U_0$	$U_{AEIE} = U_0 + 0(C_k - p_k U_0) \Rightarrow U_{IE}$	0
AEEA	$U_{AEEA} = p_k U_{EA} + (1-p_k)U_0$	$U_{AEEA} = U_0 + p_k^2(C_k - p_k U_0)$	p_k^2
AEBF	$U_{AEBF} = p_k U_{BF} + (1-p_k)U_0$	$U_{AEBF} = U_0 + p_k(C_k - p_k U_0) \Rightarrow U_{EA}$	p_k
AEGB	$U_{GB} = p_k U_{GB} + (1-p_k)U_0$	$U_{AEGB} = U_0 + (2p_k - p_k^2)(C_k - p_k U_0)$	$2p_k - p_k^2$
AECL	$U_{AECL} = p_k U_{CL} + (1-p_k)U_0$	$U_{AECL} = U_0 + 1(C_k - p_k U_0) \Rightarrow U_{BF}$	1

Why these five methods (because they form a pretty spectrum)



The Generalized Actual vs. Expected Family

(2) As a solution to the year-end roll-forward dilemma

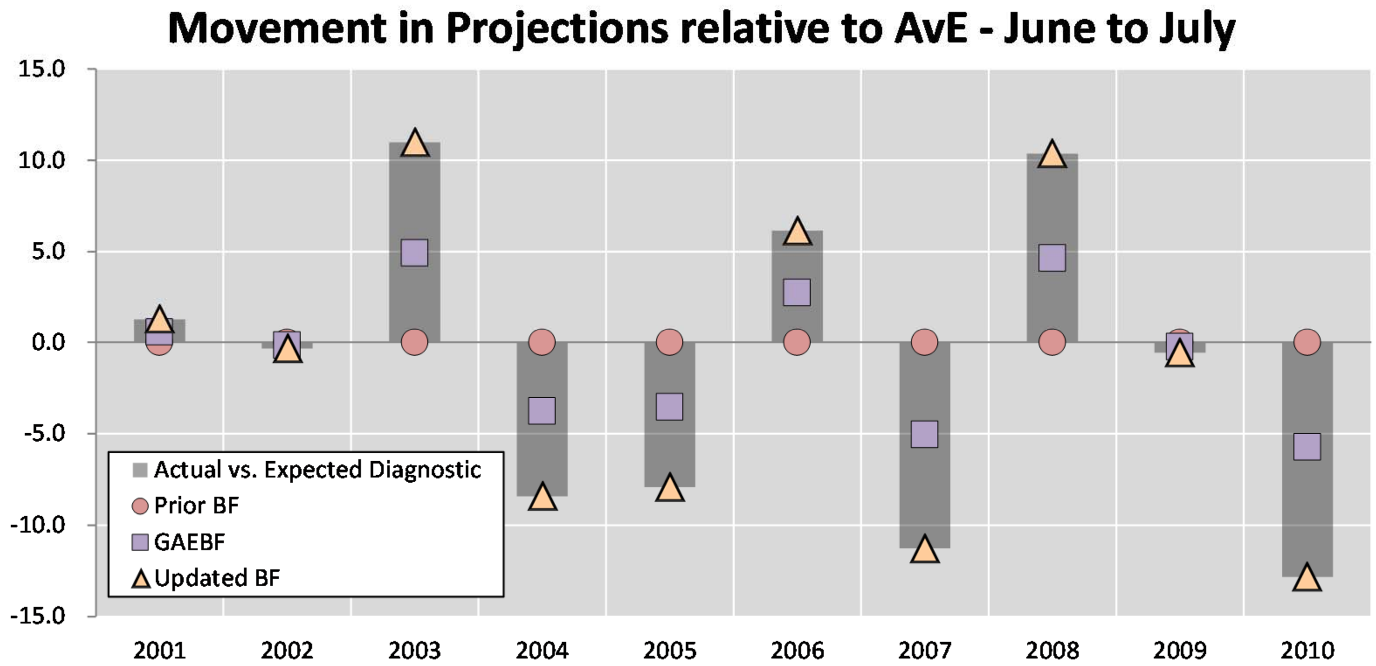
The Generalized Actual vs. Expected Family

- The trouble with year-end roll-forwards at Lloyd's (and elsewhere)
- The Generalized Actual vs. Expected Family

$$U_{AEi} = U_0 + w_i\left(C_k - p_k U_0\right)$$

$$U_{GAEBF} = U_k = U_{k-1} + \left(\frac{p_k - p_{k-1}}{1 - p_{k-1}}\right)\left(\left(C_k - C_{k-1}\right) - \left(\frac{p_k - p_{k-1}}{1 - p_{k-1}}\right)\left(U_{k-1} - C_{k-1}\right)\right)$$

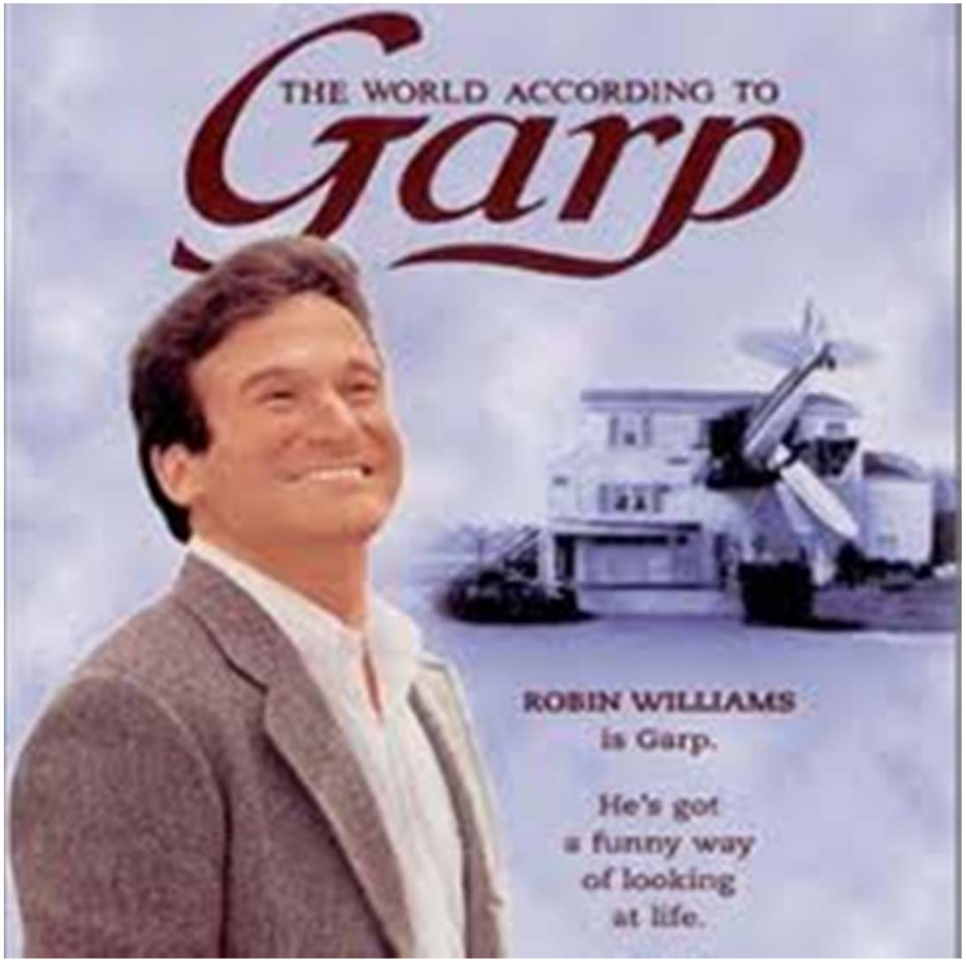
- And how does it work in practice



The Mean-Reverting Family

Or Garp’s Method

Garp

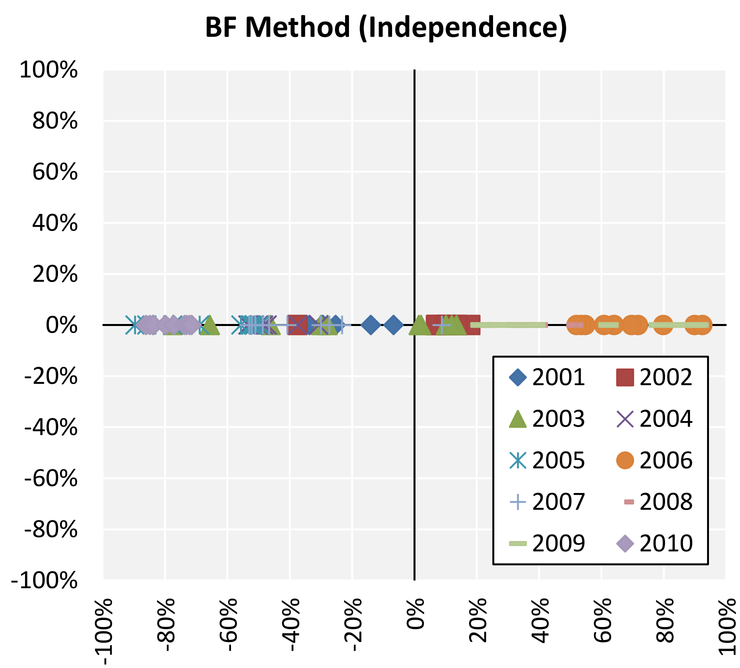

THE WORLD ACCORDING TO
Garp
ROBIN WILLIAMS
is Garp.
He's got
a funny way
of looking
at life.

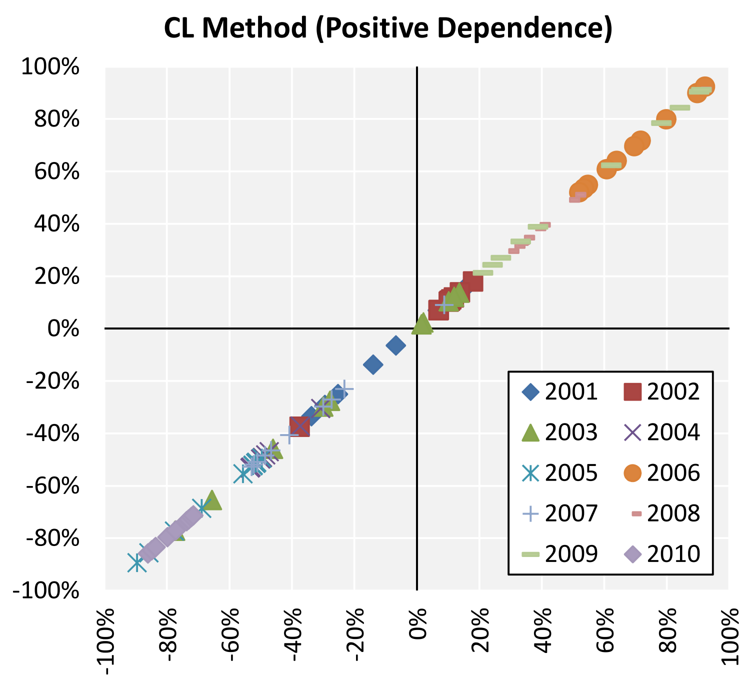
Let's talk about dependence

- Based on our prior expectations…

$$p_k U_0 \quad | \quad (1-p_k)U_0$$

- …how do we update future expectations based on experience (AvE vs. $\hat{E}$vE)





But is it possible that losses could, sometimes, maybe exhibit a degree of negative dependence

- Sure, why not. Here are some examples:
  - Crop
  - Extended Warranty
  - Construction Defect
  - Credit Disability

- Any line where the occurrence (or absence) of an event decreases (or increases) the likelihood of a future event

So what do we do – a (natural) way to introduce negative dependence

- General Formulation of the Mean-Reverting Family

Weight

$$U_{MRi} = U_i - w_i \left(C_k - p_k U_0 \right)$$

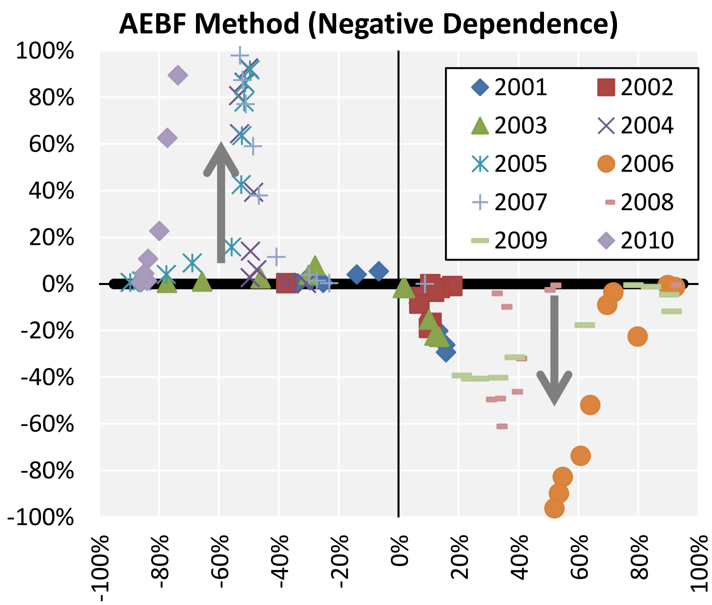
Unadjusted Method

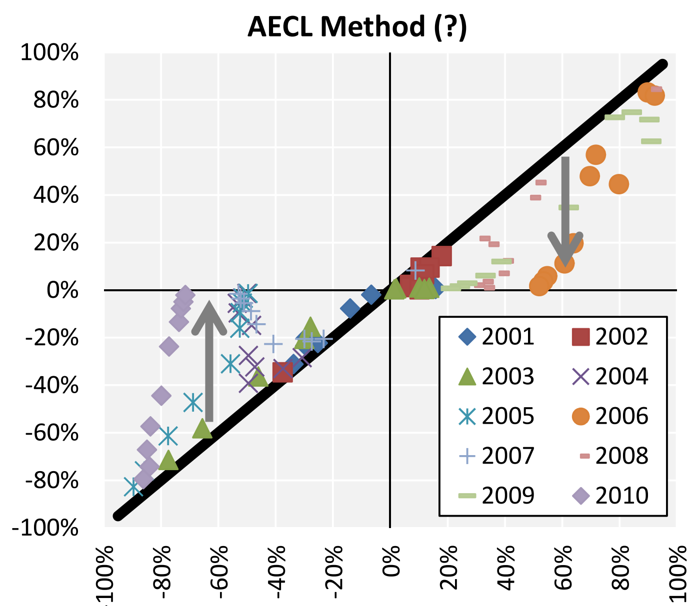
Actual vs. Expected Adjustment

- Specific Formulations

$$U_{MRBF} = U_{BF} - p_k \left(C_k - p_k U_0 \right)$$

$$U_{MRCL} = U_{CL} - 1 \left(C_k - p_k U_0 \right)$$





Absolute vs. Relative Mean-Reversion

- Unadjusted Methods

Method	Outstanding Reserve	Dependence
CL Method	$U_0(1-p_k)+\left(\frac{1-p_k}{p_k}\right)(C_k-p_kU_0)$	$p_k \in (0,1] \Rightarrow \textit{Positive}$
BF Method	$U_0(1-p_k)+(0)(C_k-p_kU_0)$	$p_k \in (0,1] \Rightarrow \textit{Independent}$

- Adjusted Methods

Method	Outstanding Reserve	Dependence
MRCL Method	$U_0(1-p_k)+\left(\frac{1-2p_k}{p_k}\right)(C_k-p_kU_0)$	$p_k \in \begin{cases}(0,0.5) & \Rightarrow \textit{Positive} \\ 0.5 & \Rightarrow \textit{Independent} \\ (0.5,1] & \Rightarrow \textit{Negative}\end{cases}$
MRBF Method	$U_0(1-p_k)-(p_k)(C_k-p_kU_0)$	$p_k \in \{(0,1] \quad \Rightarrow \textit{Negative}$

Are there any other members of the Mean-Reverting (MR) family?

- Consider the alternative [credibility] formulation of the MR family

$$U_{MRi} = p_k U_0 + (1 - p_k) U_i$$

- Plug in an actuarial method and out pops a member of the MR family

$$\begin{aligned} U_{MRi} &= p_k U_0 + (1 - p_k) U_i \\ U_{AEBF} &= p_k U_0 + (1 - p_k) U_{BF} \\ &= p_k U_0 + (1 - p_k)\left[C_k + (1 - p_k) U_0\right] \\ &= p_k U_0 + C_k + U_0 - p_k U_0 - p_k C_k - p_k U_0 + p_k^2 U_0 \\ &= U_{BF} - p_k C_k + p_k^2 U_0 \\ &= U_{BF} - p_k \left(C_k - p_k U_0\right) \end{aligned}$$

- And here are five AE methods using common actuarial methods as the plug-in

Method	Credibility Formulation	Actual vs. Expected Formulation	Weight Function
MRIE	$U_{MRIE} = p_k U_0 + (1 - p_k) U_{IE}$	$U_{MRIE} = U_{IE} - 0(C_k - p_k U_0)$	0
MREA	$U_{MREA} = p_k U_0 + (1 - p_k) U_{EA}$	$U_{MREA} = U_{AE} - p_k^2 (C_k - p_k U_0)$	p_k^2
MRBF	$U_{MRBF} = p_k U_0 + (1 - p_k) U_{BF}$	$U_{MRBF} = U_{BF} - p_k (C_k - p_k U_0)$	p_k
MRGB	$U_{MRGB} = p_k U_0 + (1 - p_k) U_{GB}$	$U_{MRGB} = U_{GB} - (2p_k - p_k^2)(C_k - p_k U_0)$	$2p_k - p_k^2$
MRCL	$U_{MRCL} = p_k U_0 + (1 - p_k) U_{CL}$	$U_{MRCL} = U_{CL} - 1(C_k - p_k U_0)$	1

Note the symmetry (and hence the name of the paper)

- Symmetry of the General Formulations

$$U_{AEi} = U_0 + w_i\left(C_k - p_k U_0\right)$$

$$U_{MRi} = U_i - w_i\left(C_k - p_k U_0\right)$$

- Symmetry of the Credibility Formulations

$$U_{AEi} = p_k U_i + (1 - p_k)U_0$$

$$U_{MRi} = p_k U_0 + (1 - p_k)U_i$$

The Adjusted Mean-Reverting Family

Fixing a problem…

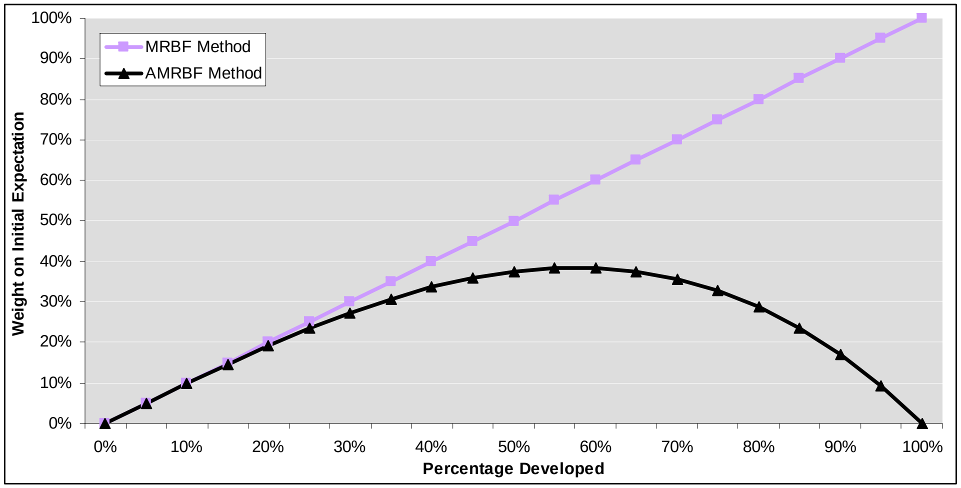
Whoops…

- A problem with the Mean-Reverting Family…

$$\text{As } p_k \to 100\% \text{ then } U_{MRi} = p_k U_0 + (1-p_k)U_i \to U_0$$

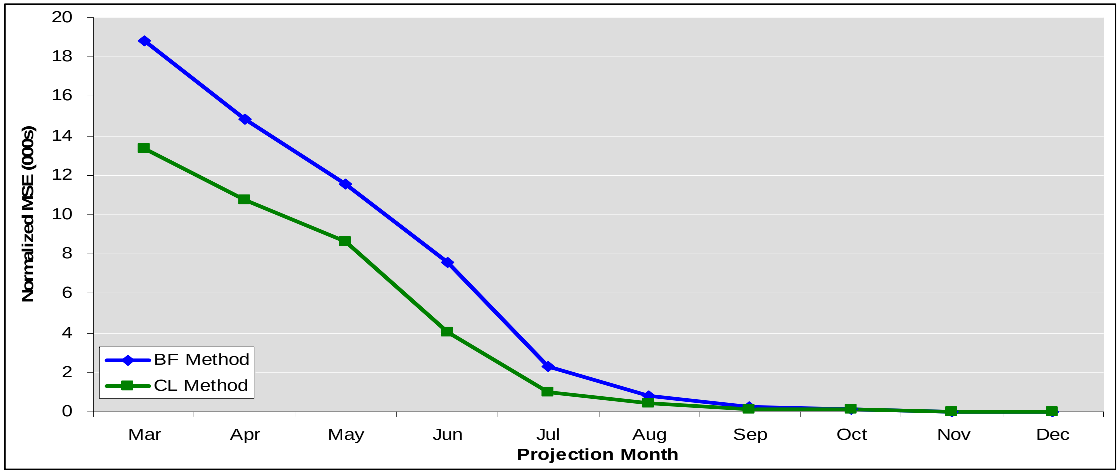
- But is it really a problem with the MR family of is it a problem with a fixed initial expectation
- So then, the natural solution might be to use a member of the AE family

$$\begin{aligned} U_{MRBF} &= U_{BF} - p_k\left(C_k - p_k U_0\right) \\ U_{AMRBF} &= U_{BF} - p_k\left(C_k - p_k U_{AEBF}\right) \\ &= U_{BF} - p_k\left(C_k - p_k\left[U_0 + p_k\left(C_k - p_k U_0\right)\right]\right) \\ &= U_{BF} - p_k C_k + p_k^2 U_0 + p_k^3 C_k - p_k^4 U_0 \\ &= U_{BF} - \left[C_k\left(p_k - p_k^3\right) - p_k U_0\left(p_k - p_k^3\right)\right] \\ &= U_{BF} - \left(p_k - p_k^3\right)\left(C_k - p_k U_0\right) \end{aligned}$$

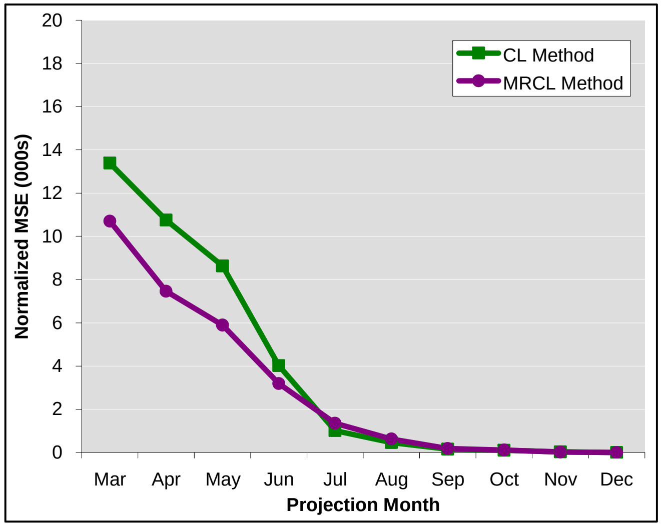


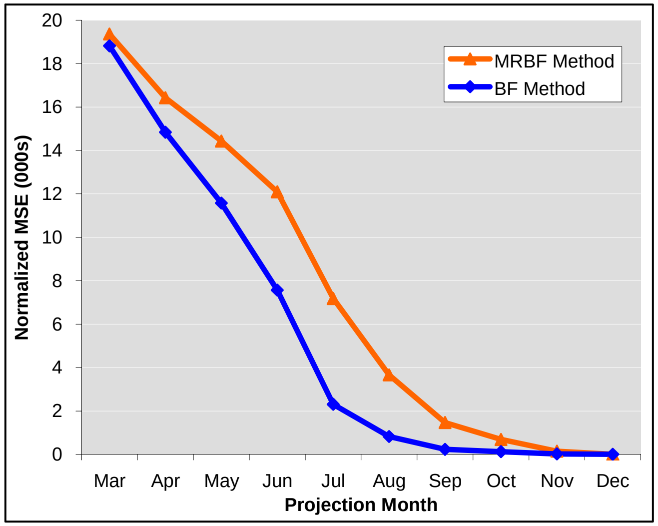
Putting it all together – hindsight testing

- Losses beget losses



- But sometimes there is a little bit of mean-reversion





Conclusions

What methods should you take home…

Take-home methods

- 4 methods which should take home

Method	Formula
AEBF	$U_{AEBF} = U_0 + p_k(C_k - p_k U_0)$
GAEBF	$U_{GAEBF} = U_k = U_{k-1} + \left(\frac{p_k - p_{k-1}}{1-p_{k-1}}\right)\left((C_k - C_{k-1}) - \left(\frac{p_k - p_{k-1}}{1-p_{k-1}}\right)(U_{k-1} - C_{k-1})\right)$
AMRBF	$U_{AMRBF} = U_{BF} - (p_k - p_k^3)(C_k - p_k U_0)$
AMRCL	$U_{AMRCL} = U_{CL} - (1 - p_k)(C_k - p_k U_0)$

  - AEBF – Used as an alternative of a fixed initial expectation
  - GAEBF – Used to credibly roll-forward prior indications
  - AMRBF – Used to introduce negative dependence
  - AMRCL – Used when losses are positively dependent with a touch of mean-reversion

Contact

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