

Determining Tail Factors

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Challenge of Tail Estimation

- Probably the most common tail estimation technique comes from:
 - R. Sherman, “Extrapolating, Smoothing, and Interpolating Development Factors”, PCAS LXXI, 1984
- But amazingly there is a hidden gem in section X of that paper. I will discuss that method today

Consider the data-

- The tail is usually considered to be the last 5/10/15% of development.
- Thus we are left with 85/90/95% already reported/settled.
- It is self-evident that the early losses are different!
- And the tail losses are different

Think about long-tail versus short-tail losses:

- Short-tail will have large Paid and small Case Reserves
- Long-tail will have smaller Paid and larger Case Reserves
- What is a good exposure measure for Tail losses?
 - Case Reserves

Let's decompose the run-off of case reserves into a paid component and a subsequent case reserve component.

For starting reserves R , in time period 1 let the paid losses equal:

$$R * p$$

and the ending case reserves equal:

$$R * c$$

The ratios p and c define the portion of case reserves that are consumed by paid losses, and subsequent closing case reserves respectively. Let's assume that these run-off ratios are constant. Then in time period $t+1$, the activity is:

Paid losses: $(R * c) * p$

Case reserves: $(R * c) * c$

And then in time $t+2, t+3, \dots$:

Paid losses: $(R * c^2) * p$
 $(R * c^3) * p$
 $(R * c^4) * p$
 etc.

Case reserves: $(R * c^2) * c$
 $(R * c^3) * c$
 $(R * c^4) * c$

Eventually the case reserves will go to zero: $R * c^\infty = 0$ because $c < 1$

So the required reserves at time zero are just the sum of all the future payments:

$$\sum_{i=0}^{\infty} R * p * c^i = R * p / (1 - c)$$

This highlights the fact that the case reserve decrement ratio must be less than one. But that is common sense – if the case reserves are continuously increasing the IBNR will be infinite.

So how should we estimate the ratios p and c ?

So how should we estimate the ratios p and c ?

No – calculate the ratio $p/(1 - c)$ instead

“IBNR” ratio is: $p/(1-c) - 1$

This algebra is simple. . . If $p = (P_{t+1} - P_t) / C_t$ and $c = C_{t+1} / C_t$, then the formula becomes:

$$R * (P_{t+1} - P_t) / (C_t - C_{t+1})$$

The time period is fungible so I just refer to the factor in the equation as $\frac{\Delta P}{\Delta C}$.

Actually it is $-\frac{\Delta P}{\Delta C}$

- The length of the exposure period is fungible as well.
- You can combine accident years and development periods!
- All you need is the paid loss movement and the case reserve (or incurred) movement!
- Cumulative data is not required!
- Accident year detail is not required!
- Data can (and should be) be combined
 - More data = more credibility!

Alternative algebra

Since $\Delta Incd = \Delta Paid + \Delta Case,$

then $\Delta Case = \Delta Incd - \Delta Paid$

And
$$\frac{\Delta Paid}{-\Delta Case} = \frac{\Delta Paid}{\Delta Paid - \Delta Incd}$$

This highlights again that cumulative inception-to-date data is not required.

Paid Loss Movements

07/07 - 06/08	07/08 - 06/09	07/09 - 06/10	07/10 - 06/11	07/11 - 06/12		UW Year
142,270	88,120	85,399	104,680	893,990		1977
262,534	50,858	95,775	142,418	154,459		1978
142,518	149,700	35,617	-1,881	74,267		1979
109,926	153,259	158,719	58,239	270,880		1980
153,155	229,764	119,349	52,495	192,542		1981
180,083	266,914	141,550	122,943	236,190		1982
291,141	142,796	129,054	205,065	168,497		1983
945,706	1,104,225	390,296	158,123	128,410		1984
4,133	-21,867	610,880	103,206	149,808		1985
64,090	2,669	82,897	11,287	459,829		1986
-80,452	22,000	1,377	-350	571,654		1987
92,122	7,598	-55,047	20,932	8,298		1988
148,809	32,635	-138,182	5,922	50,443		1989
-12,542	39,222	-486,880	3,696	92,423		1990
282,468	-29,870	-13,646	54,281	74,781		1991
618,254	42,667	287,535	-195,770	10,124		1992
336,036	-1,404,815	149,432	732	293,126		1993
161,992	541,807	121,127	465,423	86,085		1994
1,693,465	1,175,818	364,537	684,392	218,495		1995
1,104,997	689,400	1,413,675	243,826	381,683		1996
3,937,523	1,370,716	1,645,274	1,884,961	1,287,435		1997
10,338,376	2,186,629	1,915,068	2,542,054	4,618,282		1998
11,784,606	7,647,213	7,403,981	9,350,776	3,979,250		1999
5,242,466	2,852,107	2,289,430	1,478,066	1,004,565		2000
4,684,986	3,447,051	1,505,015	1,158,068	68,680		2001
42,628,662	20,786,616	18,252,232	18,653,585	15,474,194		Total

Case Reserve Movements

07/06 - 06/07	07/07 - 06/08	07/08 - 06/09	07/09 - 06/10	07/10 - 06/11	07/11 - 06/12			
-170,554	-213,701	-151,417	-15,844	56,371	-203,994			1,015,896
259,141	-193,516	-31,672	37,306	-73,319	-75,346			1,668,153
62,376	-104,963	-147,970	-65,771	-2,499	-46,293			489,917
37,147	33,795	-270,200	-247,520	21,868	-196,884			1,458,953
118,713	-173,172	-386,766	-99,237	-276,249	-183,867			1,036,059
-176,458	-175,870	-295,823	-154,889	-281,264	-378,150			1,163,407
-640,011	283,249	-215,574	-366,466	-62,154	-249,990			1,343,828
-331,392	-647,356	-1,167,972	-632,787	-97,397	-151,737			1,572,832
-177,404	529,227	-72,249	-633,810	27,529	-156,202			740,039
-87,800	81,544	29,654	65,970	70,311	-337,809			1,145,297
-66,231	-208,852	6,199	155,888	107,249	-623,958			833,742
-30,439	6,111	-59,445	72,561	-28,078	-111,058			566,958
-88,204	-151,122	-274,280	333,693	-89,994	-365,334			246,760
-359,626	57,919	-457,084	-97,373	-287,071	-190,509			286,518
451,554	300,631	-128,977	-39,534	137,867	-42,242			1,507,902
108,208	-217,355	-67,560	-409,921	21,640	65,883			649,947
-716,736	77,649	-506,515	-16,136	-230,759	-425,134			436,022
-2,003,857	244,755	-395,506	-142,001	-1,304,413	62,516			1,697,676
-406,053	-1,918,851	-768,553	-1,025,848	-526,211	-527,210			3,129,139
-4,454,731	-2,212,044	-135,646	-1,517,848	-1,293,397	-428,520			2,509,081
-5,144,621	-2,979,726	-204,671	-1,387,215	-1,060,876	-1,063,066			6,584,412
-1,436,805	-8,923,675	-2,194,232	933,639	-2,397,056	-5,296,055			9,665,545
-6,162,273	-9,036,453	-3,622,169	-4,892,160	-3,705,939	-5,312,236			13,966,415
-7,935,922	-4,219,230	-2,611,341	-2,460,227	-2,714,478	-767,753			10,298,421
-7,537,637	-4,623,467	-3,218,082	-2,207,695	-2,028,373	-33,878			1,700,216
-36,889,614	-34,384,474	-17,347,853	-14,813,224	-16,016,692	-17,038,827			65,713,132
						5 yr avg		
	1.24	1.20	1.23	1.16	0.91	1.16	10,684,352	IBNR
85 prior	3.37	0.79	0.81	1.38	1.38	1.19	85 prior	
86 later	1.20	1.27	1.30	1.16	0.86	1.16	86 later	
93 & prior	5.14	0.21	0.75	0.86	1.04	0.93		
94 & later	1.16	1.51	1.31	1.18	0.87	1.19	9,612,064	
86 - 93	27.09	0.88	2.65	0.33	0.77	0.38	86-93	