An empirical investigation of the value of finalisation count information to loss reserving

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Abstract

The purpose of the present paper has been to test whether loss reserving models that rely on claim count data can produce better forecasts than the chain ladder model (which does not rely on counts); better in the sense of being subject to a lesser prediction error.

The question at issue has been tested empirically by reference to the Meyers-Shi data set. Conclusions are drawn on the basis the emerging numerical evidence.

The chain ladder is seen as susceptible to forecast error when applied to a portfolio characterised by material changes over time in rates of claim finalisation. For this reason, emphasis has been placed here on the selection of such portfolios for testing.

The chain ladder model is applied to a number of portfolios, and so are two other models, the Payments Per Claim Incurred (PPCI) and Payments Per Claim Finalised (PPCF), that rely on claim count data. The latter model in particular is intended to control for changes in finalisation rates. Each model is used to estimate loss reserve and the associated prediction error.

A compelling narrative emerges. The chain ladder rarely performs well. Either PPCI or PPCF model produces, or both produce, superior performance, in terms of prediction error, 80% of the time.

When the chain ladder produces the best performance of the three models, this appears to be accounted for by either erratic count data or rates of claim finalisation that show comparatively little variation over time.

Keywords: bootstrap, chain ladder, count data, loss reserving, payments per claim finalised, payments per claim incurred, PPCF, PPCI, prediction error.

1. Introduction

The data set provided by Meyers and Shi (2011) makes available a large number of US claim triangles for experimentation in loss reserving. The triangles are of two types, namely:

- Paid claims; and
- Incurred claims.

Triangles of these types are suitable for analysis by the chain ladder model, and indeed this is very common in practice. Some jurisdictions across the globe are accustomed to the use of alternative loss reserving models (see e.g. Taylor (2000). Commonly, these alternatives rely on additional data, particularly triangles of counts of reported claims and finalised claims respectively.

This raises the question as to reasons Meyers and Shi did not collate count data. In private correspondence the authors advised that they had sought the views of other US actuaries on this very matter, and had been counselled not to do so.

Count data, particularly finalisation counts were said to be unreliable. There was more than one reason for this. First, some portfolios included material amounts of reinsurance, and the meaning of claim finalisation was not clear in all of these cases. But more than this, it appears that such counts are not always returned by insurers with all diligence and are unreliable on that account.

Moreover, the models that rely on count data have not received universal acclaim. Some statisticians have commented adversely, noting that these models, requiring more extensive data, also require more modelling, more parameterisation, leading to more uncertainty in forecasts.

This argument cannot be correct as a matter of logic. If claim finalisation counts followed a deterministic process, they would add no uncertainty, and the argument would fail. If they follow a process with a very small degree of stochasticity, then they would add little uncertainty, and again the argument would fail.

The evident question of relevance is whether any reduction in uncertainty in the claim payment model by conditioning on the count data is more than, or less than, offset by the additional uncertainty induced by the modelling and forecasting of the counts themselves.

The forecasts of some claim payment models that rely on finalisation count data are relatively insensitive to the distribution of finalisations over time. So any uncertainty in the forecast of this distribution will have little effect on the forecast of loss reserve in this case. These models are the operational time models, such as discussed in Section 4.3.

The debate on the merits of these models relative to the chain ladder appears fruitless. It might be preferable to allow the data to speak for themselves. That is, forecast according to both models, estimate prediction error of each, and select the model with the lesser prediction error.

Much the same argument can be applied to the issue of reliability of count data. The data may be allowed to speak for themselves by the use of prediction error as the criterion for model selection. Data unreliability should be found out through an enlarged prediction error.

2. Framework and notation

2.1 Claims data

Consider a $J \times J$ square of claims observations Y_{kj} with:

- accident periods represented by rows and labelled $k = 1,2, \dots, J$;
- development periods represented by columns and labelled by $j = 1,2, \dots, J$.

For the present the nature of these observations will be unspecified. In later sections they will be specialised to paid losses, reported claim counts, unfinalised claim counts or claim finalisation counts, or even quantities derived from these.

Within the square identify a **development triangle** of **past** observations

$$\mathfrak{D}_J = \left\{Y_{kj}: 1 \le k \le J \; and \; 1 \le j \le J-k+1\right\}$$

Let $\mathfrak{I}_J$ denote the set of subscripts associated with this triangle, i.e.

$$\mathfrak{I}_J = \{(k,j): 1 \le k \le J \; and \; 1 \le j \le J-k+1\}$$

The complement of this subset, representing **future** observations is

$$\mathfrak{D}_J^c = \left\{Y_{kj}: 1 \le k \le J \; and \; J-k+1 < j \le J\right\}$$

Also let

$$\mathfrak{D}_J^+ = \mathfrak{D}_J \cup \mathfrak{D}_J^c$$

In general, the problem is to predict $\mathfrak{D}_J^c$ on the basis of observed $\mathfrak{D}_J$.

Define the **cumulative row sums**

$$Y_{kj}^* = \sum_{i=1}^{j} Y_{ki} \tag{2.1}$$

and the full **row and column sums** (or horizontal and vertical sums) and **rectangle sums**

$$H_k = \sum_{j=1}^{J-k+1} Y_{kj}$$

$$V_j = \sum_{k=1}^{J-j+1} Y_{kj}$$

$$T_{rc} = \sum_{k=1}^{r} \sum_{j=1}^{c} Y_{kj} = \sum_{k=1}^{r} Y_{kc}^* \tag{2.2}$$

Also define, for $k = 2, \ldots, J$,

$$R_k = \sum_{j=J-k+2}^{J} Y_{kj} = Y_{kJ}^* - Y_{k,J-k+1}^* \tag{2.3}$$

$$R = \sum_{k=2}^{J} R_k \tag{2.4}$$

Note that R is the sum of the (future) observations in $\mathfrak{D}_J^c$. It will be referred to as the total amount of **outstanding losses**. Likewise, R_k denotes the amount of outstanding losses in respect of accident period k. The objective stated earlier is to forecast the R_k and R.

Let $\Sigma^{\mathcal{R}(k)}$denote summation over the entire row k of $\mathfrak{D}_J$, i.e. $\Sigma_{j=1}^{J-k+1}$ for fixed k.

Similarly, let $\Sigma^{\mathcal{C}(j)}$denote summation over the entire column of $\mathfrak{D}_J$, i.e. $\Sigma_{k=1}^{J-j+1}$ for fixed j. For example, the definition of V_j in (2.2) may be expressed as

$$V_j = \sum^{\mathcal{C}(j)} Y_{kj}$$

2.2 Generalised linear models

The present paper attempts to estimate the prediction error associated with the estimate of outstanding losses produced by various models. A stochastic model of losses is required to achieve this.

A convenient form of stochastic model, with sufficient flexibility to accommodate the various models introduced in Section 4, is the Generalised Linear Model (**"GLM"**). This type of model is defined and considered in detail by McCullagh & Nelder (1989), and its application to loss reserving is discussed by Taylor (2000).

A GLM is a regression model that takes the form:

$$\underset{n\times 1}{Y} = h^{-1}(\underset{n\times p}{X} \underset{p\times 1}{\beta}) + \underset{n\times 1}{\varepsilon} \tag{2.5}$$

where Y, X, β, ε are vectors and matrices with dimensions according to the annotations beneath them, and where:

Y is the **response (or observation) vector**;
X is the **design matrix**;

β is the **parameter vector**;
ε is a centred (stochastic) **error vector**; and
h is a one-one function called the **link function**.

The link function need not be linear (as in general linear regression). The quantity $X\beta$ is referred to as the **linear response**.

The components Y_i of the vector Y are all stochastically independent and each has a distribution belonging to the **exponential dispersion family ("EDF")** (Nelder & Wedderburn, 1972), i.e. it has a pdf of the form:

$$p(y) = exp\left[\frac{y\theta - b(\theta)}{a(\phi)} + c(y, \phi)\right] \tag{2.6}$$

where θ is a location parameter, ϕ a scale parameter, and $a(.), b(.), c(.)$ are functions.

This family will not be discussed in any detail here. The interested reader may consult one of the cited references. For present purposes, suffice to say that the EDF includes a number of well known distributions (normal, Poisson, gamma, inverse gamma, binomial, compound Poisson) and specifically that it include the **over-dispersed Poisson ("ODP")** distribution that will find repeated application in the present paper.

A random variable Z will be said to have an ODP distribution with mean μ and scale parameter ϕ (denoted $Z \sim ODP(\mu, \phi)$) if

$$Z/\ \phi \sim Poisson(\mu/\phi) \tag{2.7}$$

It follows from (2.7) that

$$E[Z] =\ \mu,\ Var[\mathrm{Y}] = \phi\mu \tag{2.8}$$

2.3 Residual plots

When the GLM -(2.6) is calibrated against a data vector $\boldsymbol{Y} = (\boldsymbol{Y_1}, \dots, \boldsymbol{Y_n})^{\boldsymbol{T}}$, let $\widehat{\boldsymbol{\beta}}$ denote the estimate of $\boldsymbol{\beta}$ and let $\widehat{\boldsymbol{Y}} = \boldsymbol{h^{-1}}(\boldsymbol{X}\widehat{\boldsymbol{\beta}}\ )$. The component $\widehat{\boldsymbol{Y}}_{\boldsymbol{i}}$ is called the **fitted value** corresponding to $\boldsymbol{Y_i}$.

Let $\boldsymbol{\ell}(\boldsymbol{Y_i}; \widehat{\boldsymbol{Y}})$ denote the log-likelihood of observation $\boldsymbol{Y_i}$ (see (2.6)) when $\boldsymbol{\beta} = \widehat{\boldsymbol{\beta}}$ (and so $\boldsymbol{E}[\boldsymbol{Y}]$=$\widehat{\boldsymbol{Y}}$). The **deviance** of the fitted model is defined as

$$D = -2\sum_{i=1}^{n} d_i = -2\sum_{i=1}^{n}\left[\ell(Y_i; \hat{Y}) - \ell(Y_i; Y)\right] \tag{2.9}$$

where $\ell(Y_i; Y)$ denotes the log-likelihood of the saturated model in which $\hat{Y} = Y$.

The **deviance residual** associated with $\boldsymbol{Y_i}$ is defined as

$$r_i^D = sgn(\boldsymbol{Y_i} - \boldsymbol{\hat{Y}_i})d_i^{½} \tag{2.10}$$

Define the **hat matrix**

$$H = X(X^TX)^{-1}X^T \tag{2.11}$$

Then the **standardised deviance residual** associated with Y_i is defined as

$$r_i^{DS} = r_i^D/(1 - H_{ii})^{½} \tag{2.12}$$

where H_{ii} denotes the (i, i) – element of H.

For a valid model -(2.6), $r_i^{DS} \sim N(0,1)$ approximately unless the data Y are highly skew. It then follows that $E[r_i^{DS}] = 0, Var[r_i^{DS}] = 1$. When the r_i^{DS} are plotted against the i, or any permutation of them, the resulting **residual plot** should contain a random scattered of positives and negatives largely concentrated in the range $(-2, +2)$, and with no left-to-right trend in dispersion (**homoskedasticity**). Homoskedastic models are desirable as they produce more reliable predictions than heteroskedastic.

2.4 Relevant development triangles

The description of a development in Section 2.1 is generic in that the nature of the observations is left unspecified. In fact, there will be a number of triangles required in subsequent sections. They are as follows:

Raw data
- Paid loss amounts;
- Reported claim counts;
- Unfinalised claim counts;

Derived data
- Finalised claim counts.

These are defined in Sections 2.2.1 to 2.2.4. Further triangles, specific to the models discussed in Sections 4.2 and 4.3, will be required and will be defined in those sections.

2.4.1 Paid loss amounts

The typical cell entry will be denoted P_{kj}. It denotes the total amount of claim payments made in cell (k, j). Payments are in raw dollars, unadjusted for inflation.

2.4.2 Reported claim counts

The typical cell entry will be denoted N_{kj}. It denotes the total number of claims reported to the insurer in cell (k, j). Let N_{kj}^* denote the cumulative count of reported claims, defined in a manner parallel to (2.1).

As $j \to \infty$, N_{kj}^* approaches the total number of claims ultimately to be reported in respect of accident period k. This will be referred to as the **ultimate claims incurred count** in respect of accident period k and will be abbreviated to N_k.

2.4.3 Unfinalised claim counts

The typical cell entry will be denoted U_{kj}. It denotes the number of claims reported to the insurer but unfinalised at the end of the time period covered by cell (k, j).

2.4.4 Finalised claim counts

The typical cell entry will be denoted F_{kj}. It denotes the number of claims reported to the insurer and finalised by the end of the time period covered by cell (k, j). It is derived from the raw data by means of the simple identity

$$F_{kj} = F_{kj}^* - F_{k,j-1}^* \quad (2.13)$$

where

$$F_{kj}^* = N_{kj}^* - U_{kj} \quad (2.14)$$

As $j \to \infty$, $N_{kj}^* \to N_k$ and $U_{kj} \to 0$, yielding the obvious result that all claims ultimately reported are ultimately finalised:

$$\lim_{j\to\infty} F_{kj}^* = N_k \quad (2.15)$$

It is possible that (2.13) will yield a result $F_{kj} < 0$. By (2.13) and (2.14),

$$F_{kj} = \left(N_{kj}^* - U_{kj}\right) - \left(N_{k,j-1}^* - U_{k,j-1}\right) = N_{kj} - \left(U_{kj} - U_{k,j-1}\right)$$
$$< 0 \text{ if } U_{kj} - U_{k,j-1} > N_{kj}$$

i.e. if an increase in the number of unfinalised claims over a development period is greater than can be explained by newly reported claims. This can occur if claims, once finalised, can be re-opened and this become unfinalised again.

3. Data

As its title indicates, this paper reports an empirical investigation. Conclusions are drawn from the analysis of real-life data sets. The triangles of paid loss amounts are those described by Meyers & Shi (2011).

Companion triangles of reported claim counts and unfinalised claim counts were provided privately by Peng Shi. The totality of all these triangles will be referred to as the **Meyers-Shi data base**. The part of the data base used by the present paper is reproduced in Appendix A.

3.1 Triangles of paid loss amounts

These are $\mathbf{10 \times 10}$ ($\boldsymbol{J = 10}$) triangles, reporting the claims history as at 31 December 1997 in respect of the 10 accident years 1988-1997. The triangles relating to these accident and development years (**"the training interval"**) will be referred to as **training triangles**. As explained by Meyers & Shi (2011), they are extracted from Schedule P of the data base maintained by the US National Association of Insurance Commissioners.

The Meyers-Shi data base contains paid loss histories in respect of six lines of business (**"LoBs"**), namely:

(1) Private passenger auto;
(2) Commercial auto;
(3) Workers compensation;
(4) Medical malpractice;
(5) Products liability;
(6) Other liability.

In each case, a triangle is provided for each of a large number of insurance companies.

The data base also contains the history of accident years 1988-97, as it developed after 31 December 1997, in each case up to the end of development year 10. These will be referred to as **test triangles**. In the notation established in Section 2.1, $\mathfrak{D}_{10}$ denotes a training triangle and $\mathfrak{D}_{10}^c$ a test triangle.

3.2 Triangles of reported claim counts and unfinalised claim counts

These are also $\mathbf{10} \times \mathbf{10}$ triangles covering the training interval. They were provided in respect of just the first three of the six LoBs listed in Section 3.1. This limited any comparative study involving claim counts to these three LoBs.

4. Models investigated

4.1 Chain ladder

4.1.1 Model formulation

This is described in many publications, including the loss reserving texts by Taylor (2000) and Wüthrich & Merz (2008). A thorough analysis of its statistical properties was given by Taylor (2011), who defines the **ODP Mack model** as a stochastic version of the chain ladder. This model is characterised by the following assumptions.

(ODPM1) Accident periods are stochastically independent, i.e. $Y_{k_1 j_1},\ Y_{k_2 j_2}$ are stochastically independent if $k_1 \neq k_2$.

(ODPM2) For each $k = 1,2, \dots, J$, the Y_{kj}^* (j varying) form a Markov chain.

(ODPM3) For each $k = 1,2, \dots, J$ and $j = 1,2, \dots, J-1$, define $G_{kj} = Y_{k,j+1}/Y_{kj}^*$ and suppose that $G_{kj} \sim ODP\left(g_j, \phi_{kj}\left(Y_{kj}^*\right)/\left(Y_{kj}^*\right)^2\right)$, where $\phi_{kj}(\cdot)$ is a function of Y_{kj}^*.

It follows from (ODPM3) that

$$E\left[Y^*_{k,j+1}/Y^*_{kj}\right] = E\left[1 + G_{kj}\right] = 1 + g_j \tag{4.1}$$

which will be denoted by $f_j (> 1)$ and referred to as an **age-to-age factor**. This will also be referred to as a **column effect**.

For the purpose of the present paper, it has been assumed that $f_j = 1$ for $j \geq J$, i.e. no claim payments after development year J. It appears that the resulting error in loss reserve will be relatively small.

4.1.2 Chain ladder algorithm

Simple estimates for the $\boldsymbol{f_j}$ are

$$\hat{f}_j = T_{J-j,j+1}/T_{J-j,j} \tag{4.2}$$

These are the conventional chain ladder estimates that have been used for many years. However, they are also known to be maximum likelihood (**"ML"**) for the above ODP Mack model (and a number of others) (Taylor, 2011) provided that $\phi_{kj}\left(Y^*_{kj}\right) = \sigma^2_j$ for quantities $\sigma^2_j > 0$ dependent on just j.

Estimator (4.2) implies a forecast of $Y^*_{kj} \in \mathfrak{D}^c_K$ as follows:

$$\hat{Y}^*_{kj} = Y^*_{k,J-k+1}\hat{f}_{J-k+1}\hat{f}_{J-k+2} \dots \hat{f}_{j-1} \tag{4.3}$$

Strictly, this forecast include claim payments only to the end of development year J. Beyond this lies outside the scope of the data, and allowance for higher development years would require additional data from some external source or some form of extrapolation.

4.1.3 GLM formulation

Regression design

The ODP Mack model may be expressed as a GLM. Since the ODP family is closed under scale transformations, (ODPM3) may be re-expressed as

$$Y_{k,j+1}|Y^*_{kj} \sim ODP\left(Y^*_{kj}g_j, \phi_{kj}\left(Y^*_{kj}\right)\right) \tag{4.4}$$

or, equivalently,

$$Y_{k,j+1}|Y^*_{kj} \sim ODP\left(\mu_{k,j+1}, \phi/w_{k,j+1}\right) \tag{4.5}$$

where

$$\mu_{k,j+1} = exp\left(ln\, Y^*_{kj} + ln\, g_j\right) \tag{4.6}$$

$$w_{k,j+1} = \phi/\phi_{kj}\left(Y^*_{kj}\right) \tag{4.7}$$

for some constant $\phi > 0$.

The weight structure (4.7), together with the ODP assumption, implies that

$$Var[Y_{k,j+1}|Y_{kj}^*] = g_j Y_{kj}^* \phi_{kj}(Y_{kj}^*) \tag{4.8}$$

The representation (4.5)-(4.7) amounts to a GLM. The link function is the natural logarithm. The linear response is seen to be $(ln\, Y_{kj}^* + ln\, g_j)$, which consists of one known term, $ln\, Y_{kj}^*$, and one, $ln\, g_j$, requiring estimation. In this case the vector β in has components $ln\, g_1, ln\, g_2, \ldots, ln\, g_9$. The vector of known values is called an **offset** vector in the GLM context.

For representation of the GLM in the form , the response vector Y consists of the observations $Y_{kj}, j \geq 2$ in dictionary order. It has dimension $9 + 8 + \cdots + 1 = 45$. Any other order will do, though the design matrix described below would require rearrangement.

The design matrix X in is of dimension 45×9, with one row for each observation and one column for each parameter. If rows are denoted by the combination (k, j) and columns by $i = 1, \ldots, 9$, then the elements of X are $X_{k,j+1,i} = \delta_{ji}$, with δ denoting the Kronecker delta.

Weights

The quantity $w_{k,j+1}$ is referred to as a **weight** as its effect is to weight the log-likelihood of the observation $Y_{k,j+1}$ in the total log-likelihood. Weights are relative in the sense that they may all be changed by the same factor without affecting the estimate of β. In this case, (4.5) shows that the estimate of ϕ will change by the same factor so that the scale parameter $\phi/w_{k,j+1}$ is unaffected.

Weights are used to correct for variances that differ from one observation to another. We do not have prior information on the structure of variance by cell. The default $w_{kj} = 1$ is therefore adopted unless there is cause to do otherwise. It then follows from (4.7) that

$$\phi_{kj}(Y_{kj}^*) = \phi \tag{4.9}$$

$$w_{k,j+1} = 1 \tag{4.10}$$

and then, by (4.8),

$$Var[Y_{k,j+1}|Y_{kj}^*] = (\phi g_j) Y_{kj}^* \tag{4.11}$$

It is interesting to note that this is a special case of the model proposed by ODP Mack model, in which $Var[Y_{k,j+1}|Y_{kj}^*] = \sigma_j^2 Y_{kj}^*$, whose ML estimates were remarked in Section 4.1.2 to be equal to those of the chain ladder algorithm. Standard software (R in the present case) calibrates GLMs according to ML. It follows that the GLM estimates will also be the same as from the chain ladder algorithm in the presence of unit weights.

ODP variates are necessarily non-negative.

4.2 Payments per claim incurred

4.2.1 Model formulation

This model, referred to as the **"PPCI model"**, is described in Taylor (2000, Section 4.2) and a very similar model in Wright (1990). It is characterised by the following assumptions.

(PPCI1) All cells are stochastically independent, i.e. $Y_{k_1 j_1}$, $Y_{k_2 j_2}$ are stochastically independent if $(k_1, j_1) \neq (k_2, j_2)$.

(PPCI2) For each $k = 1,2, \ldots, J$ and $j = 1,2, \ldots, J-1$, suppose that $Y_{kj} \sim ODP\left(N_k \pi_j \lambda(k+j-1), \phi_{kj}\right)$, where

- $\pi_j, j = 1,2, \ldots, J$ are parameters;
- $N_k, k = 1,2, \ldots, J$ are as defined in Section 2.4.2;
- $\lambda\colon [1, 2, 3, .., 2J-1] \rightarrow \mathfrak{R}$.

As in Section 4.1.1, it has been assumed that $f_j = 1$ for $j \geq J$, i.e. no claim payments after development year J.

An alternative statement of (PPCI2) is as follows:

$$Y_{kj} / N_k \sim ODP\left(\pi_j \lambda(k+j-1), \phi_{kj} / N_k^2\right) \tag{4.12}$$

The quantity on the left is the cell's amount of PPCI, with a mean of

$$E\left[Y_{kj} / N_k\right] = \pi_j \lambda(k+j-1) \tag{4.13}$$

To interpret the right side, first assume that $\lambda(k+j-1) = 1$. Then the expectation of PPCI is a quantity that depends on just development year. It is a column effect.

To interpret the function $\lambda(.)$, note that $k+j-1$ represents **experience year**, i.e. the calendar period in which the cell's payments were made. An experience year manifests itself as a diagonal of $\mathfrak{D}_K^+$, i.e. $k+j-1$ is constant along a diagonal.

Experience years are often referred to as **payment years**. However, the former terminology is preferred here because it is a more natural label in triangles of counts, which are payment-free.

Thus the function $\lambda(.)$ states how, for constant j, PPCI change with experience year. As noted in Section 2.4.1, paid loss data are unadjusted for inflation, and so $\lambda(.)$ may be thought of as a claims inflator. This reflects **claim cost escalation**, as opposed to a conventional inflation measure such as price or wage inflation.

The simplest possibility for this inflator is

$$\lambda(m) = \lambda^m, \lambda = \text{const.} > 0 \tag{4.14}$$

representing constant claim cost escalation according to a factor of λ per annum.

4.2.2 Estimation of numbers of claims incurred

The response variate in model (4.12) involves N_k, the number of claims incurred in accident year k. According to the definition in Section 2.4.2,

$$N_k = \sum_{j=1}^{J-k+1} N_{kj} + \sum_{j=J-k+2}^{J} N_{kj} \tag{4.15}$$

where the two summands relate to $\mathfrak{D}_K$ (the past) and $\mathfrak{D}_K^c$ (the future) respectively.

Naturally, the future values are unknown and estimates are required. Thus N_k is estimated by

$$\hat{N}_k = \sum_{j=1}^{J-k+1} N_{kj} + \sum_{j=J-k+2}^{J} \hat{N}_{kj} \tag{4.16}$$

where the $\hat{N}_{kj}$ are estimated by the chain ladder GLM.

Weights

Some data cells contain negative incremental numbers of reported claims (Appendix A.2). This is particularly the case for company #1538 (Appendix A.2.3). Such cells are shaded in Appendix A.2 and are assigned are assigned zero weight in the GLM.

4.2.3 Calibration

For calibration purposes the PPCI model is expressed in GLM form:

$$Y_{kj} / \hat{N}_k \sim ODP\left(\mu_{kj}, \phi_{kj}/\hat{N}_k^2\right) \tag{4.17}$$

where

$$\mu_{kj} = exp\left(ln\, \pi_j + ln\, \lambda(k + j - 1)\right) \tag{4.18}$$

and the estimates $\hat{N}_k$ are obtained as in Section 4.2.2.

In the special case of (4.14), the mean (4.18) reduces to

$$\mu_{kj} = exp\left(ln\, \pi_j + (j + k - 1)\, ln\, \lambda\right) \tag{4.19}$$

Empirical testing indicates that, as reasonable first approximation, the scale parameter in (PPCI2) may be taken as constant over all cells, i.e.

$$\phi_{kj} = \phi \widehat{N}_k^2 \tag{4.20}$$

in which case the scale parameter in (4.17) reduces to a constant (i.e. independent of k, j), implying unit weights in GLM modeling.

4.2.4 Forecasts

The GLM (4.17)-(4.18) implies the following forecast of $Y_{kj} \in \mathfrak{D}_J^c$:

$$\widehat{Y}_{kj} = \widehat{N}_k \hat{\mu}_{kj} \tag{4.21}$$

where

$$\hat{\mu}_{kj} = exp\left(ln\, \hat{\pi}_j + ln\, \hat{\lambda}(k + j - 1)\right) \tag{4.22}$$

and $ln\, \hat{\pi}_j, ln\, \hat{\lambda}(.)$ are the GLM estimates of $ln\, \pi_j, ln\, \lambda(.)$. The function $ln\, \lambda(.)$ within the GLM will necessarily be a linear combination of a finite set of basis functions, and so the estimator $ln\, \hat{\lambda}(.)$ is obtained by replacing the coefficients in the linear combination by their GLM estimates.

4.3 Payments per claim finalised

The essentials of the model appear to have been introduced by Fisher & Lange (1973) and re-discovered by Sawkins (1979).

4.3.1 Operational time

It will be useful to define the following quantity:

$$t_k(j) = F_{kj}^* / \widehat{N}_k \tag{4.23}$$

This is called the **operational time** (**"OT"**) at the end of development year j in respect of accident year k, and it is equal to the proportion of claims estimated ultimately to be reported for accident year k that have been finalised by the end of development year j. The concept was introduced into the loss reserving literature by Reid (1978).

While this definition covers only cases in which j is equal to a natural number, $t_k(j)$ retains an obvious meaning if the range of j is extended to $[0, \infty)$. In this case,

$$t_k(0) = 0 \tag{4.24}$$
$$t_k(\infty) = 1 \tag{4.25}$$

If claims, once closed, remain closed, then F_{kj}^* is an increasing function of j, and so $t_k(j)$ increases monotonically from 0 to 1 as j increases from 0 to ∞.

Also define the average operational time of cell (k, j) as

$$\bar{t}_k(j) = ½[t_k(j - 1) + t_k(j)] \tag{4.26}$$

4.3.2 Model formulation

This model, referred to as the **"PPCF model"**, is described in Taylor (2000, Section 4.3). As will be seen shortly, if one is to forecast future claim costs on the basis of PPCF, then future numbers of claim finalisations must also be forecast. The PPCF model will therefore comprise two sub-models: a payments sub-model and a finalisations sub-model.

Payments sub-model

This is characterised by the following assumptions.

(PPCF1) All cells are stochastically independent, i.e. $Y_{k_1 j_1}$, $Y_{k_2 j_2}$ are stochastically independent if $(k_1, j_1) \neq (k_2, j_2)$.

(PPCF2) For each $k = 1,2, \dots, K$ and $j = 1,2, \dots, J-1$, suppose that $Y_{kj} \sim ODP\big(F_{kj}\, \psi(\bar{t}_k(j))\, \lambda(k+j-1), \phi_{kj}\big)$, where

- $\psi: [0,1] \rightarrow \Re$;
- $\lambda(.)$ has the same interpretation as in the PPCI model described in Section 4.2.1.

As in Sections 4.1.1 and 4.2.1, it has been assumed that $f_j = 1$ for $j \geq J$, i.e. no claim payments after development year J. It would have been possible to forecast paid losses in development years beyond J because the number of claims to be finalised in those years is known $\big(= N_k - F_{kJ}^*\big)$. This was not done, however, for consistency with the chain ladder and PPCI models.

An alternative statement of (PPCF2) is as follows:

$$Y_{kj}/F_{kj} \sim ODP\big(\psi(\bar{t}_k(j))\, \lambda(k+j-1), \phi_{kj}/F_{kj}^2\big) \tag{4.27}$$

The quantity on the left is the cell's amount of PPCF, with a mean of

$$E\big[Y_{kj}/F_{kj}\big] = \psi(\bar{t}_k(j))\, \lambda(k+j-1) \tag{4.28}$$

Underlying (PPCF2) is a further assumption that mean PPCF in an infinitesimal neighbourhood of OT t, before allowance for the inflationary factor $\lambda(.)$, is $\psi(t)$. The mean PPCF for the whole of development year j is taken $\psi(\bar{t}_k(j))$, dependent on the mid-value of OT for that year.

A further few words of explanation of this form of mean are in order. It may seem that a natural extension of assumption (PPCI2) to the PPCF case would be

$$E\big[Y_{kj}/F_{kj}\big] = \psi_j\, \lambda(k+j-1)$$

i.e. with PPCF dependent on development year rather than OT.

Consider, however, the following argument, which is highly simplified in order to register its point. In most lines of business, the average size of claim settlements of an accident year increases steadily as the delay from accident year to settlement increases. Usually, if this is not the case over the whole range of claim delays, it is so over a substantial part of the range.

Now suppose that, as a result of a change in the rate of claim settlement, the OT histories of two accident years are as set out in Table 4-1.

Table 4-1 Operational times for different accident years

Accident year	**Operational time at the end of development year**					
	1	2	3	4	5	6
k	0.15	0.35	0.55	0.70	0.75	0.80
⋮	⋮	⋮	⋮	⋮	⋮	⋮
$k+r$	0.25	0.50	0.70	0.80	0.85	0.90

Suppose the claims of accident year k are viewed as forming a settlement queue, the first 15% in the queue being finalised in development year 1, the next 20% in development year 2, and so on. According to the above discussion, claims will increase in average size as one progresses through the queue.

Now suppose that the claims of accident year $k+r$ are sampled from the same distribution and form a settlement queue, ordered in the same way as for accident year k (the concept of "ordered in the same way" is left intentionally vague in the hope that the general meaning is clear enough).

Then, in the case of accident year $k+r$, the 25% of claims finalised in development year 1 will resemble the combination of:

- the claims finalised in development year 1 in respect of accident year k (15% of all claims incurred); and
- the first half of the claims finalised in development year 2 in respect of accident year k (another 10% of all claims incurred).

The latter group will have a larger average claim size than the former, and so the expected PPCF will be greater in cell $(k+r,1)$ than in $(k,1)$. The argument may be extended to show that expected PPCF will be greater in cell $(k+r,j)$ than in (k,j).

In this case the modelling of expected PPCF as a function of development year would be unjustified. On the other hand, it follows from the queue concept above that expected PPCF is a function of OT and may be modelled accordingly.

Weights for payments sub-model

Further, there are a couple of cases of cells that contain zero counts of finalisations but positive payments. These cases are shown hatched in Appendix A.3.

In such cases, claim payments have been set to zero before data analysis. As this converts assumption (PPCF2) to $Y_{kj} = 0 \sim ODP(0,\phi_{kj})$, which is devoid of information, these cells have no effect on the model calibration.

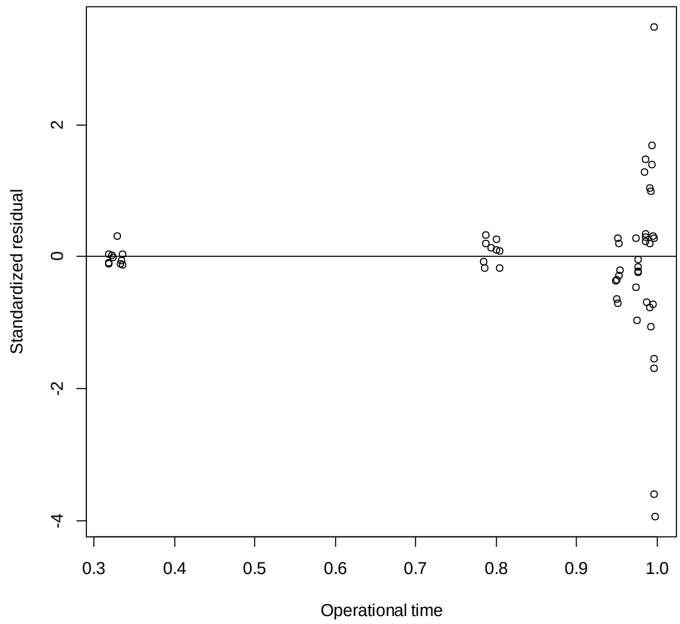
Despite this, cases of positive payments in the presence of a zero finalisation count are genuine (they indicate the existence of partial claim payments) and so omission of these cells will create some downward bias in loss reserve estimation. However, these occurrences were rare in the data sets analysed and occurred in cells that contributed comparatively little to the accident year's total incurred cost. The downward bias has been assumed immaterial.

There are also instances of negative finalisation counts, highlighted in Appendix A.3. While re-opening of finalised claims can render negative counts genuine, there was substantial evidence in the present cases that the negatives represented data errors and the associated cells were accordingly assigned zero weight.

The discussion of weights hitherto has been confined to data anomalies. However, for the PPCF model a more extensive system of weights is required. If weights are set to unity (other than the zero weighting just described), homoskedasticity is **not** obtained.

This is illustrated in Figure 4-1, which is a plot of standardised deviance residuals of PPCF against OT for Company #1538 (see the data appendix) for which the functions $ln\,\lambda(.)$ and $ln\,\psi(.)$ are quadratic and linear respectively.

Figure 4-1 Residual plot for unweighted PPCF model



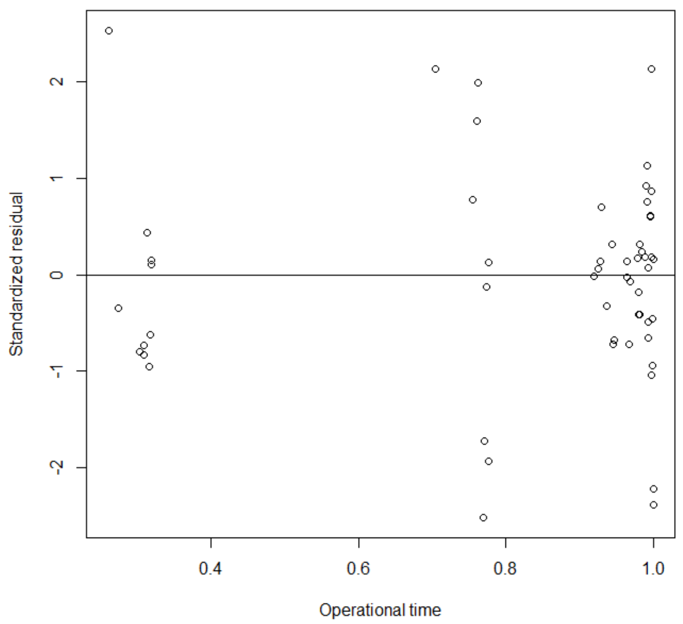
The figure clearly shows the increasing dispersion with increasing OT. This was corrected by assigning cell (k, j) the weight w_{kj}, defined by

$$w_{kj} = 1 \text{ if } \bar{t}_k(j) < 0.92$$
$$= \{5 + 100[\bar{t}_k(j) - 0.92]\}^{-2} \text{ if } \bar{t}_k(j) \geq 0.92 \quad (4.29)$$

This function exhibits a discontinuity at $\bar{t}_k(j) = 0.92$ but this is of no consequence as there are no observations in the immediate vicinity of this value of average OT. As seen in Figure 4-1, there is a clump of observation in the vicinity of OT=0.82, and then none until about OT=0.92.

On application of this weighting system, the residual plot in Figure 4-1 was modified to that appearing in Figure 4-2. A reasonable degree of homoskedasticity is seen.

Figure 4-2 Residual plot for weighted PPCF model



While the weights (4.29) were developed for specifically Company #1538, they were found reasonably efficient for all companies analysed. They were therefore adopted for all of those companies in the name of a reduced volume of bespoke modelling.

There continue to be few values of average OT in the vicinity of 0.92 when all of the companies analysed are considered. The discontinuity in (4.29) therefore remains of little consequence. Nonetheless, the PPCF modelling could probably be improved somewhat with yhe selection of weight systems specific to individual insurers.

Finalisations sub-model

This is characterised by the following assumptions.

(FIN1) All cells are stochastically independent, i.e. $F_{k_1 j_1}$, $F_{k_2 j_2}$ are stochastically independent if $(k_1, j_1) \neq (k_2, j_2)$.

(FIN2) For each $k = 1,2, \ldots, K$ and $j = 2, \ldots, J$, suppose that $F_{kj} \sim \mathrm{Bin}\left(U_{k,j-1} + N_{kj}, p_j\right)$, where the p_j are parameters.

This model is evidently an approximation as it yields the result

$$E\left[F_{kj}\right] = \left(U_{k,j-1} + N_{kj}\right) p_j$$

which is an over-statement unless all newly reported claims N_{kj} are reported at the very beginning of development year j. However, assumption (FIN2) was adopted here because the replacement of N_{kj} by κN_{kj}, with $\kappa = ½$ or ⅓ say, generated anomalous cases in which $F_{kj} > U_{k,j-1} + \kappa N_{kj}$.

4.3.3 Calibration

For calibration purposes the PPCF model is expressed in GLM form:

$$Y_{kj}/F_{kj} \sim ODP\left(\mu_{kj}, \phi/w_{kj}F_{kj}^2\right) \tag{4.30}$$

where

$$\mu_{kj} = exp\left(ln\,\psi(\bar{t}_k(j)) + ln\,\lambda(k+j-1)\right) \tag{4.31}$$

where the function $\psi(.)$ is yet to be determined. This will be discussed in Section 5.3.1.

In the special case of (4.14), the mean (4.31) reduces to

$$\mu_{kj} = exp(ln\,\psi(\bar{t}_k(j)) + (j+k-1)\,ln\,\lambda) \tag{4.32}$$

Weights w_{kj} are as set out in (4.29).

4.3.4 Forecasts

The GLM (4.27) implies the following forecast of $Y_{kj} \in \mathfrak{D}_K^c$:

$$\hat{Y}_{kj} = \hat{F}_{kj}\hat{\mu}_{kj} \tag{4.33}$$

where

$$\hat{\mu}_{kj} = exp\left(ln\,\hat{\psi}\left(\hat{\bar{t}}_k(j)\right) + ln\,\hat{\lambda}(k+j-1)\right) \tag{4.34}$$

and $ln\,\hat{\psi}(.), ln\,\hat{\lambda}(.)$ are the GLM estimates of $ln\,\psi(.), ln\,\lambda(.)$ and $\hat{F}_{kj}, \hat{\bar{t}}_k(j)$ are forecasts of $F_{kj}, \bar{t}_k(j)$ for the future cell (k,j). As explained in Section 4.2.3, the function $ln\,\lambda(.)$ within the GLM will be a linear combination of basis functions, and the estimator $ln\,\hat{\lambda}(.)$ is obtained by replacing the coefficients in the linear combination by their GLM estimates. The estimator $ln\,\hat{\psi}(.)$ is similarly constructed.

Forecasts of future operational times

The forecasts $\hat{\bar{t}}_k(j)$ are calculated, in parallel with (4.23) and (4.26), as

$$\hat{\bar{t}}_k(j) = ½[\hat{t}_k(j-1) + \hat{t}_k(j)] \tag{4.35}$$

with

$$\hat{t}_k(j) = \hat{F}_{kj}^*/\hat{N}_k \tag{4.36}$$

and the $\hat{F}_{kj}^*$ are, in turn, forecast as

$$\hat{F}_{kj}^* = \left(\hat{U}_{k,j-1} + \hat{N}_{kj}\right)\hat{p}_j \tag{4.37}$$

where the $\hat{N}_{kj}$ are the same forecasts as in (4.16), the $\hat{U}_{k,j-1}$ are forecast according to the identity

$$\hat{U}_{kj} = \hat{U}_{k,j-1} + \hat{N}_{kj} - \hat{F}_{kj} \tag{4.38}$$

initialised by

$$\hat{U}_{k,J-k+1} = U_{k,J-k+1} \text{ (known)} \tag{4.39}$$

and the $\hat{p}_j$ are estimates of the p_j in the GLM defined by (FIN1-2).

This somewhat cavalier treatment of the forecasts $\hat{F}_{kj}$ is explained by the fact that, provided they are broadly realistic, they have comparatively little effect on the forecast loss reserves R_k. The reason for this is to be found in the concept of OT described in Section 4.3.2.

If expected PPCF is described by a function $\psi(t)$ of OT t, as in (4.28) (disregarding the experience year effect for the moment), then R_k is estimated by

$$\hat{R}_k = \hat{N}_k \int_{t_k(J-k+1)}^{1} \hat{\psi}(t)\, dt = \hat{N}_k \left(\int_{t_k(J-k+1)}^{t_k(J-k+2)} + \int_{t_k(J-k+2)}^{t_k(J-k+3)} + \cdots \right) \hat{\psi}(t)\, dt \tag{4.40}$$

The second representation of $\hat{R}_k$ on the right side expresses it as the sum of its annual components, which depend on the forecasts $\hat{F}_{kj}$. However, the first representation shows that $\hat{R}_k$ depends on only $\hat{\psi}(.)$ and $\hat{N}_k t_k(J-k+1) = \hat{U}_{k,J-k+1}$ = estimated total number of claims remaining unfinalised at the end of development year $J-k+1$. There is no dependency on the partition of these $\hat{U}_{k,J-k+1}$ claims by year of finalisation.

The partition of $\hat{U}_{k,J-k+1}$ into its components $\hat{F}_{kj}$ will interact with the experience year effect $\hat{\lambda}(k+j-1)$. If $\hat{\lambda}(.)$ is an increasing function, then the more rapid the finalisation of the $\hat{\lambda}(k+j-1)$ claims, the smaller the estimate $\hat{R}_k$. However, this is a second order effect and $\hat{R}_k$ is generally relatively insensitive to the partition of $\hat{U}_{k,J-k+1}$ into components $\hat{F}_{kj}$.

4.4 Outlying observations

As pointed out in Section 2.3, the standardised deviance residuals emanating from a valid payments model should be roughly standard normal, most falling within the range $(-2,+2)$.

The residual plots for the models fitted in Section 5.3 do indeed fall mainly within this range. Those of absolute order 3 or more are relatively few but probably of rather greater frequency than justified by the above normal approximation. Those of absolute order 4 or more form a small minority but, again, occur rather more frequently than expected.

The conclusion is that the data set contains some outliers despite the weight correction, but that they are not of extreme magnitude. To have deleted these data points might have created bias. To have attempted any other form of robustification would have opened up the question of how robust reserving should be pursued, a major research initiative in its own right.

Ultimately, with these considerations weighed against the rather mild form of the outliers, no action was taken; the outliers were retained in the data for analysis (unless excluded for some other reason (see Section 5.3)).

4.5 Comparability of different models

4.5.1 Basic comparative set-up

The main purpose of the present paper is to compare the predictive power of models that make use of finalisation count data with that of the chain ladder (which does not make use of such data).

The chain ladder, in its bald form, may be reduced to a mechanical algorithm without user judgement or intervention. Objective comparisons that allow for such intervention are difficult because of the subjectivity of the adjustments.

Consequently, the comparisons made in this paper are heavily restricted to quasi-objective model forms. The specific interpretation of this is that, subject to the exceptions noted below:

- All three models (chain ladder, PPCI and PPCF) are applied mechanically in their basic forms as described in Sections 4.1 to 4.3;
- The PPCF function $\psi(.)$ is initially restricted to a simple quadratic form

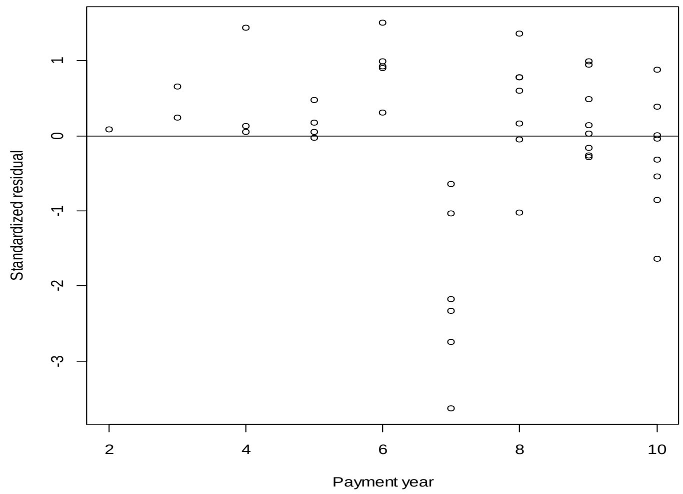
$$ln\,\psi(\bar{t}) = \beta_1\bar{t} + \beta_2\bar{t}^2 \tag{4.41}$$

- The inflation function $\lambda(.)$ is restricted to linear (constant inflation rate) or linear spline (piecewise constant inflation rate).

4.5.2 Anomalous accident and experience periods

Occasionally a residual plot will reveal an entire accident or experience year to be inconsistent with others. An example appears in Figure 4-3, which is a plot of standardised deviance residuals against experience year for the unadjusted chain ladder model applied to Company #671.

Figure 4-3 An anomalous experience year

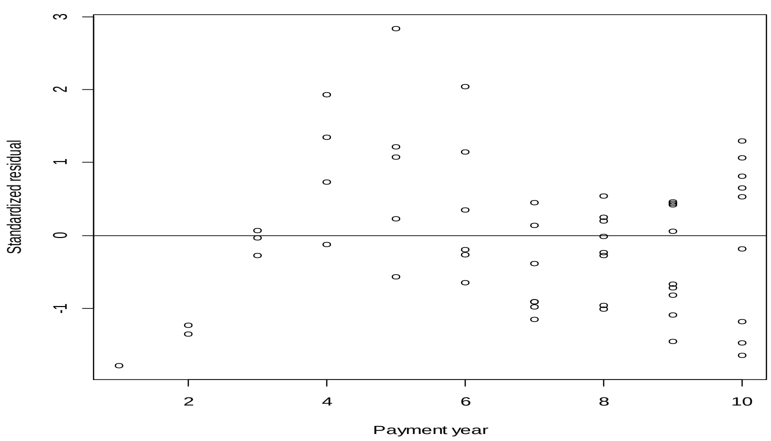


The anomalous experience of year 7 is evident. In such cases, the omission of that year from the analysis, i.e. assignment of weight zero to all observations in the year, is regarded here as admissible.

On other occasions a residual plot may reveal trending data. If the trend is other than simple, greater predictive model may be achieved by a model that excludes all but the most recent, stationary data than by a model that attempts to fit the trend.

An example appears in Figure 4-4, which is a plot of standardised deviance residuals against experience year for the unadjusted PPCI model with zero inflation, applied to Company #723. The PPCI appear to a positive inflation rate initially, followed by a negative rate, and finally an approximately zero rate. Stationarity appears to be achieved by the exclusion of all experience years other than the most recent 3 or 4.

Figure 4-4 A trending data set



4.5.3 Experience year (inflationary) effects

Allowances made

As noted in Section 2.4.1, claim payment data are unadjusted for inflation. It is therefore highly likely that they will display trends over experience years. The simple default option for incorporating this in the model is

$$ln\,\lambda(s) = \beta s \tag{4.42}$$

i.e. a constant inflation rate.

The initial versions of the PPCI and PPCF models include the experience year effect (4.42). In some cases, this simple trend is modified to a piecewise linear trend in alternative models.

This default inflationary effect is **not** incorporated in the chain ladder model for the reason that it would not materially improve the fit of the model to data. The reason for this is well known (Taylor, 2000) and is set out in Appendix B.

If a constant inflation rate added to the chain ladder model, it would add one parameter to the model while making little change to the estimated loss reserve. This amounts to over-parameterisation and the anticipated effect would be a deterioration in the prediction error associated with the loss reserve. This anticipation has been confirmed by numerical experimentation.

As mentioned, in some cases the PPCI and PPCF models have included a slightly more complex inflation structure than simple linear, whereas the chain ladder models have not done so. This perhaps gives the former models a predictive advantage over the chain ladder.

This is not viewed as introducing unfairness into the comparison of the different models' predictive powers. The chain ladder, implemented by means of the chain ladder algorithm (Section 4.1.2) as is typically the case, is incapable of such a refinement.

The inclusion of more complex modelling of experience year effects in PPCI and PPCF model but not in the chain ladder model, simply reflects the greater flexibility of GLM structures over rigid reserving algorithms.

Extrapolation to future experience years

The chain ladder model contains no explicit allowance for experience year effects though, as explained above, there is an implicit allowance for a constant inflation rate over the past and extrapolated into the future.

In the case of the PPCI and PPCF models, any allowance for experience year effects will necessarily be explicit. This necessitates decisions about the extrapolations of these effects into future experience years $(k + j - 1 > J)$. The following decision rules have been followed:

- When the past experience year trend takes the constant inflation form (4.42), the same form is extrapolated into the future, i.e. the future inflation rate is assumed constant and equal to the past rate;

- When the past experience year trend takes any other form, it is extrapolated as

$$\lambda(s) = \lambda(J + k - 1) \text{ for } s > J + k - 1 \quad (4.43)$$

i.e. nil future inflation.

4.6 Prediction error

Prediction error has been estimated in conjunction with each loss reserve estimate. This takes the form of an estimate of mean square error of prediction (**"MSEP"**) of each R and each of its components R_k. MSEP has been estimated by means of the parametric bootstrap, described in Section 4.6.1.

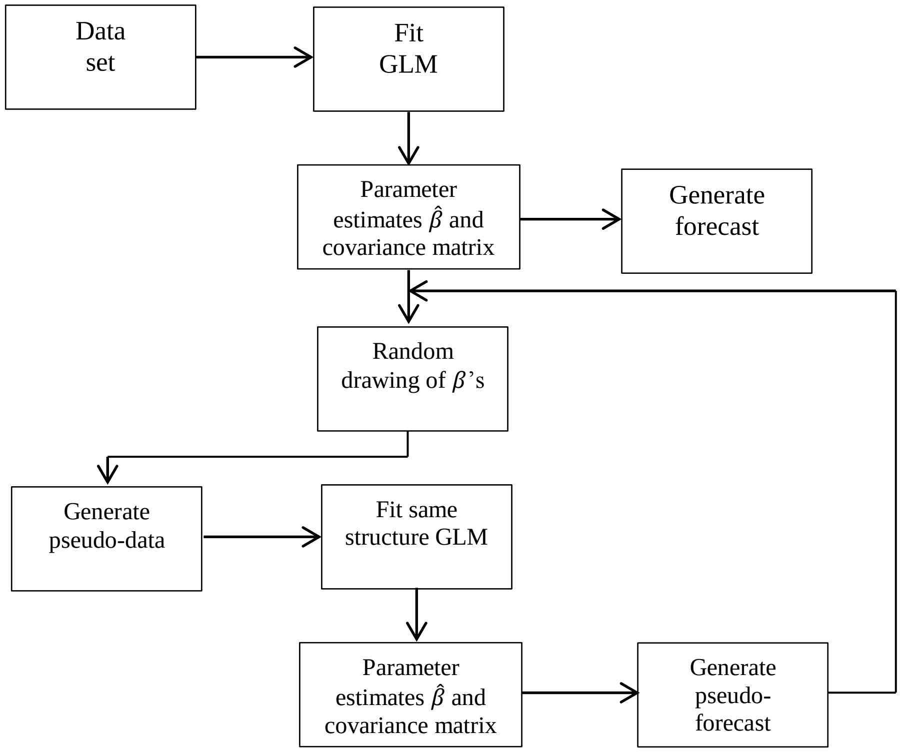
As noted in Sections 4.2 and 4.3, the PPCI and PPCF models consist of two and three sub-models respectively. These contrast with the chain ladder which is just a single model.

Each sub-model contains its own prediction error and serves to enlarge the total prediction error in the forecast loss reserve. The allowances made for the contributions of these sub-models are described in Sections 4.6.3 and 4.6.4.

4.6.1 Parametric bootstrap

A parametric bootstrap is used to estimate the distribution of the prediction of any single model. The algorithm for application of this to a GLM is as set out in Figure 4-5.

Figure 4-5 Parametric bootstrap of a GLM



A large sample of pseudo-forecasts, R in number, is generated by this means.

Assume that the GLM takes the form . The forecast in the figure is

$$\hat{Y}^{fut} = h^{-1}\left(X\hat{\beta}^{fut}\right) \tag{4.44}$$

The randomly drawn vector β, denoted $\tilde{\beta}$, satisfies

$$\tilde{\beta} \sim N(\hat{\beta}, Cov(\hat{\beta})) \tag{4.45}$$

where $Cov(\hat{\beta})$ is estimated for the GLM. The normality assumption is usually justified by the fact that the estimates $\hat{\beta}$ are ML and therefore asymptotically normal with indefinitely increasing sample size.

A **pseudo-data set** $\tilde{Y}$ is created, consistent with the model form and parameter values $\tilde{\beta}$:

$$\tilde{Y} = h^{-1}(X\tilde{\beta}\,) + \tilde{\varepsilon} \tag{4.46}$$

where $\tilde{\varepsilon}$ is a random drawing of ε, consistent with the error structure assumed for the original GLM and with scale parameter as estimated on the basis of Y.

The original model is now fitted to $\tilde{Y}$, yielding pseudo-estimates $\hat{\tilde{\beta}}$ and pseudo-forecasts $\hat{\tilde{Y}}^{fut}$.

By construction, the pseudo-forecasts, denoted $\hat{\hat{Y}}^{fut(r)}, r = 1,2, \dots, R$, are iid with the same distribution as $\hat{Y}^{fut}$. The empirical distribution associated with the sample $\left\{\hat{\hat{Y}}^{fut(r)}, r = 1,2, \dots, R\right\}$ is then taken as an approximation to the distribution of $\hat{Y}^{fut}$.

4.6.2 Chain ladder model

The parametric bootstrap described in Section 4.6.1 is applied to the GLM version of the chain ladder set out in Section 4.1.3.

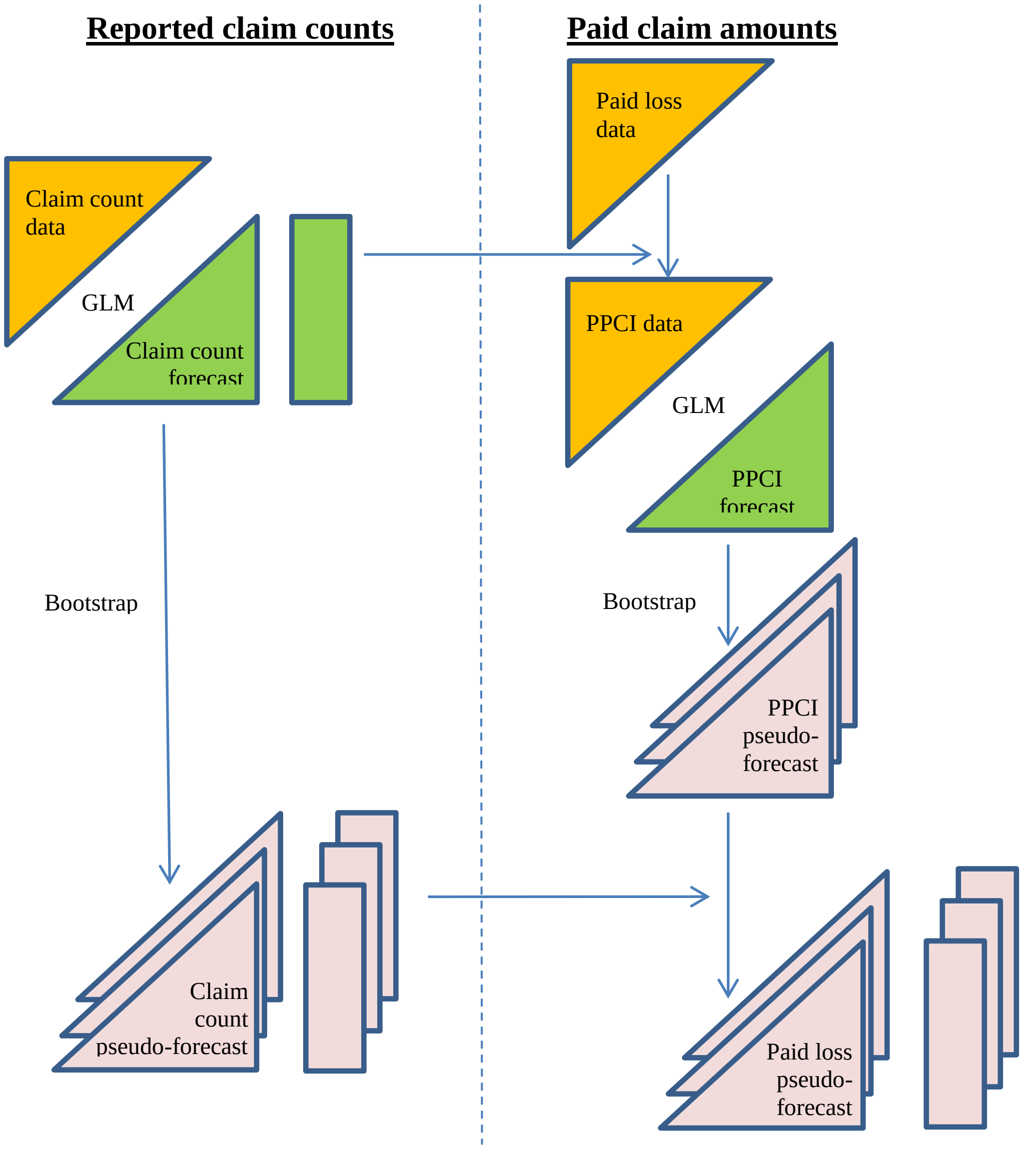
4.6.3 PPCI model

The PPCI model consists of:

- one GLM for PPCIs, as described in Section 4.2.1, dependent on the $\boldsymbol{N_k}$; and
- a second GLM to provide forecasts $\widehat{\boldsymbol{N}}_{\boldsymbol{k}}$ of the $\boldsymbol{N_k}$ (Section 4.2.2), which are then used as proxies for the $\boldsymbol{N_k}$ (Section 4.2.3).

Both of these models are bootstrapped and linked according to Figure 4-6.

Figure 4-6 Bootstrap of PPCI model



In the figure input data sets are represented as upper triangles, and output forecast arrays as lower triangles. Rectangles represent vectors that consist of row sums of forecast triangles. Thus,

- on the left side of the figure each entry of the vector represents the forecast number of claims yet to be reported in respect of an accident year;
- on the right side of the figure each entry of the vector represents the forecast amount of claims yet to be paid in respect of an accident year.

The detail of the bootstrap that appears on each side of the figure is as in Section 4.6.1. Each bundle of triangles is intended to represent the set of pseudo-forecast triangles generated by the bootstrap. Similarly, the bundles of rectangles.

A pseudo-forecast on the left is linked with its counterpart on the right. If in a notation akin to that of Section 4.6.1, $\hat{\hat{\pi}}_{kj}^{fut(r)}$ denotes the r-th forecast PPCI for cell (k,j), and $\widehat{\hat{N}}_{k}^{fut(r)}$ denotes the r-th forecast ultimate number of claims incurred for accident year k, then the r-th forecast of paid losses for cell (k,j) is calculated as $\hat{\hat{Y}}_{kj}^{fut(r)} = \hat{\hat{N}}_k^{fut(r)} \hat{\hat{\pi}}_{kj}^{fut(r)}$.

The final result at the bottom right of the diagram represents the set of pseudo-forecasts $\left\{\hat{\hat{R}}^{fut(r)}, r = 1,2, \dots, R\right\}$, where each $\hat{\hat{R}}^{fut(r)}$ is a vector of quantities $\hat{\hat{R}}_k^{fut(r)}$, denoting the r-th pseudo-loss-reserve for accident year k.

4.6.4 PPCF model

The PPCF model consists of:

- one GLM for PPCFs, dependent on the $\boldsymbol{F_{kj}}$, as described in the payments sub-model of Section 4.3.2;
- a second GLM to provide forecasts $\widehat{\boldsymbol{N}}_{\boldsymbol{k}}$ of the $\boldsymbol{N_k}$ (Section 4.2.2), which are then used as proxies for the $\boldsymbol{N_k}$ in the calculation of OTs as in (4.23); and
- a third GLM to provide forecasts of future numbers of claim finalisations, as described in the finalisations sub-model of Section 4.3.2.

All of these models are bootstrapped and linked according to Figure 4-7, most of which can be interpreted by reference to the description of Figure 4-6. Features peculiar to Figure 4-7 are as follows.

The finalisation counts are seen to be put to two different uses:

- as input to a GLM that forecasts future counts of finalisations; and
- as input to the calculation of OTs.

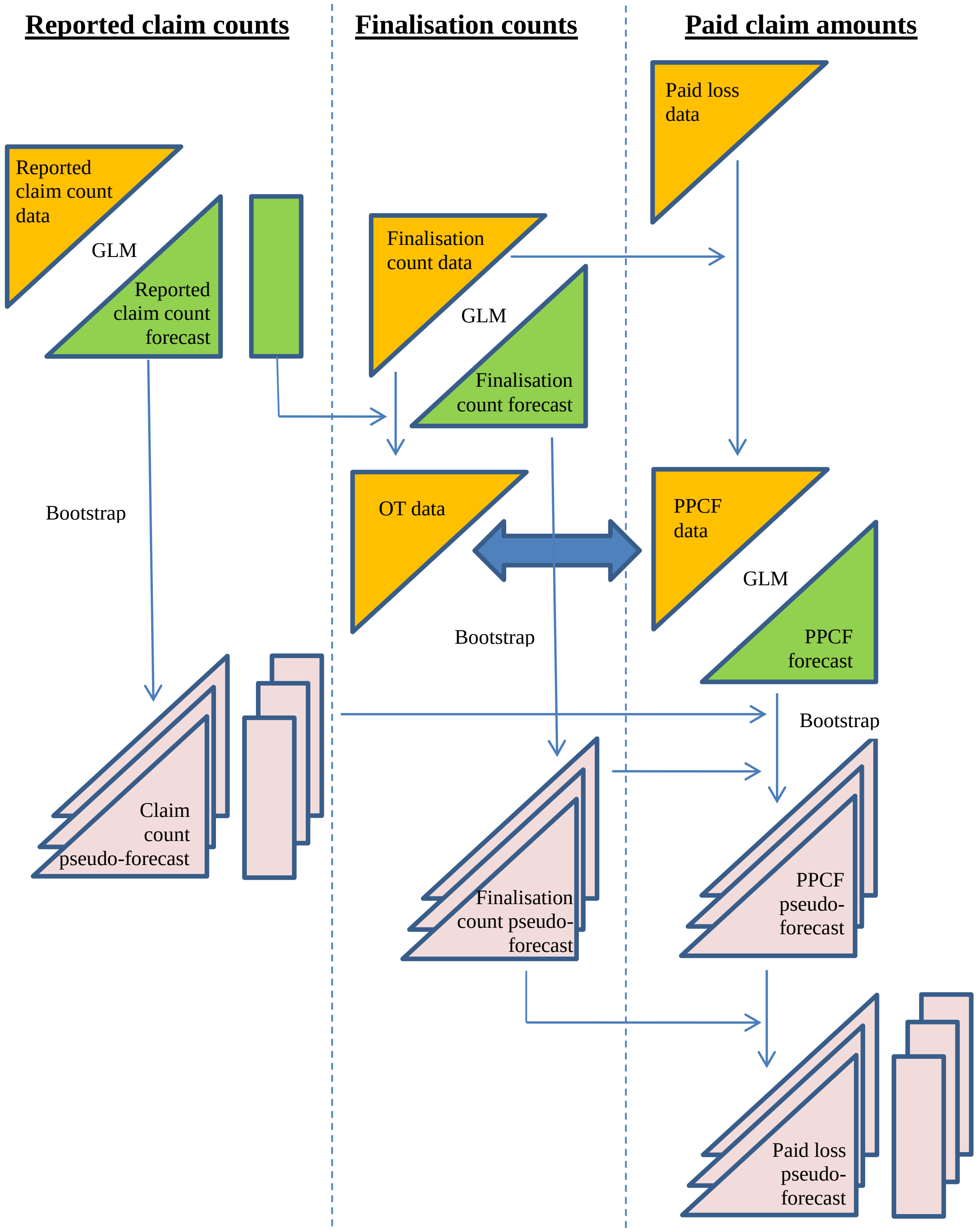
The OTs just mentioned also require estimates $\widehat{\boldsymbol{N}}_{\boldsymbol{k}}$ as inputs (see (4.23)), and these are obtained as forecasts from a GLM calibrated against the reported claim count triangle, just as in Section 4.6.3.

The block arrow connecting OT and PPCF data is intended to indicate that they form joint input to the GLM of PPCFs.

The figure clearly shows the existence of three separate bootstraps, and the links show how all three contribute to the pseudo-forecasts of PPCFs. Indeed, the pseudo-forecasts of finalisation counts contribute in two distinct ways:

- they lead to pseudo-forecasts of OTs, which are required to form the pseudo-forecasts of PPCFs; and
- they are combined with the pseudo-forecasts of PPCFs to yield pseudo-forecasts of paid losses.

Figure 4-7 Bootstrap of PPCF model



5. Results

5.1 Triangles selected for analysis

5.1.1 Lines of business selected

Although paid loss data was available in respect of 6 LoBs (Section 3.1), claim count data was available in respect of only three of these (Section 3.2), viz.

- Private passenger auto;
- Commercial auto;
- Workers compensation.

The first two of these are short tailed to the extent that one would expect elementary loss reserving procedures, such as the chain ladder, usually to be adequate. The refinement of models such as the PPCF is likely to be of greatest use in longer tailed data sets, which re characterised by:

- The potential for substantial movement over time in the OTs associated with particular development years; and
- Substantial differentiation of claim sizes by OT.

For these reasons the workers compensation LoB was selected for experimentation.

5.1.2 Companies selected

The workers compensation data base contained 166 companies. For various reasons, a PPCF model could not be applied to many of these, as detailed in Table 5-1.

Table 5-1 Defective PPCF data sets

Nature of data defect	**Number of companies**
Only small amounts of incurred losses	49
Start-up during period of training data set	14
Wind-down during period of training data set	7
Incurred loss amounts submitted only for a subset of training data set diagonals	7
No finalisation count data submitted	6
Finalisation count data submitted only for a subset of training data set diagonals	5
Virtually no paid loss data submitted	1
Reported claim count data submitted only for a subset of training data set diagonals	1
No defect	76
Total	**166**

Now consider the conditions under which the application of a PPCF model would be suitable. When a portfolio is subject to claim finalisation rates that change over accident years, i.e. the OTs represented within a given development year change from accident year to another, expected chain ladder age-to-age factors will shift with accident year. Indeed any model that does not recognise finalisation counts will be similarly distorted, i.e. subject to prediction bias.

The strength of the PPCF model is that it corrects for these shifts. The greater the degree of change in finalisation rates, the greater the value expected of the PPCF model.

Conversely, the model can be expected to add nothing in the absence of change in finalisation rates. In these cases, the PPCF model adds additional parametric structure with no expectation of improvement in prediction bias. Conventional statistical theory informs one that a model containing additional parameters that serve no apparent prediction purpose is likely to produce greater prediction uncertainty than a more parsimonious model.

Portfolios that started up or wound down during the period of the training set data contained a limited number of live accident years. In many of these cases, it appeared doubtful that volume of observations was sufficient to reveal changing rates of finalisation to the extent that the PPCF model would become superior. Consequently, cases of start-up or wind-down were excluded from consideration.

Table 5-1 indicates that more than half of the available companies were excluded from consideration for this reason and others concerned with data issues. Of those that remained, many were not realistic propositions for PPCF modelling. A substantial number of cases were fairly clearly affected by data errors.

An example is provided by Company #1066, whose triangle of counts of unfinalised reported claims appears as Table 5-2. The egregious entries along the penultimate diagonal are clearly visible.

Table 5-2 Company #1066: counts of unfinalised reported claims

Accident	Number of unfinalised reported claims at end of development year									
year	1	2	3	4	5	6	7	8	9	10
1988	0	229	162	81	43	23	11	8	-5	4
1989	876	513	231	105	42	25	16	463	7	
1990	1424	854	272	108	66	34	305	8		
1991	1693	475	182	86	42	1285	17			
1992	1476	445	204	95	1922	23				
1993	1159	384	152	4690	34					
1994	1521	336	8516	57						
1995	953	9641	114							
1996	8170	114								
1997	409									

Thus, many fewer than half of the 166 company data sets were suitable for PPCF analysis.

As pointed out earlier in the present sub-section, the PPCF model is likely to be useful only in the presence of changing claim finalisation rates. Hence the company data sets that were not excluded by Table 5-1 were reviewed in order to identify those subject to significant changes in finalisation rates.

Several simple measures were constructed for this identification. Details follow.

Variation in OTs experienced in specific development years
Consider the values of OT $t_k(j)$ defined by (4.23) and convert each of these to its complement $1 - t_k(j)$. This latter vaue is of more direct relevance to a loss reserve as it represents the proportion of claims incurred by number remaining to be finalised.

Let $m(j), d(j)$ denote the arithmetic mean and mean absolute deviation of the values of $1 - t_k(j)$ observed over development year j. Then define the first measure of variation in rates of finalisation as the following weighted average of the ratios $d(j)/m(j)\,, j = 1,2, \dots ,9$:

$$VRoF1 = \sum_{j=1}^{9} (10 - j) \frac{d(j)}{m(j)} \Bigg/ \sum_{j=1}^{9} (10 - j) \tag{5.1}$$

where each ratio has been weighted by the number of observations on which it depends.

Variation in OTs experienced in development year 1
Development year 1 usually experiences the highest incidence of claim finalisation. Any radical variation in the rate of finalisation is likely to be visible in this development year. The second measure of variation in rates of finalisation is therefore calculated in terms of the variation development year 1 rates from one accident year to the next, as follows:

$$VRoF2 = \max_{k<10} \frac{t_{k+1}(1) - t_k(1)}{1 - ½[t_{k+1}(1) + t_k(1)]} \tag{5.2}$$

Variation in OTs experienced along most recent diagonal
If, for a specific accident year, the OT attained in the most recent diagonal, i.e. at the end of claims experience to date, is approximately equal to a value typical for the development year attained by the accident year in question, then models that do not give particular recognition to finalisation counts may produce reliable loss reserves notwithstanding past changes in rates of finalisation. In short, the effects of past changes in rates of finalisation may neutralise one another.

Conversely, those same models may produce unreliable loss reserves in the presence of large departures from typical OTs along the most recent diagonal. Hence the third measure of variation in rates of finalisation (or, more precisely, the effect thereof on loss reserve) is the weighted average:

$$\boldsymbol{VRoF3} = \sum_{k=1989}^{1997} \boldsymbol{m}(\mathbf{1998} - \boldsymbol{k}) \frac{\mathbf{1} - \boldsymbol{t_k}(\mathbf{1998} - \boldsymbol{k})}{\boldsymbol{m}(\mathbf{1998} - \boldsymbol{k})} \Big/ \sum_{k=1989}^{1997} \boldsymbol{m}(\mathbf{1998} - \boldsymbol{k}) \tag{5.3}$$

Variation in premium volume

Some portfolios show very large variations in net earned premium even without actual start-up or wind-down. Such large changes may induce changes in the nature of the business under written and, as a result, in its settlement patterns. Such cases will generate small values of the statistic:

$$VNEP = \min_{k<11} P_k \Big/ \max_{k<11} P_k \tag{5.4}$$

where P_k denotes the net earned premium for year k.

It is not suggested that the PPCF model is less able than other models to deal with this situation. However, such cases do not form the best test bed for model comparison, and so portfolios displaying low values of $VNEP$ have not been selected for test data sets.

Ultimately 9 insurers were selected. They are listed in Table 5-3, along with the selection measures $VRoF1 - 3$ and $VNEP$. According to the above discussion, a data set becomes more suitable for inclusion as:

- $VRoF3$ departs from 100%;
- Either of $VRoF1$ or $VRoF2$ increases;

and provided that $VNEP$ is not too small.

Table 5-3 Selected test data sets

Company	Value of selection measure			
	VRoF1	VRoF2	VRoF3	VNEP
	%	%	%	%
#671	16	27	92	29
#723	13	7	97	38
#1538	28	25	118	54
#1694	6	31	102	37
#1767	8	6	98	42
#3360	58	95	81	67
#4731	6	9	101	25
#4740	20	29	76	46
#38733	31	49	92	43

Companies #1767, #4731 appear not to meet the selection criteria. However, examination of the constituents of the selection measures reveals the following:

- **Company #1767.** Displays values of the constituent ratio $[\mathbf{1} - \boldsymbol{t_k}(\mathbf{1998} - \boldsymbol{k})]/\boldsymbol{m}(\mathbf{1998} - \boldsymbol{k})$ (see (5.3)) for a couple of accident years materially different from 100%, despite the closeness of the average $\boldsymbol{VRoF3}$ to this value.

- **Company #4731.** Similar comment and, in addition, some relatively large values of the constituent ratio $d(j)/m(j)$ (see (5.1)) for several development years, despite the smallness of the average $\boldsymbol{VRoF}1$.

5.2 Model assessment

A major purpose of the compilation of the Meyers & Shi data base was the retrospective testing of loss reserve models. Accordingly, one is expected to apply the following procedure to the data base or a subset of it:

- Calibrate a model by reference to the training triangle(s), as defined in Section 3.1;
- Forecast loss reserve from the calibration
- Compare forecast with the actual outcomes, as given by the test triangle(s), i.e. symbolically, compare R, as defined by (2.4), with its forecast $\hat{R}$, and also perhaps compare R_k with $\hat{R}_k$.

While this approach, applied to a collection of models, will certainly determine which model produced the closest forecasts to subsequent outcomes, this will not necessarily equate to testing the general forecasting qualities of the models.

Strictly, the forecast $\hat{R}$ should be written $\hat{R}|\mathfrak{D}_J$, and this should be tested against some value of R that is consistent with $\mathfrak{D}_J$, i.e. one seeks to answer the question "Was $\hat{R}$ a good forecast on the basis of the information that existed at the end of year J?". Or, expressed another and slightly more precise way, "Was $\hat{R}$ a tight forecast (small prediction error) under the condition that the state(s) of the world existing over the training interval $\mathfrak{I}_J$ persist through the test interval?"

If $\mathfrak{D}_J^c$ is inconsistent with $\mathfrak{D}_J$, then the difference $R - \left(\hat{R}|\mathfrak{D}_J\right)$ will reflect this fact and will not necessarily be informative on the questions just posed. An example will illustrate.

Suppose that wage inflation is consistently 4% per annum throughout the training interval but falls to nil immediately at the end of that interval and remains there throughout the test interval. Suppose this causes the outcome R to be 10% less than would have occurred had the 4% inflation regime endured.

Now consider Models A and B. The former estimates claims inflation to be 4% per annum over $\mathfrak{I}_J$. It is sufficiently flexible to be able to produce forecasts on the basis of any desired set of future inflation rates. However, on the basis of $\mathfrak{D}_J$, a future rate of 4% is inserted into it. The resulting forecast is equal to $R/0.9$. If the future inflation rate had been known from some external source to be nil, the forecast could have been corrected to precisely the correct value.

Model B contains a purely implicit, and non-estimable, allowance for claims inflation. Its forecast is precisely equal to R. It is asserted here that this is **not** a reasonable estimate on the basis of the facts at the time of its formulation. The same forecast would have remained R had the inflation rate increased rather than decreased. Its equality to its estimand is fortuitous rather than informative.

5.3 Numerical results

5.3.1 Adopted models and results

Table 5-3 lists the company data sets selected for analysis. Each of these has been modelled by chain ladder, PPCI and PPCF models. In most cases, several variations of each of these models have been tested, and the best in each category selected for comparison with the other categories.

The families of specific model forms are as follows:

Chain ladder model

The model is as set out in (4.5)-(4.7) where weights take the form:

$$
\begin{aligned}
w_{k,j+1} &= 0 \text{ if } k+j+2 \in \mathcal{Y}_{CL} \\
&= 1 \text{ otherwise}
\end{aligned}
\qquad (5.5)
$$

with $\mathcal{Y}_{CL} \subset \{1988, \ldots, 1997\}$, a set of experience years specific to the company and model.

PPCI model

The model is as set out in (4.17)-(4.18) where values of the scale parameter take the form (4.20) but with some exceptions, as follows:

$$
\begin{aligned}
\phi_{kj} &= \infty \text{ (cell weight = 0) if } k+j+1 \in \mathcal{Y}_{PPCI} \\
&= \phi \hat{N}_k^2 \text{ otherwise}
\end{aligned}
\qquad (5.6)
$$

where $\mathcal{Y}_{PPCI} \subseteq \mathcal{Y}_{CL}$.

While the member of (4.18) involving the function $\lambda(.)$ was included in a number of test models, in no case did its inclusion produce a model that was materially superior (to that which excluded it). So this member does not feature in the PPCI models summarised in Table 5-5.

PPCF model

The model is as set out in (4.27) where values of the scale parameter take the form:

$$
\begin{aligned}
\phi_{kj} &= \infty \text{ (cell weight = 0) if } k+j+1 \in \mathcal{Y}_{PPCF} \\
&= \phi / w_{kj} \text{ otherwise } (w_{kj} \text{ from (4.29))}
\end{aligned}
\qquad (5.7)
$$

where $\mathcal{Y}_{PPCI} \subseteq \mathcal{Y}_{CL}$, and

$$\psi(t) = \beta_{OT1} t + \beta_{OT2} t^2 \textbf{ OR} \qquad (5.8)$$
$$\psi(t) = \beta_{OT1} ln(1-t) + \beta_{OT2} [ln(1-t)]^2 \textbf{ OR} \qquad (5.9)$$
$$\psi(t) = \beta_{OT1} (1-t)^{0.35} + \beta_{OT2} \min(0.8, t) \qquad (5.10)$$

$$ln\, \lambda(i) = \sum_{h=1}^{H} \beta_{Yh} max\big(0, min(y_h, i - y_{h-1})\big) \qquad (5.11)$$

for a defined set of values $\{y_0, \ldots, y_H\}$ subject to $y_0 = 1988, y_H = 1997$. Some coefficients β_{Yh} were set to zero before model fitting commenced.

Equation (5.11) represents the experience year effect as a **linear spline** with $H-1$ knots $\{y_1, \ldots, y_{H-1}\}$. The gradient of the spline segment over the interval $i \in (y_{h-1}, y_h)$ is β_{Yh}. The following special cases occur:

- $\boldsymbol{H = 1}$**:** (5.11) reduces to a simple linear function over the interval $i \in (1988,1997)$ (constant rate of claim cost escalation, as in (4.14)).
- $\boldsymbol{H = 0}$**:** By convention, (5.11) is taken to be null.

Table 5-4 sets out the specific model choices adopted and whose results are reported in Table 5-5.

Table 5-4 Selected models

Company	**Chain ladder model**	**PPCI model**	**PPCF model**			
	$\mathcal{Y}_{CL}$	$\mathcal{Y}_{PPCI}$	$\mathcal{Y}_{PPCF}$	$\psi(.)$ **from**	H	**Knots** y_h
#671	{1993,1994}	$\{\leq 1994\}$	{1993,1994}	(5.8)	2	{1992}
#723	$\{\leq 1994\}$	$\{\leq 1994\}$	$\emptyset$	(5.8)	3	{1991,1993}
#1538	$\{\leq 1994\}$	$\{\leq 1994\}$	$\emptyset$	(5.9)	3	{1991,1994}
#1694	$\{\leq 1991\}$	$\{\leq 1994\}$	$\emptyset$	(5.9)	4	{1991,1993,1995}
#1767	$\{\leq 1993\}$	$\{\leq 1993\}$	$\{\leq 1993\}$	(5.8)	0	
#3360	$\{\leq 1993\}$	$\{\leq 1992\}$	$\emptyset$	(5.8)	0	
#4731	$\{\leq 1993\}$	$\{\leq 1993\}$	$\emptyset$	(5.10)	2	{1995}
#4740	$\emptyset$	$\{\leq 1993\}$	$\{\leq 1993\}$	(5.8)	0	
#38733	{1993,1994}	$\{\leq 1993\}$	$\emptyset$	(5.8)	3	(a)

Note: (a) This case is exceptional. It does not involve a linear spline, but instead the PPCF is constant across all experience years except 1993, for which it assumes a different value.

Table 5-5 displays the principal results obtained from the application of the models described in Table 5-4. Detail underlying the table appears in Appendix C.

The left part of the table reports the "CoV", or **coefficient of variation** of the forecast loss reserve, defined as:

$$CoV = \frac{MSEP^{½}}{Forecast\ loss\ reserve} \tag{5.12}$$

where both numerator and denominator are obtained from the bootstrapped empirical distribution of outstanding losses described in Section 4.6.1.

The right part of the table reports the ratio of forecast loss reserve to the actual claim cost outcome from the test triangle.

Table 5-5 Forecast results

Company	CoV (%)			Ratio to actual (%)		
	Chain ladder	PPCI	PPCF	Chain ladder	PPCI	PPCF
#671	18	***11***	***11***	120	***106***	***91***
#723	12	***8***	9	94	***101***	118
#1538	24	14	***11***	105	95	***138***
#1694	***6***	***6***	8	***83***	***87***	94
#1767	5	5	***4***	93	109	***106***
#3360	***6***	12	22	***52***	64	81
#4731	***8***	***8***	***8***	***123***	***96***	***109***
#4740	7	***6***	7	104	***94***	82
#38733	10	***9***	22	88	***89***	289

For each company in Table 5-5, the smallest CoV(s) are displayed in bold italic font. The associated model(s) are the "winner(s)" for that company. Table 5-6 records the score of each model, where the score is equal to the number of wins out of the 9 cases, with a score of ½ in the case of a two-way tie, and a score of ⅓ in the case of a three-way tie.

Table 5-6 Model scores

Model	Number of wins	Percentage of wins
Chain ladder	1.8	20%
PPCI	4.3	48%
PPCF	2.8	31%
Total	**9**	**100%**

It is seen that the use of count data improves prediction error in 7.1 cases out of 9, i.e. 80% of the cases.

5.3.2 Discussion of results

It is instructive to examine the circumstances in which the different models produce superior predictive performance. This may be done by examining Table 5-3 and Table 5-5 in conjunction.

Company #3360

The chain ladder is clear winner in only one case, namely company #3360. VRoF1 and VRoF2 in Table 5-3 indicate that this portfolio is characterised by extremely variable rates of claim finalisation. The details of this appear in Table 5-7, which displays the company's triangle of OTs (actually complements thereof).

If rates of finalisation had been constant, then entries in this table would have been constant within each column. Evidently, this is far from the case.

Table 5-7 Company #3360: operational times

Accident year	Complement of operational time attained by end of development year 1	2	3	4	5	6	7	8	9	10
1988		0.119	0.075	0.046	0.032	0.025	0.015	0.008	0.005	0.004
1989	0.483	0.149	0.093	0.062	0.045	0.025	0.015	0.008	0.006	
1990	0.473	0.109	0.049	0.008	-0.036	-0.051	0.018	0.014		
1991	0.557	0.222	0.168	0.088	0.057	0.033	0.024			
1992	0.561	0.225	0.122	0.058	0.025	0.011				
1993	0.567	0.172	0.091	0.029	0.015					
1994	0.576	0.182	0.052	0.032						
1995	0.485	0.114	0.069							
1996	0.273	0.092								
1997	0.498									

A number of cells are shaded in Table 5-7, indicating likely disruptions to, or errors in, the data.

- **Accident year 1988.** There is no entry for development year 1. This is because no claims were reported for this cell, rendering calculation of numbers of finalisations impossible. It appears that the number of claims reported as received in development year 2 was actually the total for development years 1 and 2.
- **Accident year 1990.** The entries for development years 5 and 6 indicate that cumulative numbers of finalisations to those years exceeded the total number of claims estimated as incurred ($\hat{N}_k$) for 1990, which in turns exceeds the total number reported to the end of the relevant development year. This indicates the presence of data errors. Examination of the source data enables this anomaly to be traced to a large and negative number of claims reported in development year 6 (see Appendix A.2.6).
- **Accident year 1996.** This year is subject to dramatic increase in the rate of finalisation over accident year 1995, and one that is not sustained into accident year 1997. Reference once again to the source data for reported claims in Appendix A.2.6 reveals a dramatic increase in claim counts in accident year 1996, followed by a reversal of this in accident year 1997. Net earned premium did not change markedly over this period. To all appearances, either:
  - the data for the accident year are erroneous; or
  - the nature of the claims incurred changed abruptly, and temporarily, around 1996.

It is evident that the reliability of the models depending on claim counts (PPCI and PPCF) will be a function of the reliability of those counts. In the present case, there is clear evidence of errors in the counts and other cause to view them with suspicion.

In the case of clearly erroneous data (Appendices A.2.6 and A.3.6), the offending cells have been assigned zero weight in any modelling. However, it is possible (probable?) that adjacent cells at least carry similar anomalies that are not manifestly errors, e.g. quantities (1-OT) are under-stated but not actually negative.

The conclusion of this reasoning is that the application of PPCI and PPCF models to company #3360 was dubious from the start, and it is perhaps not surprising that the chain ladder forecast appears superior.

Company #1694

For this company the chain ladder is involved in a two-way tie with the PPCI model as the best predictor.

Reference to Table 5-3 indicates little overall variation in rates of claim finalisation (VRoF1), and OTs at the end of 1997 reasonably close to average values (VRoF3), though some appreciable movement in OTs observed in development year 1 (VRoF2). The detail appears in Table 5-8.

Table 5-8 Company #1694: operational times

Accident	Complement of operational time attained by end of development year									
year	1	2	3	4	5	6	7	8	9	10
1988	0.261	0.065	0.031	0.018	0.010	0.006	0.005	0.003	0.002	0.001
1989	0.260	0.064	0.032	0.018	0.012	0.008	0.005	0.004	0.003	
1990	0.191	0.060	0.031	0.019	0.012	0.008	0.005	0.004		
1991	0.197	0.061	0.032	0.019	0.012	0.009	0.006			
1992	0.197	0.061	0.030	0.017	0.011	0.008				
1993	0.200	0.060	0.031	0.018	0.012					
1994	0.242	0.063	0.033	0.018						
1995	0.219	0.060	0.028							
1996	0.225	0.060								
1997	0.232									

The single large shift in OTs occurs in development year 1 in the transition from accident year 1989 to 1990. One may conclude then that the finalisation count data adds little information. In this case it is unsurprising that PPCF model is outperformed by the other two.

Company #4731

For this company the chain ladder is involved in a three-way tie with the PPCI and PPCF models as the best predictor.

It was noted earlier in relation to Table 5-3 that this company appeared to have experience relatively stable rates of claim finalisation by all three criteria VRoF1-3. However, reference was made to the fact that some of the ratios in $d(j)/m(j)$ in (5.1) were material. Specifically, these were development years 6, 7 and 8. The individual development year contributions to VRoF1 were as shown in Table 5-9.

Table 5-9 Company #4731: development year contributions to VRoF1

Development year j	Ratio $d(j)/m(j)$
1	4%
2	3%
3	4%
4	4%
5	6%
6	13%
7	14%
8	16%
9	2%

The instability of rates of claim finalisation In development years 6 and later suggests that the PPCF model may produce loss reserve forecasts of superior reliability in accident years whose liability relates mainly to these development years.

Table 5-10 gives the CoVs of loss reserve separately by accident year for each of the three models. The loss reserve for accident year 1989 and 1990 do not involve development years 6 to 8, only 9 and 10. The PPCF model is **not** superior here.

On the other hand, loss reserves for accident years 1991 to 1993 are dominated by development years 6 to 8, and accident year 1994 is heavily affected by them. And here the PPCF model does produce superior performance.

The influence of these development years steadily diminishes with accident year increasing from 1994. And, sure enough, the PPCF model loses it superiority in these accident years.

Table 5-10 Company #4731: loss reserve prediction errors by accident year

Accident year	Estimated CoV of loss reserve (%)		
	chain ladder	PPCI	PPCF
1989	76	**73**	93
1990	**38**	**38**	44
1991	29	**28**	**28**
1992	21	21	**19**
1993	17	16	**14**
1994	12	12	**10**
1995	**9**	**9**	**9**
1996	**7**	**7**	**7**
1997	**5**	**5**	6
Total	**8**	**8**	**8**

.

Company #1538

Table 5-3 showed this company to have exhibited a consistently high degree of variation in rates of claim finalisation. The detail appears in Table 5-11.

Table 5-11 company #1538: rates of claim finalisation

Accident year	Complement of operational time attained by end of development year 1	2	3	4	5	6	7	8	9	10
1988	0.367	0.081	0.030	0.012	0.0057	0.0024	0.0003	0.0003	0.0000	0.0000
1989	0.393	0.097	0.043	0.021	0.0091	0.0042	0.0022	0.0017	0.0015	
1990	0.373	0.102	0.047	0.024	0.0116	0.0034	0.0025	0.0015		
1991	0.478	0.112	0.048	0.023	0.0131	0.0085	0.0074			
1992	0.381	0.078	0.028	0.011	0.0057	0.0035				
1993	0.382	0.079	0.029	0.010	0.0048					
1994	0.362	0.086	0.039	0.026						
1995	0.363	0.090	0.054							
1996	0.364	0.114								
1997	0.450									

Thus, company #1538 appears *a priori* to be a good candidate application of the PPCF model. And so it proves in Table 5-5, where that model outperforms its two rivals and, in particular, outperforms the chain ladder by a large margin.

It may be noted that there is some uncertainty concerning the numbers of claims incurred, and hence the OTs, for the company due to the high error rate in the triangle of numbers of claims reported (Appendix A.2.3).

Company #38733

Table 5-3 also indicates a consistently high degree of variation in rates of claim finalisation of this company. In an apparent paradox, however, the PPCF model performs extremely poorly.

Part or all of the explanation in this case appears to lie in faulty data. The triangle of finalisation counts appears in Table 5-12, in which anomalous observations have been shaded.

The entry of 282 in accident year 1989, development year 8 appears most peculiar and seems likely to be a mis-statement. It arises from a recorded number of 281 claims reported in the cell, whereas the expected number would have been 1 or 2. In addition, there are systematic anomalies in accident years 1988 and 1989. One may be forgiven for considering these data of dubious integrity.

Table 5-12 company #38733: finalisation counts

Accident year	Finalisation count in development year 1	2	3	4	5	6	7	8	9	10
1988	2,057	**1,520**	84	**18**	27	**1**	14	7	**37**	1
1989	3,524	834	111	64	**7**	13	10	**282**	3	
1990	4,438	836	178	**6**	24	15	4	4		
1991	4,577	821	**111**	62	30	18	3			
1992	5,656	913	142	55	25	10				
1993	6,067	1,011	143	46	29					
1994	5,760	940	120	46						
1995	5,487	820	113							
1996	5,190	734								
1997	4,908									

A version of the PPCF model was produced in which all observations associated with either or both of accident year 1989 and experience year 1993 were assigned zero weight, but without improvement in prediction error. The reason for this may be as follows.

If there were data errors in the shaded cells, there might be sympathetic errors in other cells. For example, finalisation counts in experience year 1993 appear low for a number of accident years. If this derives from some systematic mis-reporting whereby some finalisations from that experience year have been assigned to others, then a large number of entries in the table may be incorrect.

All in all, it is difficult to assess the quality of finalisation count data for this company and the applicability of the PPCF model.

6. Model extensions

It is explained in Section 4.5.1 that, for comparability with the chain ladder model, the PPCI and PPCF models are restricted to relatively simple and mechanical forms. No attempt has been made to optimise these model forms. It is likely that further investigation would lead to improved model forms, with accompanying reduction in their respective prediction errors.

6.1 PPCI model

Some simple possibilities can be outlined. First, recall assumption (PPCI2) in Section 4.2.1, leading to (4.13). According to this, the expected PPCI in cell (k,j) takes the form $\pi_j \lambda(k+j-1)$. The development year effect π_j is treated here as a categorical variable, and so estimates are required of the 10 parameters $\pi_1, \dots, \pi_{10}$.

This is done for comparability with the chain ladder model, which similarly specifies age-to age factors as the categorical variable $ln\ g_j$ in (4.6). It represents, however, parametric profligacy, as it is likely that some parametric form $\pi(j)$ could be found that would represent the development year effect almost as accurately as π_j and with considerably fewer parameters. This would reduce prediction error.

For example, Hoerl curves, as used by De Jong & Zehnwirth (1983), are sometimes used to represent the development year effect. These take the gamma-like parametric form:

$$ln\,\pi(j) = \beta_1 ln\,j + \beta_2 j \tag{6.1}$$

represented by just two parameters instead of 10.

6.2 PPCF model

One of the distinctions between the PPCI and PPCF models is that the latter contains an OT effect that is already expressed parametric form (see (5.8) to (5.10)). However, one of the requirements of the model in Section 4.5.1 is that initially $\psi(.)$ take the same form for all insurers.

This restriction is relaxed later, but it is still fair to say that the parametric form of $\psi(.)$ has been only lightly researched. Further investigation might lead to improved prediction error of the PPCF model.

6.3 Hybrid forecasts

Table 5-10 raises the possibility of hybrid forecasts. For example, one might base the loss reserve on say:

- the PPCF model for the middle accident years 1991-1995; and
- the PPCI model for the early and late accident years 1989-1990 and 1996-1997.

The effect is close to optimisation of the CoV of the total loss reserve. This would be less than the CoV from any one of the models. Note that this diversification from a single model is likely to reduce correlation across accident years, which will also contribute to reduction in the CoV of the total loss reserve.

Hybrid forecasts are discussed further in Chapter 12 of Taylor (2000).

6.4 Incurred losses

This paper has concentrated on incremental paid claim data, its analysis, and subsequent forecast. The same data source also provided triangles of **incurred claims** (defined in a cell as equal to paid claims adjusted by the increase in case estimates of unpaid claims over the interval from beginning to end of the cell).

The incurred claims data has not been used here. However, it could have been subjected to analysis by means of the chain ladder and other models. Those other models would not have been PPCI or PPCF, but would need to have been adapted to case estimate data. Some of the issues associated with such models are aired in Section 4.4 of Taylor (2000).

How the chain ladder would have fared in competition with these other models remains to be seen. This exercise is left for other investigators.

7. Conclusion

The purpose of the present paper has been to test whether loss reserving models that rely on claim count data can produce better forecasts than the chain ladder model (which does not rely on counts); better in the sense of being subject to a lesser prediction error.

A couple of commonly cited arguments against the use of count data have been canvassed in Section 1. It is suggested here that the data be allowed to speak for themselves, and that count data be used if doing so reduces prediction error, and not used otherwise.

The question at issue has been tested empirically by reference to the Meyers-Shi data set. While this includes data from a large number of portfolios, many of these are unsuitable for various reasons.

Ultimately the empirical investigation relies on only 9 workers compensation portfolios. This is limited and it is unlikely that the results can be considered conclusive. On the other hand, a consistent and coherent narrative emerges from the results to the point where they may be considered at least compelling.

Some trouble has been taken in attempting to ensure comparability between the different models. The chain ladder is a largely mechanical algorithm without user judgement or intervention. The formulation and calibration of the competing models (PPCI and PPCF) are much more flexible. Taking advantage of this flexibility, with detailed statistical modelling determining the algebraic form of the model might be considered to confer an unfair advantage on these models. For this reason, the competing models have also been largely constrained to relatively mechanistic versions.

This approach may hobble the competing models unduly. It may well be argued that the inflexibility of the chain ladder is an intrinsic shortcoming, and that imposing the same shortcoming on other models, where it does not naturally exist, does not in fact promote fairness of comparison.

Nonetheless, this is the approach taken here, and so to the extent that favourable aspects of the chain ladder that emerge from this study, they are probably over-stated. Conversely, any favourable findings in respect of the competing models probably apply *a fortiori*.

The 9 selected data sets were chosen according to a number of criteria (detail in Section 5.1), including material changes in rate of claim finalisation over the training interval. These are the circumstances in which the PPCF model in particular is, on *a priori* considerations, likely to perform well for, in the event of finalisation rates that remained strictly constant over time, finalisation counts would add no information to the loss process and forecast based on them would be expected to be inferior.

The first finding is that, for the selected data sets, the chain ladder rarely performs well. Either PPCI or PPCF model produces, or both produce, superior performance, in terms of prediction error, 80% of the time (Section 5.3.2).

When the chain ladder produces the best performance of the three models, the reasons are evident. Either count data contain erratic entries (companies #3360, #38733), or rates of claim finalisation are less variable than at first appeared (company #1694).

The first case is one in which the data speak for themselves; the second is a demonstration of the conclusion already reached that the chain ladder is likely to produce reliable estimates, relative to the PPCF model at least, in the presence of a high degree of stability in rates of claim finalisation.

For a portfolio characterised by consistently high variation in finalisation rates (company #1538), the PPCF model is likely to produce the forecast of loss reserve that has the lowest prediction error.

Sometimes variation in finalisation rates is seen to affect some accident years particularly, and other less so (company #4731). In these cases it is likely that the PPCF model will produce superior forecasts for the accident years affected, and inferior forecasts for the others.

Of the three LoBs for which count data were available, two (Private Passenger Auto and Commercial Auto) were short tailed. Here there is comparatively little scope for the chain ladder to under-perform, and so rival models are likely to be less useful.

The remaining LoB, on which the present study has relied, is only medium tailed. Typical experience is that the advantage of PPCI, and particularly PPCF, models over the chain ladder increases with tail length, since the longer tailed LoBs (e.g. Auto Bodily Injury, Public Liability) bring rates of claim finalisation more into play.

Moreover, the PPCF model is best adapted to claims whose payments are concentrated close to the finalisation date. This is typical of claims subject to settlement under the law of tort. The long tailed LoBs cited above satisfy the condition but the workers compensation LoB usually would not.

Certainly, there is a sentiment in some jurisdictions that failure to consider count data in the long tailed cases creates a serious risk of mis-estimation of loss reserve, and could expose the actuary to liability for negligence. An example of such mis-estimation is given in the data set investigated by Taylor (2000).

From this follows an expectation that the conclusions reached in this paper in connection with the workers compensation LoB would be likely to emerge in starker relief if a long tailed LoB were investigated.

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We wish to express our gratitude to Dr Peng Shi who privately made available the triangles of count data exemplified in Appendices A.2 and A.3.

Appendix A
Data

A.1 Paid loss amounts

A.1.1 Company #671

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	1346	2043	1277	665	388	378	29	166	76	47
1989	1411	2230	1088	623	583	97	29	161	70	
1990	1424	3036	1331	1158	17	358	272	134		
1991	2355	3853	1983	177	527	267	402			
1992	2544	5010	454	1676	953	188				
1993	3512	3233	2428	1113	503					
1994	2708	4652	2724	1350						
1995	2609	3631	2040							
1996	2652	2680								
1997	2192									

A.1.2 Company #723

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	3888	4193	2194	1432	546	417	176	191	14	15
1989	3708	4358	1912	1380	647	280	168	165	120	
1990	5176	6713	3711	2066	762	500	149	65		
1991	6792	6838	3213	1134	850	635	460			
1992	6091	5602	2252	1209	825	698				
1993	5374	5076	2090	1221	716					
1994	4533	4270	1586	707						
1995	4399	3993	1450							
1996	4361	4489								
1997	3392									

A.1.3 Company #1538

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	3178	3225	2420	1063	733	334	121	23	27	19
1989	3708	4982	3039	1300	970	285	285	54	13	
1990	5220	5771	2628	1566	770	458	42	260		
1991	4198	4874	2040	1148	669	328	128			
1992	3597	3878	1578	794	616	244				
1993	4281	4134	1855	1233	442					
1994	5329	5401	2171	626						
1995	4631	4475	1641							
1996	4217	4530								
1997	4169									

A . 1 . 4 Company #1694

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	30515	38802	22027	14027	8984	4990	3034	2230	1173	817
1989	39708	45440	23634	15867	8236	6422	4398	2311	1896	
1990	46048	49806	28397	16800	9427	6885	3937	2731		
1991	48445	42578	25783	18192	10272	5330	3832			
1992	41470	40719	21793	12222	7492	5412				
1993	34998	27593	14256	8414	4734					
1994	25756	24086	11352	5085						
1995	23079	21427	10770							
1996	20902	18046								
1997	19712									

A . 1 . 5 Company #1767

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	22190	38644	24270	15047	8661	6155	3823	2768	1934	1557
1989	26542	51256	28609	16015	10937	5240	4430	2683	1646	
1990	32977	67517	34392	22872	11233	9074	4722	4973		
1991	38604	75824	42675	24219	16089	11393	4592			
1992	42466	83354	38956	24269	15332	9527				
1993	46447	70317	38133	24522	14257					
1994	41368	58976	31677	19060						
1995	35719	47497	28052							
1996	28746	37287								
1997	25265									

A . 1 . 6 Company #3360

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	18283	24201	16193	10435	5555	2649	1985	1014	778	585
1989	21346	26631	21455	12389	5778	3530	1765	1295	621	
1990	24771	40101	26546	14214	7747	4569	3304	1983		
1991	26946	37927	24474	15733	8112	4584	2824			
1992	24419	30621	23272	13656	8142	4047				
1993	20715	26516	16343	11434	6439					
1994	22416	29599	21498	11096						
1995	27696	36055	25378							
1996	26637	53871								
1997	32509									

A.1.7 Company #4731

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	3272	3682	1805	1571	1111	590	298	338	258	161
1989	3818	4052	2184	1314	923	611	296	194	339	
1990	5375	5488	2783	1903	1407	661	466	235		
1991	5871	6103	4019	1771	1015	531	563			
1992	7701	8970	4540	2312	1483	642				
1993	8383	8828	3954	2312	1132					
1994	9304	11264	5231	3056						
1995	8409	8465	3674							
1996	6985	6924								
1997	6602									

A.1.8 Company #4740

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	27711	32029	23699	14910	5194	5948	2959	2101	1254	2996
1989	29975	37522	28278	17738	13028	5876	2369	2484	3139	
1990	31555	40802	26001	17864	8860	3573	2707	3956		
1991	32667	37919	27630	16571	8074	4134	3868			
1992	35529	47377	27673	14712	8217	7650				
1993	34690	46254	28256	16389	10883					
1994	39147	44837	26988	20761						
1995	34362	44470	30695							
1996	33710	45883								
1997	32206									

A.1.9 Company #38733

Accident	Amount of paid losses during development year ($000)									
year	1	2	3	4	5	6	7	8	9	10
1988	4386	3526	1756	834	389	290	158	83	71	20
1989	5321	4902	1939	889	484	199	194	94	86	
1990	4775	5819	2930	1313	516	204	184	141		
1991	6731	8442	3378	1841	876	610	347			
1992	9166	9711	3291	1091	690	350				
1993	8321	8235	2983	1479	1140					
1994	7045	7389	2739	1458						
1995	7332	7890	3228							
1996	6599	6271								
1997	7048									

A.2 Reported claim counts

A.2.1 Company #671

Accident	Number of claims reported during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	1810	274	10	16	4	2	1	1	0	0
1989	2185	293	35	13	6	0	2	2	2	
1990	2287	358	38	23	6	2	0	0		
1991	2725	321	60	4	2	6	2			
1992	2778	346	21	17	0	0				
1993	2892	436	43	2	3					
1994	2935	477	58	17						
1995	2702	495	65							
1996	2361	345								
1997	2087									

A.2.2 Company #723

Accident	Number of claims reported during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	3525	161	3	1	2	0	0	0	0	0
1989	3269	142	3	2	0	1	0	0	0	
1990	4181	198	10	4	1	1	0	0		
1991	4720	204	11	6	0	1	0			
1992	4927	216	13	3	1	1				
1993	4821	143	12	0	2					
1994	3841	112	6	1						
1995	3915	125	7							
1996	3920	99								
1997	2976									

A.2.3 Company #1538

Accident	Number of claims reported during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	3230	414	71	-6	1	-2	-1	0	0	-1
1989	3541	487	6	31	13	0	-7	-1	-1	
1990	3735	352	77	20	13	-7	-10	1		
1991	2391	366	42	19	-3	-2	-1			
1992	2731	357	41	4	-10	-5				
1993	3065	353	-4	-16	-9					
1994	3333	54	-26	-12						
1995	2755	-30	-29							
1996	2580	-54								
1997	2373									

A.2.4 Company #1694

Accident year	Number of claims reported during development year									
	1	2	3	4	5	6	7	8	9	10
1988	38217	6364	673	183	92	45	12	12	7	8
1989	42362	6986	627	225	82	32	31	20	23	
1990	53186	4554	683	216	75	52	28	30		
1991	49899	4337	360	128	58	42	29			
1992	45517	3829	358	113	55	21				
1993	35630	2964	281	141	59					
1994	29286	2660	297	131						
1995	27468	2400	314							
1996	24791	1946								
1997	23925									

A.2.5 Company #1767

Accident year	Number of claims reported during development year									
	1	2	3	4	5	6	7	8	9	10
1988	38132	6591	873	416	214	113	81	40	28	12
1989	39706	7330	1381	490	214	97	63	66	22	
1990	44502	9193	1508	637	255	178	74	46		
1991	57097	9973	1667	533	355	143	43			
1992	63734	11019	1508	565	285	81				
1993	63791	9933	1572	551	220					
1994	58580	8984	1452	487						
1995	50833	7269	1152							
1996	46234	6150								
1997	40703									

A.2.6 Company #3360

Accident year	Number of claims reported during development year									
	1	2	3	4	5	6	7	8	9	10
1988	0	26020	219	105	68	47	125	123	63	13
1989	22646	1137	281	102	82	242	145	95	6	
1990	19781	8632	247	368	521	78	-2053	13		
1991	21455	2828	320	733	234	242	18			
1992	19577	1896	1345	612	331	36				
1993	17748	3954	991	872	18					
1994	20867	3871	2579	49						
1995	23411	3992	145							
1996	58993	5164								
1997	27274									

A.2.7 Company #4731

Accident	Number of claims reported during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	5079	696	77	33	15	5	2	1	0	0
1989	5880	843	112	22	12	2	4	2	3	
1990	7334	1171	81	32	15	5	2	1		
1991	7974	1064	127	45	16	7	4			
1992	8455	1117	130	35	17	16				
1993	9008	1167	119	47	17					
1994	9714	1370	152	46						
1995	8476	1075	118							
1996	5988	754								
1997	4908									

A.2.8 Company #4740

Accident	Number of claims reported during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	23459	1571	85	17	6	1	0	1	0	0
1989	26658	1521	62	13	0	1	0	1	1	
1990	27319	1415	61	14	3	0	0	0		
1991	28735	1354	60	7	1	6	0			
1992	30645	1602	54	13	6	5				
1993	30960	1191	61	27	-84					
1994	31958	1144	70	17						
1995	28949	886	60							
1996	28475	860								
1997	23997									

A.2.9 Company #38733

Accident	Number of claims reported during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	3483	230	11	5	0	1	1	1	35	0
1989	4138	402	19	7	3	0	2	281	1	
1990	5076	414	18	8	3	0	1	1		
1991	5247	355	19	6	3	1	0			
1992	6405	369	17	14	4	4				
1993	6830	438	34	13	5					
1994	6456	418	43	8						
1995	6137	373	22							
1996	5766	336								
1997	5522									

A.3 Finalised claim counts

A.3.1 Company #671

Accident year	Number of claims finalised during development year 1	2	3	4	5	6	7	8	9	10
1988	1317	598	88	45	17	16	14	10	5	5
1989	1312	936	146	54	33	26	8	3	6	
1990	1134	1288	155	71	30	11	8	3		
1991	1739	1055	185	79	23	13	11			
1992	1677	1180	168	77	26	16				
1993	1677	1342	241	56	17					
1994	1598	1507	239	61						
1995	1592	1371	196							
1996	1647	872								
1997	1344									

A.3.2 Company #723

Accident year	Number of claims finalised during development year 1	2	3	4	5	6	7	8	9	10
1988	2469	1004	100	53	30	12	12	3	3	1
1989	2336	877	99	38	28	8	7	6	4	
1990	2940	1151	162	67	24	17	9	8		
1991	3204	1423	147	58	26	17	5			
1992	3469	1420	117	54	20	22				
1993	3413	1287	130	46	24					
1994	2712	1033	115	31						
1995	2729	1142	86							
1996	2752	1054								
1997	2144									

A.3.3 Company #1538

Accident year	Number of claims finalised during development year 1	2	3	4	5	6	7	8	9	10
1988	2347	1060	189	64	25	12	8	0	1	0
1989	2470	1205	217	92	47	20	8	2	1	
1990	2619	1132	232	94	54	34	4	4		
1991	1468	1029	179	70	28	13	3			
1992	1926	945	154	52	18	7				
1993	2090	1023	170	64	17					
1994	2132	922	156	45						
1995	1717	734	98							
1996	1617	633								
1997	1424									

A.3.4 Company #1694

Accident	Number of claims finalised during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	33730	8916	1574	591	362	153	75	65	49	32
1989	37293	9886	1611	711	301	198	122	75	49	
1990	47611	7695	1743	711	413	229	139	48		
1991	44065	7500	1557	748	382	180	118			
1992	40135	6798	1522	655	296	131				
1993	31337	5465	1148	518	238					
1994	24610	5843	968	497						
1995	23748	4815	970							
1996	21087	4490								
1997	20622									

A.3.5 Company #1767

Accident	Number of claims finalised during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	26643	13343	2771	1547	820	481	332	166	121	93
1989	27159	14468	3476	1802	1035	538	343	192	85	
1990	29392	17029	4133	2484	1421	762	364	296		
1991	36469	20485	5948	3026	1574	830	453			
1992	40383	24554	5940	2941	1420	716				
1993	41422	23412	5577	2629	1366					
1994	37862	22109	4933	2073						
1995	32895	18763	4003							
1996	29518	17141								
1997	26136									

A.3.6 Company #3360

Accident	Number of claims finalised during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	-9426	33013	1196	755	375	188	291	169	92	30
1989	12788	8280	1387	758	409	507	246	175	54	
1990	14557	10060	1680	1129	1220	397	-1912	125		
1991	11490	8712	1389	2080	791	637	237			
1992	10329	7903	2419	1492	790	318				
1993	10151	9245	1896	1445	328					
1994	11622	10804	3548	560						
1995	14445	10394	1241							
1996	48927	12186								
1997	18412									

A . 3 . 7 Company #4731

Accident	Number of claims finalised during development year								
year	1	2	3	4	5	6	7	8	9	10
1988	3965	1579	174	76	56	21	14	3	4	2
1989	4588	1837	252	84	46	24	16	9	6	
1990	5500	2601	276	128	56	44	8	10		
1991	6085	2504	388	99	78	24	17			
1992	6228	2899	333	134	72	41				
1993	6621	3074	364	136	79					
1994	7291	3256	407	165						
1995	6550	2575	335							
1996	4602	1820								
1997	3687									

A . 3 . 8 Company #4740

Accident	Number of claims finalised during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	17276	6397	466	321	222	147	72	34	22	13
1989	19901	6604	641	415	230	149	46	51	32	
1990	21006	6109	709	403	215	74	43	46		
1991	22297	6208	782	377	141	101	51			
1992	24716	6000	860	256	162	102				
1993	26531	4267	729	319	78					
1994	27680	4058	722	380						
1995	25132	3357	764							
1996	24830	3212								
1997	21006									

A . 3 . 9 Company #38733

Accident	Number of claims finalised during development year									
year	1	2	3	4	5	6	7	8	9	10
1988	2057	1520	84	18	27	1	14	7	37	1
1989	3524	834	111	64	7	13	10	282	3	
1990	4438	836	178	6	24	15	4	4		
1991	4577	821	111	62	30	18	3			
1992	5656	913	142	55	25	10				
1993	6067	1011	143	46	29					
1994	5760	940	120	46						
1995	5487	820	113							
1996	5190	734								
1997	4908									

Appendix B
Chain ladder and claim cost inflation

Suppose the data are unadjusted for inflation but that the true annual inflation rate is $\theta, const.$ Inflation adjustment of the observations to money values of experience year J would take the form

$$Y_{kj}^{(J)} = Y_{kj}(1+\theta)^{J-k-j+1} \qquad \text{(B.1)}$$

Now consider the application of the chain ladder forecast of loss reserve to each of the data sets $\{Y_{kj}\}$ and $\{Y_{kj}^{(J)}\}$. Consider $\{Y_{kj}\}$ first.

It is known that the ODP Mack model produces the same ML fitted values and loss reserve forecasts as the alternative model (referred to as the ODP cross-classified chain ladder model) defined by the following conditions:

(ODPCC1) All observations Y_{kj} are stochastically independent.
(ODPCC2) $Y_{kj} \sim ODP(\alpha_k \beta_j, \phi_j)$ for unknown parameters $\alpha_k, \beta_j, \phi_j$.

The equivalence was demonstrated for the simple Poisson case $(\phi_j = 1)$ by Hachemeister & Stanard (1975) and Renshaw & Verrall (1998), and for the more general ODP by England & Verrall (2002).

Let the ML estimates of α_k, β_j be $\hat{\alpha}_k, \hat{\beta}_j$ respectively. By the equivalence of fitted values of the two methods, the fitted value $\hat{Y}_{kj}$ associated with observation Y_{kj} can be expressed in the following form irrespective of whether one or other of the two models is used:

$$\hat{Y}_{kj} = \hat{\alpha}_k \hat{\beta}_j \tag{B.2}$$

It is also known (Schmidt & Wünsche, 1998) that for the Poisson case ML estimates are obtained by marginal sum estimation, i.e.

$$\sum^{\mathcal{R}(k)} Y_{kj} = \hat{\alpha}_k \sum^{\mathcal{R}(k)} \hat{\beta}_j \tag{B.3}$$

$$\sum^{\mathcal{C}(j)} Y_{kj} = \hat{\beta}_j \sum^{\mathcal{C}(j)} \hat{\alpha}_k \tag{B.4}$$

This result extends easily from the simple Poisson to the ODP case.

Now consider application of the chain ladder to the data set $\{Y_{kj}^{(J)}\}$. Marginal sum estimation equation parallel to (B.3) and (B.4) hold:

$$\sum^{\mathcal{R}(k)} Y_{kj}^{(J)} = \hat{\alpha}_k^{(J)} \sum^{\mathcal{R}(k)} \hat{\beta}_j^{(J)} \tag{B.5}$$

$$\sum^{\mathcal{C}(j)} Y_{kj}^{(J)} = \hat{\beta}_j^{(J)} \sum^{\mathcal{C}(j)} \hat{\alpha}_k^{(J)} \tag{B.6}$$

Now, by (B.1), the left side of (B.5) may be expressed as

$$\sum^{\mathcal{R}(k)} Y_{kj}^{(J)} = (1+\theta)^{J+1} \sum^{\mathcal{R}(k)} \frac{Y_{kj}}{(1+\theta)^{k+j}} \cong (1+\theta)^{J+1} \sum^{\mathcal{R}(k)} \frac{\hat{\alpha}_k \hat{\beta}_j}{(1+\theta)^{k+j}} \tag{B.7}$$

if it is valid to approximate the middle summation by the same expression with Y_{kj} replaced by $\hat{Y}_{kj}$. This is equivalent to assuming that the dispersions $Var[Y_{kj}]$ are suitably small.

The following result is equivalent to (B.7):

$$\sum^{\mathcal{R}(k)} Y_{kj}^{(J)} \cong \frac{\hat{\alpha}_k (1+\theta)^{J+1}}{(1+\theta)^k} \sum^{\mathcal{R}(k)} \frac{\hat{\beta}_j}{(1+\theta)^j} \tag{B.8}$$

This is the same as (B.5) provided that

$$\hat{\alpha}_k^{(J)} \cong \frac{\hat{\alpha}_k (1+\theta)^{J+1}}{(1+\theta)^k} \tag{B.9}$$

$$\hat{\beta}_j^{(J)} \cong \frac{\hat{\beta}_j}{(1+\theta)^j} \tag{B.10}$$

Similarly, it is possible to produce an equation that is the same as (B.6) if (B.9) and (B.10) hold. This demonstrates that $\hat{\alpha}_k^{(J)}, \hat{\beta}_j^{(J)}$ are approximately the ML parameter estimates for the data set $\{Y_{kj}^{(J)}\}$.

Now consider fitted values of the two models. For the raw data, the fitted value is given by (B.2). For the inflation-adjusted data, the fitted value is

$$\hat{Y}_{kj}^{(J)} = \hat{\alpha}_k^{(J)} \hat{\beta}_j^{(J)} \cong \hat{\alpha}_k \hat{\beta}_j (1+\theta)^{J-k-j+1} \tag{B.11}$$

This is in money values of experience year J and, if converted to raw values (i.e. of experience year $k+j-1$, yields simply $\hat{\alpha}_k \hat{\beta}_j$, the same (approximately) as (B.2).

This proves that the chain ladder fitted values for raw claim payments are approximately unaffected by whether one uses raw payment or inflation-adjusted data provided that the following two conditions hold:

(a) the annual inflation rate is constant over experience years $1,2,\dots,2J-1$; and
(b) dispersions $Var[Y_{kj}]$ are moderate.

Forecast values are obtained merely by applying the fitted value formulas (B.2) and (B.11) to future cells $(k+j-1>J)$, and so the result quoted for fitted values extends to forecasts also.

Appendix C
Results of numerical experiments

Table 5-3 lists the company data sets selected for analysis. Section 5.3 describes the models according to which the analysis is carried out. The results are reported in Tables C.1 to C.9. In these tables "CoV" means "coefficient of variation", defined as:

$$CoV = \frac{MSEP^{1/2}}{Forecast\ loss\ reserve} \tag{C.1}$$

C . 1 Company #671

Accident year	Actual claim cost ($000)	Loss reserve forecast by								
		Chain ladder model			PPCI model			PPCF model		
		Amount ($000)	CoV (%)	Ratio to actual (%)	Amount ($000)	CoV (%)	Ratio to actual (%)	Amount ($000)	CoV (%)	Ratio to actual (%)
1989	65	102	201	156	85	140	130	117	73	180
1990	178	243	116	136	190	78	107	232	56	131
1991	322	548	70	170	431	46	134	375	44	116
1992	656	988	49	151	703	33	107	590	33	90
1993	969	1,364	38	141	1,070	25	110	973	24	100
1994	2,081	2,356	27	113	1,842	19	89	1,508	19	72
1995	2,538	3,182	18	125	3,097	14	122	2,425	15	96
1996	3,885	4,563	13	117	4,579	11	118	3,784	11	97
1997	7,034	8,011	10	114	6,842	9	97	6,162	8	88
Total	**17,728**	**21,356**	**18**	**120**	**18,839**	**11**	**106**	**16,166**	**11**	**91**

C . 2 Company #723

Accident year	Actual claim cost ($000)	Loss reserve forecast by								
		Chain ladder model			PPCI model			PPCF model		
		Amount ($000)	CoV (%)	Ratio to actual (%)	Amount ($000)	CoV (%)	Ratio to actual (%)	Amount ($000)	CoV (%)	Ratio to actual (%)
1989	50	49	282	98	30	223	61	90	101	181
1990	30	189	120	630	133	77	442	314	47	1,047
1991	686	396	64	58	343	38	50	674	30	98
1992	496	598	39	121	667	24	134	1,009	23	203
1993	1,308	1,041	22	80	1,277	15	98	1,466	18	112
1994	1,197	1,407	16	118	1,647	11	138	1,742	16	146
1995	1,935	2,196	12	114	2,571	9	133	2,841	13	147
1996	3,934	3,987	9	101	4,143	7	105	4,950	9	126
1997	7,510	6,249	7	83	6,468	5	86	7,081	6	94
Total	**17,146**	**16,113**	**12**	**94**	**17,279**	**8**	**101**	**20,169**	**9**	**118**

C . 3 Company #1538

Accident year	Actual claim cost ($000)	Loss reserve forecast by								
		Chain ladder model			PPCI model			PPCF model		
		Amount ($000)	CoV (%)	Ratio to actual (%)	Amount ($000)	CoV (%)	Ratio to actual (%)	Amount ($000)	CoV (%)	Ratio to actual (%)
1989	20	120	383	598	69	323	345	0	3,436	0
1990	96	195	277	203	112	206	117	138	81	144
1991	162	275	161	170	159	106	98	228	53	141
1992	414	338	106	82	314	61	76	534	39	129
1993	536	705	61	132	699	36	130	1,252	29	234
1994	1,480	1,545	35	104	1,332	22	90	2,271	23	153
1995	2,635	2,216	23	84	1,819	18	69	3,081	19	117
1996	3,485	3,939	15	113	3,288	15	94	4,872	16	140
1997	7,692	8,018	10	104	7,977	12	104	10,495	13	136
Total	**16,520**	**17,351**	**24**	**105**	**15,769**	**14**	**95**	**22,872**	**11**	**138**

C . 4 Company #1694

Accident year	Actual claim cost ($000)	Loss reserve forecast by: Chain ladder model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCI model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCF model Amount ($000)	CoV (%)	Ratio to actual (%)
1989	1,359	1,070	68	79	976	56	72	2,235	43	164
1990	4,905	3,099	34	63	3,045	28	62	5,523	31	113
1991	7,316	5,644	21	77	5,460	18	75	7,787	26	106
1992	10,862	8,322	14	77	8,714	13	80	10,640	21	98
1993	9,178	9,974	10	109	11,080	11	121	12,078	17	132
1994	17,410	12,467	7	72	14,190	10	82	14,758	14	85
1995	24,059	18,894	5	79	19,515	9	81	20,503	12	85
1996	28,746	27,096	5	94	27,141	9	94	27,624	10	96
1997	52,335	43,623	4	83	45,829	8	88	45,761	8	87
Total	**156,170**	**130,190**	**6**	**83**	**135,950**	**6**	**87**	**146,909**	**8**	**94**

C . 5 Company #1767

Accident year	Actual claim cost ($000)	Loss reserve forecast by: Chain ladder model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCI model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCF model Amount ($000)	CoV (%)	Ratio to actual (%)
1989	1,894	1,952	41	103	1,774	48	94	1,603	22	85
1990	8,332	5,055	25	61	4,217	29	51	3,783	18	45
1991	16,246	10,922	16	67	9,981	18	61	8,882	13	55
1992	21,927	17,148	11	78	17,316	13	79	16,088	9	73
1993	31,192	26,018	8	83	27,734	10	89	25,657	8	82
1994	39,491	34,068	6	86	39,692	8	101	38,700	6	98
1995	42,031	45,763	4	109	53,081	6	126	53,213	5	127
1996	60,370	57,026	4	94	74,317	5	123	73,145	4	121
1997	86,327	87,934	3	102	107,234	5	124	104,185	3	121
Total	**307,810**	**285,886**	**5**	**93**	**335,347**	**5**	**109**	**325,255**	**4**	**106**

C . 6 Company #3360

Accident year	Actual claim cost ($000)	Loss reserve forecast by: Chain ladder model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCI model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCF model Amount ($000)	CoV (%)	Ratio to actual (%)
1989	-369	895	107	-243	844	138	-229	1,434	118	-389
1990	1,596	2,289	63	143	1,886	86	118	5,057	99	317
1991	5,838	4,104	39	70	3,313	55	57	9,438	90	162
1992	6,122	6,205	25	101	5,474	39	89	14,309	74	234
1993	12,931	8,307	17	64	9,183	31	71	20,394	56	158
1994	43,026	15,993	11	37	19,237	25	45	33,709	43	78
1995	76,828	34,810	8	45	35,046	22	46	49,561	34	65
1996	173,663	76,040	6	44	144,899	15	83	161,500	22	93
1997	198,122	120,962	5	61	109,861	18	55	126,246	20	64
Total	**517,757**	**269,607**	**6**	**52**	**329,743**	**12**	**64**	**421,648**	**22**	**81**

C . 7 Company #4731

Accident year	Actual claim cost ($000)	Loss reserve forecast by Chain ladder model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCI model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCF model Amount ($000)	CoV (%)	Ratio to actual (%)
1989	294	198	76	67	213	73	72	120	93	41
1990	396	704	38	178	686	38	173	454	44	115
1991	714	1,142	29	160	1,091	28	153	930	28	130
1992	2,338	2,195	21	94	1,674	21	72	1,664	19	71
1993	1,774	3,010	17	170	2,533	16	143	3,027	14	171
1994	4,034	5,572	12	138	4,283	12	106	5,546	10	137
1995	6,206	6,725	9	108	5,951	9	96	7,279	9	117
1996	6,751	9,063	7	134	7,107	7	105	8,226	7	122
1997	13,780	15,964	5	116	11,192	5	81	12,242	6	89
Total	**36,287**	**44,571**	**8**	**123**	**34,730**	**8**	**96**	**39,487**	**8**	**109**

C . 8 Company #4740

Accident year	Actual claim cost ($000)	Loss reserve forecast by Chain ladder model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCI model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCF model Amount ($000)	CoV (%)	Ratio to actual (%)
1989	3,277	3,800	46	116	3,470	37	106	710	45	22
1990	7,745	6,152	32	79	5,947	26	77	2,148	24	28
1991	8,884	9,099	23	102	9,370	19	105	4,616	17	52
1992	12,292	13,473	18	110	13,539	15	110	8,106	15	66
1993	17,580	19,698	14	112	19,185	12	109	14,927	13	85
1994	26,352	30,824	10	117	29,495	9	112	25,983	11	99
1995	48,915	48,412	7	99	42,641	7	87	40,280	10	82
1996	70,153	77,265	5	110	68,266	6	97	65,274	8	93
1997	111,150	111,064	5	100	95,148	5	86	89,300	6	80
Total	**306,348**	**319,786**	**7**	**104**	**287,060**	**6**	**94**	**251,343**	**7**	**82**

C . 9 Company #38733

Accident year	Actual claim cost ($000)	Loss reserve forecast by Chain ladder model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCI model Amount ($000)	CoV (%)	Ratio to actual (%)	PPCF model Amount ($000)	CoV (%)	Ratio to actual (%)
1989	44	66	215	150	57	197	130	128	154	290
1990	36	189	96	524	180	84	501	1,939	75	5,386
1991	337	450	60	133	326	54	97	7,249	32	2,151
1992	380	850	38	224	707	32	186	9,269	32	2,439
1993	1,286	1,206	26	94	1,210	23	94	10,575	30	822
1994	1,623	1,814	17	112	2,038	16	126	11,017	27	679
1995	4,141	3,333	11	80	3,419	12	83	11,843	24	286
1996	5,654	5,187	8	92	5,962	10	105	14,088	19	249
1997	16,054	12,884	5	80	12,370	8	77	19,373	13	121
Total	**29,555**	**25,978**	**10**	**88**	**26,270**	**9**	**89**	**85,481**	**22**	**289**

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