# Do finalisation counts improve loss reserves?

**Greg Taylor** Taylor Fry Consulting Actuaries Sydney Australia Jing Xu University of New South Wales Sydney Australia

Casualty Loss Reserve Seminar, Boston USA 15-17 September 2013

#### More formal title

"An empirical investigation of the value of finalisation count information to loss reserving"

Formal paper at <u>https://cas.confex.com/cas/clrs13/webprogram/S</u>ession6523.html

## Partially funded by Institute of Actuaries of Australia

Taylor & Xu Finalisation counts

#### Purpose

- Consider a data set consisting of triangles of:
  - Paid loss amounts
  - Reported claim counts
  - Finalisation counts
- The objective is to forecast the amount of outstanding paid losses
  - Together with an estimate of prediction error
- Is the prediction error likely to be larger or smaller by virtue of the recognition of the claim counts in the loss reserving model?

#### Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

#### Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

#### Meyers-Shi data set (1)

- Produced around 2011 by Glenn Meyers & Peng Shi
  - See

<u>http://www.casact.org/research/index.cfm?fa=loss</u> <u>reserves\_data</u>

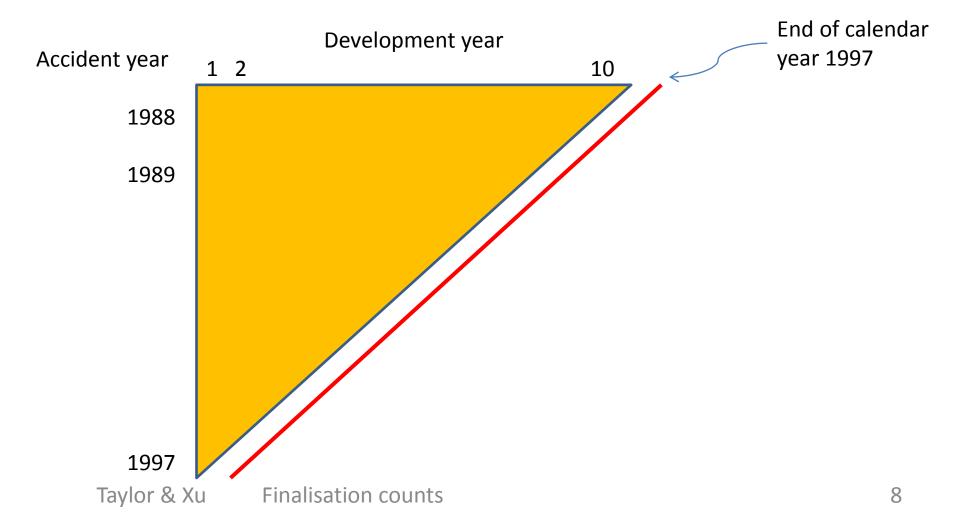
• Extracted from Schedule P returns to NAIC

#### Meyers-Shi data set (2)

- Separate data set for each of 6 LoBs:
  - Private passenger auto liability/medical
  - Commercial auto/truck liability/medical
  - Workers' compensation
  - Medical malpractice Claims made
  - Other liability Occurrence
  - Products liability Occurrence
- For each LoB, a number of companies

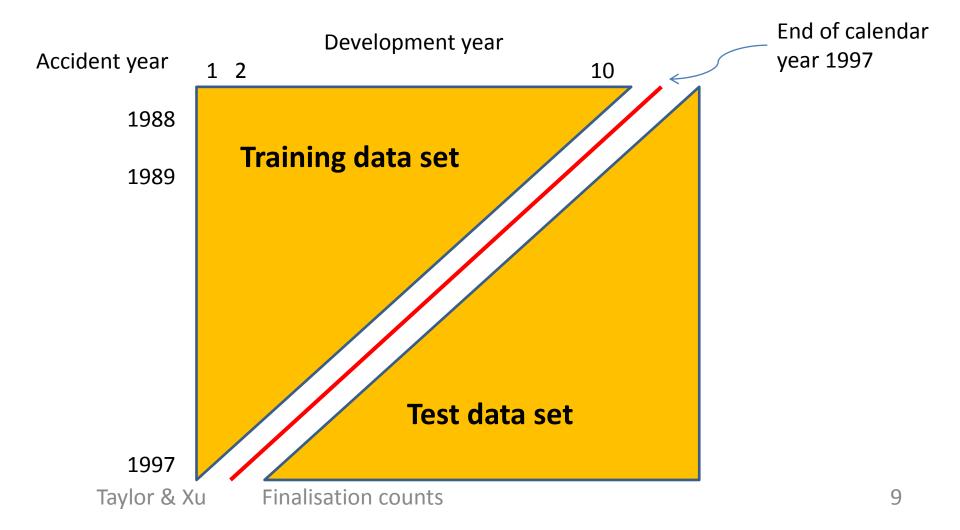
#### Meyers-Shi data set (3)

#### • Form of data



#### Meyers-Shi data set (3)

#### • Form of data



#### Meyers-Shi data set (4)

- Content of triangles
  - Paid amounts
  - Incurred amounts

### Meyers-Shi data set (4)

- Content of triangles
  - Paid amounts
  - Incurred amounts
  - No count data!
    - Notifications
    - Finalisations
      - Would these not assist?

### Finalisation count data (1)

- Reasons given to explain why these counts would not be helpful
  - Generally they are unreliable
    - They may be gross or net of reinsurance
    - The basis may differ from one year to another
    - Some companies simply "make them up"
      - e.g. divide the triangle of paid losses through by some notional claim size

### Finalisation count data (2)

- It get's worse
  - Finalisation counts, though available from Schedule P, are actually derived data
  - Raw claim count data comprise
    - Reported claims
    - Unfinalised claims

### Finalisation count data (2)

- It get's worse
  - Finalisation counts, though available from Schedule P, are actually derived data
  - Raw claim count data comprise
    - Reported claims
    - Unfinalised claims

*#finalisations in a cell = opening #unfinalised + #reported — closing #unfinalised* 

 So if reported or unfinalised counts are inaccurate, then so will the derived counts of finalisations be inaccurate

Taylor & Xu Finalisation counts

### Finalisation count data (3)

- It is an empirical fact that some triangles of finalisation counts are inaccurate
- Example

Accident	Number of unfinalised reported claims at end of development year									
year	1	2	3	4	5	6	7	8	9	10
1988	0	229	162	81	43	23	11	8	-5	4
1989	876	513	231	105	42	25	16	463	7	
1990	1424	854	272	108	66	34	305	8		
1991	1693	475	182	86	42	1285	17			
1992	1476	445	204	95	1922	23				
1993	1159	384	152	4690	34					
1994	1521	336	8516	57						
1995	953	9641	114							
1996	8170	114								
1997	409									
or & Xu	Finalisation counts									

### Finalisation count data (4)

- An alternative view
  - We cannot assume that all companies returned counts are inaccurate
  - Perhaps we should allow the data to speak for themselves
    - If a company's counts are inaccurate, then either:
      - This will be manifest in the data (certainly reject them then); or
      - It will create more subtle distortions of any model based on the counts

### Finalisation count data (5)

- Suggested general procedure for forecasting loss reserve
  - Apply a number of models to the data
    - Some dependent on counts
    - Some independent of counts
  - For each model generate:
    - Forecast
    - Associated uncertainty, e.g. mean square error of prediction ("MSEP")
  - Select model with smallest MSEP
    - For current purposes we omit the possibility of a further reduction in MSEP by means of a combination of models
  - Any inaccuracies in the count data can be expected to enlarge the MSEP of models depending on them

#### Finalisation count data (5)

- On the basis of these arguments, Glenn and Peng agreed to extract triangles of Schedule P count data for 3 of the 6 LoBs:
  - Private passenger auto liability/medical
  - Commercial auto/truck liability/medical
  - Workers' compensation
- We chose to experiment with workers' compensation data since:
  - This would be the longest tailed of the 3 LoBs
  - We considered that it would carry the greatest forecast uncertainty
- In fact, workers' compensation is not especially long tailed
  - We would expect the methods we develop here to produce even stronger results for Auto Bodily Injury or General Liability

#### Final data set

- Workers' compensation
  - Training triangle and test triangle for:
    - Paid amounts
    - Incurred amounts
    - Reported counts
    - Unfinalised (equivalently, finalised) counts

#### Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

#### Models selected for experimentation

- Chain ladder: widely used, independent of claim counts
- 2. A model that recognises claim counts, especially finalisation counts
  - Sensitive to changes in the rate of claim finalisation
- 3. An intermediate model that recognises **counts of reported claims** but not finalisations
  - Insensitive to changes in the rate of claim finalisation;
     BUT
  - Not based on age-to-age ratios, so forecasts for latest accident years possibly less volatile than chain ladder

### Chain ladder: preliminaries (1)

#### Notation

k denotes accident year

*j* denotes development year

 $Y_{kj}$  denotes incremental claim amount in (k, j) cell

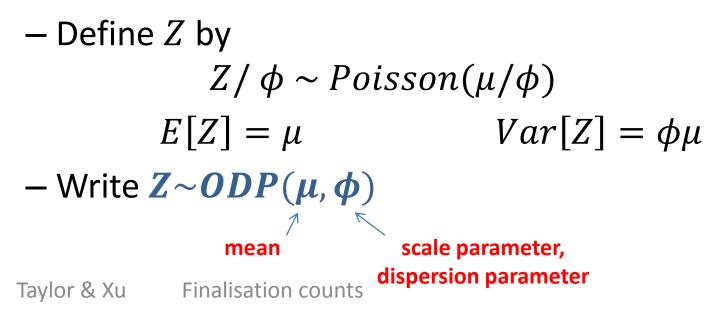
 $Y_{kj}^* = \sum_{i=1}^{j} Y_{ki}$  denotes cumulative claim amount to (k, j) cell

### Chain ladder: preliminaries (2)

• Poisson distribution

 $Y \sim Poisson(\mu)$  $E[Y] = \mu \qquad Var[Y] = \mu$ 

• Over-dispersed Poisson (ODP) distribution



#### Chain ladder (ODP Mack form)

- 1) Accident periods are stochastically independent, i.e.  $Y_{k_1j_1}$ ,  $Y_{k_2j_2}$  are stochastically independent if  $k_1 \neq k_2$ .
- 2) For each k = 1, 2, ..., J, the  $Y_{kj}^*$  (*j* varying) form a Markov chain.
- 3) For each k = 1, 2, ..., J and j = 1, 2, ..., J 1, define  $G_{kj} = Y_{k,j+1}/Y_{kj}^*$  and suppose that  $G_{kj} \sim ODP\left(g_j, \phi/(Y_{kj}^*)^2\right)$ , where  $\phi$  is independent of k, j $1 + G_{kj}$  = conventional age-to-age factor

### Why the ODP Mack form?

- This project is concerned with loss reserving with estimated prediction error
- Estimation of prediction error requires a stochastic model
- The chain ladder is not conventionally formulated as a stochastic model
- **BUT** the ODP Mack version:
  - Is stochastic; and
  - Its maximum likelihood estimates of age-to-age factors coincide with the conventional ones

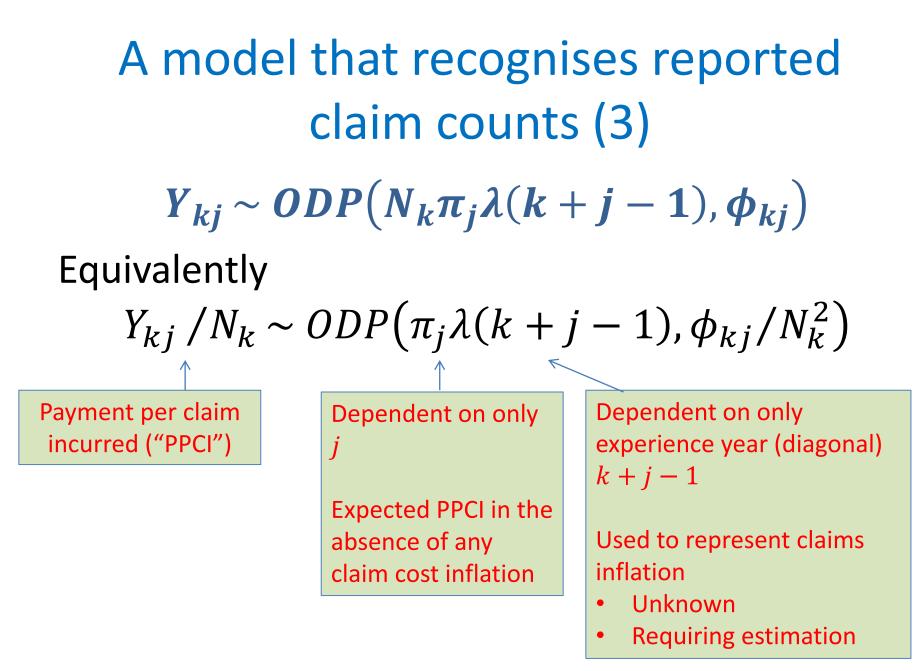
## A model that recognises reported claim counts (1)

- Notation
  - $N_{kj}$  denotes incremental reported claim count in (k, j) cell
  - $N_k = \sum_{j=1}^{\infty} N_{kj}$  = number of claims incurred in accident year k
  - $\widehat{N}_k = \sum_{j=1}^{\infty} \widehat{N}_{kj}$  = an estimate of the number incurred, from a chain ladder model
- Note that k + j 1 denotes experience year, often called payment year, situated on the (k + j - 1)-th diagonal of a triangle

A model that recognises reported claim counts (2)

1) All  $Y_{kj}$  are stochastically independent

- 2) For each k = 1, 2, ..., J and j = 1, 2, ..., J, suppose that  $Y_{kj} \sim ODP(N_k \pi_j \lambda (k + j - 1), \phi_{kj})$ where
  - $\pi_j$ , j = 1, 2, ..., J are parameters
  - $\lambda: [1, 2, 3, \dots, 2J 1] \rightarrow \Re$



## A model that recognises reported claim counts (4)

 $Y_{kj}/N_k \sim ODP(\pi_j\lambda(k+j-1),\phi_{kj}/N_k^2)$ 

• Express in GLM form

 $Y_{kj}/\widehat{N}_k \sim ODP(\mu_{kj}, \phi/\widehat{N}_k^2)$ 

where

$$\mu_{kj} = exp\left(\ln \pi_j + \ln \lambda(k+j-1)\right)$$

#### Default assumption:

- $\phi_{kj}$ constant over all cells
- Empirically reasonable
- Inflation function λ(.) needs to be expressed in a form that is linear in a set of parameters
  - Simplest available form is

$$\lambda(m) = \lambda^{m}, \lambda = \text{const.} > 0$$
  
$$\mu_{kj} = \exp(\ln \pi_{j} + (j + k - 1) \ln \lambda)$$

Though there will be experimentation with other forms as necessary

## A model that recognises claim finalisation counts (1)

- First introduce the concept of **operational time**
- Notation
  - $F_{kj}$  denotes incremental finalised claim count in (k, j) cell
  - $-F_{kj}^*$  denotes cumulative finalised claim count in (k, j) cell
- Define operational time at the end of development year *j* in respect of accident year *k* as

$$t_k(j) = F_{kj}^* / \widehat{N}_k$$

i.e. proportion of incurred claims finalised

$$t_k(0) = 0 \qquad t_k(\infty) = 1$$

• Average operational time associated with (k, j) cell is  $\bar{t}_k(j) = \frac{1}{2}[t_k(j-1) + t_k(j)]$ 

A model that recognises claim finalisation counts: payments sub-model (1)

1) All  $Y_{kj}$  are stochastically independent

- 2) For each k = 1, 2, ..., J and j = 1, 2, ..., J, suppose that  $Y_{kj} \sim ODP(F_{kj} \psi(\bar{t}_k(j)) \lambda(k+j-1), \phi_{kj})$ where  $-\psi: [0,1] \rightarrow \Re;$ Based on concept of "settlement queue"
  - $-\lambda(.)$  is an inflation function of the same type as previously

A model that recognises claim finalisation counts: payments sub-model (2)

 $\begin{array}{c} Y_{kj} \sim ODP(F_{kj} \psi(\bar{t}_k(j)) \lambda(k+j-1), \phi_{kj}) \\ \text{Equivalently} \\ Y_{kj} / F_{kj} \sim ODP(\psi(\bar{t}_k(j))\lambda(k+j-1), \phi/w_{kj}F_{kj}^2) \\ \uparrow \\ \text{Payment per claim} \\ \text{finalised ("PPCF")} \\ \end{array}$ 

Expected PPCF in the absence of any claim cost inflation experience year (diagonal) k + j - 1

Used to represent claims inflation

- Unknown
- Requiring estimation

#### A model that recognises claim finalisation counts: payments sub-model (3)

 $Y_{kj}/F_{kj} \sim ODP(\psi(\bar{t}_k(j))\lambda(k+j-1),\phi/w_{kj}F_{kj}^2))$ 

Residual plot for  $w_{kj} = 1$ 

Set  $w_{kj} = {5 + 100[\bar{t}_k(j) - 0.92]}^{-2}$  for  $\bar{t}_k(j) \ge 0.92$ 

???

**°** °

0

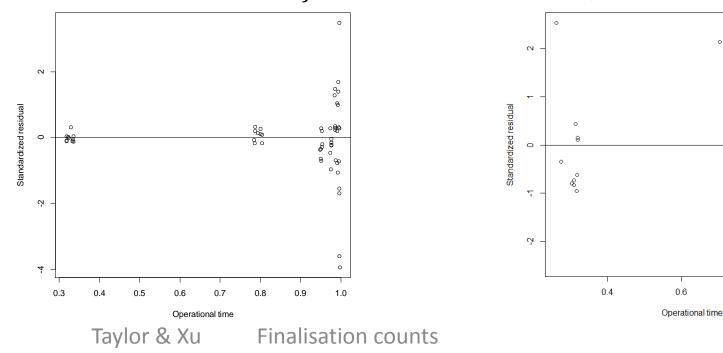
0

1.0

ಿಹಿತಿ

0

0.8



A model that recognises claim finalisation counts: finalisations sub-model (1)

 Consider forecasts of future claim amounts  $Y_{ki} \sim ODP(F_{kj} \psi(\bar{t}_k(j)) \lambda(k+j-1), \phi_{kj}))$  $E[Y_{kj}] = F_{kj} \psi(\overline{t}_k(j)) \lambda(k+j-1)$  $\widehat{Y}_{kj} = \widehat{F}_{kj}\widehat{\psi}(\widehat{\overline{t}}_k(j))\,\widehat{\lambda}(k+j-1)$ Estimated by payments sub-**Requires a sub-model** of finalisation counts model GLM

A model that recognises claim finalisation counts: finalisations sub-model (2)

- Notation
  - Let  $U_{kj}$  denote the number of reported but unclosed claims at the end of development year j in respect of accident year k
- Sub-model
- 1) All  $F_{kj}$  are stochastically independent
- 2) For each k = 1,2, ..., J and j = 1,2, ..., J, suppose that

$$F_{kj} \sim \operatorname{Bin}(U_{k,j-1} + N_{kj}, p_j)$$

where the  $p_j$  are parameters (finalisation probabilities)

## A model that recognises claim finalisation counts: forecast algorithm

$$\widehat{F}_{kj} = (\widehat{U}_{k,j-1} + \widehat{N}_{kj})\widehat{p}_j \widehat{U}_{kj} = \widehat{U}_{k,j-1} + \widehat{N}_{kj} - \widehat{F}_{kj}$$

$$\hat{t}_{k}(j) = \hat{F}_{kj}^{*} / \hat{N}_{k}$$
$$\hat{\bar{t}}_{k}(j) = \frac{1}{2} [\hat{t}_{k}(j-1) + \hat{t}_{k}(j)]$$

$$\widehat{Y}_{kj} = \widehat{F}_{kj} \widehat{\psi} (\widehat{\overline{t}}_k(j)) \widehat{\lambda}(k+j-1)$$

A model that recognises claim finalisation counts: payments sub-model re-visited (1)

$$Y_{kj} \sim ODP(F_{kj} \psi(\overline{t}_k(j)) \lambda(k+j-1), \phi_{kj})$$

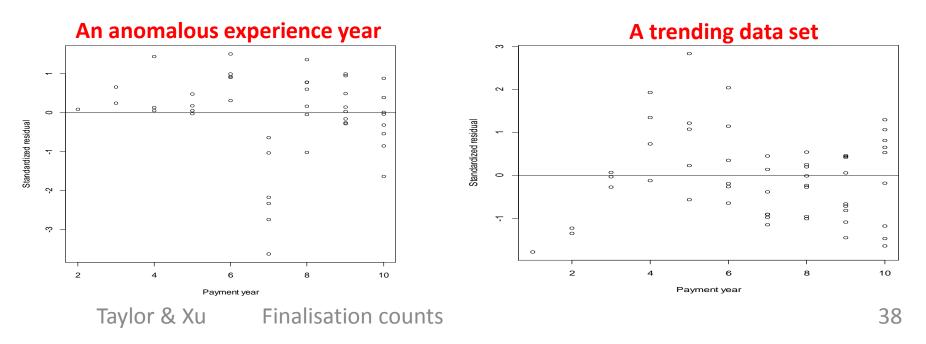
 $\overline{t} \in [0,1]$ 

Estimating a function of a continuous variable, not development year

- Initial model form:
  - $ln \psi(\bar{t}) = \beta_1 \bar{t} + \beta_2 \bar{t}^2$
  - λ(.) restricted to linear (constant inflation rate) or linear spline (piecewise constant inflation rate).

A model that recognises claim finalisation counts: payments sub-model re-visited (2)

- Initial model form:
  - $ln \psi(\bar{t}) = \beta_1 \bar{t} + \beta_2 \bar{t}^2$
- Anomalies identified from residual plots and modelled as necessary



A model that recognises claim finalisation counts: extrapolation of inflation

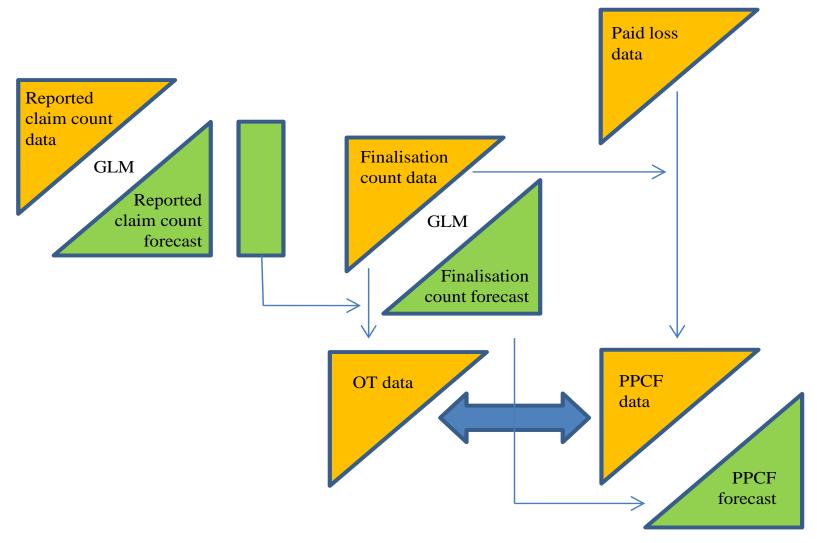
$$\widehat{Y}_{kj} = \widehat{F}_{kj}\widehat{\psi}(\widehat{\overline{t}}_k(j))\widehat{\lambda}(k+j-1)$$

Forecasts require future values of inflation index

It has been assumed as a normative measure that

$$\lambda(s) = \lambda(J + k - 1) \text{ for } s > J + k - 1$$
Nil future inflation
Last observed
diagonal

### **PPCF model schematic summary**



## Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

# Prediction error (1)

- Estimated by means of **parametric bootstrap** 
  - Details in paper
- Every sub-model of a model involves estimation and forecast
  - Hence a source of prediction error
  - This must be accounted for in the bootstrap
- Hence

Model	Number of sources of prediction error
Chain ladder	1
PPCI	2
PPCF	3

# Prediction error (2)

- Some statisticians have argued that models such as PPCF are bound to predict less efficiently than other simpler models because of the drag of the additional sub-models
- But in situations where the simpler models are poor representations of reality the greater accuracy of the more complex models may overcome this drag

# **Comparison between models**

- MSEP estimated by bootstrap
- Converted to coefficient of variation ("CoV") = MSEP/forecast
- Model with lowest CoV regarded as the producing the most efficient forecast
- N.B. models are NOT assessed by reference to the closeness of their predictions to the outcomes of the test data sets

## Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

# Quality of companies' data sets

Nature of data defect	Number of companies
Only small amounts of incurred losses	49
Start-up during period of training data set	14
Wind-down during period of training data set	7
Incurred loss amounts submitted only for a subset of training data	
set diagonals	7
No finalisation count data submitted	6
Finalisation count data submitted only for a subset of training data	
set diagonals	5
Virtually no paid loss data submitted	1
Reported claim count data submitted only for a subset of training	
data set diagonals	1
No defect	76
Total	166

# Selection of data sets for experimentation

- Meyers-Shi data set was searched for companies exhibiting:
  - Variations in OT profiles by accident year (changes in rates of claim finalisation)
    - i.e.  $t_k(j)$  as a function of j varies as k varies
    - Especially, variations in  $t_k(1)$
    - Or variations in  $t_k(1998 k)$  (final diagonal of training data set)
  - Companies exhibiting large changes in premium volume over the 10 accident years in the training data set were excluded
- All of these criteria were expressed in terms of objective measures
  - Details in paper
- On this basis, selected 9 companies for detailed analysis
  - 7 of the 9 exhibited fairly clear changes in rates of claim finalisation over past years

## Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

#### Results: prediction errors of models

Company	C	oV (%)		Ratio	to actual (9	%)	Model	Number	Percentage
	Chain	PPCI	PPCF	Chain	PPCI	PPCF		of wins	of wins
_	ladder			ladder			Chain ladder	1.8	20%
#671	18	11	11	120	106	91			
#723	12	8	9	94	101	118	PPCI	4.3	48%
#1538	24	14	11	105	95	138	PPCF	2.8	31%
							Total	9	100%
#1694	6	6	8	83	87	94	Total		100/0
#1767	5	5	4	93	109	106	<	imited ev	idence of
#3360	6	12	22	52	64	81	i	nstability	in rates of
#4731	8	8	8	123	96	109	f	inalisatior	า
#4740	7	6	7	104	94	82			
#38733	10	9	22	88	89	289			

Company	C	oV (%)		Ratio	%)	
_	Chain	PPCI	PPCF	Chain	PPCI	PPCF
_	ladder			ladder		
#1538	24	14	11	105	95	138

Accident	C	ompleme	ent of op	erationa	l time att	ained by	end of d	evelopm	ent year	
year	1	2	3	4	5	6	7	8	9	10
1988	0.367	0.081	0.030	0.012	0.0057	0.0024	0.0003	0.0003	0.0000	0.0000
1989	0.393	0.097	0.043	0.021	0.0091	0.0042	0.0022	0.0017	0.0015	
1990	0.373	0.102	0.047	0.024	0.0116	0.0034	0.0025	0.0015		
1991	0.478	0.112	0.048	0.023	0.0131	0.0085	0.0074			
1992	0.381	0.078	0.028	0.011	0.0057	0.0035				
1993	0.382	0.079	0.029	0.010	0.0048					
1994	0.362	0.086	0.039	0.026						
1995	0.363	0.090	0.054							
1996	0.364	0.114								
1997	0.450									

Persistently high degree of variation in finalisation rates

Classical case in favour of PPCF model

- Other cases of high performance by PPCF model are similar
- We don't dwell on them
- More interesting to examine cases of poor performance by PPCF model

Company	C	oV (%)		to actual (%	6)	
	Chain	PPCI	PPCF	Chain	PPCI	PPCF
	ladder			ladder		
#3360	6	12	22	52	64	81

Accident	C	ompleme	ent of op	erational	time att	ained by	end of de	evelopm	ent year		
year	1	2	3	4	5	6	7	8	9	10	
1988		0.119	0.075	0.046	0.032	0.025	0.015	0.008	0.005	0.004	
1989	0.483	0.149	0.093	0.062	0.045	0.025	0.015	0.008	0.006		Apparen
1990	0.473	0.109	0.049	0.008	-0.036	-0.051	0.018	0.014			erroneou
1991	0.557	0.222	0.168	0.088	0.057	0.033	0.024				count da
1992	0.561	0.225	0.122	0.058	0.025	0.011					
1993	0.567	0.172	0.091	0.029	0.015						
1994	0.576	0.182	0.052	0.032							
1995	0.485	0.114	0.069								
1996	0.273	0.092									
1997	0.498										

		Com	bany		CoV (%	6)			Ratio to	o actual (	%)
				Chain Iadder	PF	PCI	PPCF		hain dder	PPCI	PPCF
		#3	8733	10		9	22		88	89	289
Accident			Fina	lisation c	ount in	develo	pment y	vear			
year	1	2	3	4	5	6	7	8	9	10	
1988	2,057	1,520	84	18	27	1	14	7	37	1	<b>A</b>
1989	3,524	834	111	64	7	13	10	282	3		Арр
1990	4,438	836	178	6	24	15	4	4			erro
1991	4,577	821	111	62	30	18	3				cour
1992	5,656	913	142	55	25	10					agai
1993	6,067	1,011	143	46	29						U
1994	5,760	940	120	46							
1995	5,487	820	113								
1996	5,190	734									
1997	4,908										

Company	C	oV (%)		Ratio	to actual (%	%)
	Chain ladder	PPCI	PPCF	Chain ladder	PPCI	PPCF
<b>#</b> 1694	6	6	8	83	87	94

Accident	Co	Complement of operational time attained by end of development year								
year	1	2	3	4	5	6	7	8	9	10
1988	0.261	0.065	0.031	0.018	0.010	0.006	0.005	0.003	0.002	0.001
1989	0.260	0.064	0.032	0.018	0.012	0.008	0.005	0.004	0.003	
1990	0.191	0.060	0.031	0.019	0.012	0.008	0.005	0.004		
1991	0.197	0.061	0.032	0.019	0.012	0.009	0.006			
1992	0.197	0.061	0.030	0.017	0.011	0.008				
1993	0.200	0.060	0.031	0.018	0.012					
1994	0.242	0.063	0.033	0.018						
1995	0.219	0.060	0.028							
1996	0.225	0.060								
1997	0.232									

Only a single noticeable change in rates of finalisation

Company	C	oV (%)		Ratio	to actual (%	%)
	Chain	PPCI	PPCF	Chain	PPCI	PPCF
	ladder			ladder		
#4731	8	8	8	123	96	109

- Already noted limited evidence of instability in rates of finalisation
- Any evident instability appeared in development years 6 to 8
- These development years
  - Do not affect accident years 1989 and 1990
  - Crucially affect accident years 1991 to 1993
  - Have steadily decreasing effect on accident years 1994 and later

Taylor & Xu	Finalisation counts
-------------	---------------------

Accident	Estimated CoV of loss reserve (%)		
year	chain	PPCI	PPCF
	ladder		
1989	76	73	93
1990	38	38	44
1991	29	28	28
1992	21	21	19
1993	17	16	14
1994	12	12	10
1995	9	9	9
1996	7	7	7
1997	5	5	6
Total	8	8	<b>8</b> <sup>5</sup>

## Overview

- Meyers-Shi data set and data issues
- Models selected for experimentation
- Prediction error and model comparison
- Companies selected for experimentation
- Results
- Conclusions

# Conclusions (1)

- Let the data speak for themselves
  - The use of count data is justified if this leads to lower prediction error
- Results based on only a small selection (9) of workers compensation portfolios
  - However, consistent and coherent narrative emerges
  - Would expect improved relative performance of PPCF model in some longer tailed lines, e.g. Auto Bodily Injury, Public Liability
- For objectivity, all three models have been applied in as mechanistic fashion
  - The chain ladder may be seen as inherently more mechanistic than the others
  - So the models based on count data may be at a disadvantage in this comparison

# Conclusions (2)

- When an insurer data set shows material and persistent changes in the rate of claim finalisation
  - There is an *a priori* expectation that claim count data might be of value
  - And, empirically, they are
  - The chain ladder is usually out-performed by PPCI or PPCF model, particularly the latter
  - Sometimes this conclusion may apply only to selected accident years