Mixed Model Application to Reserving for Enterprise Risk Management

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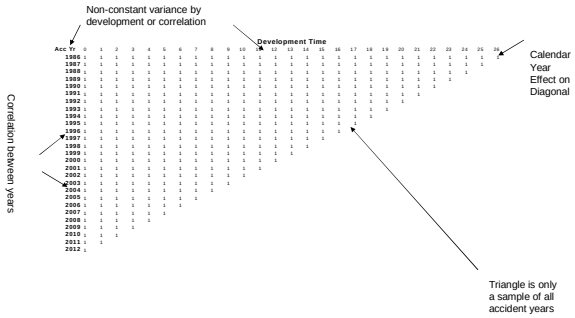
Agenda

- Reserve model requirements for Economic Capital (DFA) model
- Describe modeling problems for reserving
- Why use a Mixed Model when estimating reserves?
- What is a Mixed Model?
- Example of application of Linear Mixed Model to Work Comp
- Conclusion

Economic Capital Reserve Model Application

- Input to financial simulation model
- Economic Capital Model requirements
  - Distribution functions that describe variability
    - By payment period
    - For each accident year total (excluding inflation)
  - Linkage between interest rates, CPI & loss costs
    - One year spot rates linked to CPI change (separate model)
    - Formula linking calendar year loss cost change to CPI (developed within reserve model)
  - Correlation between lines estimate

Modeling Loss Triangle Issues



Bootstrapping Option for Reserve Payment Distribution

- Bootstrapping paid link ratio option
  - Re-sample triangle error terms from paid link ratios
  - Repeat re-sampling and projection process many times
  - Create empirical forecast distribution
- Bootstrapping assumption
  - Model (paid link ratios) is a good fit to the data
  - Error terms are randomly distributed
  - Assumptions may not be met leading to questions on bootstrap results

Mixed Model vs. GLM

- **GLM assumes**
  - Variance function of mean
  - No correlation between observations
  - Only covers fixed effect explanatory variables
  - Linear model (after link function)
  - Data distribution is part of exponential family
  - Assumptions don't match reserve modeling requirements
- **Mixed Model**
  - Models variance explicitly
  - Handles correlation between observations
  - Includes both random and fixed effects
  - Linear model (after link function)
  - Data distribution is part of exponential family
  - Assumptions may match reserve modeling requirements

What is a Mixed Model

- Includes Fixed & Random effects
  - Fixed effect completely contains variable levels
  - Random effect recognizes one has a sample of possible values for a variable
- Two Covariance Matrixes
  - Fixed & Random each have own matrix (matrices are interconnected with each other)
  - Iterative approach to solve: freeze one and optimize other until convergence
- Matrix structure options
  - Correlation description options
  - Grouping effects (related to modeling varying variance)

Work Comp Linear Mixed Model Example

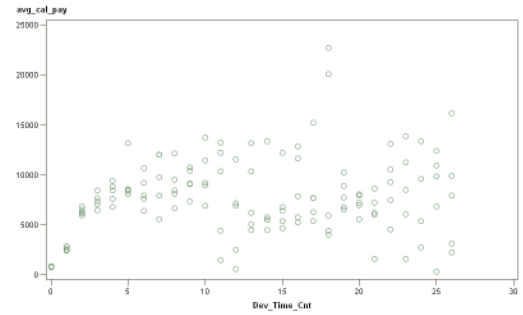
Live Work Comp Example Description

- Sample of live experience
- Review model set up
- Examine graphs of experience
- Walk through SAS EG screen shots from setting up Linear Mixed Model run

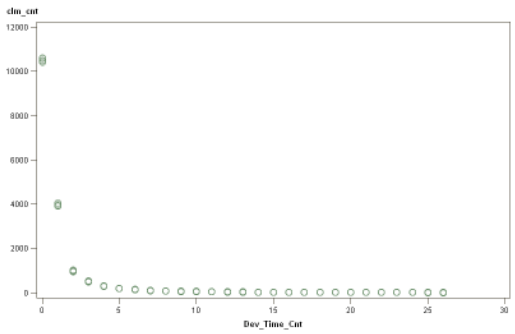
Linear Mixed Model (LMM) Example Description

- Model incremental counts X amounts
  - Count is number of claims with positive payment in one development period by accident year (Poisson Distribution used with GLM)
  - Amount is Natural Log of average incremental payment by accident year & development (transformed to Normal & use LMM)
- Exposure Base
  - Reported claim count at 12 months for each accident year
- Random vs. Fixed Effects
  - Fixed Effects: development time & log of CPI change
  - Random Effect: accident year
- Variance Model
  - Grouped effect: development time to solve for variance
- Model by payment type (gross, ceded, salvage & subrogation)
- Sample groups
  - Split claim numbers into 20 random groups

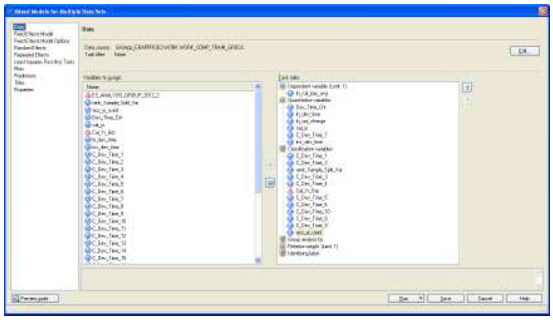
Incremental Calendar Year Average Accident Year 1986



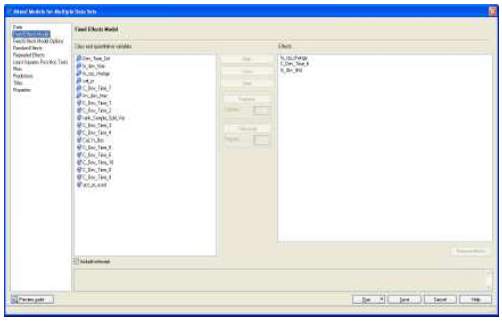
Incremental Payment Claim Count Accident Year 1986



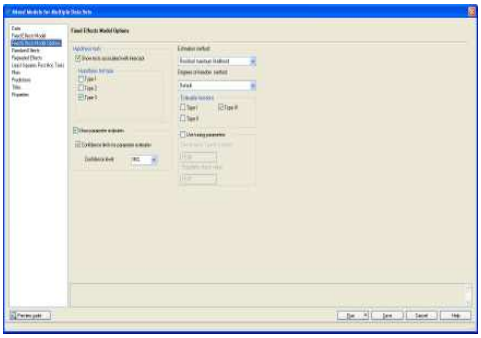
Data Screen for LMM



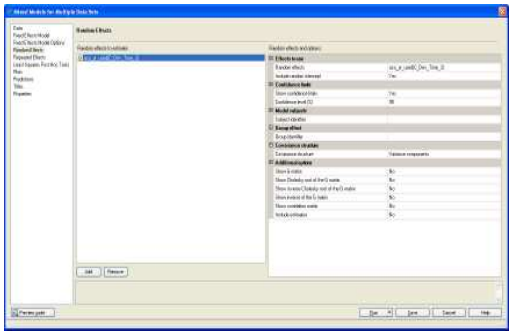
Define Fixed Effect LMM



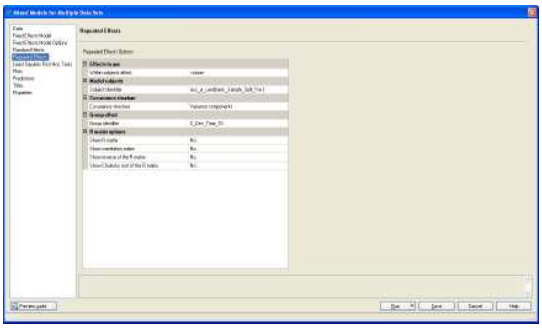
Fixed Effect Model Options LMM



Define Random Effects Covariance



Defined Fixed Effect Covariance



Linear Mixed Model
Variance by Development Time

Covariance Parameter Estimates			
Cov Parm	Subject	Group	Estimate
acc_yr_us(C_Dev_Tim)			0.02273
Intercept			0.001182
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 1	0.002638
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 2	0.003172
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 3	0.01198
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 4	0.01462
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 5	0.03927
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 6	0.1026
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 7	0.07856
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 8	0.2159
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 9	0.2138
Residual	acc_yr_us(rank_Samp)	C_Dev_Time_10 10	0.5016

Linear Mixed Model
Fixed Effect Parameters

Solution for Fixed Effects

Effect	C_Dev_Time_6	Estimate	Error	DF	t Value	Pr > |t|	Alpha	Lower	Upper
Intercept		7.3641	0.1607	0	45.82	.	0.05	.	.
ln_cpi_change		1.3578	0.08442	1799	16.08	<.0001	0.05	1.1922	1.5233
C_Dev_Time_6	1	-2.533	0.09613	1799	-26.35	<.0001	0.05	-2.7216	-2.3445
C_Dev_Time_6	2	-1.4547	0.07233	1799	-20.11	<.0001	0.05	-1.5966	-1.3129
C_Dev_Time_6	3	-0.5871	0.04403	1799	-13.33	<.0001	0.05	-0.6735	-0.5008
C_Dev_Time_6	4	-0.2645	0.03453	1799	-7.66	<.0001	0.05	-0.3322	-0.1968
C_Dev_Time_6	5	-0.1352	0.03145	1799	-4.3	<.0001	0.05	-0.1969	-0.07348
C_Dev_Time_6	6	0	.	.	.	.	.	.	.
ln_dev_time		-0.5609	0.04276	1799	-13.12	<.0001	0.05	-0.6447	-0.477

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Random Effects
Partial Solution (1st Dvlp Period)

Solution for Random Effects

Effect	C_Dev_Time_3	acc_yr_used	Estimate	Std Err Pred	DF	t Value	Pr > |t|	Alpha	Lower	Upper
acc_yr_us	1	1986	-0.2755	0.04994	1799	-5.52	<.0001	0.05	-0.3735	-0.1776
acc_yr_us	1	1987	-0.2409	0.04801	1799	-5.02	<.0001	0.05	-0.3351	-0.1468
acc_yr_us	1	1988	-0.1527	0.04593	1799	-3.32	0.0009	0.05	-0.2428	-0.06263
acc_yr_us	1	1989	-0.06135	0.04371	1799	-1.4	0.1606	0.05	-0.1471	0.02437
acc_yr_us	1	1990	-0.0101	0.04149	1799	-0.24	0.8076	0.05	-0.09147	0.07126
acc_yr_us	1	1991	0.015	0.03999	1799	0.38	0.7076	0.05	-0.06343	0.09344
acc_yr_us	1	1992	-0.03069	0.03906	1799	-0.79	0.4321	0.05	-0.1073	0.04592
acc_yr_us	1	1993	-0.08784	0.03828	1799	-2.29	0.0219	0.05	-0.1629	-0.01276
acc_yr_us	1	1994	-0.0503	0.03771	1799	-1.33	0.1824	0.05	-0.1243	0.02366
acc_yr_us	1	1995	-0.08642	0.03722	1799	-2.32	0.0204	0.05	-0.1594	-0.01342
acc_yr_us	1	1996	-0.07962	0.03686	1799	-2.16	0.0309	0.05	-0.1519	-0.00733
acc_yr_us	1	1997	-0.04298	0.03668	1799	-1.17	0.2414	0.05	-0.1149	0.02896
acc_yr_us	1	1998	-0.0354	0.03661	1799	-0.97	0.3337	0.05	-0.1072	0.0364
acc_yr_us	1	1999	0.02819	0.03659	1799	0.77	0.4411	0.05	-0.04357	0.09995
acc_yr_us	1	2000	0.0274	0.03673	1799	0.75	0.4558	0.05	-0.04463	0.09943
acc_yr_us	1	2001	0.01208	0.037	1799	0.33	0.744	0.05	-0.06048	0.08464
acc_yr_us	1	2002	0.03257	0.03721	1799	0.88	0.3816	0.05	-0.04042	0.1056
acc_yr_us	1	2003	0.07121	0.03776	1799	1.89	0.0584	0.05	-0.00254	0.145
acc_yr_us	1	2004	0.005488	0.03816	1799	0.14	0.8857	0.05	-0.06936	0.08034
acc_yr_us	1	2005	-0.01259	0.03903	1799	-0.32	0.747	0.05	-0.08914	0.06395
acc_yr_us	1	2006	0.06507	0.04001	1799	1.63	0.1041	0.05	-0.01341	0.1435
acc_yr_us	1	2007	0.05198	0.04101	1799	1.27	0.2052	0.05	-0.02846	0.1324
acc_yr_us	1	2008	0.1305	0.0425	1799	3.07	0.0022	0.05	0.04718	0.2139
acc_yr_us	1	2009	0.1766	0.04236	1799	4.17	<.0001	0.05	0.09355	0.2597
acc_yr_us	1	2010	0.1732	0.04306	1799	4.02	<.0001	0.05	0.08874	0.2576
acc_yr_us	1	2011	0.1917	0.04446	1799	4.31	<.0001	0.05	0.1045	0.2789
acc_yr_us	1	2012	0.1854	0.04544	1799	4.08	<.0001	0.05	0.09627	0.2745

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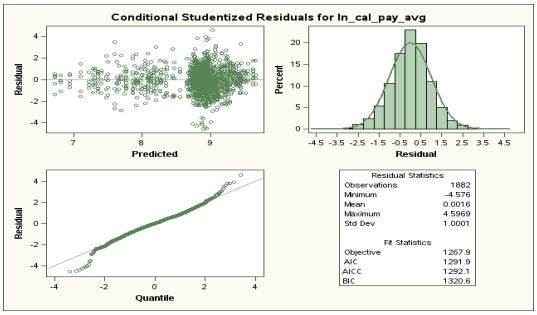
Linear Mixed Model
Residual Diagnostics



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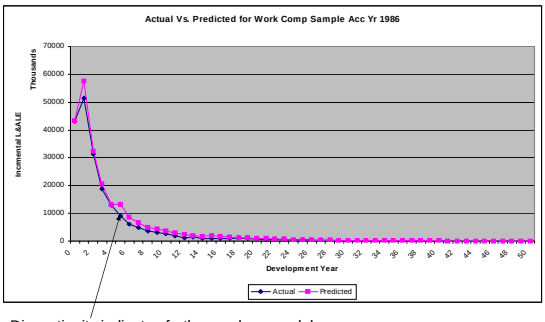
Apply Frequency & Severity Model



Discontinuity indicates further work on model

Concluding Remarks

- Technology is changing
- Move beyond Linear Mixed Model
  - Distributions may be more skewed than Lognormal
  - Underlying may not be linear
  - Simplest form of Hierarchical Model (good starting point)
- Bayesian MCMC
  - Offers wider range of distributions
  - Facilitates updating model estimate vs. ignoring last model and refitting
  - More complex environment