


**VR-6: Incorporating Model Error into the Actuary's Estimate**

Dave Otto & Jamie Mackay  
CAS Loss Reserving Seminar  
September 15-17 2014

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### Session Description

- Within the property/casualty insurance industry, increased interest is being placed on understanding the **variability** inherent in a point estimate of unpaid claims
- The session will begin with a **dilemma** that confronts actuaries when relying upon a *single* model to measure the variability around a central estimate based on *multiple* models
- We will then provide an overview of the basic building blocks to estimating reserve variability and will then address a component of reserve variability that is often overlooked: **model uncertainty**
- This session will present **practical methodologies** for incorporating model uncertainty into the actuary's estimate of uncertainty and will use a case study to demonstrate their use



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## Dilemma

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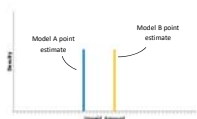
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**Dilemma**

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:



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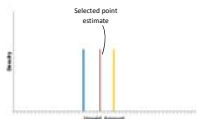
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**Dilemma**

- Consider a situation where we have two models, Model A & Model B, that each produce a point estimate:
- Assume the actuary selects the central estimate to be the average of the point estimates from the two models:

$$\text{Central Estimate} = \frac{(\text{Model A} + \text{Model B})}{2}$$

- How do we estimate the uncertainty in our central estimate?



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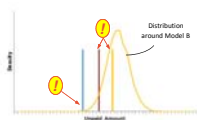
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**Dilemma**

- One way might be to estimate uncertainty using one of our underlying models as the basis
  - Using Model B as the basis for estimating uncertainty:
- This raises two issues:
  - Central estimate (red) is not "central" within distribution
  - Model A point (blue) estimate appears unlikely yet given 50% weight



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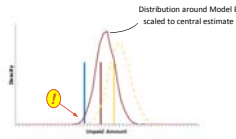
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### Dilemma

- The first issue can be resolved by scaling:

*Additive Scaling* =  $\hat{y}_p = \hat{y}_p + [\text{central estimate} - \bar{y}]$

*Multiplicative Scaling* =  $\hat{y}'_p = \hat{y}_p \frac{[\text{central estimate}]}{\bar{y}}$



- The central estimate (red) is now "central" with distribution
- However, the second issue remains:
  - Model A point estimate (blue) still appears unlikely yet given 50% weight

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### Dilemma

- It is common to estimate unpaid claims using more than one model
- It is rare for different models to produce point estimates that are equivalent
- Current approaches to estimating uncertainty tend to derive variability within the context of a single model
- Central estimate is often not equivalent to any single model.

*How do we derive a suitable distribution of variability?*

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### Incorporating Model Uncertainty

#### Overview of Approach

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### Uncertainty in an Actuarial Central Estimate

- Measuring uncertainty is a challenge in our profession because the unpaid claim process is unknown and the output from this process is not a repeatable exercise
- Many approaches exist to estimating the uncertainty in an unpaid claim estimate
  - Mack, Bootstrapping, MCMC, practical stochastic simulation
- Two common themes in these approaches:
  - Prediction error is comprised of parameter error and process error
  - A single model is assumed to be representative of the unpaid claim process

Parameter Error

Process Error

=

Prediction Error

Selected Unimodal	Paid CL	Method
32,037	32,037	
28,387	28,387	
27,752	27,752	
32,809	32,809	
32,308	32,308	
30,938	30,938	
28,343	28,343	
25,494	25,494	
23,308	23,308	

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### Uncertainty in an Actuarial Central Estimate

- However, this is rarely the case, and actuaries will commonly employ multiple models
- We therefore need some way in which to reflect the additional implied uncertainty among the models

Parameter Error

Process Error

Model Error

+

Prediction Error

Selected Unimodal	Paid CL	Method	Insured CL	Paid EP	Method
32,238	32,237		32,490	32,037	
28,343	28,337		28,499	28,387	
28,388	27,752		28,690	27,754	
32,326	32,809		32,865	32,822	
32,038	32,308		32,732	32,322	
32,342	30,938		32,892	30,823	
28,708	28,343		28,254	28,490	
27,769	28,494		28,241	28,204	
25,520	23,308		26,178	25,546	

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### Our Approach

Generate a distribution comprised of simulations about each model using current approaches:

- Bootstrapping; simulation from an assumed distribution; simulation from analytical models, simulating and scaling, etc.

↓ Weighted sample

Parameter Error

Process Error

Model Error

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Prediction Error

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### Our Approach

Generate a distribution comprised of simulations about each model using current approaches:

- Bootstrapping; simulation from an assumed distribution; simulation from analytical models; simulating and scaling, etc.

**Weighted sample**

Aggregating results across multiple years requires additional rigor:

- **Rank Tying** and **Model Tying** approaches are available to generate aggregate distributions

The diagram shows three boxes labeled 'Parameter Error', 'Process Error', and 'Model Error' with arrows pointing to a central box labeled 'Prediction Error'. To the right of the 'Prediction Error' box is a small bar chart with a red arrow pointing to it, labeled 'Total'.

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### Weighted Sampling

Single Years

The bar chart shows a single bar for 'Single Years' under the 'Weighted Sampling' category.

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### Sampling of methods

- Start by creating simulated distributions for each of Model A and B:

The diagram shows two vertical bars representing 'Model A point estimate' and 'Model B point estimate'. An arrow labeled 'Simulation methodology' points to two overlapping bell curves representing simulated distributions for Model A and Model B.

Model A Simulations		Model B Simulations	
Min	Value	Min	Value
1	1.8	1	1.8
2	2.5	2	4.9
3	3.8	3	5.2
4	3.8	4	4.4
5	4.4	5	5.4
6	5.0	6	5.5
7	5.0	7	6.4
8	6.0	8	5.9
9	6.7	9	6.4
10	6.4	10	6.0

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### Sampling of methods

- Adjusting our underlying weights will shift the resulting distribution accordingly:

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### Sampling of methods Multi-modal distributions

- Weighted Sampling may produce 'lumpy', or multi-modal, probability density distributions
- However, the probabilities across a range of outcomes may be more easily interpreted using the associated cumulative probabilities graph
- Further adjustments could be made to the simulated results if such an outcome was deemed problematic

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### Sampling of methods

- So far, we have considered a scenario with just a single set of simulations
- What if we have multiple sets of predictions?
  - Multiple accident years, for example

Year	1-1-0	1-1-0	1-1-0
1	3.0	12.0	28.0
2	2.5	12.5	28.0
3	3.0	12.0	28.0
4	3.5	11.5	27.5
5	4.0	11.0	27.0
6	3.5	11.5	27.5
7	3.0	12.0	28.0
8	2.5	12.5	28.0
9	3.0	12.0	28.0
10	3.5	11.5	27.5

Year	1-1-0	1-1-0	1-1-0
1	3.0	12.0	28.0
2	4.0	11.0	27.0
3	3.5	11.5	27.5
4	4.0	11.0	27.0
5	3.5	11.5	27.5
6	3.0	12.0	28.0
7	4.0	11.0	27.0
8	3.5	11.5	27.5
9	3.0	12.0	28.0
10	3.5	11.5	27.5

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## Weighted Sampling

### Multiple Years

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## Sampling of methods Multiple Year Aggregations

- Again, for each time period, we can create a 'Model Matrix' based on the selected weighting

Year	0-1	1-2	2-3
1	0.4	0.8	20.0
2	23.0	0.0	20.0
3	0.0	0.0	20.0
4	0.0	0.0	20.0
5	0.0	0.0	20.0
6	0.0	0.0	20.0
7	0.0	0.0	20.0
8	0.0	0.0	20.0
9	0.0	0.0	20.0
10	0.0	0.0	20.0

Year	0-1	1-2	2-3
1	0.4	0.8	20.0
2	0.0	0.0	20.0
3	0.0	0.0	20.0
4	0.0	0.0	20.0
5	0.0	0.0	20.0
6	0.0	0.0	20.0
7	0.0	0.0	20.0
8	0.0	0.0	20.0
9	0.0	0.0	20.0
10	0.0	0.0	20.0

Year	0-1	1-2	2-3
1	0.4	0.8	20.0
2	0.0	0.0	20.0
3	0.0	0.0	20.0
4	0.0	0.0	20.0
5	0.0	0.0	20.0
6	0.0	0.0	20.0
7	0.0	0.0	20.0
8	0.0	0.0	20.0
9	0.0	0.0	20.0
10	0.0	0.0	20.0

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## Sampling of methods Multiple Year Aggregations

A note on simulation tying

- Typically, the methods that are used to generate the simulations around each of the underlying models do not treat each accident year in isolation, but rather produce year-by-year results that are intrinsically related to each other
- This is reflected in each and every simulation, which we can think of as 'strings'
- This means that we are able to calculate the total unpaid amount for each simulation by simply summing across each row

Sim	t = 1	t = 2	t = 3	Total
#1	3.4	6.6	26.0	36.0
#2	2.5	3.1	26.0	31.6

- In this manner, any accident year correlation that is inherent to the model can be maintained

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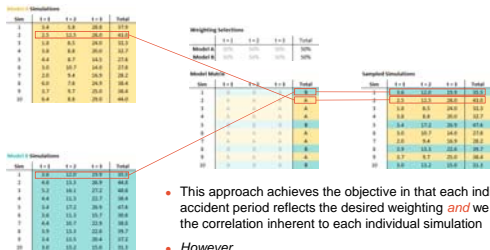
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### Sampling of methods Multiple Year Aggregations

- With this example (equal weighting for each accident period), we can get around the problem by sampling just one time and ensuring that we pick the same simulation for every time period



- This approach achieves the objective in that each individual accident period reflects the desired weighting *and* we maintain the correlation inherent to each individual simulation
- However...

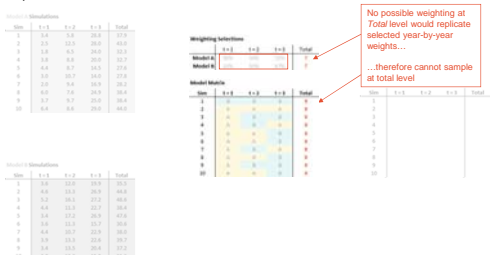
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### Sampling of methods Multiple Year Aggregations

- ...what if our selected weightings vary for each origin year?
- In this case, we need to sample independently to maintain appropriate year-by-year representation



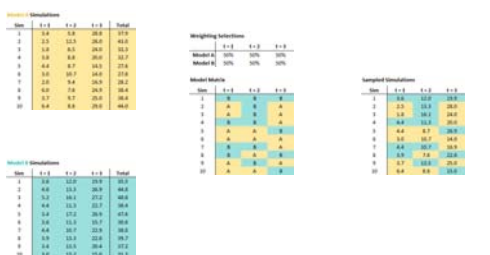
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### Sampling of methods Multiple Year Aggregations

- We require some manner of rearranging our simulations to reflect underlying correlations
- Going back to our earlier example, sampling individually by years, we suggest 2 ways in which to achieve this....



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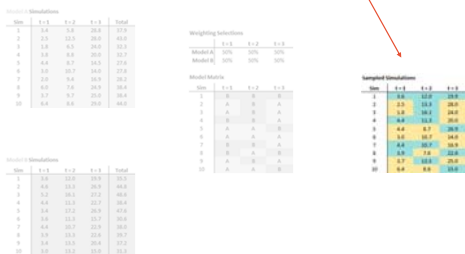
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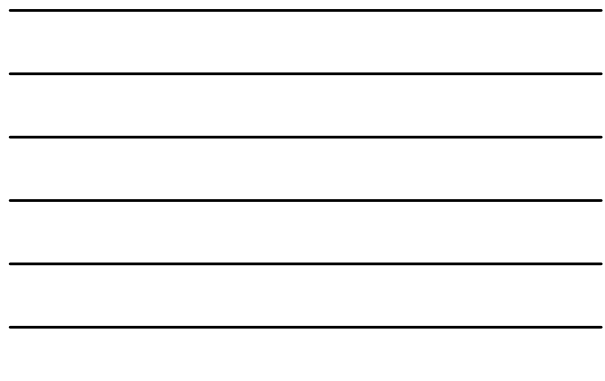
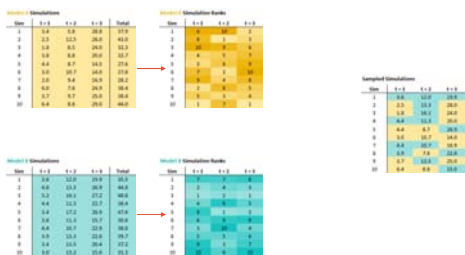
### Aggregating Results Rank Tying

- This approach involves rearranging the sampled, simulated reserves



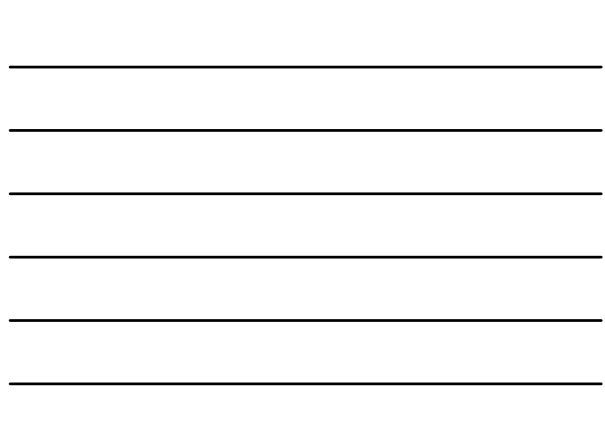
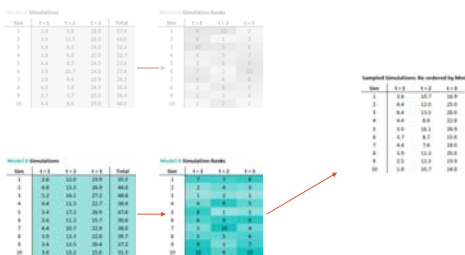
### Aggregating Results Rank Tying

- We can 'borrow' a correlation matrix from one of the underlying models
- We do this by calculating the reserve ranks for each year of the underlying models



### Aggregating Results Rank Tying

- We then select which model to use as the basis for our rank-tying (in this case, Model B)...
- ...and reorder the sampled simulations accordingly on a year-by-year basis













### Aggregating Results Summary

- All three approaches are scalable to allow for the incorporation of multiple models and multiple accident years in the estimate of reserve uncertainty
- Furthermore, the Rank Tying and Method Tying approaches involve sampling at the individual year level and therefore also support the ability to apply weights specific to each accident year
- This allows actuaries to reflect the same weighting philosophy in their uncertainty estimate as employed in their selection of the central estimate




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### Case Study

#### Application of Approach

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### Case Study Underlying Models

- Three models are investigated
- For the central estimate, each model is given equal weight (for each accident year)

Central Estimate: Selected Weighting			
	Model A	Model B	Model C
2009	33.3%	33.3%	33.3%
2010	33.3%	33.3%	33.3%
2011	33.3%	33.3%	33.3%

Central Estimate: Reserves				
	A	B	C	Reference
2009	3.8	3.8	3.9	3.8
2010	7.1	12.9	13.5	11.8
2011	19.9	21.5	22.8	21.2

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