Loss Reserve Variability and Reserve Ranges – Part II

Presented by
Matthew G. Killough, FCAS, MAAA

Casualty Loss Reserve Seminar
September 15-17, 2014
San Diego, CA

Milliman

"I cannot rest, I cannot stay, I cannot linger anywhere. My spirit never walked beyond our counting-house -- mark me! -- in life my spirit never roved beyond the narrow limits of our money-changing hole; and weary journeys lie before me!"

MARLEY'S GHOST

"I cannot rest, I cannot stay, I cannot linger anywhere. My spirit never walked beyond our deterministic methods -- mark me! -- in life my spirit never roved beyond the narrow limits of our chain ladder hole; and weary journeys lie before me!"

MARLEY'S GHOST

Audience and Context

Range of Estimates or Range of Outcomes?

- Auditors
  - Justify a range of reserve estimates
- Regulators
  - Australia: Hold reserves at 75th percentile
  - Solvency II: 99.5% probability of sufficient capital over a 1 year time horizon
  - ORSA requirements
- Accountants: IFRS risk margins
- Risk Managers: quantify reserving risk
- Other Management: risk measurement for capital allocation

Stochastic Reserving Models

- Move beyond "point estimates"
- Treat reserves as random variables
- Goal:
  - Quantify variability in unpaid liabilities
  - Mathematically / statistically rigorous
- Different way to think about reserve ranges:
  - More about statistics
  - Less about actuarial judgment

Methods vs. Models

Actuarial Literature	Most of the Rest of the World
▪ "Method" = deterministic – Chain ladder – Bornhuetter-Ferguson – Etc. ▪ "Model" = stochastic – Statistical bootstrap – Generalized linear model – Bayesian	A mathematical model is any description of a system using mathematical concepts and language.

Milliman

Sources of Uncertainty

Range of Possible Outcomes

Model Risk	Parameter Risk	Process Risk
• Begin by postulating some form of mathematical description of future claim payments • Something mathematically convenient (normal, lognormal, Pareto, Poisson, etc.) • Almost surely not a perfectly accurate description of the actual system	• Most models require the selection (or estimation) of parameters: • Loss development factors, • Trend rates, • Expected Value • Variance • Etc. • Often the exact value is not knowable	• The inherent randomness in the process • Flipping coins and rolling dice are pure process risk

Sources of Uncertainty

Range of Reasonable Estimates

Model Risk

- Begin by postulating some form of mathematical description of future claim payments
- Something mathematically convenient (normal, lognormal, Pareto, Poisson, etc.)
- Almost surely not a perfectly accurate description of the actual system

Parameter Risk

- Most models require the selection (or estimation) of parameters:
- Loss development factors,
- Trend rates,
- Expected Value
- Variance
- Etc.
- Often the exact value is not knowable

Process Risk

- [redacted]

Milliman

``Who, and what are you?" Scrooge demanded.

``I am the Ghost of Christmas Past."

``Long past?" inquired Scrooge: observant of its dwarfish stature.

``No. Your past."

THE FIRST OF THE THREE ~~SPIRITS~~ APPROACHES

Mack

- Mack, T., "Measuring the Variability of Chain Ladder Reserve Estimates." CAS *Forum*, Spring 1994.
- Attempt to turn the Chain Ladder Method into a Stochastic Model

Mack Modeling Assumptions

1. E[future cumulative loss] = f × {current cumulative loss}
2. Actual future cumulative losses for different accident years (at any given age) are independent random variables
3. Variance of future cumulative loss is proportional to the current cumulative loss
2. Actual future age-to-age factors are all independent random variables
3. Variance of future age-to-age factors is inversely proportional to the current cumulative loss

Milliman

Mack Parameter Fitting

12	24	36
81	162	177
23	27	
5		
12-24	24-36	
2.012	1.088	
1.130		

- MSE at 12-24 = $(81 \times f - 162)^2 + (23 \times f - 27)^2$
- Minimized by weighted average: $$f = 1.817$$
- Key is assumption about variance
- Different variance assumptions result in different development factors

Select development factors to minimize mean square error

Milliman

Mack Measure of Variability

- Definition of variance:

$$\mathrm{Var}[X] = \mathrm{E}[(X - \mathrm{E}[X])^2]$$

- Estimate of the expected value:

$$\mathrm{E}[X] \approx M$$

- Mean square error in estimate:

$$\mathrm{E}[(X - M)^2] = \mathrm{Var}[X] + (\mathrm{E}[X] - M)^2$$

- Process Variance + Estimation Error (Parameter Risk)
- Mack does the math and derives a closed form expression for the mean square error in the reserve estimate

From Mack Variability to a Range

- Central Limit Theorem:
  Total reserve is approximately normally distributed?
- If variance is too large then normal distribution is clearly nonsense
  - Non-zero probability of negative ultimate losses
  - Mack's solution: Use lognormal instead.
  - Why lognormal? Why not.
  - Model Risk
- Is this bad?

Mack

Range of Estimates or Range of Outcomes?

- Auditors
  - Justify a range of reserve estimates
  - Risk transfer analysis
- Regulators
  - Australia: Hold reserves at 75th percentile
  - Solvency II: 99.5% probability of sufficient capital over a 1 year time horizon
  - ORSA requirements
- Accountants: IFRS risk margins
- Risk Managers: quantify reserving risk
- Other Management: risk measurement for capital allocation

Milliman

"Spirit," said Scrooge submissively, "conduct me where you will. I went forth last night on compulsion and I learnt a lesson which is working now. To-night, if you have aught to teach me, let me profit by it."

THE SECOND OF THE THREE ~~SPIRITS~~ <u>APPROACHES</u>

England & Verrall GLM

- England, P. & Verrall, R., “Analytic and Bootstrap Estimates of Prediction Errors in Claims Reserving.” Insurance: Mathematics and Economics 25 (1999)
- Still focused on chain ladder
- Fit a GLM to the triangle instead of just measuring variance
- **Explicit assumptions about parametric probability distributions**

England & Verrall Basics

Cumulative Loss

12	24	36
164	224	243
120	192	
102		

Incremental Loss

12	24	36
164	60	20
120	72	
102		

- Fit GLM to Incremental Triangle
- Expected incremental loss is a combination of:
  - AY expected ultimate loss
  - Development pattern

England & Verrall Parameterization

- **Expected Incrementals:**

	b_1	b_2	b_3
a_1	a_1b_1	a_1b_2	a_1b_3
a_2	a_2b_1	a_2b_2	
a_3	a_3b_1		

- **Log Transform:**

$\ln(a_1)+\ln(b_1)$	$\ln(a_1)+\ln(b_2)$	$\ln(a_1)+\ln(b_3)$
$\ln(a_2)+\ln(b_1)$	$\ln(a_2)+\ln(b_2)$	
$\ln(a_3)+\ln(b_1)$		

- **Variance Assumption:**
  Variance proportional to expected value
- **Incrementals all mutually independent**

England & Verrall Output

- Expected value of incremental loss
  - Each accident year and development period
  - Compare to chain ladder method
- Complete parametric distribution for process risk:
  - "Over-dispersed Poisson" with expected value $a_i b_j$
  - Process variance is proportional to expected value

$$\text{Var}[X] = \varphi a_i b_j$$

Milliman

England & Verrall Range of Outcomes

- We just did process variability
- Need to add parameter uncertainty:

$$E\left[\left(X - a_i b_j\right)^2\right] = Var[X] + (E[X] - a_i b_j)^2$$

- In theory: the GLM framework allows for calculation of parameter uncertainty contribution
- Not so easy in practice
- Full distribution vs. variance

Milliman

GLM Limitations

- Computationally challenging to calculate the variance
- Does not give the complete probability distribution
- Logarithmic transformation means you can never have a negative incremental
- Over-parameterized
- Model risk: over-dispersed Poisson

Circumvent all of this by letting go of the GLM framework
Bootstrap it instead.

GLM Bootstrapping (Oversimplified!)

Incremental Loss

12	24	36
148	57	16
172	51	
222		

Fitted Incremental

12	24	36
154	52	16
167	56	
222		

Raw Residual

12	24	36
6	-6	0
-6	6	
0		

Cumulative Loss

12	24	36
148	205	222
172	223	
222		

Fitted Cumulative

12	24	36
154	205	222
167	223	
222		

Development Factors

1.385	1.080
1.295	
1.337	1.080

Development Factors

1.337	1.080
1.337	
1.337	1.080

Milliman

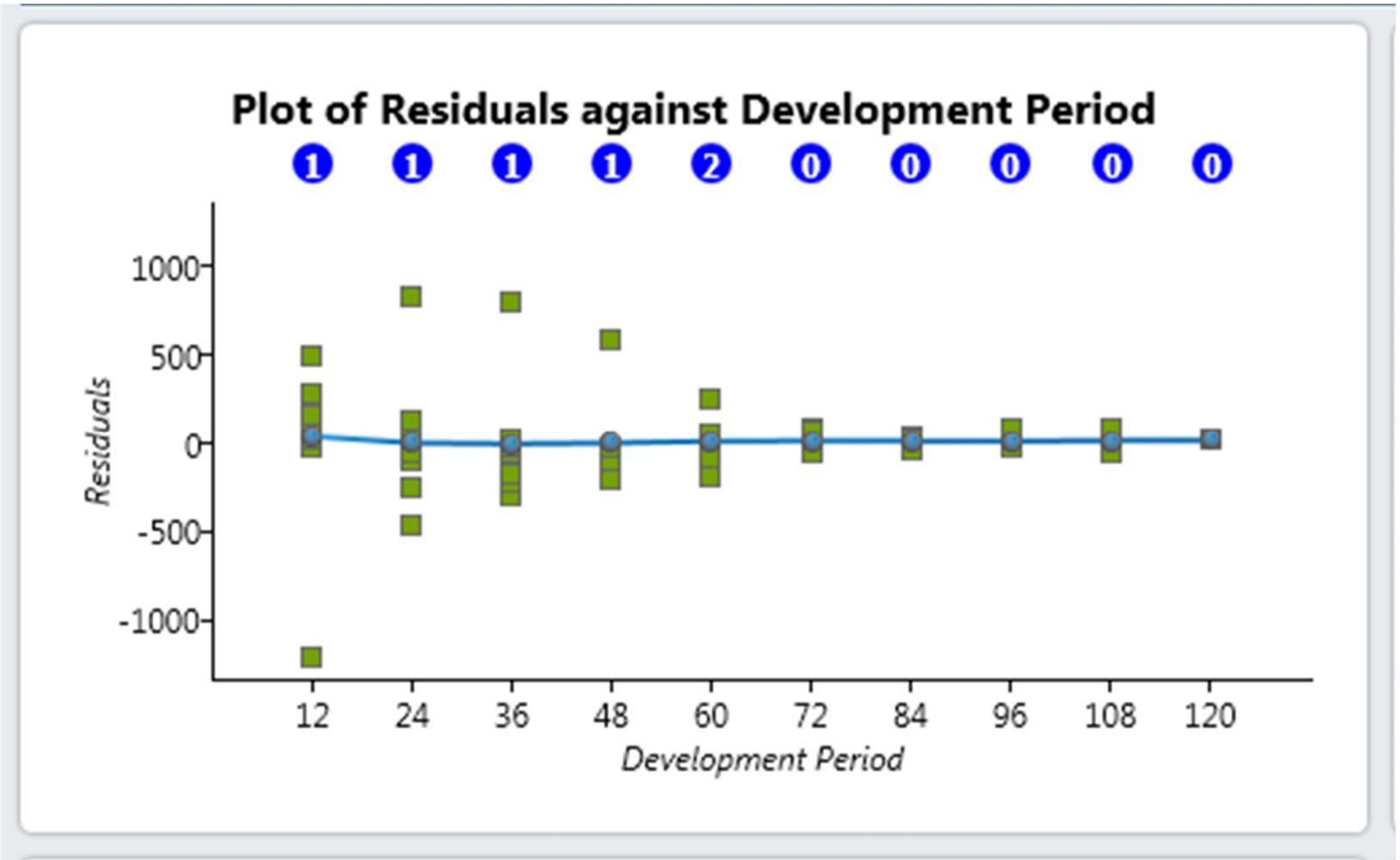
GLM Bootstrapping Cautions

ODP Bootstrap Standardized Pearson Residuals – Incurred Loss

Accident Period	12	24	36	48	60	72	84	96	108	120
2003	261.17	-267.04	5.90	-50.75	23.76	-16.15	-51.38	-35.95	-63.48	0.00
2004	67.77	-37.37	-85.56	-56.94	40.51	64.22	14.55	-41.78	63.48	
2005	-1221.69	814.72	785.55	570.31	230.84	54.19	24.89	65.15		
2006	57.62	0.95	-62.99	-108.25	-22.31	-42.46	5.31			
2007	226.89	-59.40	-309.80	-78.09	-200.70	-64.08				
2008	-31.39	108.22	-41.09	-150.51	-96.56					
2009	259.71	-121.17	-242.89	-220.10						
2010	142.62	-73.68	-189.77							
2011	485.45	-485.45								
2012	0.00									

Fundamental bootstrapping assumption:
All residuals have the same underlying probability distribution

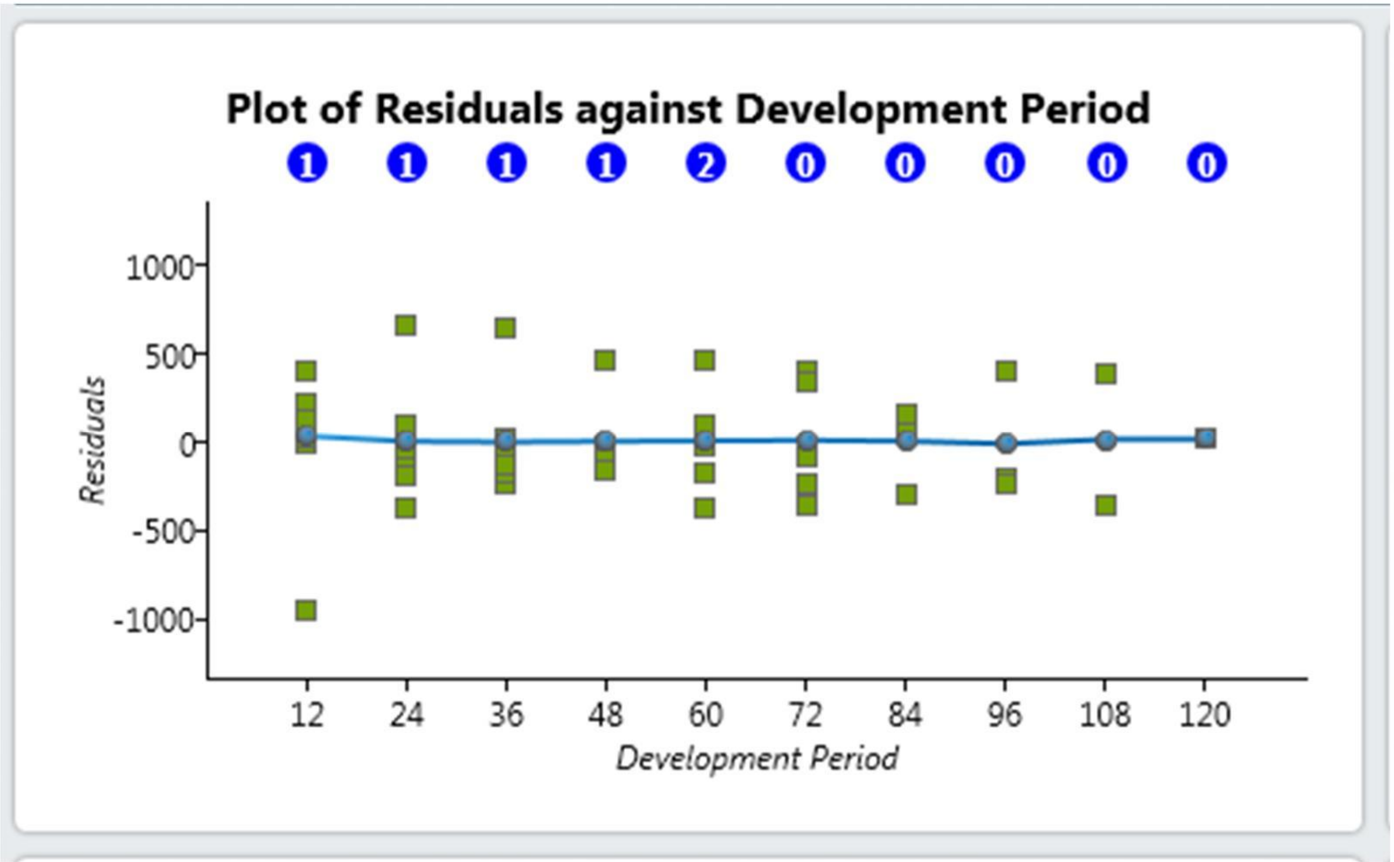
GLM Bootstrapping Cautions



Fundamental bootstrapping assumption:
All residuals have the same underlying probability distribution

Milliman

GLM Bootstrapping Cautions



Residuals will never be truly identically distributed
But you can at least adjust them so they are approximately homoskedastic

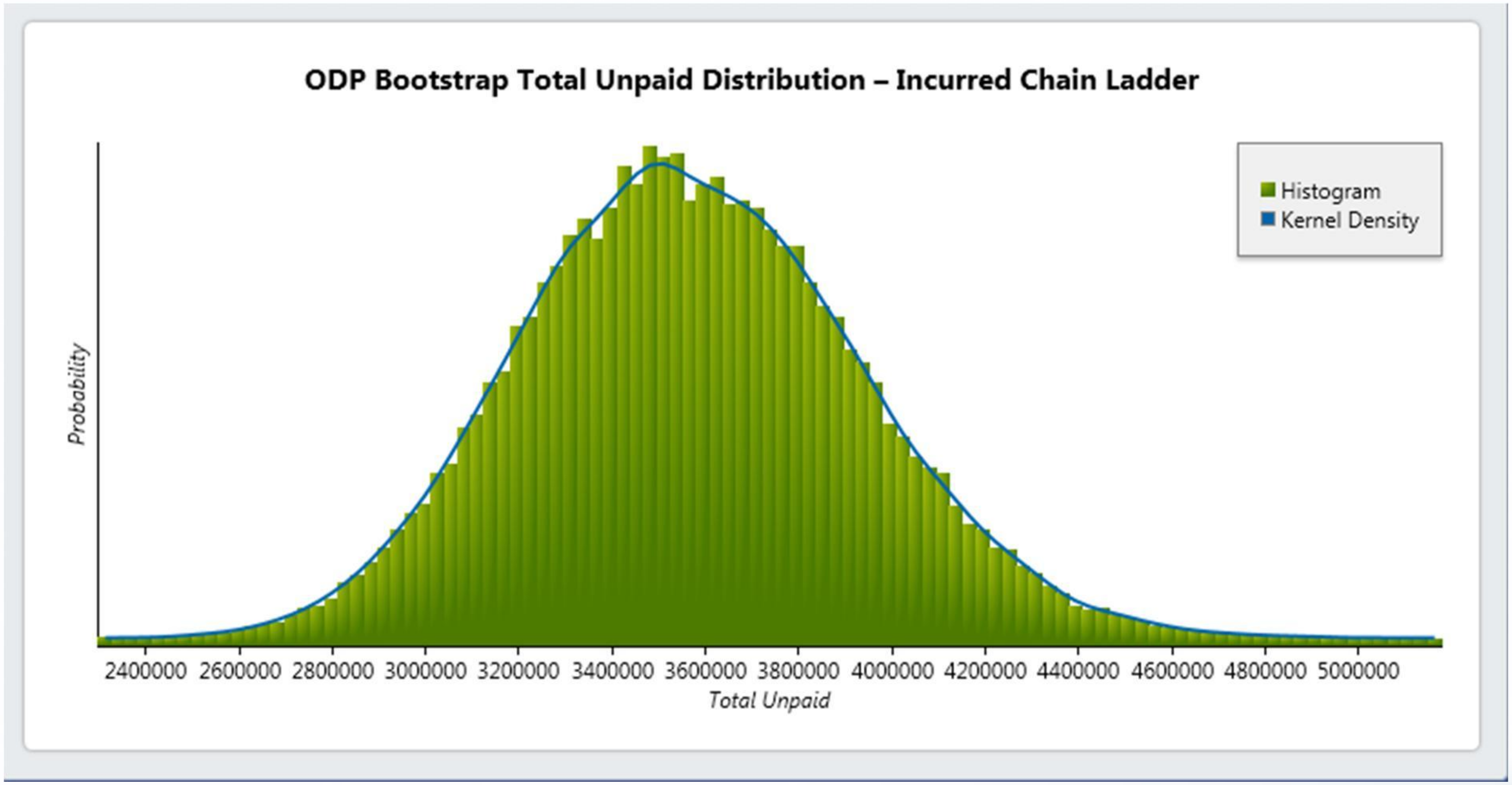
Milliman

Bootstrapping Output

Range of Estimates	Range of Outcomes
▪ Calculate link ratios ▪ Use link ratios and latest diagonals to get the "fitted" triangle ▪ Calculate residuals ▪ Sample residuals and construct a re-sampled triangle ▪ Calculate re-sampled link ratios ▪ Project to ultimate	▪ Calculate link ratios ▪ Use link ratios and latest diagonals to get the "fitted" triangle ▪ Calculate residuals ▪ Sample residuals and construct a re-sampled triangle ▪ Calculate re-sampled link ratios ▪ Square the re-sampled triangle ▪ Adjust future projected incrementals using sampled residuals

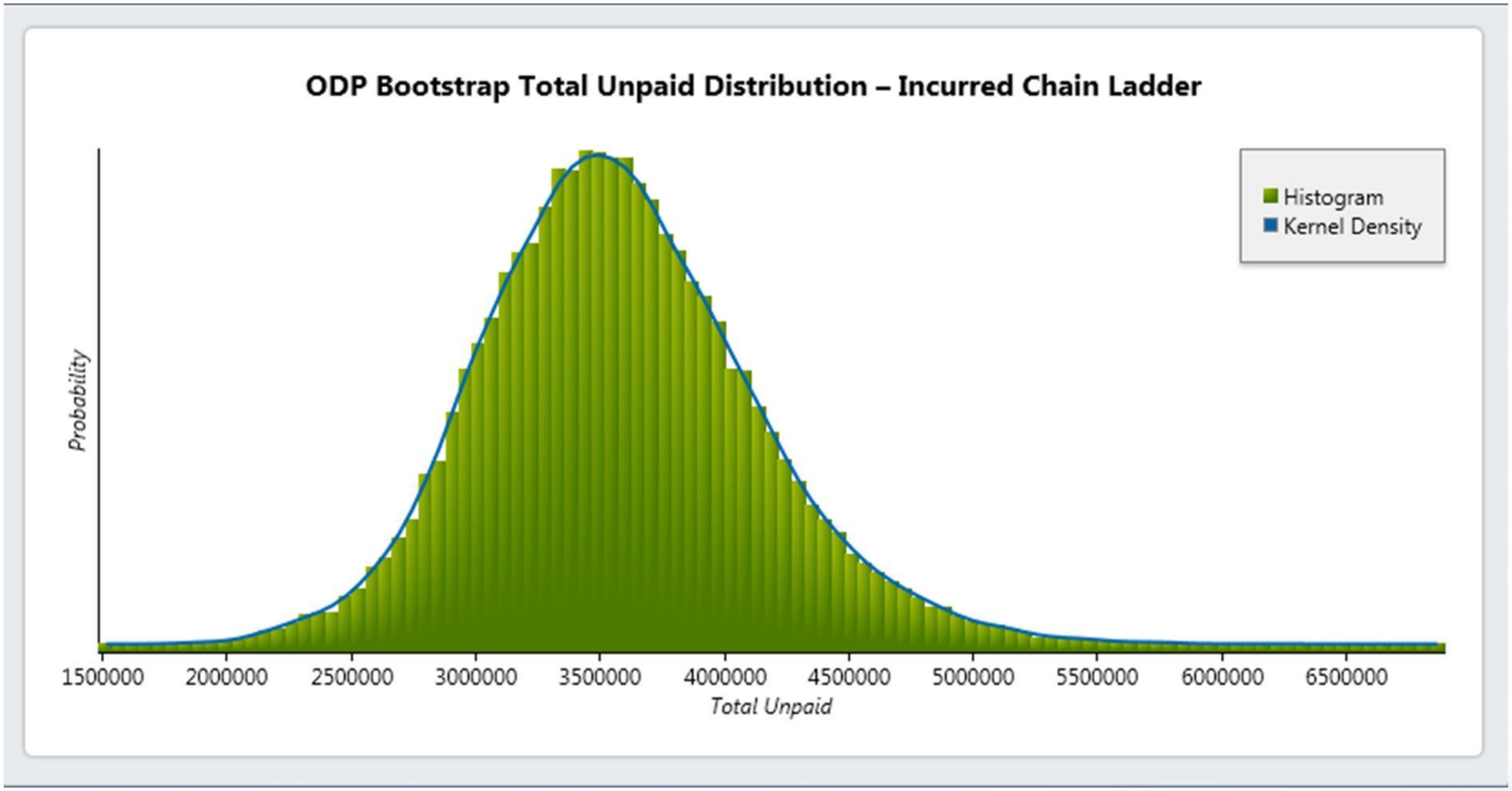
Milliman

Bootstrapping Range of Estimates



Milliman

Bootstrapping Range of Outcomes



Milliman

Bootstrap Summary

- Statistical measurement of the variability observed in the historical triangle
  1. Creates uncertainty in selected LDFs
  2. Can also be used to estimate process uncertainty in future development periods
- No issues with negative development
- Easy to use and understand – no complicated math

Bootstrap Limitations

- Statistical measurement of the variability observed in the historical triangle
  1. If an event occurs in a ten-year triangle, bootstrapping implicitly assumes there is a 1-in-10 chance of it happening in any given year
  2. If an event does not occur in a 10-year triangle, bootstrapping implicitly assumes that it will never happen
- Independence of all incrementals
  1. No calendar year effects
  2. No adjustment for changes in case reserving
  3. Etc.

``I am in the presence of the Ghost of Christmas Yet To Come?" said Scrooge. The Spirit answered not, but pointed onward with its hand. ``You are about to show me shadows of the things that have not happened, but will happen in the time before us," Scrooge pursued. ``Is that so, Spirit?"

THE LAST OF THE ~~SPIRITS~~ APPROACHES

Bayesian Loss Reserving

- Move beyond statistical measurements of past variability
- Incorporate professional knowledge and expertise
- Force the actuary to state assumptions in complete detail
- Complete shift in mindset from frequentist approach
- Extremely difficult modeling exercise
- The math used to be prohibitively difficult, but not any more
  - Ubiquitous high-power computing
  - Steal MCMC methods from the statistical physicists
  - Packages currently available in R can get you started at relatively low cost

One Bayesian Vision

- Start with an explicit statement of the distribution of possible results before you see any of the data
  - Ultimate loss ratios
  - Trend factors
  - Loss development patterns
  - Etc
- Then each observation in the triangle adds a little information, and leads to an adjustment in the distribution

Milliman