USE OF GLM TO SMOOTH AND EXTRAPOLATE DEVELOPMENT PATTERNS

David R. Clark, Terry Fang

Casualty Actuaries of the Mid-Atlantic Region (CAMAR)

October 9, 2014

Munich RE

Agenda

1. GLM Background
2. GLM in Reserving
3. Methods for Smoothing and Extrapolating Patterns

GLM background

The birth of GLM came in 1972 with the paper "Generalized Linear Models" by J. A. Nelder and R. W. M. Wedderburn (Journal of the Royal Statistical Society).

Their insight was that several types of existing regression models shared common elements and could be solved with the same – very efficient – algorithm.

Normal Regression (least squares):	$y \in (-\infty, +\infty)$
Poisson Regression (for counts):	$y \in 0, 1, 2, 3, \cdots$
Logistic Regression (binary):	$y \in 0, 1$

GLM Background

Common Elements:

- All are members of the "exponential family" of distributions
- All use Maximum Likelihood Estimation for the parameters
- All include a linear function of explanatory variables **X**·β

Normal:	$E(y_i) = \sum x_{ij} \cdot \beta_j$	$Var(y_i) = \sigma^2$
Poisson:	$E(y_i) = exp\left(\sum x_{ij} \cdot \beta_j\right)$	$Var(y_i) = E(y_i)$
Logistic:	$E(y_i) = \left[1 + exp\left(\sum x_{ij} \cdot \beta_j\right)\right]^{-1}$	$Var(y_i) = E(y_i) - E(y_i)^2$

The link functions and variance structures can be mixed and matched!

GLM Background

Actuaries begin using GLMs:

For Ratemaking:

- "Minimum Bias With Generalized Linear Models" Brown, Robert PCAS 1988
- "A Systematic Relationship Between Minimum Bias and Generalized Linear Models" Mildenhall, Stephen J.; PCAS 1999

For Reserving:

- "A Stochastic Method for Claims Reserving in General Insurance," Wright, Tom S.; Journal of the Institute of Actuaries 1990
- "A Stochastic Model Underlying the Chain-Ladder Technique" Renshaw, A.E.; Verrall, Richard; B.A.J. 1998

GLM for Loss Development

Incremental Paid Loss Development Triangle

	12	24	36	48	60	72	84	96
AY 1	1,505,051	1,005,474	1,119,085	588,359	877,344	923,280	510,300	66,491
AY 2	1,807,245	643,161	984,398	610,562	879,658	835,344	393,825	
AY 3	874,882	542,660	1,154,021	891,272	929,420	623,740		
AY 4	1,163,296	595,137	1,259,306	694,994	792,775			
AY 5	768,573	1,186,021	944,448	714,014				
AY 6	1,703,433	595,596	935,374					
AY 7	1,304,653	1,747,887						
AY 8	1,351,247							

	β_{12}	β_{24}	β_{36}	β_{48}	β_{60}	β_{72}	β_{84}	β_{96}
α_1	$\alpha_1 \cdot \beta_{12}$	$\alpha_1 \cdot \beta_{24}$	$\alpha_1 \cdot \beta_{36}$	$\alpha_1 \cdot \beta_{48}$	$\alpha_1 \cdot \beta_{60}$	$\alpha_1 \cdot \beta_{72}$	$\alpha_1 \cdot \beta_{84}$	$\alpha_1 \cdot \beta_{96}$
α_2	$\alpha_2 \cdot \beta_{12}$	$\alpha_2 \cdot \beta_{24}$	$\alpha_2 \cdot \beta_{36}$	$\alpha_2 \cdot \beta_{48}$	$\alpha_2 \cdot \beta_{60}$	$\alpha_2 \cdot \beta_{72}$	$\alpha_2 \cdot \beta_{84}$	
α_3	$\alpha_3 \cdot \beta_{12}$	$\alpha_3 \cdot \beta_{24}$	$\alpha_3 \cdot \beta_{36}$	$\alpha_3 \cdot \beta_{48}$	$\alpha_3 \cdot \beta_{60}$	$\alpha_3 \cdot \beta_{72}$		
α_4	$\alpha_4 \cdot \beta_{12}$	$\alpha_4 \cdot \beta_{24}$	$\alpha_4 \cdot \beta_{36}$	$\alpha_4 \cdot \beta_{48}$	$\alpha_4 \cdot \beta_{60}$			
α_5	$\alpha_5 \cdot \beta_{12}$	$\alpha_5 \cdot \beta_{24}$	$\alpha_5 \cdot \beta_{36}$	$\alpha_5 \cdot \beta_{48}$				
α_6	$\alpha_6 \cdot \beta_{12}$	$\alpha_6 \cdot \beta_{24}$	$\alpha_6 \cdot \beta_{36}$					
α_7	$\alpha_7 \cdot \beta_{12}$	$\alpha_7 \cdot \beta_{24}$						
α_8	$\alpha_8 \cdot \beta_{12}$							

GLM for Loss Development

Munich RE

Let the incremental paid loss in the triangle be set as the cross product of an accident year (AY) factor and a development age factor.

$$\text{Expected Incremental Loss } (Y_{i,j}) = \alpha_{AY\,i} \cdot \beta_{Age\,j}$$

If we use a GLM with a "log-link" and a Poisson variance structure, then the resulting parameters reproduce the familiar chain ladder reserve.

$$E(Y_{i,j}) = \exp(\ln(\alpha_{AY\,i}) + \ln(\beta_{Age\,j}))$$ [log-link]

$$Var(Y_{i,j}) = \phi \cdot E(Y_{i,j})$$ [Poisson variance]

GLM for Loss Development-Design Matrix

RESPONSE | **DESIGN MATRIX (X)**

Incr. Paid	Yr1	Yr2	Yr3	Yr4	Yr5	Yr6	Yr7	12	24	36	48	60	72	84	96
1,505,051	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1,807,245	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0
874,882	0	0	1	0	0	0	0	1	0	0	0	0	0	0	0
1,163,296	0	0	0	1	0	0	0	1	0	0	0	0	0	0	0
768,573	0	0	0	0	1	0	0	1	0	0	0	0	0	0	0
1,703,433	0	0	0	0	0	1	0	1	0	0	0	0	0	0	0
1,304,653	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0
923,280	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0
835,344	0	1	0	0	0	0	0	0	0	0	0	0	1	0	0
623,740	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0
510,300	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0
393,825	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0
66,491	1	0	0	0	0	0	0	0	0	0	0	0	0	0	1

Munich RE

For a ten year development triangle we need to estimate ten development period parameters:

$\beta_{12}, \beta_{24}, \beta_{36}, \beta_{48}, \beta_{60}, \beta_{72}, \beta_{84}, \beta_{96}, \beta_{108}, \beta_{120}$

Can we reduce the number of parameters?

Methods for Smoothing and Extrapolation

Munich RE

Proposal #1: make all later development periods equal

$\beta_{60+} = \beta_{60} = \beta_{72} = \beta_{84} = \beta_{96} = \beta_{108} = \beta_{120}$

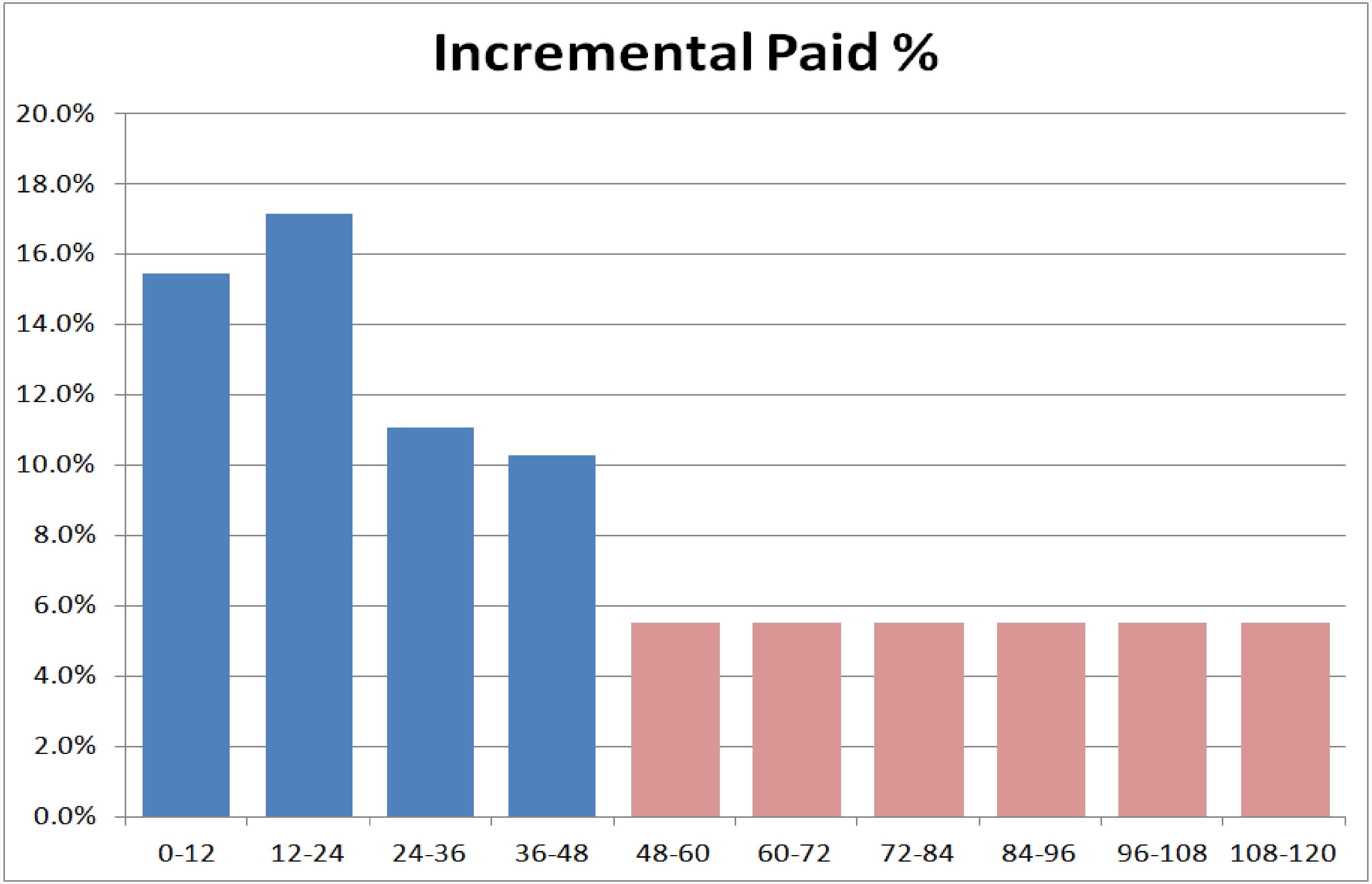
$\beta_{12}, \beta_{24}, \beta_{36}, \beta_{48}, \beta_{60+}$

When in development period 60-120:

$E(Y_{i,j}) = exp(\ ln(\alpha_i) + ln(\beta_{60+})\ )$

Methods for Smoothing and Extrapolation

Munich RE



GLM for Loss Development

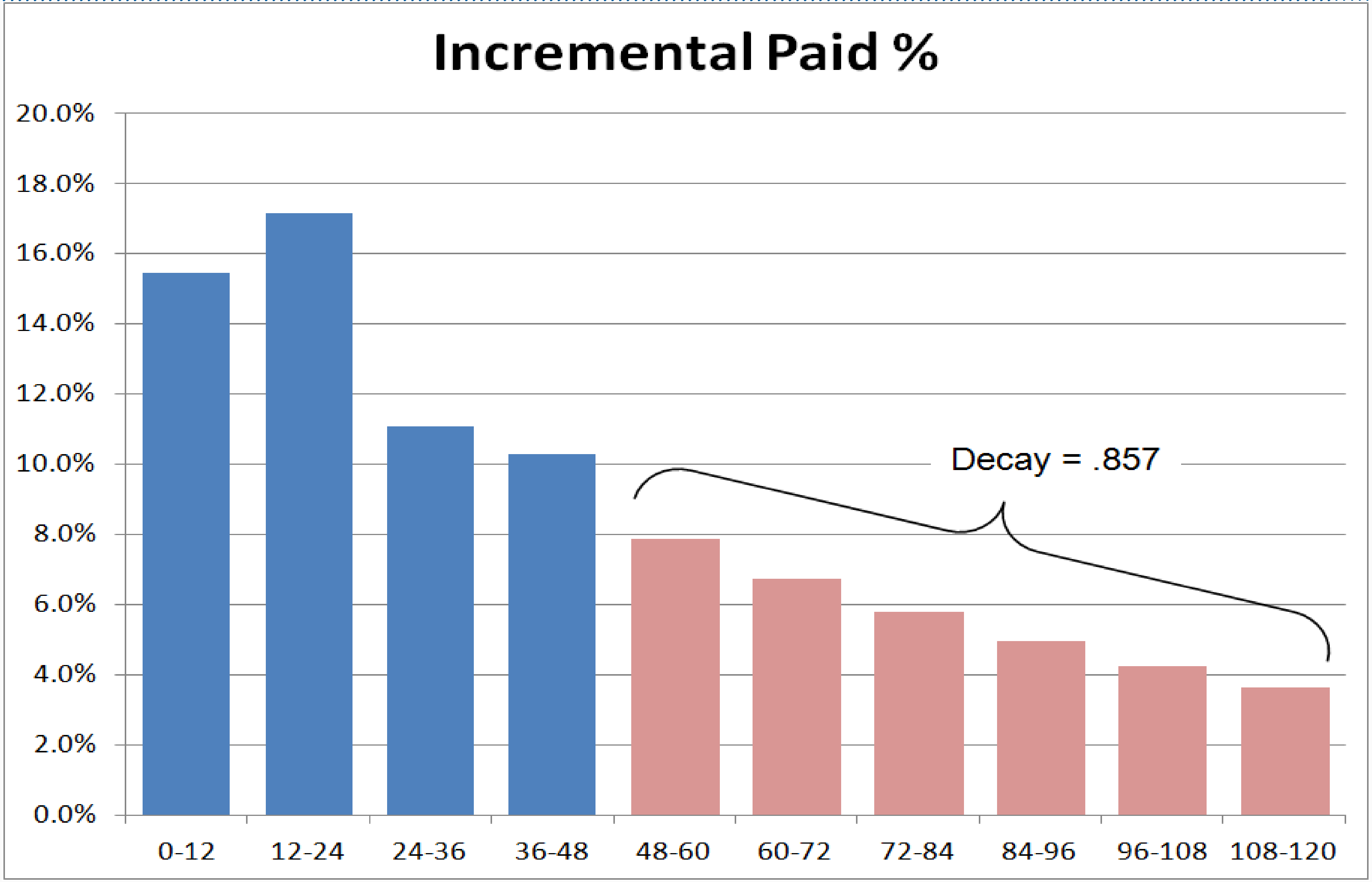
	β_{12}	β_{24}	β_{36}	β_{48}	β_{60}			
α_1	$\alpha_1 \cdot \beta_{12}$	$\alpha_1 \cdot \beta_{24}$	$\alpha_1 \cdot \beta_{36}$	$\alpha_1 \cdot \beta_{48}$	$\alpha_1 \cdot \beta_{60}$	$\alpha_1 \cdot \beta_{60}$	$\alpha_1 \cdot \beta_{60}$	$\alpha_1 \cdot \beta_{60}$
α_2	$\alpha_2 \cdot \beta_{12}$	$\alpha_2 \cdot \beta_{24}$	$\alpha_2 \cdot \beta_{36}$	$\alpha_2 \cdot \beta_{48}$	$\alpha_2 \cdot \beta_{60}$	$\alpha_2 \cdot \beta_{60}$	$\alpha_2 \cdot \beta_{60}$	
α_3	$\alpha_3 \cdot \beta_{12}$	$\alpha_3 \cdot \beta_{24}$	$\alpha_3 \cdot \beta_{36}$	$\alpha_3 \cdot \beta_{48}$	$\alpha_3 \cdot \beta_{60}$	$\alpha_3 \cdot \beta_{60}$		
α_4	$\alpha_4 \cdot \beta_{12}$	$\alpha_4 \cdot \beta_{24}$	$\alpha_4 \cdot \beta_{36}$	$\alpha_4 \cdot \beta_{48}$	$\alpha_4 \cdot \beta_{60}$			
α_5	$\alpha_5 \cdot \beta_{12}$	$\alpha_5 \cdot \beta_{24}$	$\alpha_5 \cdot \beta_{36}$	$\alpha_5 \cdot \beta_{48}$				
α_6	$\alpha_6 \cdot \beta_{12}$	$\alpha_6 \cdot \beta_{24}$	$\alpha_6 \cdot \beta_{36}$					
α_7	$\alpha_7 \cdot \beta_{12}$	$\alpha_7 \cdot \beta_{24}$						
α_8	$\alpha_8 \cdot \beta_{12}$							

GLM for Loss Development

Munich RE

RESPONSE	DESIGN MATRIX (X)											
Incr. Paid	Yr1	Yr2	Yr3	Yr4	Yr5	Yr6	Yr7	12	24	36	48	60 & above
1,505,051	1	0	0	0	0	0	0	1	0	0	0	0
1,807,245	0	1	0	0	0	0	0	1	0	0	0	0
874,882	0	0	1	0	0	0	0	1	0	0	0	0
1,163,296	0	0	0	1	0	0	0	1	0	0	0	0
768,573	0	0	0	0	1	0	0	1	0	0	0	0
1,703,433	0	0	0	0	0	1	0	1	0	0	0	0
1,304,653	0	0	0	0	0	0	1	1	0	0	0	0
877,344	1	0	0	0	0	0	0	0	0	0	0	1
879,658	0	1	0	0	0	0	0	0	0	0	0	1
929,420	0	0	1	0	0	0	0	0	0	0	0	1
792,775	0	0	0	1	0	0	0	0	0	0	0	1
923,280	1	0	0	0	0	0	0	0	0	0	0	1
835,344	0	1	0	0	0	0	0	0	0	0	0	1
623,740	0	0	1	0	0	0	0	0	0	0	0	1
510,300	1	0	0	0	0	0	0	0	0	0	0	1
393,825	0	1	0	0	0	0	0	0	0	0	0	1
66,491	1	0	0	0	0	0	0	0	0	0	0	1

Methods for Smoothing and Extrapolation



Methods for Smoothing and Extrapolation

Munich RE

Proposal #2: Exponential Decay

$\beta_{72} = \beta_{60} \cdot D \quad \beta_{84} = \beta_{60} \cdot D^2 \quad \beta_{96} = \beta_{60} \cdot D^3$

$\beta_{108} = \beta_{60} \cdot D^4 \quad \beta_{120} = \beta_{60} \cdot D^5$

$\beta_{12}, \beta_{24}, \beta_{36}, \beta_{48}, \beta_{60}, D$

When in development period 60-120:

$E(Y_{i,j}) = exp(\ ln(\alpha_i) + ln(\beta_{60}) + ln(D) \cdot t\ )$

GLM for Loss Development

	β_{12}	β_{24}	β_{36}	β_{48}	β_{60}, D			
α_1	$\alpha_1 \cdot \beta_{12}$	$\alpha_1 \cdot \beta_{24}$	$\alpha_1 \cdot \beta_{36}$	$\alpha_1 \cdot \beta_{48}$	$\alpha_1 \cdot \beta_{60}$	$\alpha_1 \beta_{60} D$	$\alpha_1 \beta_{60} D^2$	$\alpha_1 \beta_{60} D^3$
α_2	$\alpha_2 \cdot \beta_{12}$	$\alpha_2 \cdot \beta_{24}$	$\alpha_2 \cdot \beta_{36}$	$\alpha_2 \cdot \beta_{48}$	$\alpha_2 \cdot \beta_{60}$	$\alpha_2 \beta_{60} D$	$\alpha_2 \beta_{60} D^2$	
α_3	$\alpha_3 \cdot \beta_{12}$	$\alpha_3 \cdot \beta_{24}$	$\alpha_3 \cdot \beta_{36}$	$\alpha_3 \cdot \beta_{48}$	$\alpha_3 \cdot \beta_{60}$	$\alpha_3 \beta_{60} D$		
α_4	$\alpha_4 \cdot \beta_{12}$	$\alpha_4 \cdot \beta_{24}$	$\alpha_4 \cdot \beta_{36}$	$\alpha_4 \cdot \beta_{48}$	$\alpha_4 \cdot \beta_{60}$			
α_5	$\alpha_5 \cdot \beta_{12}$	$\alpha_5 \cdot \beta_{24}$	$\alpha_5 \cdot \beta_{36}$	$\alpha_5 \cdot \beta_{48}$				
α_6	$\alpha_6 \cdot \beta_{12}$	$\alpha_6 \cdot \beta_{24}$	$\alpha_6 \cdot \beta_{36}$					
α_7	$\alpha_7 \cdot \beta_{12}$	$\alpha_7 \cdot \beta_{24}$						
α_8	$\alpha_8 \cdot \beta_{12}$							

Munich RE

Proposal #3: Pareto-ish Decay (inverse power, heavy-tail)

$\beta_{72} = \beta_{60} / 2^{\theta}$ $\beta_{84} = \beta_{60} / 3^{\theta}$ $\beta_{96} = \beta_{60} / 4^{\theta}$

$\beta_{108} = \beta_{60} / 5^{\theta}$ $\beta_{120} = \beta_{60} / 6^{\theta}$

$\beta_{12}, \beta_{24}, \beta_{36}, \beta_{48}, \beta_{60}, \theta$

When in development period 60-120:

$E(Y_{i,j}) = exp(ln(\alpha_i) + ln(\beta_{60}) - \theta \cdot ln(1+t))$

GLM for Loss Development

	β_{12}	β_{24}	β_{36}	β_{48}	β_{60}, θ			
α_1	$\alpha_1 \cdot \beta_{12}$	$\alpha_1 \cdot \beta_{24}$	$\alpha_1 \cdot \beta_{36}$	$\alpha_1 \cdot \beta_{48}$	$\alpha_1 \cdot \frac{\beta_{60}}{1^\theta}$	$\alpha_1 \cdot \frac{\beta_{60}}{2^\theta}$	$\alpha_1 \cdot \frac{\beta_{60}}{3^\theta}$	$\alpha_1 \cdot \frac{\beta_{60}}{4^\theta}$
α_2	$\alpha_2 \cdot \beta_{12}$	$\alpha_2 \cdot \beta_{24}$	$\alpha_2 \cdot \beta_{36}$	$\alpha_2 \cdot \beta_{48}$	$\alpha_2 \cdot \frac{\beta_{60}}{1^\theta}$	$\alpha_2 \cdot \frac{\beta_{60}}{2^\theta}$	$\alpha_2 \cdot \frac{\beta_{60}}{3^\theta}$	
α_3	$\alpha_3 \cdot \beta_{12}$	$\alpha_3 \cdot \beta_{24}$	$\alpha_3 \cdot \beta_{36}$	$\alpha_3 \cdot \beta_{48}$	$\alpha_3 \cdot \frac{\beta_{60}}{1^\theta}$	$\alpha_3 \cdot \frac{\beta_{60}}{2^\theta}$		
α_4	$\alpha_4 \cdot \beta_{12}$	$\alpha_4 \cdot \beta_{24}$	$\alpha_4 \cdot \beta_{36}$	$\alpha_4 \cdot \beta_{48}$	$\alpha_4 \cdot \frac{\beta_{60}}{1^\theta}$			
α_5	$\alpha_5 \cdot \beta_{12}$	$\alpha_5 \cdot \beta_{24}$	$\alpha_5 \cdot \beta_{36}$	$\alpha_5 \cdot \beta_{48}$				
α_6	$\alpha_6 \cdot \beta_{12}$	$\alpha_6 \cdot \beta_{24}$	$\alpha_6 \cdot \beta_{36}$					
α_7	$\alpha_7 \cdot \beta_{12}$	$\alpha_7 \cdot \beta_{24}$						
α_8	$\alpha_8 \cdot \beta_{12}$							

Munich RE

LIVE EXAMPLE(S)

[switch to excel]

Conclusions – Advantages of GLM

- GLM with log-link can be used to smooth and extrapolate development factors
- Shape of smoothing can include
  - Exponential decay
  - Pareto tail (inverse power)
  - Other forms such as benchmark patterns
- Easy to select period over which smoothing is performed
- Standard error of the fits can be use to evaluate the model
- Results can be transformed back into age-to-age factors if needed

Dave Clark
daveclark@munichreamerica.com

Terry Fang
TFang@munichreamerica.com

Munich RE SM