The Case for Case: Case-Reserve Development

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Outline

I. Introduction
II. Loss Statics
    Ratio methods
    Aggregation principle
    Paid and case duality
III. Loss Dynamics
    Loss development as ΔIncd(t)
    ΔIncd to what? To paid, incurred, or case?
    Considerations and Findings
IV. Conclusion

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I. Introduction

- Question 3.14159 from CAS Exam 2.71828:

Assume that the average car costs \$30,000 and weighs 3,000 pounds. What is the expected cost of a car that weighs 2,000 pounds?

A) \$20,000
B) Greater than \$20,000 and less than \$30,000
C) \$30,000
D) Greater than \$30,000
E) Cannot be determined

- Bonus Question: Which is greater, e^{π} or π^{e}?

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I. Introduction (cont'd)

- "Actuarial Science" – Is loss reserving scientific?

Any intelligent fool can make things bigger and more complex. It takes a touch of genius – and a lot of courage – to move in the opposite direction. Albert Einstein

- Is ASOP 36 §3.6.3 (Expected Value Estimate) science, proto-science, or pseudoscience?
- Methods versus Models? A red herring.
  "Model risk": Are actuaries making a science out ignorance?
- The paid and incurred goalposts – Do they span the whole field? Are you a "paidist" or an "incurredist"?
- NAIC versus CAS terminologies:
  Reported vs. Incd = Paid+Case
  Bulk+IBNR vs. IBNER+IBNYR = IBNR

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II. Loss Statics

- Ratio Methods – an example:

$100 has been paid on loss X
Adjusters have set case reserves at $50

The loss portions of losses similar to X are:
α = ½ or 50% paid
β = ⅙ or 16.7% case
γ = ⅓ or 33.3% IBNR

Estimate the IBNR, or the ultimate amount, of X.

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II. Loss Statics (cont'd)

- Notation for a systematic analysis of the ratio methods:

	Paid	Case	IBNR	Ultimate
Amount X	P	C	I	$U = P+C+I$
Portion ξ	α	β	γ	$1 = \alpha+\beta+\gamma$

- Binary representation of loss types:
  1 = 001 = Paid 2 = 010 = Case 4 = 100 = IBNR
  Define: 1 + 2 = 3 = 011 = Incd
  1 + 4 = 5 = 101 = Paid + IBNR = ¬Case
  2 + 4 = 6 = 110 = ¬Paid
  1 + 6 = 7 = 111 = Paid + ¬Paid = Ultimate
  2 + 5 = 7 = 111 = Case + ¬Case = Ultimate
  3 + 4 = 7 = 111 = Incd + IBNR = Ultimate

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II. Loss Statics (cont'd)

- Ratio method *i*::*j* yields the IBNR that satisfies:

$$\frac{X_i}{\xi_i} = \frac{X_j}{\xi_j}$$

- E.g., method 1::7: $\frac{P}{\alpha} = \frac{U}{1} \Rightarrow IBNR_{1::7} = \frac{P}{\alpha} - P - C$
  - Equivalent to the paid CL method

- *i*::*j* ≡ *j*::*i*, and *i* ≠ *j*. So let *i* < *j*. (6×7)/2 = 21 methods. But 1::2, 1::3, and 2::3 have no unknowns; so really 18 methods. But 18 = 5×3 + 3×1. Five triads and three monads, for 8 algebraically different methods.

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II. Loss Statics (cont'd)

- Discuss workbook [Ratio Methods.xlsx]

Type	1	2	3	4	5	6	7
Name	Paid	Case	Incd	IBNR	-Case	-Paid	Ultimate
Amount	100	50	150	#N/A	#N/A	#N/A	#N/A
Portion	0.5	0.166667	0.666667	0.333333	0.333333	0.5	1

Ratio	Description				Ultimate	
1::2	Paid to Case				#N/A	
1::3	Paid to Incd				#N/A	
1::4	Paid to IBNR	66.66667			216.6667	
1::5	Paid to -Case		166.6667		216.6667	
1::6	Paid to -Paid			100	200	
1::7	Paid to Ultimate				200	
2::3	Case to Incd				#N/A	
2::4	Case to IBNR	100			250	
2::5	Case to -Case		250		300	
2::6	Case to -Paid			150	250	
2::7	Case to Ultimate				300	
3::4	Incd to IBNR	75			225	
3::5	Incd to -Case		187.5		237.5	
3::6	Incd to -Paid			112.5	212.5	
3::7	Incd to Ultimate				225	
4::5	IBNR to -Case	66.66667	166.6667		216.6667	0
4::6	IBNR to -Paid	100		150	250	0
4::7	IBNR to Ultimate	75			225	0
5::6	-Case to -Paid		125	75	175	0
5::7	-Case to Ultimate		250		300	0
6::7	-Paid to Ultimate			100	200	0

Equations: 7 = 1 + 2 + 4 = 1 + 6 = 2 + 5 = 3 + 4
6 = 2 + 4
5 = 1 + 4
3 = 1 + 2

Ultimate	Ratios		
175	5::6		
200	1::6	6::7	1::7
212.5	3::5		
216.6667	1::4	4::5	1::5
225	3::4	4::7	3::7
237.5	3::5		
250	2::4	4::6	2::6
300	2::5	5::7	2::7

If xk = xi + xj (same for (), then i::j = j::k = i::k
Six triads; 3 + 5 + 6; so 3::5, 3::6, and 5:6 are monadic.

Method Relations:
225 = 200(0.75)+300(0.25), where 0.75 : 0.25 = paid : case
225 = 216⅔(0.75)+250(0.25), where 0.75 : 0.25 = paid : case

225 = average(212.5, 237.5): 1 : 1 < paid : case
175 is extreme (5::6)

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II. Loss Statics (cont'd)

- Five Triadic methods:
  - If $x_i + x_j \equiv x_{k=i+j}$, then methods *i*::*j*, *i*::*k*, and *j*::*k* give the same result. For:

$$IBNR_{i::j} \Leftrightarrow \begin{vmatrix} x_i & x_j \\ \xi_i & \xi_j \end{vmatrix} = 0$$

  - So:

$$1::6 \equiv 1::7 \equiv 6:7 \quad Ult\ /\ Paid$$
$$3::4 \equiv 3::7 \equiv 4:7 \quad Ult\ /\ Incd$$
$$2::5 \equiv 2::7 \equiv 5:7 \quad Ult\ /\ Case$$
$$1::4 \equiv 1::5 \equiv 4:5 \quad IBNR\ /\ Paid$$
$$2::4 \equiv 2::6 \equiv 4:6 \quad IBNR\ /\ Case$$

- Three Monadic methods 3::5, 3::6, 5::6
  - Binary complements of the unlawful 1::2, 1::3, and 2::3

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II. Loss Statics (cont'd)

- Define weights $w_1 = \frac{\alpha}{\alpha+\beta}$ $w_2 = \frac{\beta}{\alpha+\beta}$
- Let ~ be relation in $\frac{P}{\alpha} \sim \frac{C}{\beta}$
- Incd mediates between paid and case because:

$$\frac{P+C}{\alpha+\beta} = \frac{P}{\alpha}\frac{\alpha}{\alpha+\beta} + \frac{C}{\beta}\frac{\beta}{\alpha+\beta} = \frac{P}{\alpha}w_1 + \frac{C}{\beta}w_2$$

  - The goalposts are Paid and Case; Incd is the 50-yard line
- The five triadic methods order as:

$$IBNR_{1:7} \sim IBNR_{1:4} \sim IBNR_{3:4::3:7} \sim IBNR_{2:4} \sim IBNR_{2:7}$$

$$\frac{Ult}{Paid} \sim \frac{IBNR}{Paid} \sim \left(\frac{IBNR}{Incd} \equiv \frac{Ult}{Incd}\right) \sim \frac{IBNR}{Case} \sim \frac{Ult}{Case}$$

  - Incd IBNR is the (w_1, w_2) weighted average of both sets of Paid and Case methods.

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II. Loss Statics (cont'd)

- As for the monadic methods:

$$IBNR_{1:7} \sim IBNR_{3:6} \sim IBNR_{3:4::3:7} \sim IBNR_{3:5} \sim IBNR_{2:7}$$

$$\frac{Ult}{Paid} \sim \frac{\neg Paid}{Incd} \sim \left(\frac{IBNR}{Incd} \equiv \frac{Ult}{Incd}\right) \sim \frac{\neg Case}{Incd} \sim \frac{Ult}{Case}$$

  - Incd IBNR mediates here as the simple average of $IBNR_{3::5}$ and $IBNR_{3::6}$. $IBNR_{5::6}$ is unruly, often an extremum.
- The Actuarial Central Estimate (ASOP 43 §3.3a) can be none other than the Incd chain-ladder.
  - Incd methods mediate between paid and case methods
  - Not obvious for two reasons
    - Actuaries aren't acquainted with dynamic forms of case methods
    - The AY orderings obey the ~. Some cancellation in total order.

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II. Loss Statics (cont'd)

- Aggregation Principle:

$$\text{If } \forall \eta : \frac{x_{\eta,i}}{\xi_i} = \frac{x_{\eta,j}}{\xi_j}, \text{ then :}$$

$$\frac{X_i}{\xi_i} = \frac{\sum_\eta x_{\eta,i}}{\xi_i} = \sum_\eta \frac{x_{\eta,i}}{\xi_i} = \sum_\eta \frac{x_{\eta,j}}{\xi_j} = \frac{\sum_\eta x_{\eta,j}}{\xi_j} = \frac{X_j}{\xi_j}$$

  - a very desirable and appealing property for a loss-development method
  - really just means that multiplication distributes over addition
  - no implication that the method is unbiased
    - Can a method's results may be biased by AY, but not in total?

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II. Loss Statics (cont'd)

- Implications
  - Incurred mediates between paid and case
    - Implications to the underwriting cycle?
  - Develop only open claims, for which C and $\beta > 0$
  - Reopenings and IBNYR better treated separately
  - How to handle this annoying situation?

    A self-insured changes its claim adjuster. The new adjuster tracks only the claims left open from the old one.

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II. Loss Statics (cont'd)

- Paid and Case Duality
  - Abstractly, paid = 1 = 001, case = 2 = 010
  - Methods don't favor paid or case
  - If paid and case equally well suited for loss reserving, then probably their sum (incd = 3 = 011) will be well suited
  - If one is better suited than the other, then incd is not likely to be as good as the better method.
- Concretely, how do paid and case losses differ?
  - Paid is a fact of the past; case is an expectation of the future
  - Retrospective vs prospective

    "Actuaries drive by looking in their rearview mirrors," said a CEO. True?
  - Open claims tend to be the largest and most idiosyncratic

    Lazy [unethical] to invoke the car fallacy ($10 per pound)
  - Are future payments on a few open claims related to the payments of many closed claims? NPV analogy

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III. Loss Dynamics

- Dynamics: Changes at regular intervals of time
  - The point of the loss triangle
- Loss reaches ultimate in steps:
  - Ultimately paid: Ult = ΔPaid(1) + ΔPaid(2) + ...
  - Ultimately incd: Ult = ΔIncd(1) + ΔIncd(2) + ...
  - But ΔIncd = ΔPaid + ΔCase
- Loss development as a vector: $\Delta Incd(t) = \begin{bmatrix} \Delta Paid(t) \\ \Delta Case(t) \end{bmatrix}$
- Which development factor?

$$\frac{\Delta Incd(t)}{Paid(t-1)} \quad \frac{\Delta Incd(t)}{Incd(t-1)} \quad \frac{\Delta Incd(t)}{Case(t-1)}$$

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III. Loss Dynamics (con't)

- Incd method mediates between paid and case:

$$\frac{\Delta Incd(t)}{Incd(t-1)} = \frac{\Delta Paid(t)}{Paid(t-1)} \cdot \left(w_1 = \frac{Paid(t-1)}{Incd(t-1)} \right) + \frac{\Delta Case(t)}{Case(t-1)} \cdot \left(w_2 = \frac{Case(t-1)}{Incd(t-1)} \right)$$

- The static arguments for case apply here
  - Compare (paid + remaining case) with (÷) initial case
- The so-called "IBNR-to-Case" method:

$$IBNR = Case \cdot \frac{1 - 1/IncdLDF}{1/IncdLDF - 1/PaidLDF}$$

  - Formally unobjectionable as $IBNR_{2::4}$
  - But its LDFs derived separately, not "codeveloped"
  - This "generally accepted" method is defective!

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III. Loss Dynamics (con't)

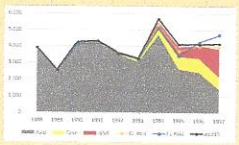
- Discuss workbook [Meyers‡ Example.xlsx]

Comparison of Methods

Losses as of 12/31/1997

AY	Paid	Case	Incd	IBNR	CRD	CL Incd	CL Paid
1988	3,512	5	3,517	0	3,917	3,917	3,912
1989	2,531	7	2,538	0	2,538	2,538	2,531
1990	4,155	15	4,170	2	4,168	4,167	4,152
1991	4,332	11	4,343	21	4,364	4,364	4,307
1992	3,491	72	3,563	110	3,673	3,594	3,556
1993	3,094	156	3,150	103	3,293	3,233	3,306
1994	4,714	462	5,176	344	5,520	5,339	5,145
1995	2,607	775	3,382	587	3,969	3,744	3,424
1996	2,412	895	3,307	763	4,070	4,087	4,279
1997	1,813	790	2,603	1,644	3,847	4,026	4,726
Total	32,601	3,188	35,799	3,570	39,399	38,939	39,363

IBNR = CRD - Incd



‡ Meyers [2015], p. 5, Tables 2 and 3

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III. Loss Dynamics (con't)

- The development equation:

$$\begin{bmatrix} Paid_{t+1} \\ Case_{t+1} \end{bmatrix} = \begin{bmatrix} Paid_t \\ 0 \end{bmatrix} + \begin{bmatrix} a \\ b \end{bmatrix} Case_t = \begin{bmatrix} 1 & a \\ 0 & b \end{bmatrix} \begin{bmatrix} Paid_t \\ Case_t \end{bmatrix}$$

- Matrices of the form $\begin{bmatrix} 1 & a \in \Re \\ 0 & b \in \Re^+ \end{bmatrix}$ constitute a group
  - Closed under matrix multiplication
  - Identity matrix $I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, inverse matrix $\begin{bmatrix} 1 & -a/b \\ 0 & 1/b \end{bmatrix}$
  - Matrix development is not commutative in general
  - Development matrices commute ⇔ collinear with I_2
  - Development is order-dependent

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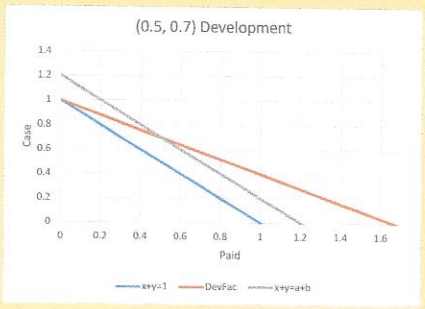
III. Loss Dynamics (con't)

- Development graph



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III. Loss Dynamics (con't)

- Considerations and Findings
  - n^{th}-to-Ult development
    If $0 < b < 1$: $\lim_{n\to\infty}\begin{bmatrix}1 & a\\ 0 & b\end{bmatrix}^n = \begin{bmatrix}1 & a/(1-b)\\ 0 & 0\end{bmatrix}$
  - If Incd < Paid, fine. Ignoring Case may aggravate the underwriting cycle.
  - The mixing of paid and case is like the feet of iron and clay in Daniel 2.43. But which is iron and which is clay?
  - Loss development = loss individuation or "idiosyncratization"
    No tolerance for "average" statistics (car fallacy)
  - For Bulk (IBNER) actuaries are corrective lenses for the eyes of claim adjusters
  - A scientific hypothesis: Underwriting ~ loss in suspension; Reserving ~ loss precipitating into claim crystals

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IV. Conclusion

- The Case for Case
  - Reserving data consists of claim counts, paid, and case
  - Case is the only prospective quantity.
  - Bulk (IBNER) differs from IBNYR and reopenings
  - Case-reserve development started w Marker & Mohl (1980)
    But not limited to claims-made exposures
- The "methods" have nothing to do with sampling
  - The "Central Estimate" is the incurred.
- Wanted: a scientific theory of how loss is incurred.
  - No loss without a claim. Precipitation?
  - Ratemaking : Reserving :: Incurrable : Incurred
    A loss incurred jumps off the exposure track into the claim adjuster's lap!
- Who ya gonna call? Reserve busters!

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References

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- Jing, Y., Lebens, J. R., and Lowe, S. P., "Claim Reserving: Performance Testing and the Control Cycle," *Variance* 2009, *Vol.3, Issue 2*, 161-193, www.variancejournal.org/issues/03-02/161.pdf.
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- Wiser, R. F., Loss Reserving, *Foundations of Casualty Actuarial Science* (3rd edition, 1996), "case reserve" development method, pp 201-212.

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Ranking	Session	Title	Response Percent	Response Count
1	PD-8	PD - Reserving War Stories	54%	52
2	PD-7	PD - Professionalism in Reserve Setting	49%	47
3	ST-8	ST - You've Got to See it to Believe it: Data Visualization Techniques in Reserving	49%	47
4	VR-1	VR - Beyond the Point Estimate: An Introduction on How to Understand and Communicate Reser	47%	46
5	AR-5	AR - The Case for Case: Case-Reserve Development	46%	45
6	VR-4	VR - How Do Companies Develop a Range of Reserves, from Theory to Practice	46%	45
7	AR-2	AR - Improving Actuarial Reserve Analysis through Claim-Level Predictive Analytics	44%	43
8	LOB-2	LOB - Cyber Risk - Industry Impact of Cyber Risk and Aggregation/Accumulation Management	42%	41
9	FR-5	FR - Reserving Disclosures in Financial Reports;Current & Proposed US GAAP & SEC Disclosur	34%	34
10	ERM-1	ERM - A Capital Modeler's View of Reserving Ranges	32%	31
11	AR-3	AR - Introduction to GLM with Application to Smoothing and Extrapolating Development Patterns	31%	30
12	PD-5	PD - Peering into Peer Reviews	30%	29
13	AR-1	AR - A Deep Exploration of Loss Adjustment Expense Reserving	29%	28
14	WC-1	WC - Impact of claim Level Predictive Modeling on Actuarial Reserve Analysis	29%	28
15	PD-1	PD - ABCD - Case Studies	27%	26
16	ERM-2	ERM - AM Best Stochastic BCAR model	25%	24
17	ERM-6	ERM - ERM Best Practices: Assisting the CFO And Chief Actuary In Establishing Loss Reserves	25%	24
18	LOB-3	LOB - The third wave of asbestos liabilities	25%	24
19	ST-2	ST - Fracking: An Emerging Resource and Source of New Risk	24%	23
20	AR-4	AR - Reserve Mixology 201	23%	22
21	WC-3	WC - NCCI Studies - Unexpected Impact of WC Medical Fee Schedules and Prescription Drug Fe	22%	21
22	R-4	RE - Reserving for Non-Property Catastrophes	21%	20
23	FR-1	FR - IASB insurance accounting standard for property/casualty contracts â€“ latest update	20%	19
24	FR-6	FR - Tax Issues for P&C Actuaries	20%	19
25	PD-10	PD - Take 3: Lights! Camera! Professionalism!	20%	19
26	ST-1	ST - Actuarial Functions: What Kind of Improvements to Face Future Challenges?	19%	18
27	R-2	RE - Ceded Reserves - Setting and Monitoring	18%	17
28	VR-5	VR - Incorporating Model Error into the Actuary's Estimate of Uncertainty	18%	17
29	VR-6	VR - Past the Bootstrap - Bayesian Reserving	18%	17
30	ERM-4	ERM - Bringing It All Together - Comparing Risk & Capital Regimes Globally	17%	16
31	PD-3	PD - Mock Deposition - 1	17%	16
32	WC-4	WC - Workers Compensation Presumptions: A Double Edged Sword	17%	16
33	WC-6	WC - WC and the Intersection with Asbestos Bankruptcy Trust Claims and Other Emerging OD Ti	16%	15
34	PD-2	PD - Communication in Multiple Environments	14%	14
35	FR-7	FR - Updated AAA Issues Brief: An Overview for P/C Insurer's Audit Committees: Effective Use c	13%	13
36	PD-9	PD - Survey Says - Professionalism Edition	13%	13
37	SI-3	SI - How is your TPA Performing?	13%	13
38	VR-3	VR - GLM's and Bayesian Models	13%	13
39	SI-1	SI - Captive Feasibility Studies	12%	12
40	ST-6	ST - Reserving in High Inflation (International)	12%	12
41	PD-6	PD - Professionalism Considerations in Captive Insurance	11%	11
42	R-5	RE - Run Off Solutions for Legacy Liabilities	11%	11
43	ERM-3	ERM - Applying Fuzzy Logic to Risk Assessment and Decision-Making	10%	10
44	FR-4	FR - Economic Balance Sheet: what is it for Bermuda General Insurers?	10%	10
45	HC-1	HC - Have You Thought About These Issues in Medical Professional Liability?	10%	10
46	ST-4	ST - Property Casualty Specialty Insurance Markets - Survival of the Fittest	10%	10
47	FR-3	FR - Opinion Writers Coffee Klatsch	9%	9
48	HC-2	HC - Health Care Professional Liability Claim Trends: The Perspective of 3	9%	9
49	HC-3	HC - Today's Medical Professional Liability Market: Challenges and Opportunities	9%	9
50	LOB-1	LOB - Credit and Surety - Business Changing with New Exposures	9%	9
51	WC-2	WC - Medicare Secondary Payer Status: The Impact of Section 111 Reporting Requirements	9%	9
52	PD-4	PD - Mock Deposition - 2	8%	8
53	R-1	RE - Alternative Capital	8%	8
54	WC-5	WC - State of WC in Pennsylvania and TBD - Old, New and Potential Trends	8%	8
55	ERM-5	ERM - ERM ASOP's	7%	7
56	ST-5	ST - Reserving for Loyalty Rewards Programs	7%	7
57	FR-2	FR - Know Your Measurement Basis	5%	5
58	SI-2	SI - How Does Your Captive Rate?	5%	5
59	VR-2	VR - Extrapolating Co-Linear Payment Trends for Development Triangle GLMs	5%	5
60	R-3	RE - Losses, Contracts and Money - oh my!	4%	4
61	ST-3	ST - NICB's Focus in Combating Insurance Fraud and Crime	4%	4
62	FR-8	FR - How To Fit a Quart Into a Pint	3%	3
63	SI-4	SI - Working Effectively with Corporate Risk Managers	3%	3
64	VR-7	VR - Alimi	1%	1
65	ST-7	ST - RMarkdown and GIT for Collaborative Actuarial Research and Analysis	0%	0
		VR - Reserve Variability and ERM - bridging the gap	17%	16
		HC - Leading Databases	7%	7