Applying Fuzzy Logic to Risk Assessment and Decision-Making

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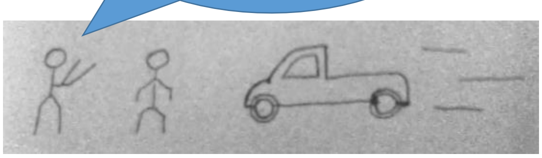
Fuzzy Logic vs. Probability

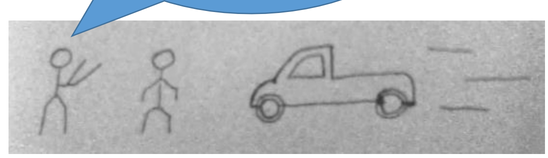
Probability Measure

A pick-up truck is coming from your right side at a speed of 60 mph. If you do not run now, you will get hit at a probability of 90%.



Fuzzy Logic

Look Out!!!



Agenda

1. Background
2. Fuzzy Logic and Fuzzy Set Theory
3. Numerical Example
4. Risk Assessment Framework Based on Fuzzy Logic
5. Key Considerations
6. Recap

Background

Background

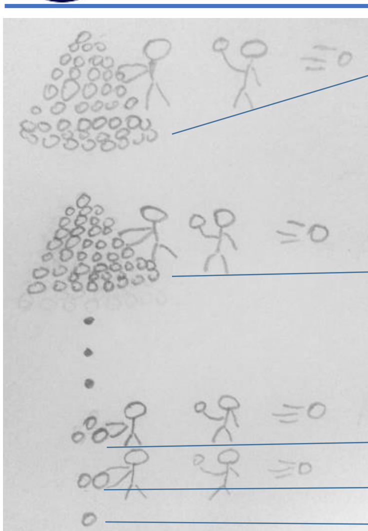
- The research report titled “Applying Fuzzy Logic to Risk Assessment and Decision-Making” was sponsored and published by the JRMS of the CAS, the CIA and the SOA in Nov. 2013.
- Two co-authors and 11 POG members.
- An EXCEL tool was also built that is capable of implementing simple fuzzy logic models.
- The report and the tool can be accessed at http://www.soa.org/Research/Research-Projects/Risk-Management/research-2013-fuzzy-logic.aspx.

Fuzzy Logic and Fuzzy Set Theory

Why do we need fuzzy logic?

- Lack of experience data
- Entangled cause-and-effect relationships
- Imprecise data
- It can easily incorporate non-numerical information.

Sorites Paradox



A heap

A heap

A heap?

A heap?

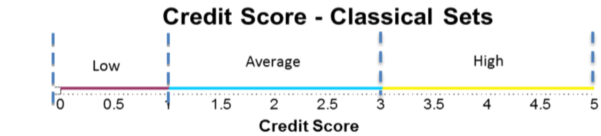
A heap?

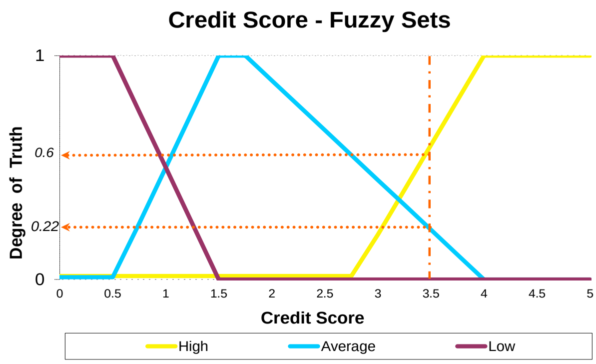
Paradox of the heap

1. 1,000,000 grains of sand is a heap of sand;
2. A heap of sand minus one grain is still a heap.

Vagueness in natural language.

Fuzzy Set Theory vs. Classical Set Theory





Fuzzy Set Membership Functions: define the degree of truth.

$$\mu^{High}(x) = \begin{cases} 0 & x \le 2.75 \\ {(x-2.75)}/{1.25} & 2.75 < x \le 4 \\ 1 & x > 4 \end{cases}$$

$$\mu^{\text{Average}}(x) = \begin{cases} 0 & x \le 0.5 \\ {(x-0.5)}/{1} & 0.5 < x \le 1.5 \\ 1 & 1.5 < x \le 1.75 \\ {(4-x)}/{2.25} & 1.75 < x \le 4 \\ 0 & x > 4 \end{cases}$$

$$\mu^{Low}(x) = \begin{cases} 1 & x \le 0.5 \\ {(1.5-x)}/{1} & 0.5 < x \le 1.5 \\ 0 & x > 1.5 \end{cases}$$

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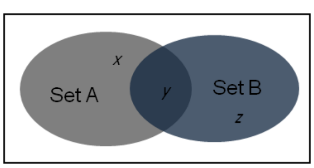
Fuzzy Set Operation vs. Classical Set Operation

Operation on Classical Sets



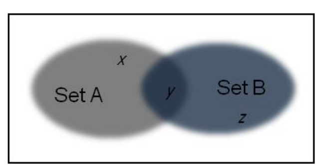
	$A \cup B$	$A \cap B$	$\overline{A}$
$=$	$\{x, y, z\}$	$\{y\}$	$\{z\}$
x	$1 \cup 0 = 1$	$1 \cap 0 = 0$	$1 - 1 = 0$
y	$1 \cup 1 = 1$	$1 \cap 1 = 1$	$1 - 1 = 0$
z	$0 \cup 1 = 1$	$0 \cap 1 = 0$	$1 - 0 = 1$

Where

$1 : \in \quad e.g.\ x \in A$

$0 : \notin \quad e.g.\ x \notin A$

Operation on Fuzzy Sets



$\mu_A(x) = 0.5 \quad \mu_B(x) = 0.1$

$\mu_A(y) = 0.6 \quad \mu_B(y) = 0.4$

$\mu_A(z) = 0.1 \quad \mu_B(z) = 0.7$

	$A \cup B = \max(\mu_A, \mu_B)$	$A \cap B = \min(\mu_A, \mu_B)$	$\overline{A} = 1 - \mu_A$
x	0.5	0.1	0.5
y	0.6	0.4	0.4
z	0.7	0.1	0.9

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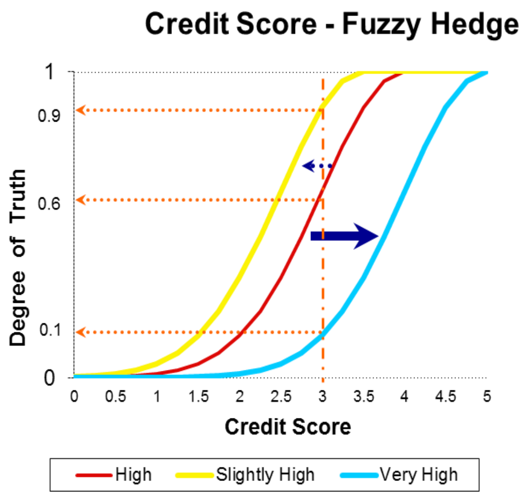
Fuzzy Hedge



Rule 1. If a credit score is slightly high, the chance of getting a mortgage rate discount is high.

Rule 2. If a credit score is high, the chance of getting a mortgage rate discount is high.

Rule 3. If a credit score is very high, the chance of getting a mortgage rate discount is high.

Inference Rules

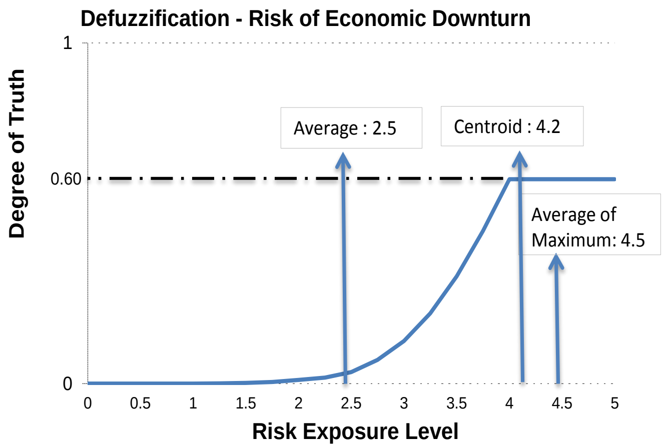
1. *If A and B, then C.*
   The maximum degree of truth for C is the lesser of the degree of truth for A and that for B.
2. *If A or B, then C.*
   The maximum degree of truth for C is the greater of the degree of truth for A and that for B.
3. *If not A, then C.*
   The maximum degree of truth for C is one deducted by the degree of truth for A.

If the term premium is small and investors' confidence level is low, the risk of economic downturn in the near future is high.

Term premium: μ_{small}(2 percent) = 0.6
Condidence Index: μ_{low}(65) = 0.72

The maximum degree of truth that there is a high risk of economic downturn is 0.6.

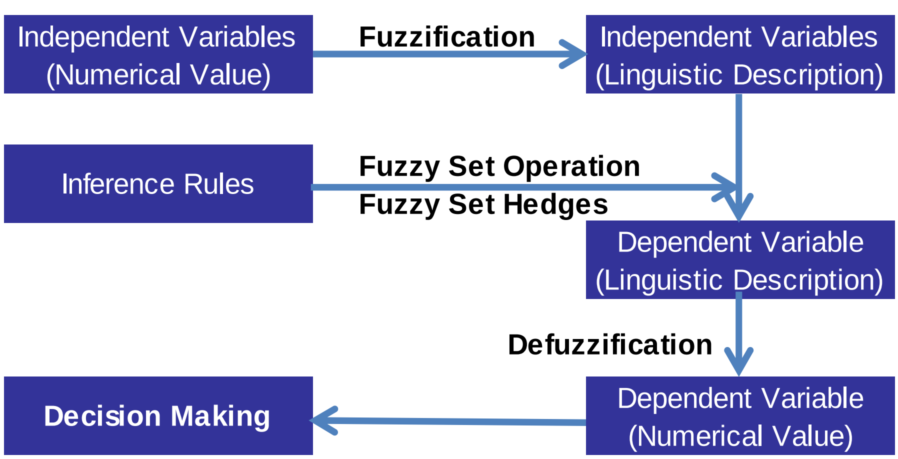
Defuzzification



Defuzzification is the process of estimating the value of the dependent variable based on the resulting fuzzy set after applying the fuzzy inference rule.

Avg. method: The avg. value of the dependent variable.
Avg. of maximum method: The average value of the dependent variable with the maximum degree of truth.
Centroid method: The weighted avg. value of the dependent variable. The weight is the degree of truth.

Fuzzy Logic System



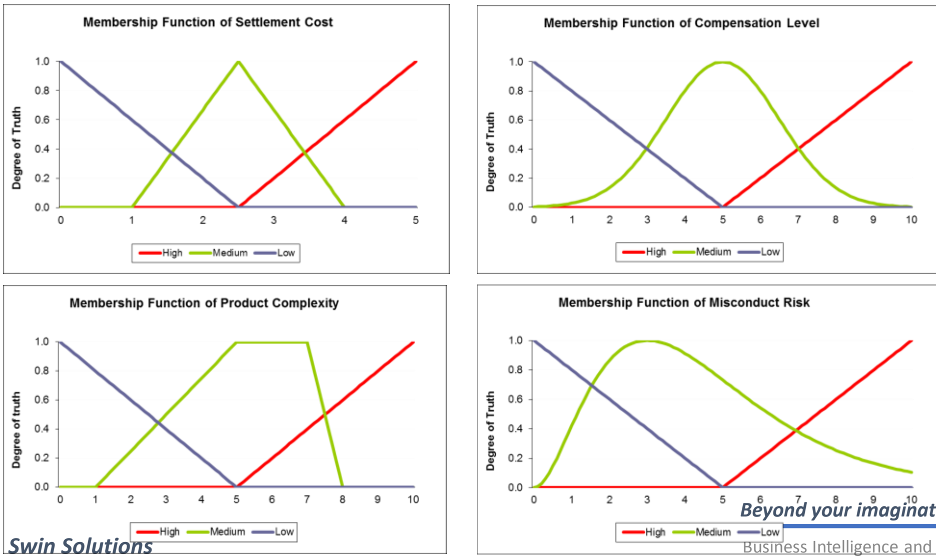
Numerical Example

Advisor's Misconduct Risk

Key Risk Indicators

1. **Settlement cost** over the past year due to misleading or deceptive advertising.
2. **Product complexity**, which measures how difficult it is for clients or advisers to understand the product being sold.
3. **Compensation level** of advisers.

Membership Functions



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Inference Rules

1. If (product complexity is not low or compensation level is very high) and settlement cost is not low, then misconduct risk is high.
2. If (product complexity is high or settlement cost is high) and compensation level is high, then misconduct risk is high.
3. If (product complexity is not high and settlement cost is not high) and compensation level is not high, then misconduct risk is medium.

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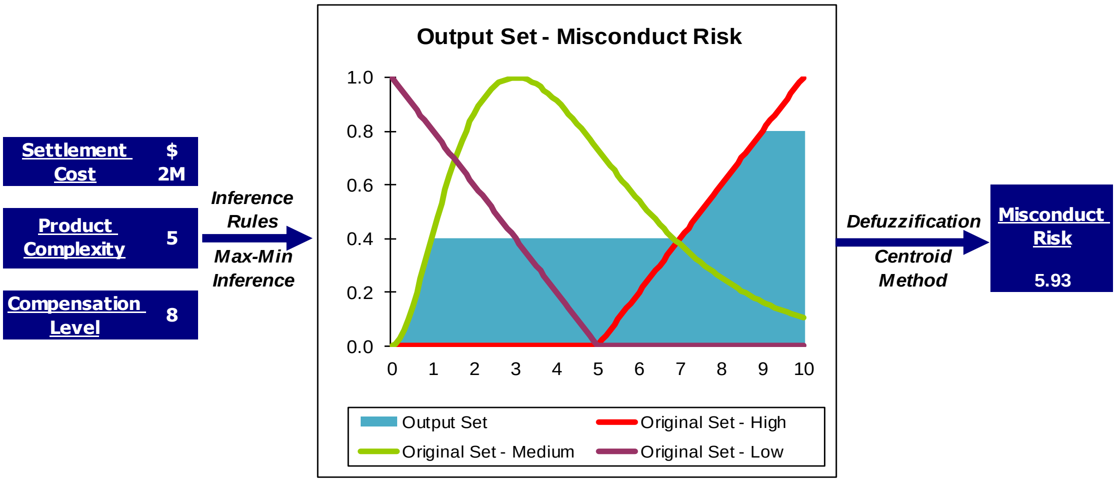
An Example



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Misconduct Risk Monitoring

Product	Misconduct Risk Level (1)	New Business Volume ($M) (2)	Misconduct Risk Exposure = (1) × (2)	Rank
A	8.3	2	16.6	4
B	7.6	13	98.8	1
C	5.9	5	29.5	3
D	5.1	7	35.7	2
E	3.5	3	10.5	5

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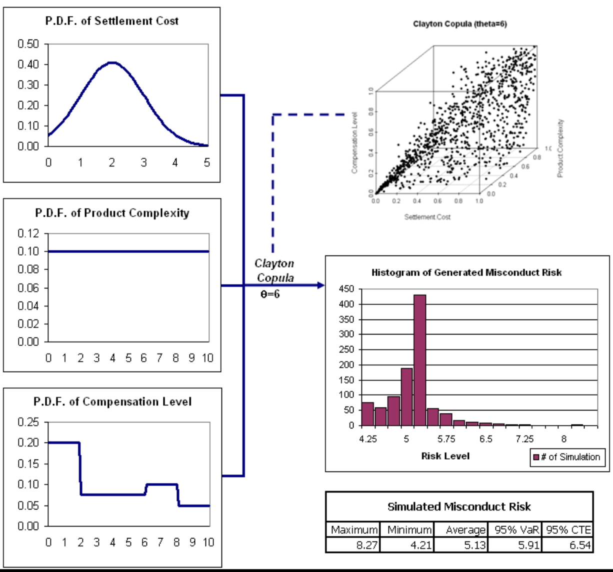
Output Variable Simulation



Simulated Misconduct Risk				
Maximum	Minimum	Average	95% VaR	95% CTE
8.27	4.21	5.13	5.91	6.54

Marginal Distribution + Correlation/ Copula

Settlement Cost

Product Complexity

Compensation Level

= Joint Distribution

Simulation

Misconduct Risk

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Risk Assessment Framework Based on Fuzzy Logic

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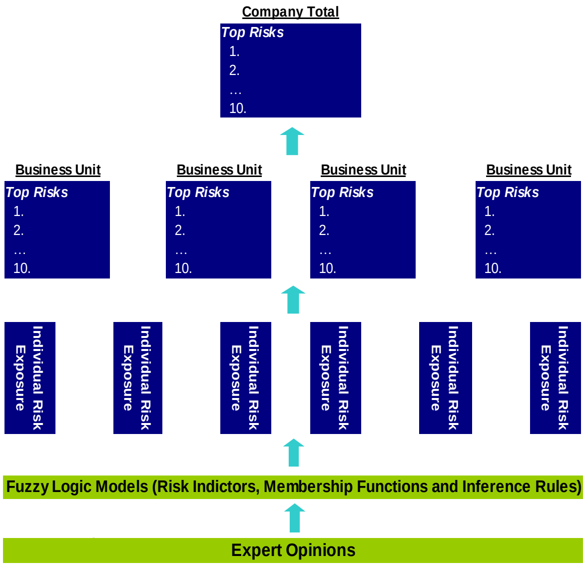
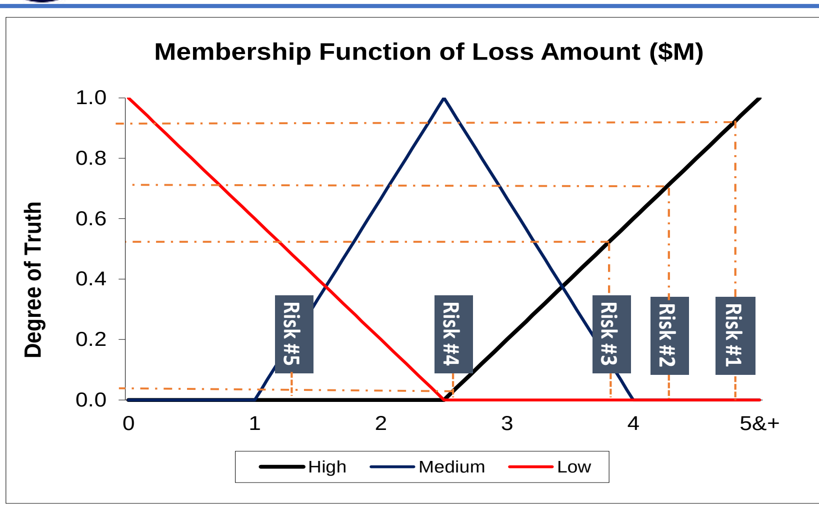
Risk Assessment Hierarchy Structure



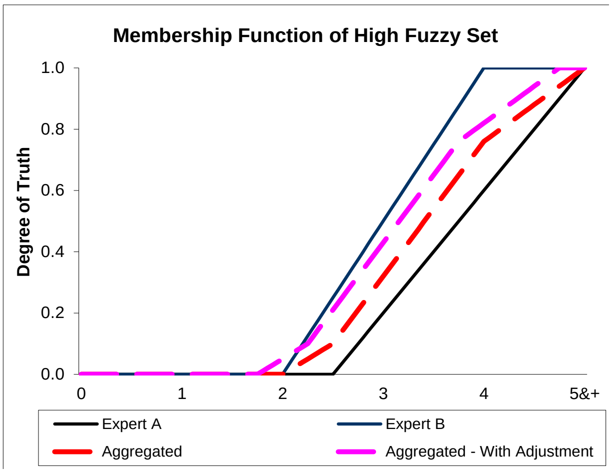
Risk Measure:
Estimated amount of loss under extreme events.

Risk Ranking Based on Loss Amount



Aggregation of Opinions

Adjust Membership Functions and Inference Rules



Expert A and Expert B have different membership functions and inference rules for the high fuzzy set.

Expert A: If X is high, then Y is high.
Expert B: If X is not low (medium or high), then Y is high.

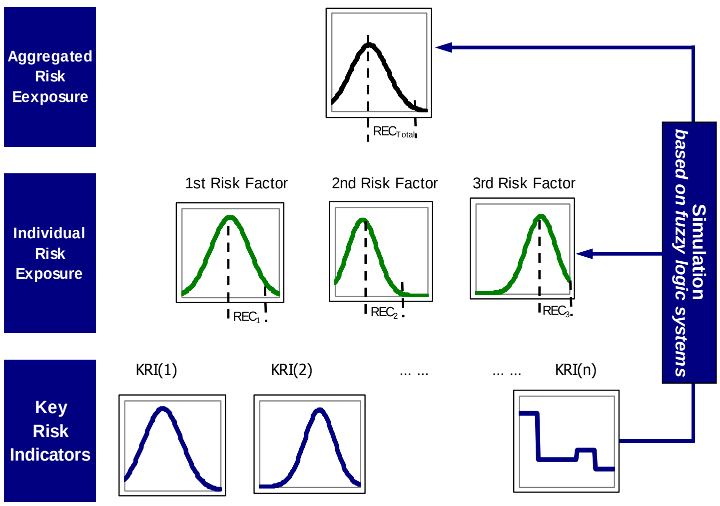
Step 1: Aggregate the membership functions.
Step 2: Adjust the aggregated membership function to consolidate the two inference rules into one.

Aggregation of Opinions

Weighted Average of the Results

1. Each expert may have his/her own fuzzy logic model with unique membership functions and inference rules.
2. The aggregated risk assessment result is simply the weighted average of the results generated from the different individual models.
3. A specific case of the second approach is to assign an equal weight to all opinions.

Required Economic Capital



Notations

KRI: Key risk indicators

REC_{Total}: Aggregated required economic capital

REC_i: Required economic capital for risk factor i

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Key Considerations

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Expert Opinions: Collection and Analysis

Main information source of a fuzzy logic system

1. Request and collection.
    - Key factors that may cause any risk event.
    - The value of each factor for existing business.
    - Any known cause-and-effect relationship.
    - Any risk measures that could be used and any relationship with other risk types.
2. Aggregation and analysis.
3. Feedback.
4. Data collection and risk monitoring.

Membership Function Selection

1. Ask the subject-matter experts to provide inputs.

   "any score greater than 1.5 is not low"

   "any score less than 0.5 is absolutely low"

$$\mu^{Low}(x) = \begin{cases} 1 & x \le 0.5 \\ (1.5-x)/1 & 0.5 < x \le 1.5 \\ 0 & x > 1.5 \end{cases}$$

2. May be partially calibrated to experience data.
3. May be fully calibrated to experience data.

The Role of Experience Data

1. Experience data collected may not be statistically credible.
2. The fuzzy logic model does not change solely based on experience data.
3. It can enhance our knowledge of the risks and improve the accuracy of the fuzzy logic model.
4. It helps adjust the membership function, the weight on experts' opinions, and their understanding.

Fuzzy Logic System Review

New risks

New understanding

New strategy

Linkage to Decision-Making

Risk identification and ranking.

Individual risk: The major contributors to the risk exposure may be identified by the fuzzy logic model.

Business unit: The top risks can be identified by the fuzzy logic system.

Overall company: It provides a more holistic view of the company's risks when planning its future.

Recap

Recap

- Fuzzy logic models can be applied to assess risks for which there is insufficient data and incomplete knowledge.
- Fuzzy logic models is a complement to probability models.
- Using a fuzzy logic system, it is possible to consistently analyze multiple risks that are not well understood.
- Inference rules in a fuzzy logic model may help identify the cause of a certain risk and design efficient and effective mitigation plans.
- The systems keep risk managers and subject matter experts free from the inference part for many risks.

Q&A

Disclaimer

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