Hierarchical Compartmental Models for Loss Reserving

Jake Morris

19 September 2016

Liberty Specialty Markets

Agenda

Overview

– Motivations

Methodology

– Single accident year

Case study

– Multiple accident years

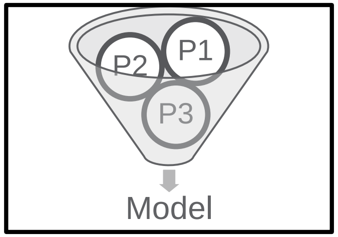
Bayesian implementation

Conclusions

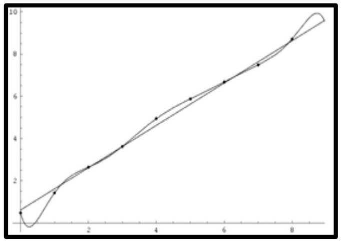
Overview

Motivations

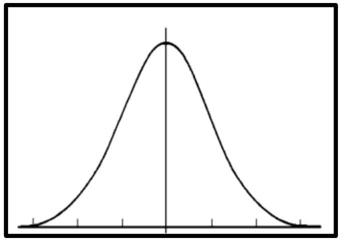
1. **Interpretability & Extensibility**
   – meaningful parameters
   – option to capture specific process features



2. **Parsimony**
   – extract signal from noise
   – description of individual cohort vs. average



3. **Quantification of reserve uncertainty**
   – incorporate multiple information sources
   – isolate drivers of uncertainty



Overview

Features

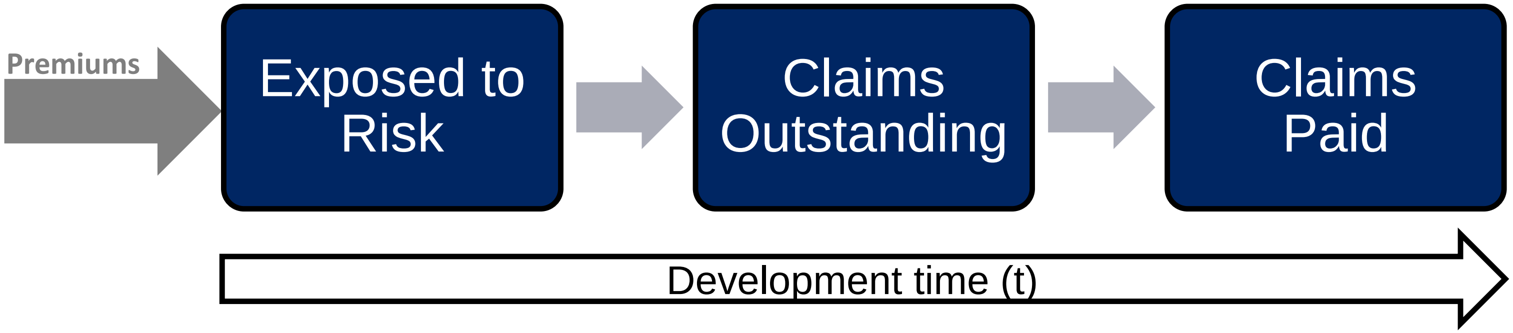
- Intuitive parameters including **case reserve robustness measure**
- Provides coherent measure of **reserve uncertainty**
- Supports **negative development**
- Can capture **calendar effects**
- **Independent** of DFM / BF
- Incorporates **judgement**

Models the claims generation *process*

Methodology

Compartmental reserving model

Structural model



*ODEs: a collection of simultaneous Ordinary Differential Equations

Methodology

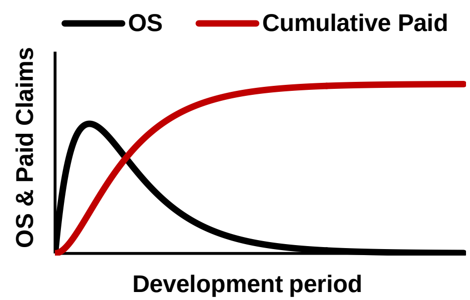
Compartmental reserving model

Structural model

- Cash flows between compartments governed by ODEs*

- Fit to paid <u>and</u> outstanding triangles
  - Simultaneously
  - Explicitly estimating tails

Supports negative development

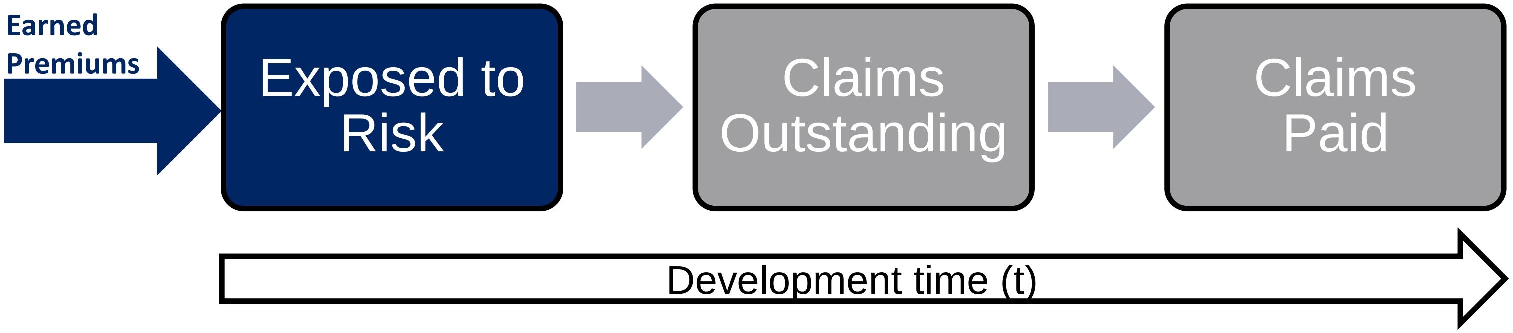


*ODEs: a collection of simultaneous Ordinary Differential Equations

Methodology

Parameters

Parameters have natural interpretations

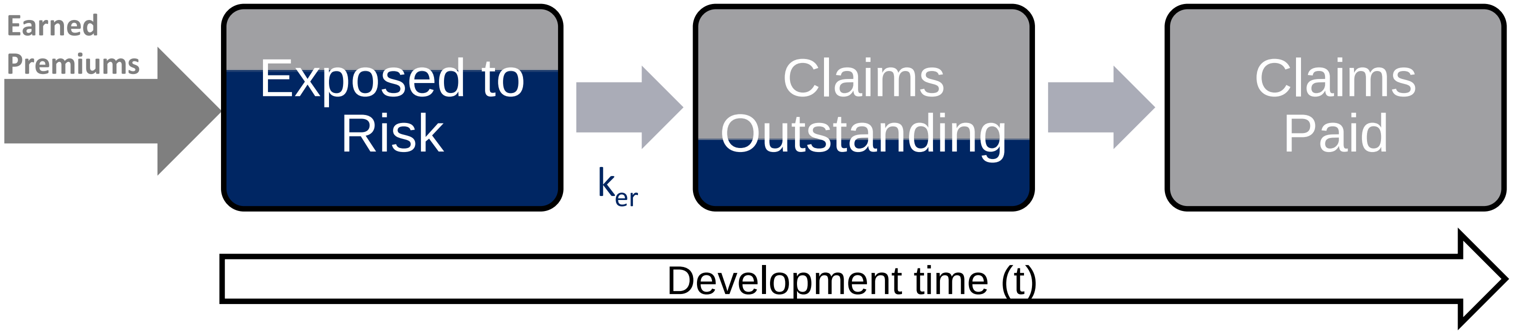


Base model parameters for a single accident year

Methodology

Parameters

Parameters have natural interpretations



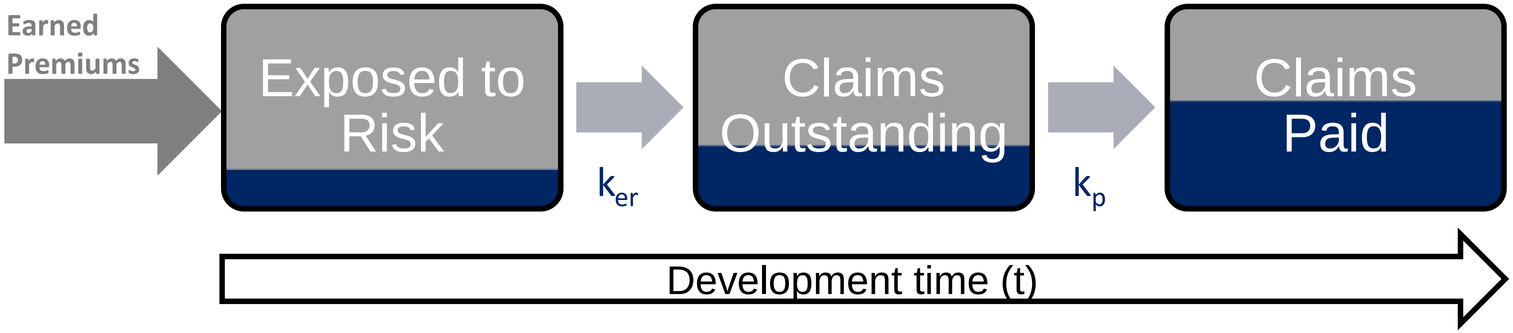
Rate of earning + reporting (“$\mathbf{k_{er}}$”)

Base model parameters for a single accident year

Methodology

Parameters

Parameters have natural interpretations



Rate of earning + reporting (“$\mathbf{k_{er}}$”)

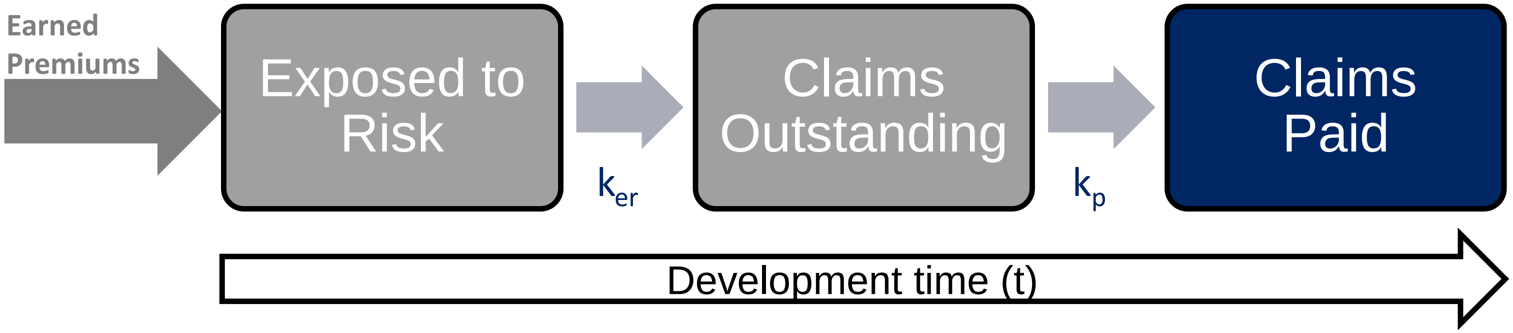
Rate of payment (“$\mathbf{k_p}$”)

Base model parameters for a single accident year

Methodology

Parameters

Parameters have natural interpretations



Rate of earning + reporting (“$\mathbf{k_{er}}$”)

Rate of payment (“$\mathbf{k_p}$”)

Base model parameters for a single accident year

Methodology

Parameters

Parameters have natural interpretations

Reported loss ratio ("**RLR**")

Rate of earning + reporting ("$\mathbf{k_{er}}$")

Rate of payment ("$\mathbf{k_p}$")

Base model parameters for a single accident year

Methodology

Parameters

Parameters have natural interpretations

Reported loss ratio ("**RLR**")

Rate of earning + reporting ("$\mathbf{k_{er}}$")

Reserve robustness factor ("**RRF**")

Rate of payment ("$\mathbf{k_p}$")

Base model parameters for a single accident year

Methodology

Parameters

Parameters have natural interpretations

Reported loss ratio ("**RLR**")

Rate of earning + reporting ("$\mathbf{k_{er}}$")

Reserve robustness factor ("**RRF**")

Rate of payment ("$\mathbf{k_p}$")

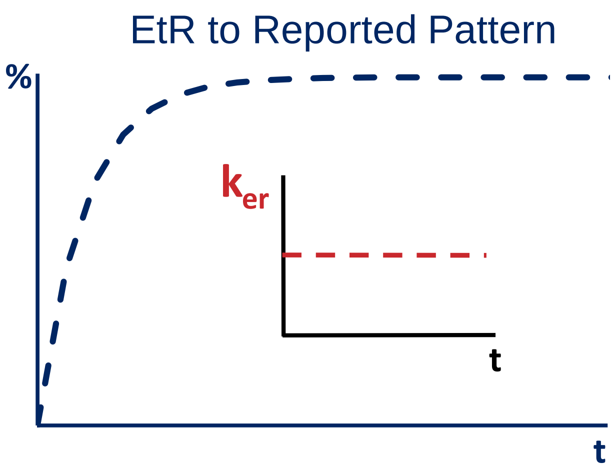
$$\text{ULR} = \text{RLR} \cdot \text{RRF}$$

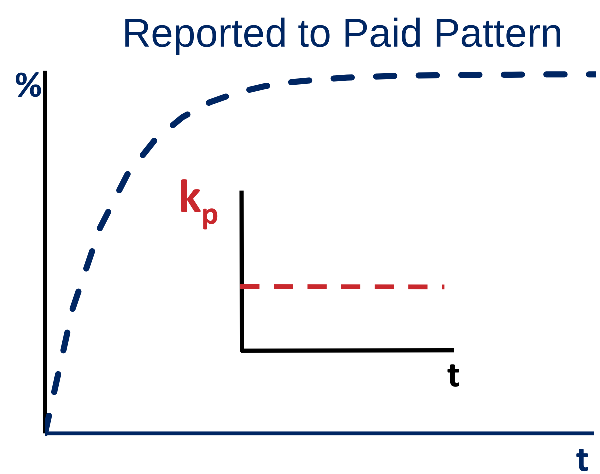
Base model parameters for a single accident year

Methodology

Rates → Patterns

Pattern % = $1 - e^{-rate*t}$



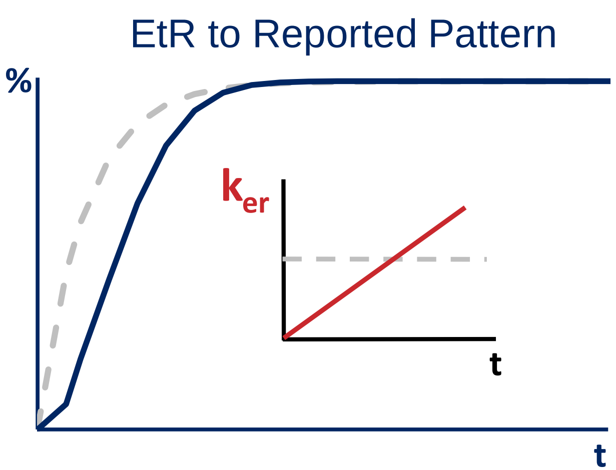


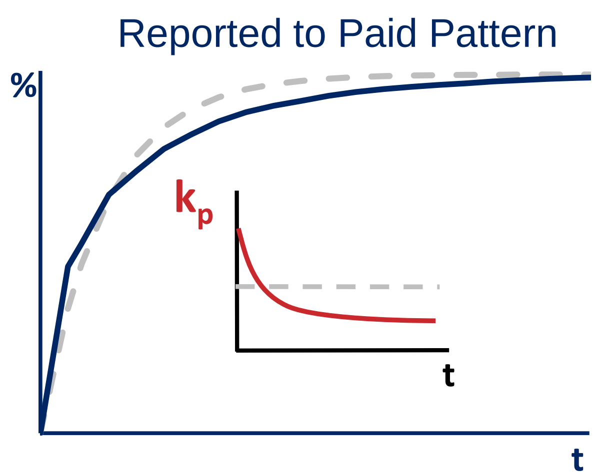
t = development time

Methodology

Rates → Patterns

Pattern % = $1 - e^{-\int rate(t)dt}$



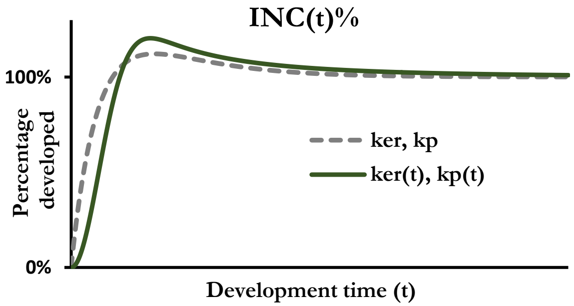


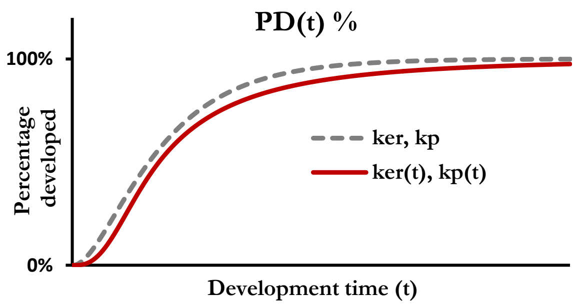
t = development time

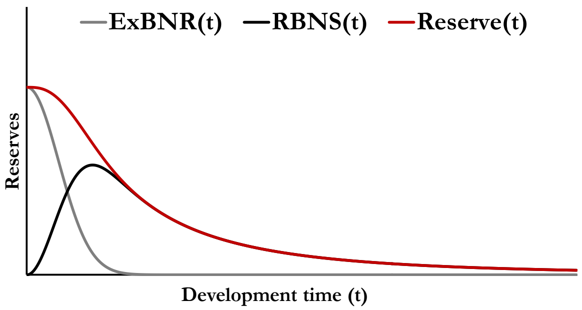
Illustration spreadsheet

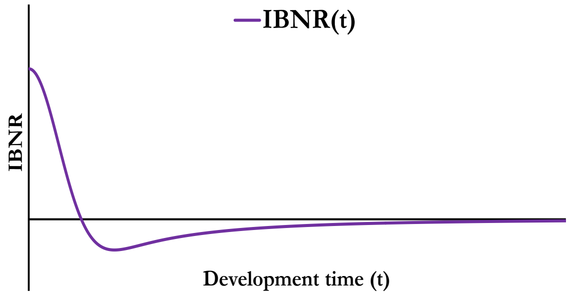
Discretized compartmental model

Premium	RLR	ker	RRF	kp	ULR
100	1.2	1.5	0.7	0.75	84.00%





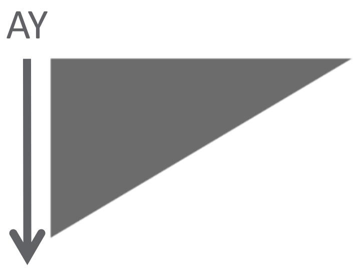




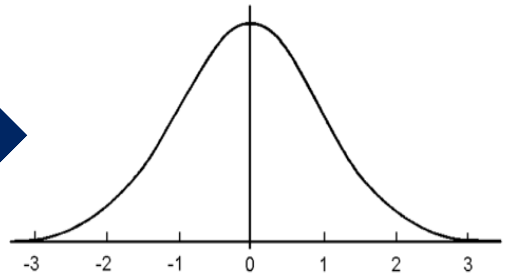
Multiple accident years

Hierarchical ("mixed-effects") models

Hierarchical compartmental models



Parameters a *mixture* of those varying across AY and those not*



AY	RLR	k_{er}	RRF	k_p
1	RLR_1	k_{er}	RRF_1	k_p
2	RLR_2		RRF_2	
…	…		…	
N	RLR_N		RRF_N	

Only estimate mean and s.d. of the variable parameters

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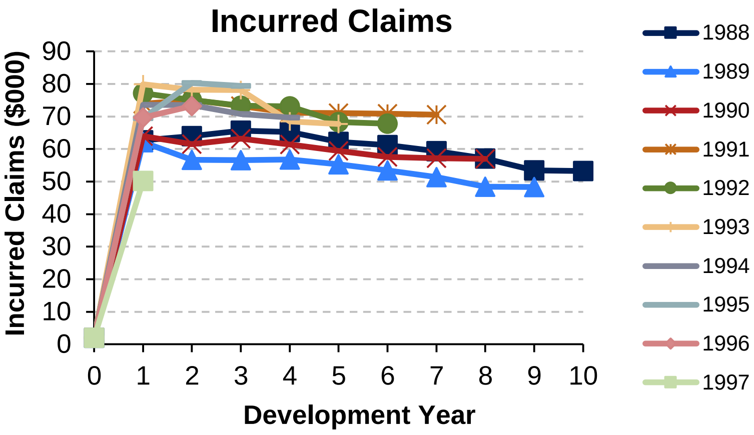
*Also known as a mixture of *random effects* and *fixed effects*

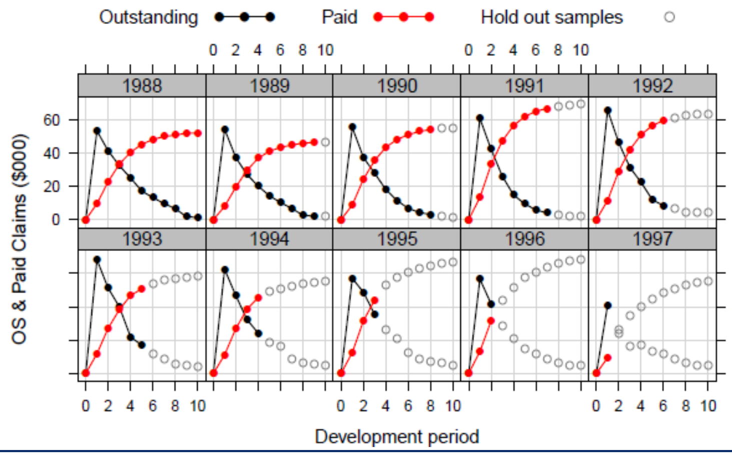
Case study

Data & Objectives

- **Workers' Comp Schedule P data**
  - Accident year cohorts (1988 – 1997)
  - Earned premiums
  - Paid and incurred claims development



- **Objectives**
  - Fit **frequentist** compartmental model
  - Refine model and interpret parameters
  - Compare projections to lower triangles

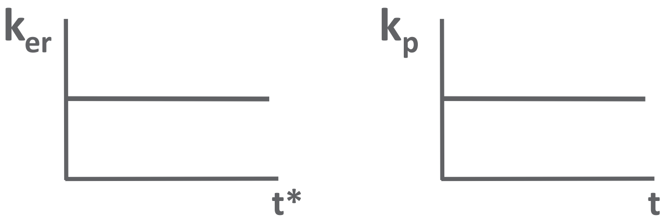


Case study

Model 1

Base model:

Constant rates



2 random effects

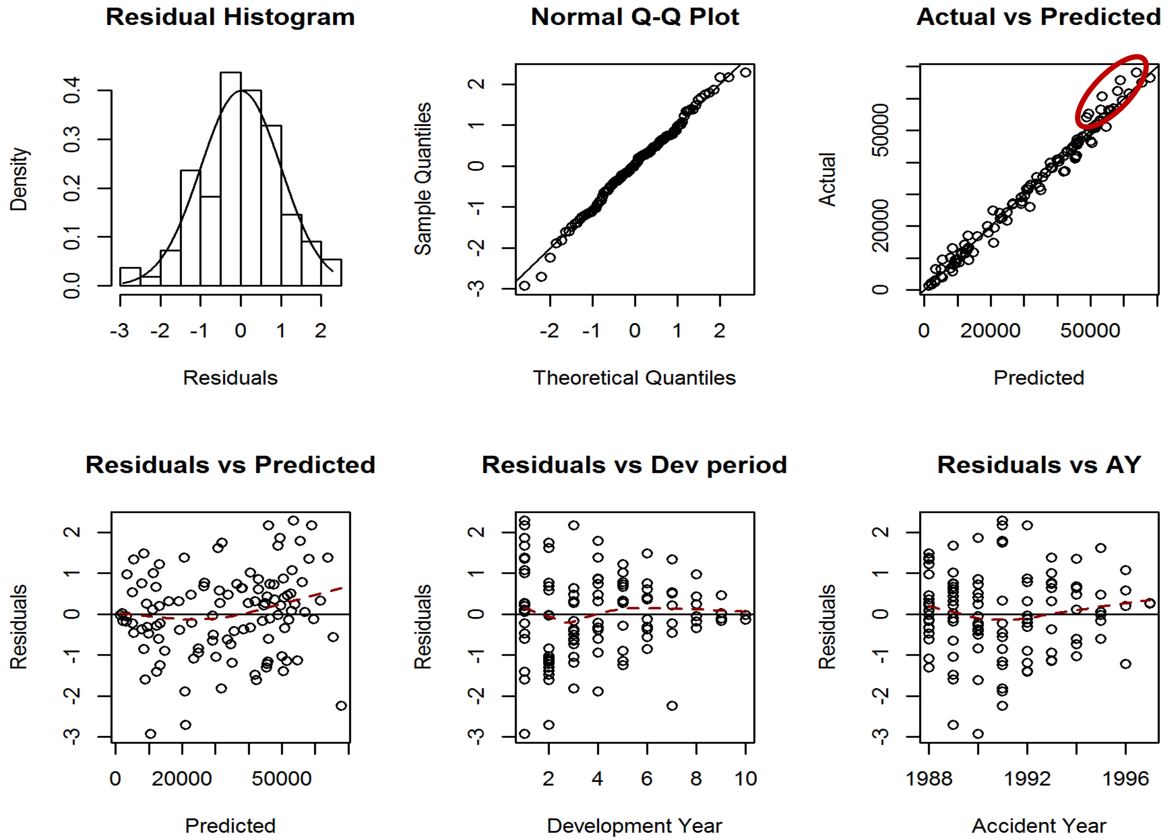
AY	RLR	k_{er}	RRF	k_p
1988	RLR_1	k_{er}	RRF_1	k_p
1989	RLR_2		RRF_2	
…	…		…	
1997	RLR_{10}		RRF_{10}	

Judgementally select parameter starting values

*Development time

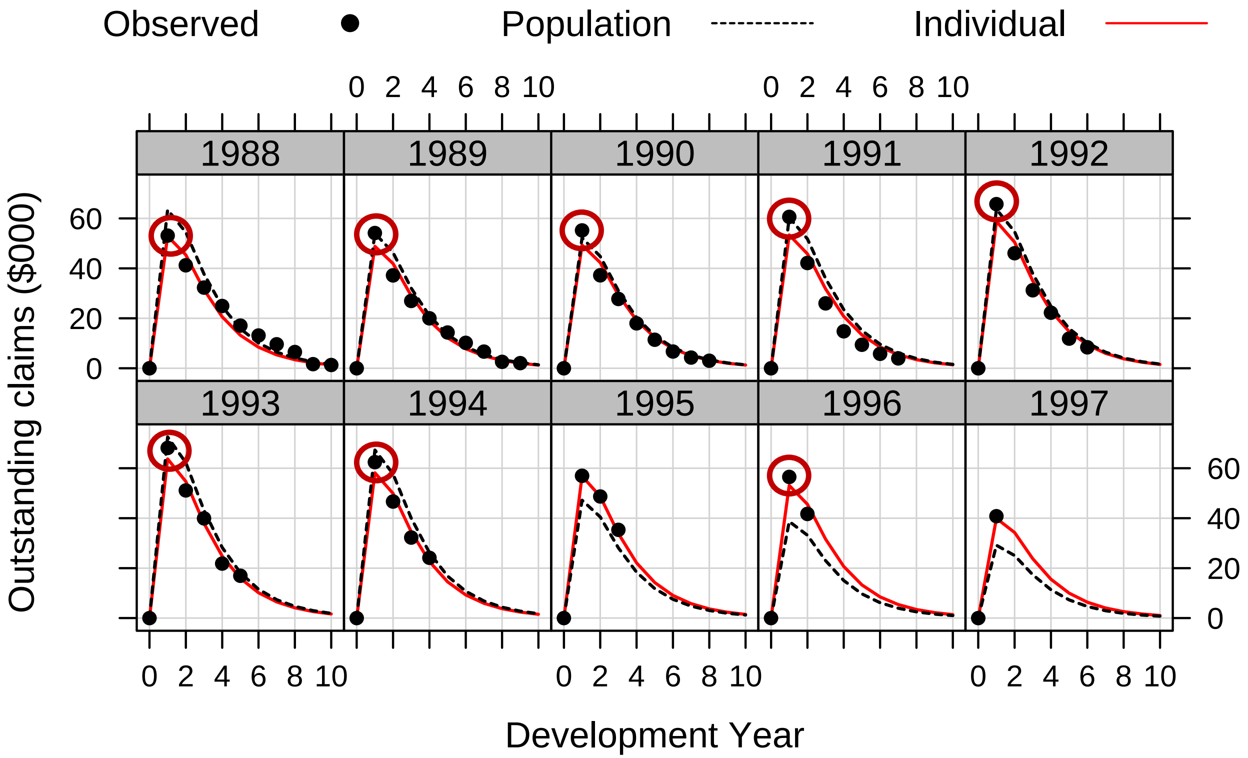
Case study

Model 1 Diagnostics



Case study

Model 1 Diagnostics

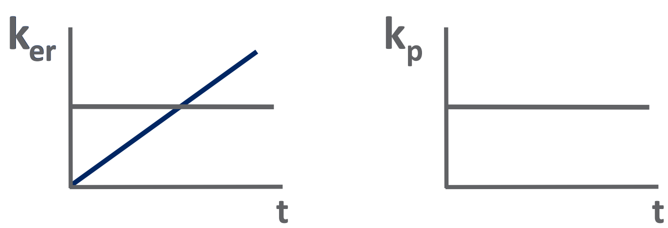


Case study

Model 2

Base model (extended):

~~Constant rates~~



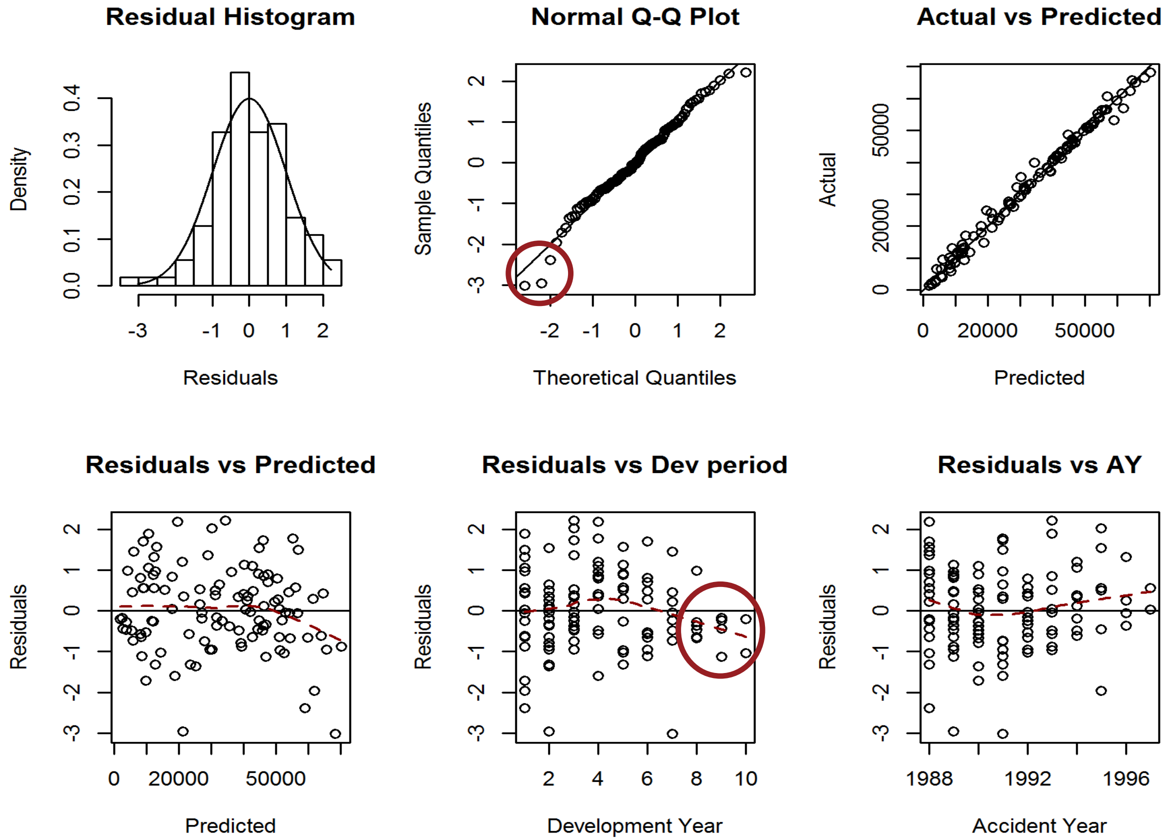
2 random effects

AY	RLR	k_{er}	RRF	k_p
1988	RLR_1	k_{er}	RRF_1	k_p
1989	RLR_2		RRF_2	
…	…		…	
1997	RLR_{10}		RRF_{10}	

Fit model and explore diagnostics…

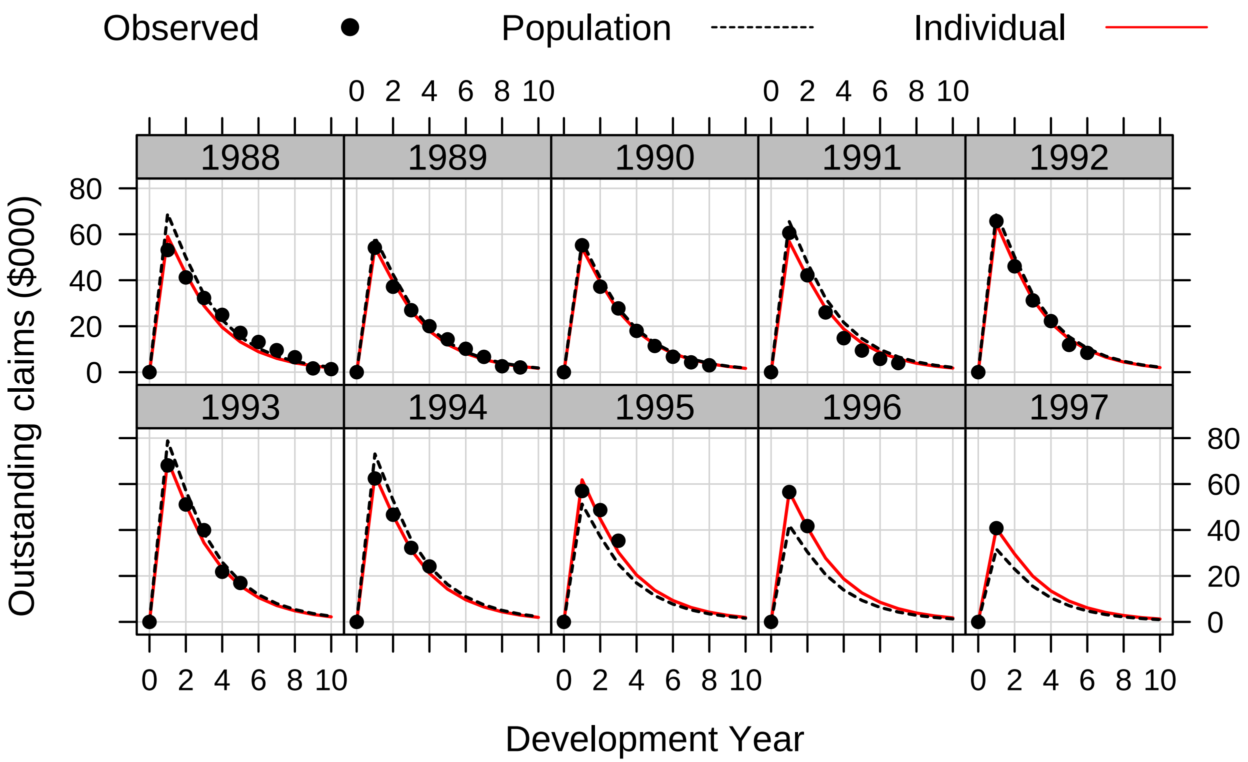
Case study

Model 2 Diagnostics



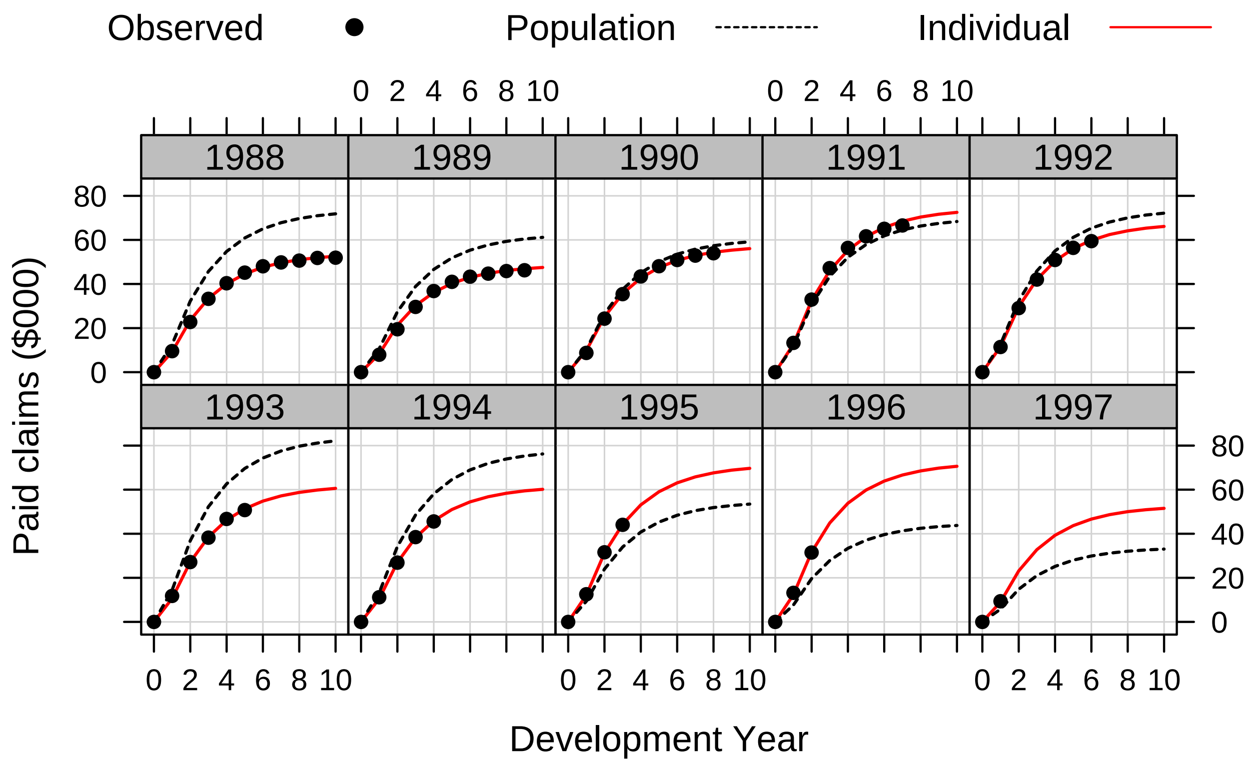
Case study

Model 2 Diagnostics



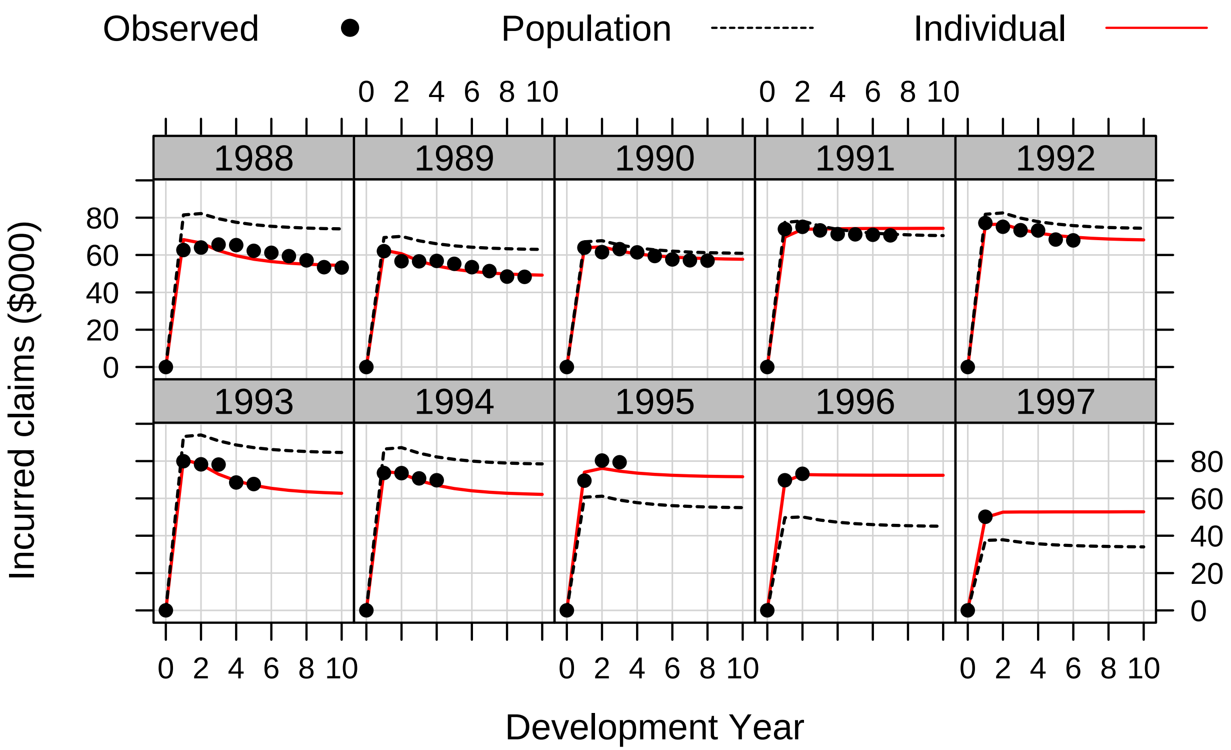
Case study

Model 2 Diagnostics



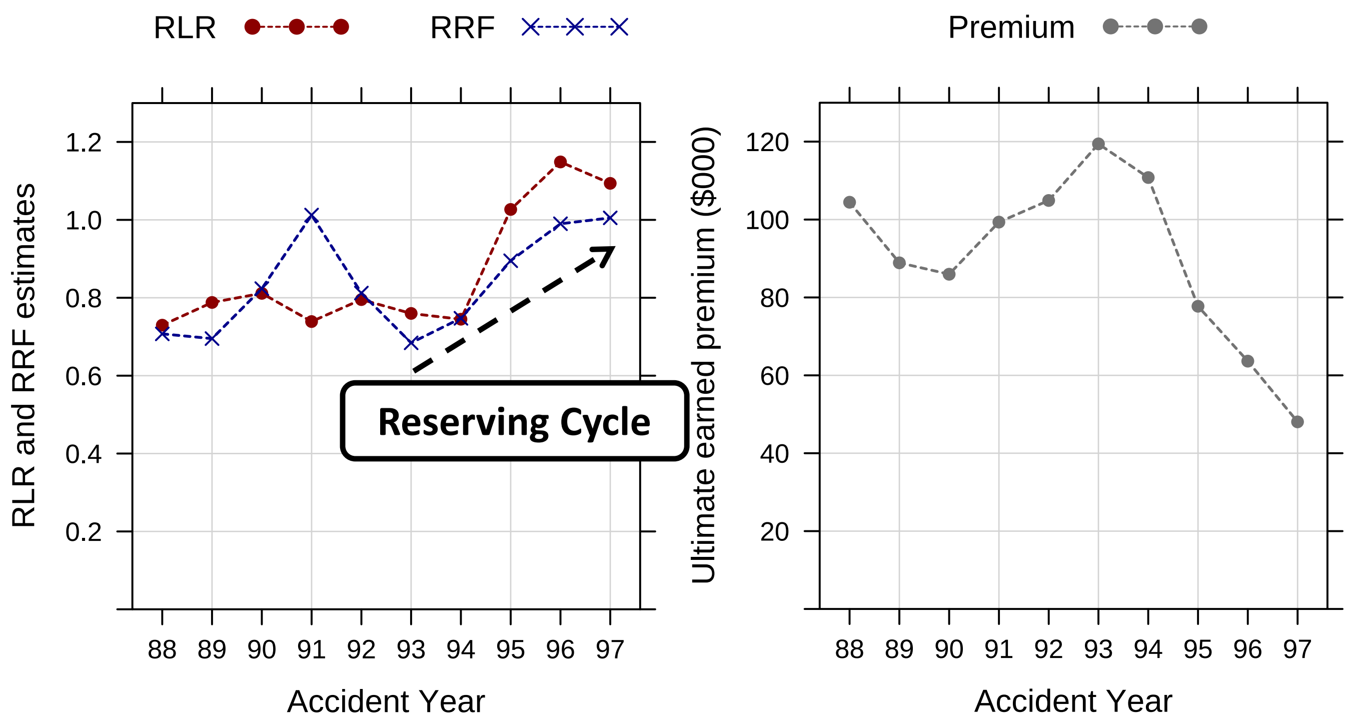
Case study

Model 2 Diagnostics



Case study

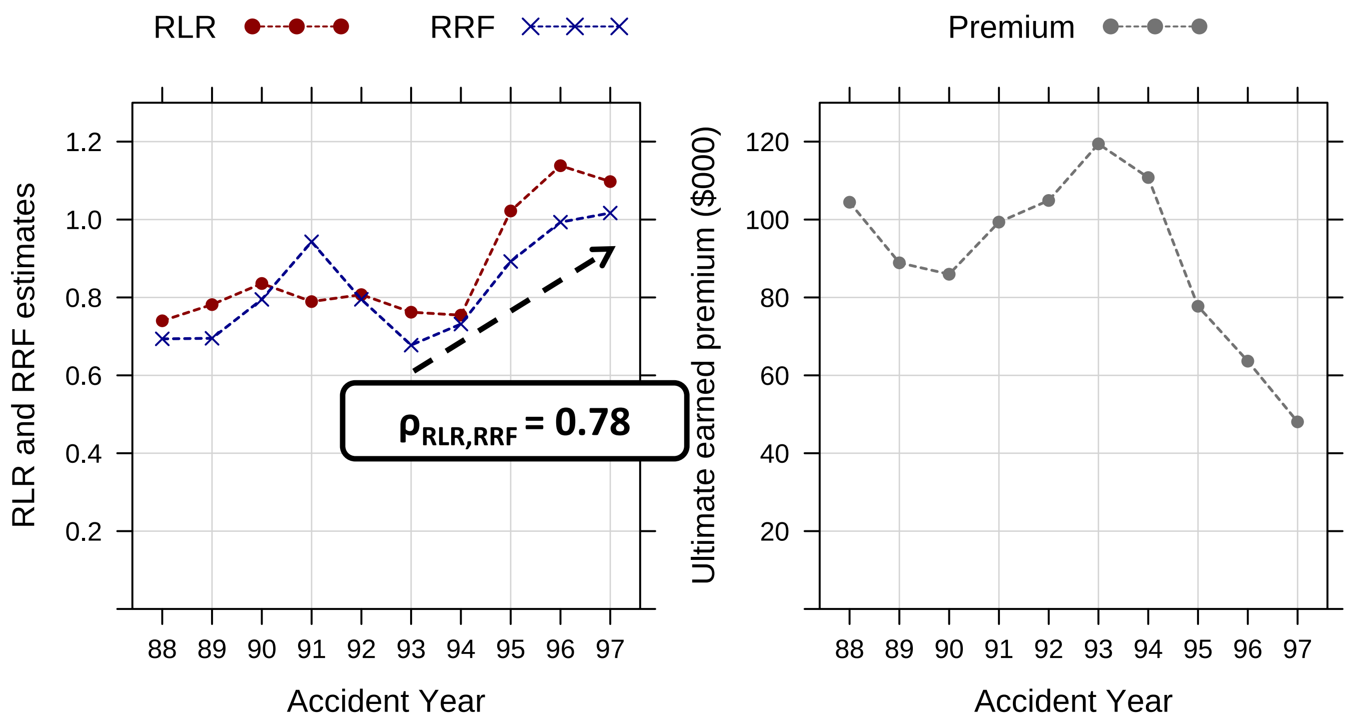
Model 2 Parameter Estimates



Update model to estimate correlation

Case study

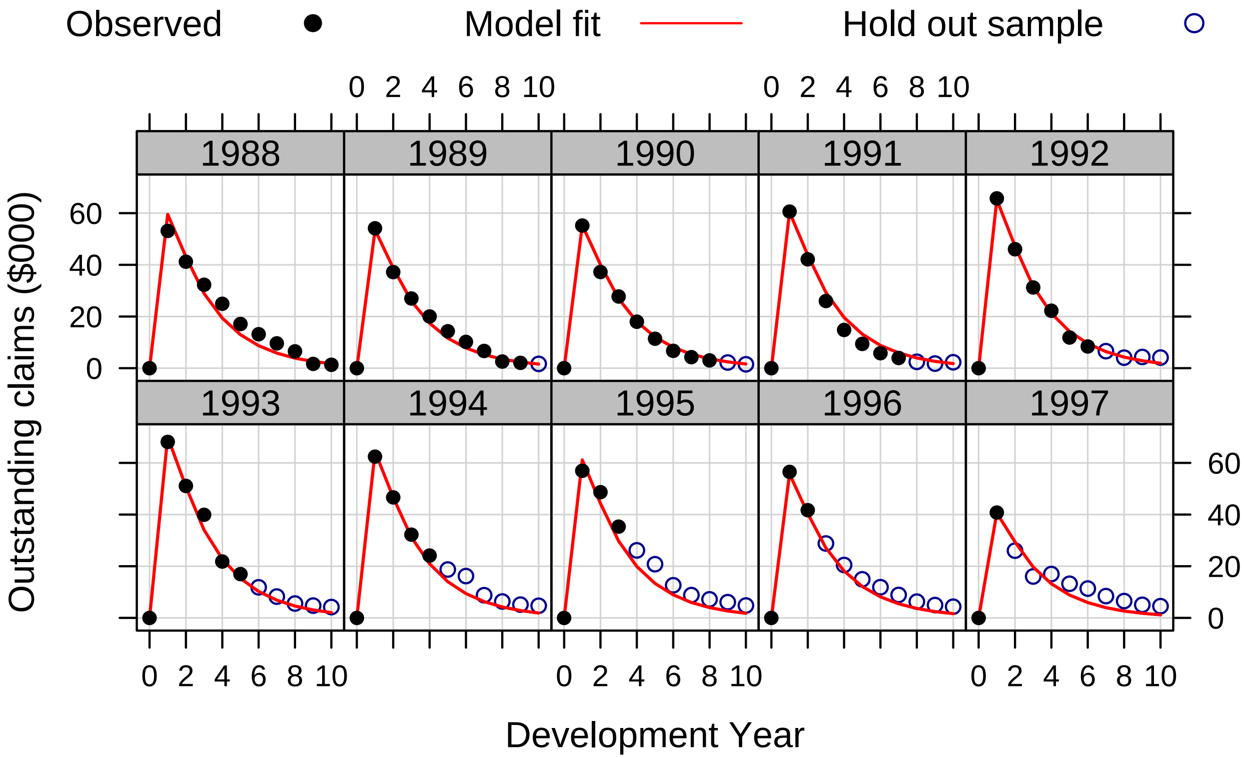
Model 3 Parameter Estimates



Compare model extrapolations to hold out samples…

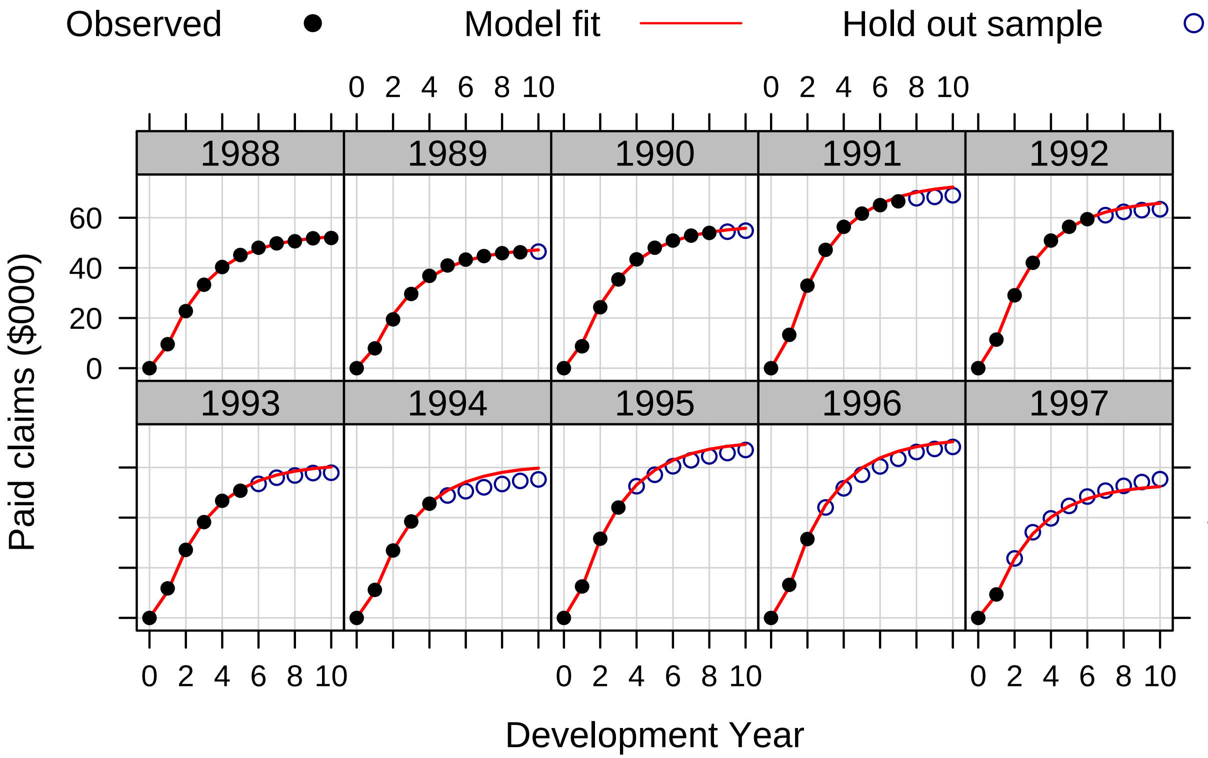
Case study

Model 3 Extrapolations



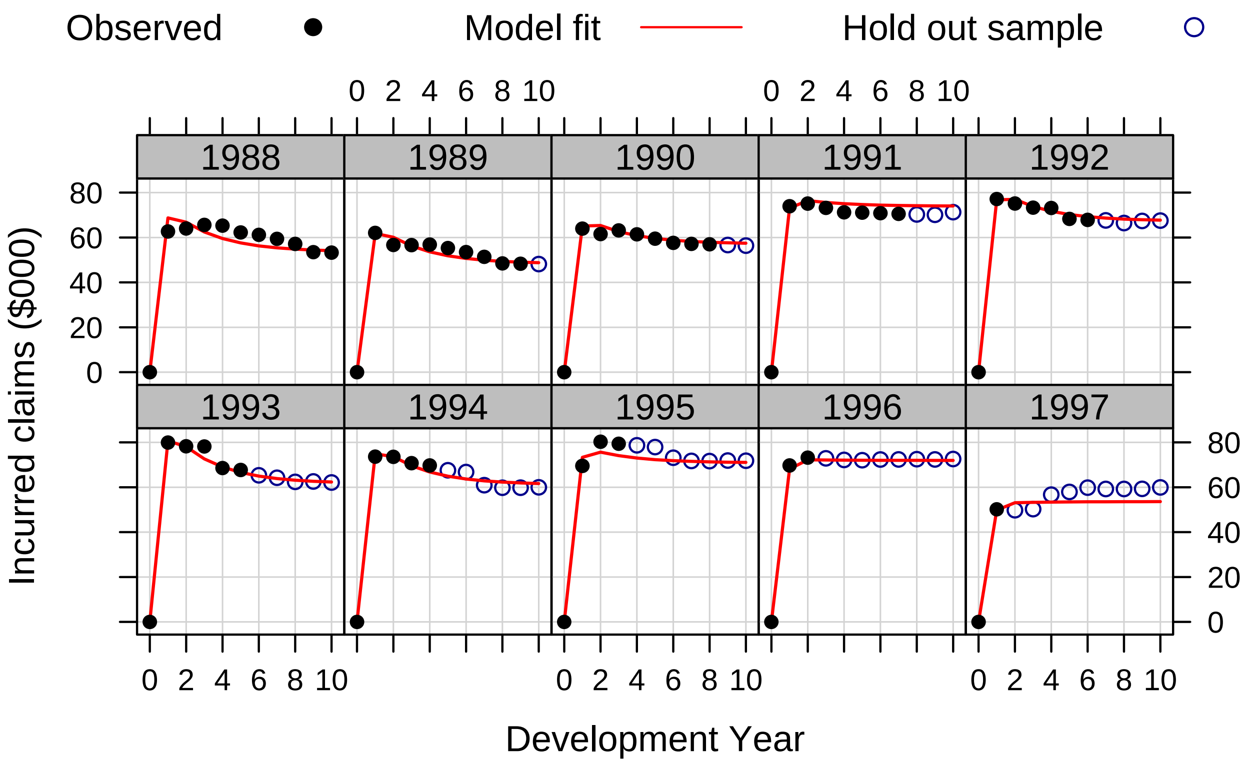
Case study

Model 3 Extrapolations



Case study

Model 3 Extrapolations



Bayesian implementation

Why bother?

Objective:

*"**Given** any value (estimate of future payments) and **our current state of knowledge**, what is the probability that final payments will be no larger than the given value?"*

- Casualty Actuarial Society (2004)
Working Party on Quantifying Variability in Reserve Estimates

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Bayes' theorem:

$$p(\theta \mid y) \propto L(\theta; y)\, p(\theta)$$

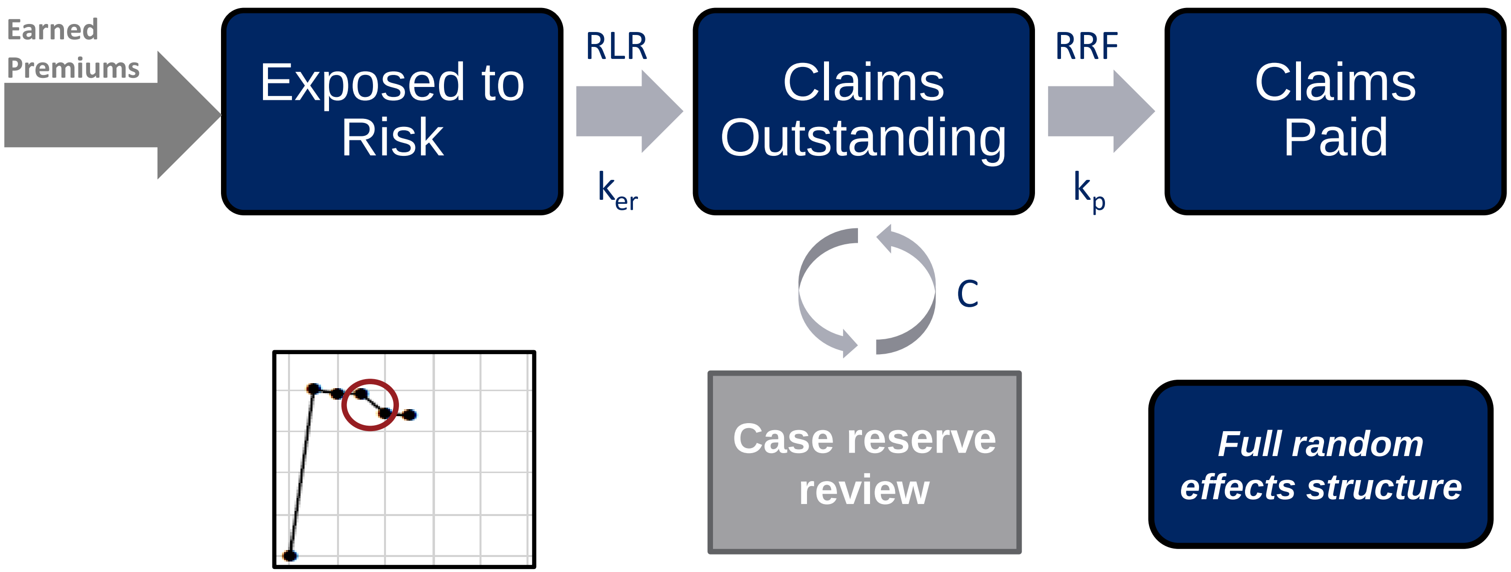
Posterior $\propto$ Likelihood x Prior

$$\boxed{p(ULR \mid incurred) \propto L(ULR; incurred)\, p(ULR)}$$

Bayesian implementation

With added complexity…

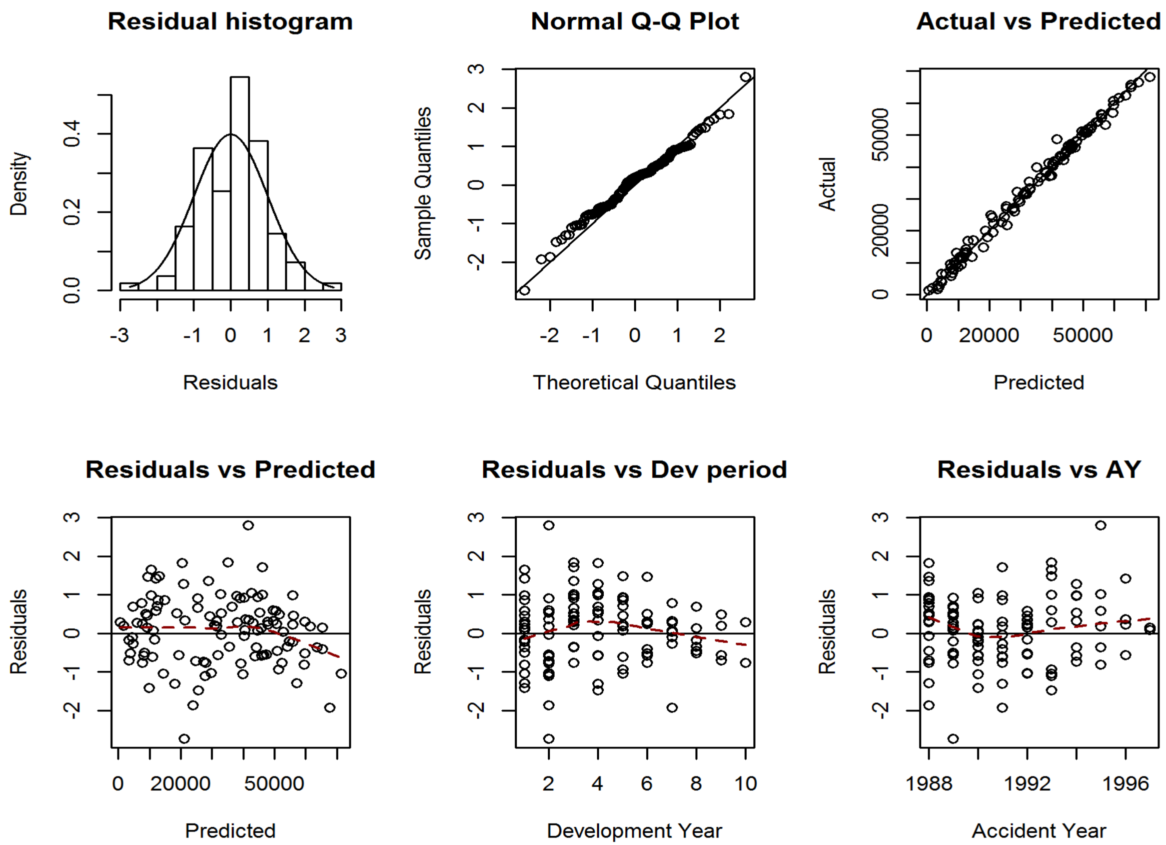
Explicitly model calendar shock (& autocorrelation)**:**



Estimate case reserve % increases/decreases

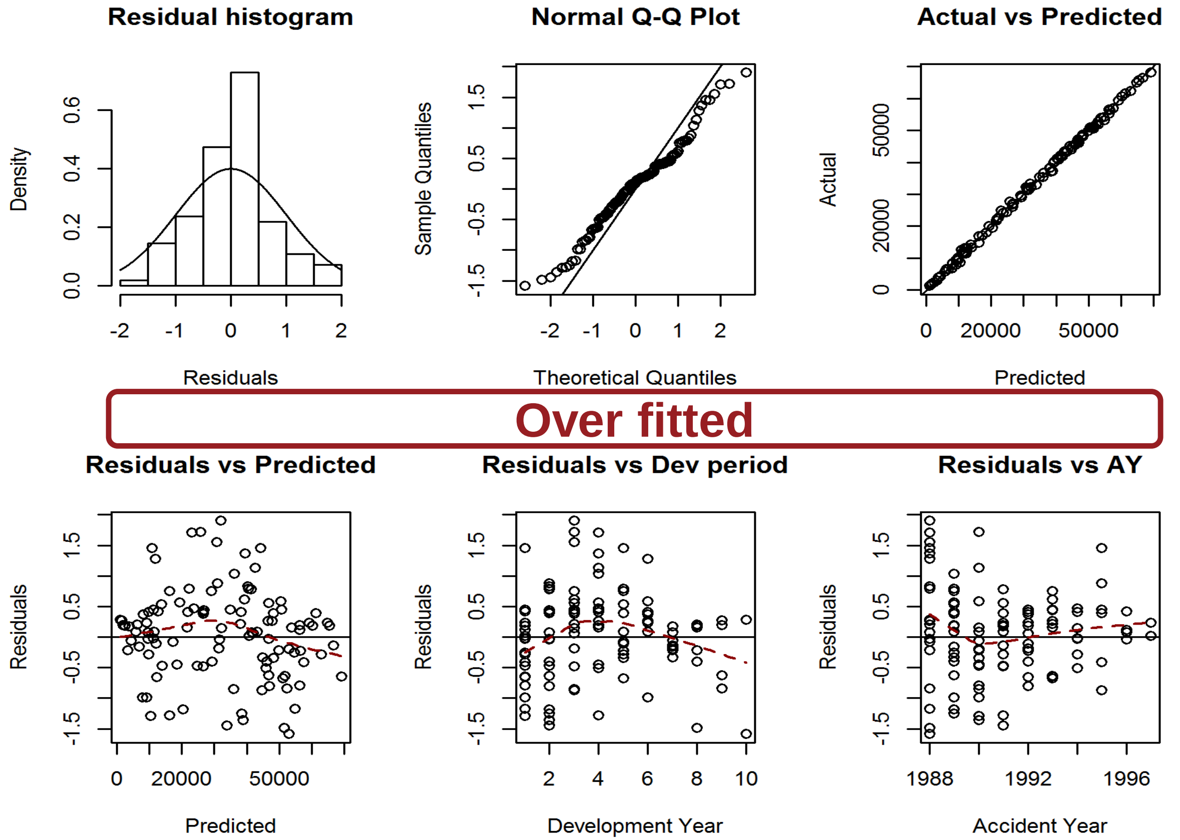
Bayesian implementation

*Diagnostics: **frequentist equivalent** vs. calendar shock model*



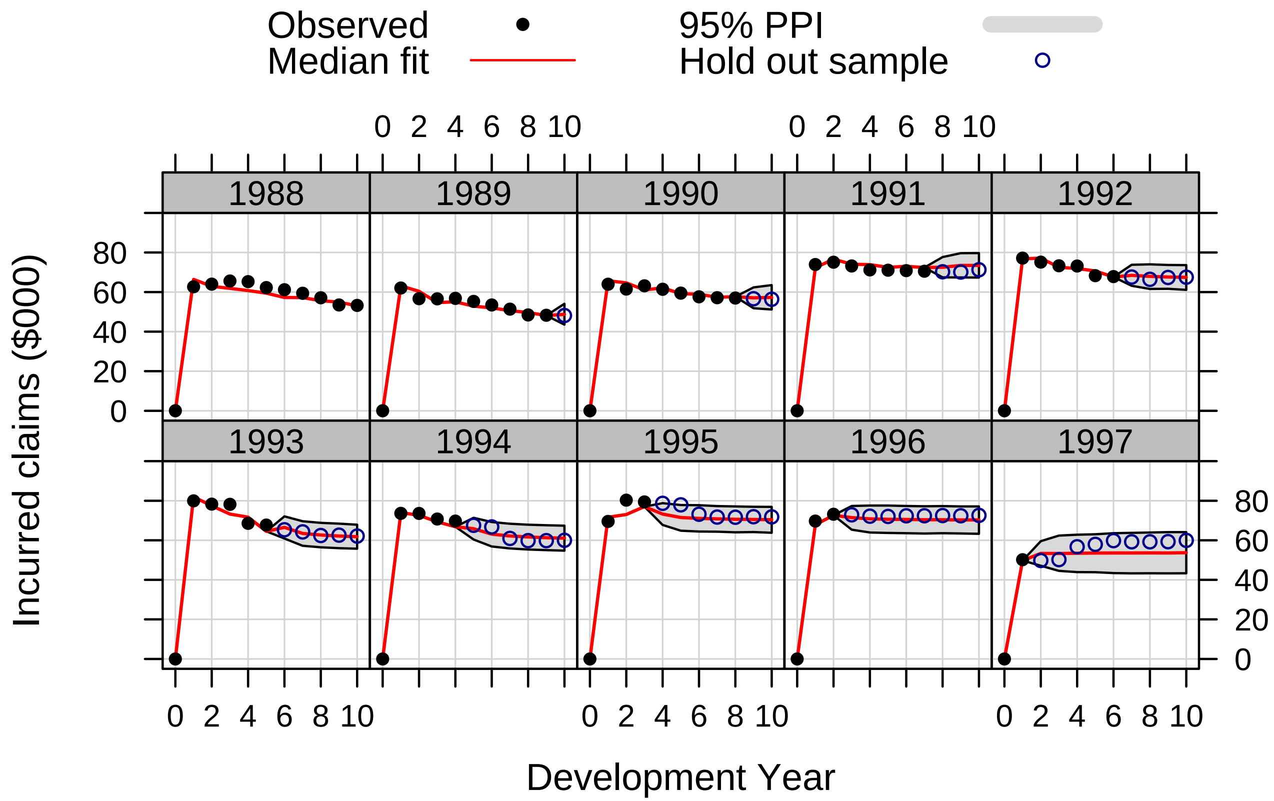
Bayesian implementation

*Diagnostics: frequentist equivalent vs. **calendar shock model***



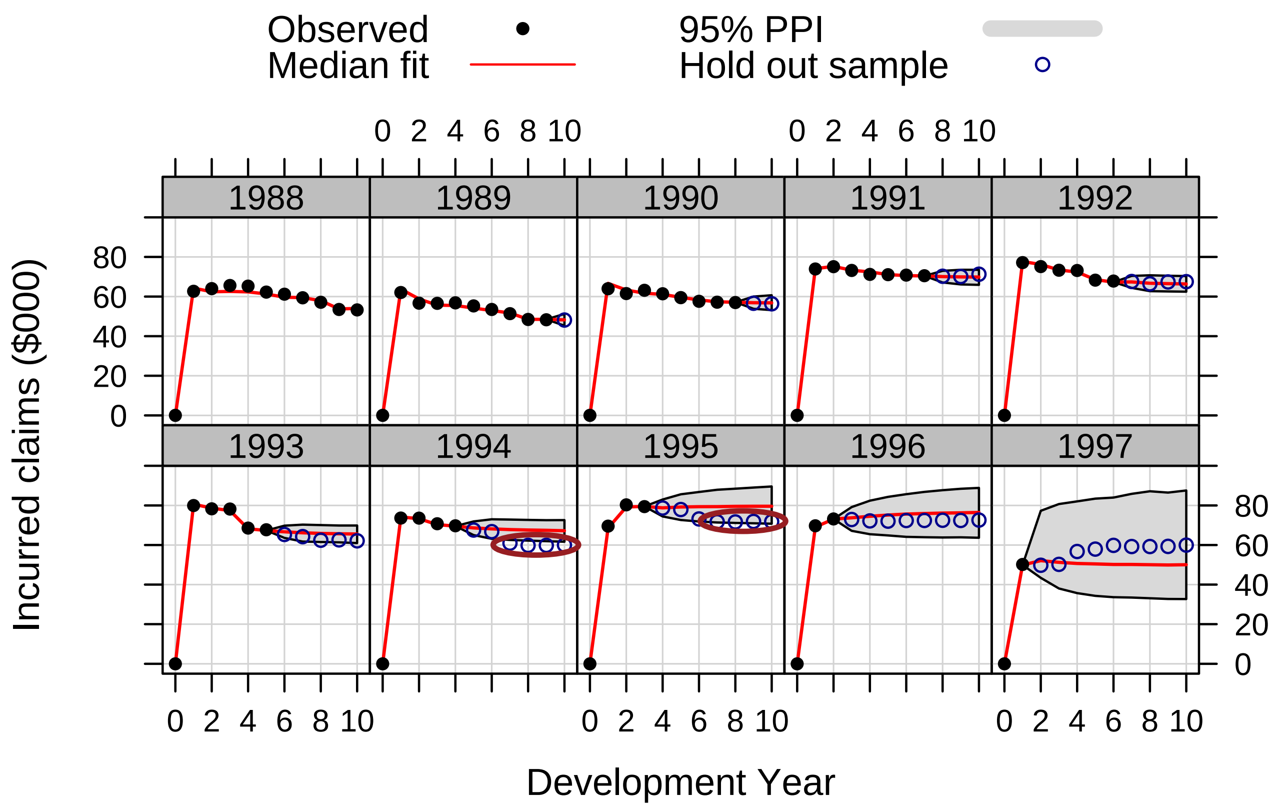
Bayesian implementation

*Diagnostics: **frequentist equivalent** vs. calendar shock model*



Bayesian implementation

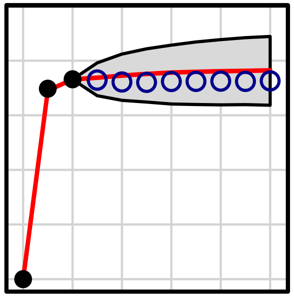
Diagnostics: frequentist equivalent vs. ***calendar shock model***



Conclusions

Hierarchical compartmental reserving

$$p(ULR \mid incurred) \propto L(ULR; incurred)\, p(ULR)$$



Conclusions

Hierarchical compartmental reserving

- **Strengths of compartmental reserving:**
  - Independent stochastic method — ***supports negative incurred development***
  - Meaningful parameters — ***including measure of reserve robustness***
  - Parsimonious yet extensible — ***can capture calendar effects***

- **Weaknesses of compartmental reserving:**
  - Model shape constraints with volatile data — ***try SDEs?***
  - Sensitivity to starting values / priors — ***strength!***
  - Learning curve — ***paper and materials…***

Try it out for yourself!

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The paper…

- **Full case study analysis**
  - Mathematics and assumptions
  - MCL and BCL comparisons
  - Data, R and OpenBUGS code
- **Reserve derivations**
  - ExBNR vs. RBNS
- **Non-steady-state exposure**
- **Patterns of development**
- **Parameter starting value algorithm**
- **SDE exploration**

Hierarchical Compartmental Models for Loss Reserving

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19 September 2016

Liberty Specialty Markets