AR-6
Practical Application of Development Trend Extrapolation for Triangle GLMs

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9/20/2016

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AR-6 – Development Trend Extrapolation

Overview.

- **Introduction** (5 minutes)
- **Q & A on review material*** (10 minutes)
- **Getting started with the GLM template** (10 minutes)
- **Example I – Reproducing 2d results with 2d+1 model** (10 minutes)
- **Example II – Mean reverting pattern vs long term average** (10 minutes)
- **Example III – Using offsets** (10 minutes)
- **Example IV – Choice of variance function** (10 minutes)
- **Example V – Stochastic tail factors** (10 minutes)

***It is assumed that participants have studied the separate deck of review slides <u>prior</u> to attending the session!**

AR-6 – Development Trend Extrapolation

Learning Objectives.

Participants will explore and gain familiarity with the following concepts

- **Reserve projection with 2d model** (extend triangle using fitted parameters)
- **Snag with 2d+1 model** (need future payment period parameters)
- **Extrapolating trend parameters** (make sure weights add up to one)
- **2d+1 model can reproduce 2d model** (constant payment period trend)
- **Modeling exogenous assumptions using weights** (e.g. mean reversion)
- **Modeling exogenous assumptions using offsets** (e.g. cyclical pattern)
- **Impact of variance function** (high dispersion factor may indicate poor fit)
- **Stochastic tail factors** (payment period extrapolation is a must, but the method can also be used to model development beyond triangle)

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Reserve projection with 2d model.

$$\eta_{ij} = \alpha_i + \beta_j,$$

where $\beta_2 = 0$.

$\alpha_1 + \beta_1$	α_1	$\alpha_1 + \beta_3$	$\alpha_1 + \beta_4$	$\alpha_1 + \beta_5$
$\alpha_2 + \beta_1$	α_2	$\alpha_2 + \beta_3$	$\alpha_2 + \beta_4$	
$\alpha_3 + \beta_1$	α_3	$\alpha_3 + \beta_3$		
$\alpha_4 + \beta_1$	α_4			
$\alpha_5 + \beta_1$				

Start with fitting a GLM to the triangle to get the parameters, …

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Reserve projection with 2d model.

$$\eta_{ij} = \alpha_i + \beta_j,$$

where $\beta_2 = 0.$

$\alpha_1 + \beta_1$	α_1	$\alpha_1 + \beta_3$	$\alpha_1 + \beta_4$	$\alpha_1 + \beta_5$
$\alpha_2 + \beta_1$	α_2	$\alpha_2 + \beta_3$	$\alpha_2 + \beta_4$	$\alpha_2 + \beta_5$
$\alpha_3 + \beta_1$	α_3	$\alpha_3 + \beta_3$	$\alpha_3 + \beta_4$	$\alpha_3 + \beta_5$
$\alpha_4 + \beta_1$	α_4	$\alpha_4 + \beta_3$	$\alpha_4 + \beta_4$	$\alpha_4 + \beta_5$
$\alpha_5 + \beta_1$	α_5	$\alpha_5 + \beta_3$	$\alpha_5 + \beta_4$	$\alpha_5 + \beta_5$

… and extend the triangle using the fitted parameters. The reserve is the sum of all expected values in the bottom half of the “squared” triangle.

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Snag with 2d+1 model.

$$\eta_{ij} = \alpha_i + \beta_j + \gamma_{i+j-1},$$

where $\alpha_1 = \beta_1 = \gamma_3 = 0.$

		β_3	$\beta_4 + \gamma_4$	$\beta_5 + \gamma_5$
	$\alpha_2 + \beta_2$	$\alpha_2 + \beta_3 + \gamma_4$	$\alpha_2 + \beta_4 + \gamma_5$	$\alpha_2 + \beta_5 + \gamma_6$
α_3	$\alpha_3 + \beta_2 + \gamma_4$	$\alpha_3 + \beta_3 + \gamma_5$	$\alpha_3 + \beta_4 + \gamma_6$	$\alpha_3 + \beta_5 + \gamma_7$
$\alpha_4 + \gamma_4$	$\alpha_4 + \beta_2 + \gamma_5$	$\alpha_4 + \beta_3 + \gamma_6$	$\alpha_4 + \beta_4 + \gamma_7$	$\alpha_4 + \beta_5 + \gamma_8$
$\alpha_5 + \gamma_5$	$\alpha_5 + \beta_2 + \gamma_6$	$\alpha_5 + \beta_3 + \gamma_7$	$\alpha_5 + \beta_4 + \gamma_8$	$\alpha_5 + \beta_5 + \gamma_9$

Want to extend triangle using fitted parameters as for 2d model …

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Snag with 2d+1 model.

$$\eta_{ij} = \alpha_i + \beta_j + \gamma_{i+j-1},$$

where $\alpha_1 = \beta_1 = \gamma_3 = 0.$

		β_3	$\beta_4 + \gamma_4$	$\beta_5 + \gamma_5$
	$\alpha_2 + \beta_2$	$\alpha_2 + \beta_3 + \gamma_4$	$\alpha_2 + \beta_4 + \gamma_5$	$\alpha_2 + \beta_5 + \cancel{\gamma_6}$
α_3	$\alpha_3 + \beta_2 + \gamma_4$	$\alpha_3 + \beta_3 + \gamma_5$	$\alpha_3 + \beta_4 + \cancel{\gamma_6}$	$\alpha_3 + \beta_5 + \cancel{\gamma_7}$
$\alpha_4 + \gamma_4$	$\alpha_4 + \beta_2 + \gamma_5$	$\alpha_4 + \beta_3 + \cancel{\gamma_6}$	$\alpha_4 + \beta_4 + \cancel{\gamma_7}$	$\alpha_4 + \beta_5 + \cancel{\gamma_8}$
$\alpha_5 + \gamma_5$	$\alpha_5 + \beta_2 + \cancel{\gamma_6}$	$\alpha_5 + \beta_3 + \cancel{\gamma_7}$	$\alpha_5 + \beta_4 + \cancel{\gamma_8}$	$\alpha_5 + \beta_5 + \cancel{\gamma_9}$

Want to extend triangle using fitted parameters as for 2d model …
… but we don't know the values of γ_6, γ_7, γ_8, and γ_9!

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Extrapolating trend parameters.

We can fix the "snag" using linear extrapolation with weights that add up to one:

$$\eta_{ij} = \alpha_i + \beta_j + \gamma_{i+j-1},$$

where $\alpha_1 = \beta_1 = \gamma_3 = 0.$

		β_3	$\beta_4 + \gamma_4$	$\beta_5 + \gamma_5$
	$\alpha_2 + \beta_2$	$\alpha_2 + \beta_3 + \gamma_4$	$\alpha_2 + \beta_4 + \gamma_5$	$\alpha_2 + \beta_5 + \gamma_6$
α_3	$\alpha_3 + \beta_2 + \gamma_4$	$\alpha_3 + \beta_3 + \gamma_5$	$\alpha_3 + \beta_4 + \gamma_6$	$\alpha_3 + \beta_5 + \gamma_7$
$\alpha_4 + \gamma_4$	$\alpha_4 + \beta_2 + \gamma_5$	$\alpha_4 + \beta_3 + \gamma_6$	$\alpha_4 + \beta_4 + \gamma_7$	$\alpha_4 + \beta_5 + \gamma_8$
$\alpha_5 + \gamma_5$	$\alpha_5 + \beta_2 + \gamma_6$	$\alpha_5 + \beta_3 + \gamma_7$	$\alpha_5 + \beta_4 + \gamma_8$	$\alpha_5 + \beta_5 + \gamma_9$

$$\gamma_k = \sum_{\ell=3}^{k-1} \ddot{\gamma}_\ell$$

fitted

where $\ddot{\gamma}_i = \gamma_{i+1} - \gamma_i$ **for** $i = 1, \ldots, 4$, **and**

$$\ddot{\gamma}_i = \kappa_i + \sum_{\ell=1}^{4} \omega_{i\ell} \cdot \ddot{\gamma}_\ell, \text{ for } i = 5, \ldots, 8, \text{ and } \sum_{\ell=1}^{4} \omega_{i\ell} = 1.$$

extrapolated

This is just the mechanics – the practical implications will be explored with hands-on examples during the concurrent session.

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Getting started with the GLM template.

1) Open workbook "CLRS 2016 – Triangle GLM Template.xlsm".
2) Explore how to navigate and set up model structure.
3) Pay attention to extrapolation for payment period parameters.
4) Ask many questions!
5) Will do guided tour for those who want it

AR-6 – Development Trend Extrapolation

Example I – Reproducing 2d results with 2d+1 model.

1) Make sure (on “Input” tab) Taylor & Ashe data is used.
2) Make sure (on “2d+1” tab) whole triangle is included.
3) Make sure (on “2d+1” tab) you use the max number of exposure and development period parameters.
4) Make sure (on “2d+1” tab) you group all payment period parameters.
5) Make sure (on “2d+1” tab) Poisson variance function is selected.
6) Make sure (on “2d+1” tab) No is selected for Use Tail Factor.
7) Make sure (on “2d+1” tab) Split-Linear bootstrap is selected.
8) Run bootstrap with 5,000 iterations (more if your laptop is fast).
9) Compare your results with the table on next slide.

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Example I – Reproducing 2d results with 2d+1 model.

		SEP		
Year	Projected Reserves	Pinheiro $B = 1{,}000$	$2d$ $B = 50{,}000$	$2d+1$ $B = 50{,}000$
2	94,634	110,936	111,210	111,860
3	469,511	213,571	217,674	217,079
4	709,638	257,996	263,366	262,645
5	984,889	301,476	304,886	306,053
6	1,419,459	370,270	377,209	377,420
7	2,177,641	498,900	496,718	496,648
8	3,920,301	771,798	798,489	797,221
9	4,278,972	1,029,730	1,051,448	1,053,632
10	4,625,811	2,039,736	2,046,448	2,031,011
Total	18,680,856	2,915,885	2,994,376	3,000,853

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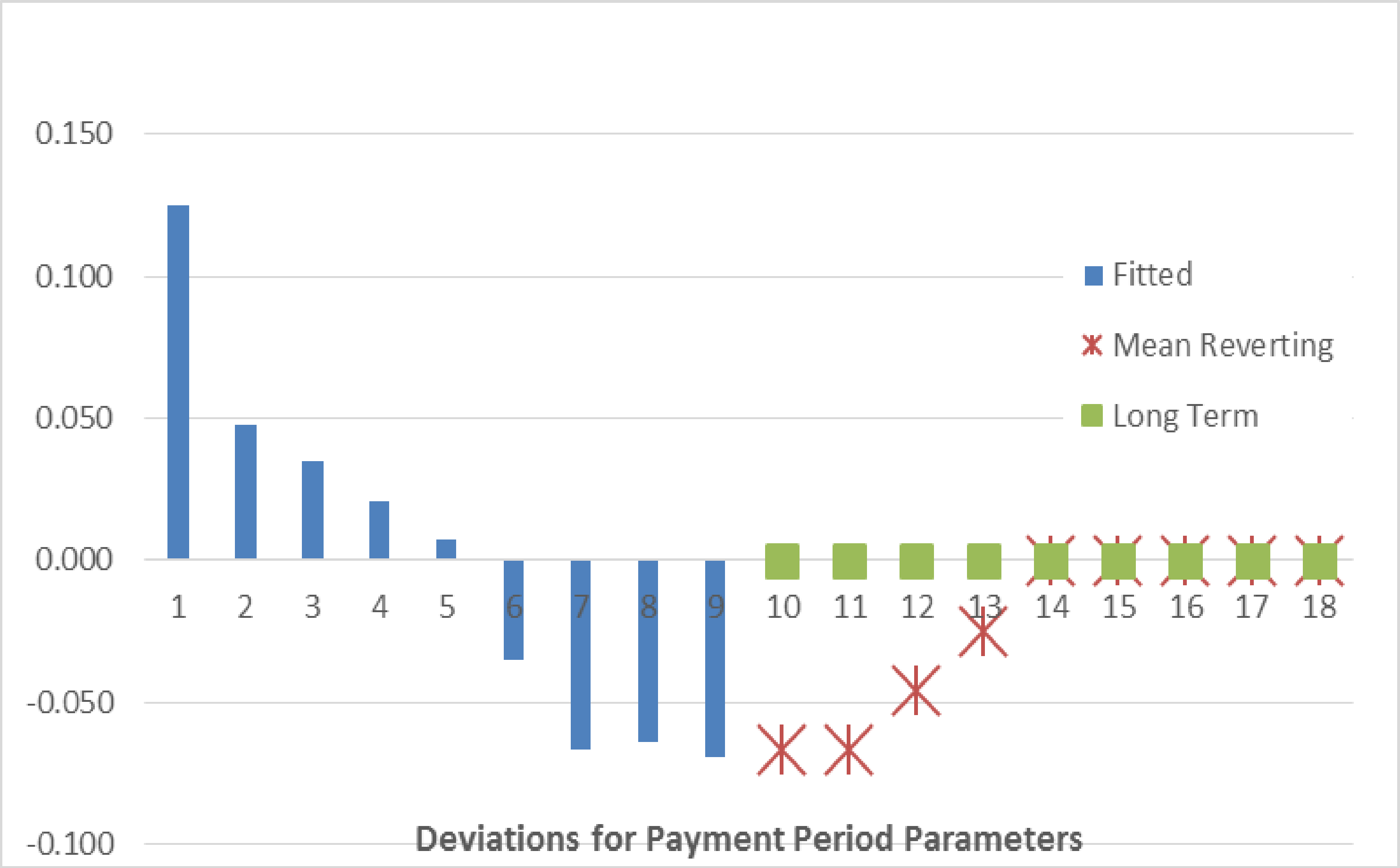
Example II – Mean reverting pattern vs long term average.



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Example II – Mean reverting pattern vs long term average.

1) Find a buddy (both of you will run a scenario).
2) Make sure the Xie & Wu data is used.
3) Make sure the whole triangle is included.
4) Make sure you use the max number of exposure, development, and payment period parameters.
5) Make sure the Poisson variance function is selected.
6) Each of you should do one of the extrapolation scenarios indicated on the next slide (so you cover both scenarios in your team).
7) Run bootstrap with 5,000 iterations (more if your laptop is fast).
8) Compare your results with the table on 2nd next slide.

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Example II – Mean reverting pattern vs long term average.

Mean Reverting

$$
\begin{aligned}
{}_{\gamma}\omega_{10k} &= 1/3\,, && 7 \le k \le 9 \\
{}_{\gamma}\omega_{11k} &= 1/3\,, && 7 \le k \le 9 \\
{}_{\gamma}\omega_{12k} &= 1/5\,, && 5 \le k \le 9 \\
{}_{\gamma}\omega_{13k} &= 1/7\,, && 3 \le k \le 9 \\
{}_{\gamma}\omega_{\ell k} &= 1/9\,, && 14 \le \ell \le 18 \\
& && 1 \le k \le 9
\end{aligned}
$$

No offsets (${}_{\gamma}\kappa_{\ell} = 0$)

Long Term Average

$$
\begin{aligned}
{}_{\gamma}\omega_{\ell k} &= 1/9\,, && 10 \le \ell \le 18 \\
& && 1 \le k \le 9
\end{aligned}
$$

No offsets (${}_{\gamma}\kappa_{\ell} = 0$)

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Example II – Mean reverting pattern vs long term average.

	$2d + 1$ Mean Reverting		$2d + 1$ Long Term	
Year	Mean	SEP	Mean	SEP
2	9,349	6,062	9,994	6,372
3	24,248	8,826	26,540	9,424
4	60,621	13,298	67,071	14,235
5	132,427	19,529	147,617	20,781
6	286,223	29,402	320,247	30,737
7	599,539	45,186	671,546	46,079
8	1,218,878	69,045	1,366,538	69,717
9	2,431,639	106,309	2,729,028	106,023
10	5,565,928	186,352	6,224,017	190,714
Total	10,328,852	307,009	11,562,598	284,060

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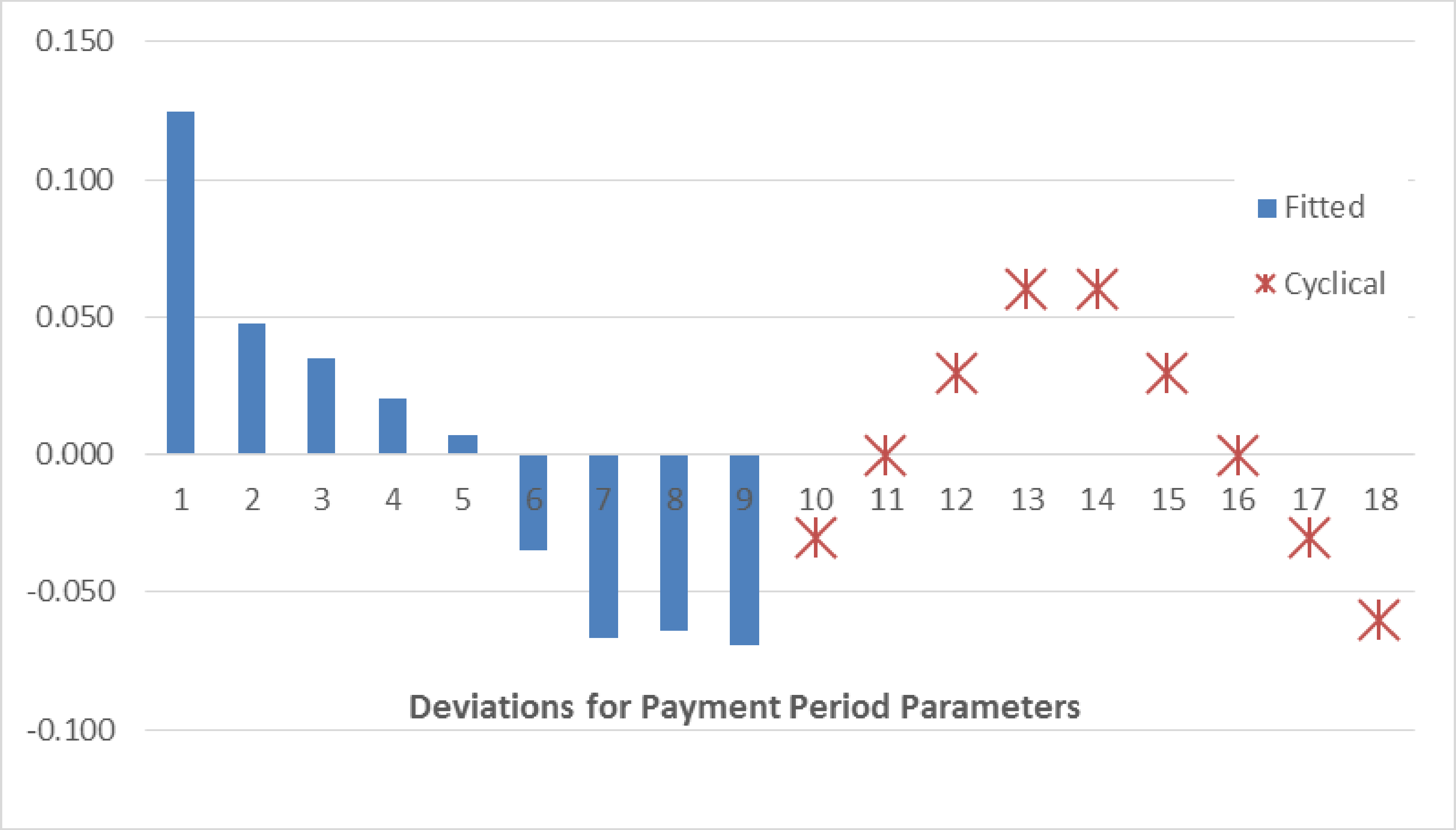
Example III – Using offsets.



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Example III – Using offsets.

1) Find a buddy (both of you will run a scenario).
2) Make sure the Xie & Wu data is used.
3) Make sure the whole triangle is included.
4) Make sure you use the max number of exposure, development, and payment period parameters.
5) Make sure the Poisson variance function is selected.
6) Each of you should do one of the extrapolation scenarios indicated on the next slide (so you cover both scenarios in your team).
7) Run bootstrap with 5,000 iterations (more if your laptop is fast).
8) Compare your results with the table on 2nd next slide.

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Example III – Using offsets.

Long Term Average & Offsets

$$_{\gamma}\omega_{\ell k} = 1/9\,, \qquad 10 \le \ell \le 18$$
$$1 \le k \le 9$$

Offsets:

$$_{\gamma}\kappa_{10} = {}_{\gamma}\kappa_{17} = -0.03$$
$$_{\gamma}\kappa_{18} = -0.06$$
$$_{\gamma}\kappa_{11} = {}_{\gamma}\kappa_{16} = \ \ 0.00$$
$$_{\gamma}\kappa_{12} = {}_{\gamma}\kappa_{15} = +0.03$$
$$_{\gamma}\kappa_{13} = {}_{\gamma}\kappa_{14} = +0.06$$

Repeat Past Trends

$$\ddot{\gamma}_{13} = \ddot{\gamma}_{14} = 0.15\ddot{\gamma}_{1} + 0.85\ddot{\gamma}_{2}$$
$$\ddot{\gamma}_{10} = \ddot{\gamma}_{17} = \ddot{\gamma}_{6}$$
$$\ddot{\gamma}_{11} = \ddot{\gamma}_{16} = \ddot{\gamma}_{5}$$
$$\ddot{\gamma}_{12} = \ddot{\gamma}_{15} = \ddot{\gamma}_{3}$$
$$\ddot{\gamma}_{18} = \ddot{\gamma}_{7}$$

No offsets ($_{\gamma}\kappa_{\ell} = 0$)

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Example III – Using offsets.

	$2d + 1$ Cyclical Offset		$2d + 1$ Cyclical Repeat	
Year	Mean	SEP	Mean	SEP
2	9,699	6,271	9,651	6,212
3	25,755	9,332	25,693	9,235
4	65,393	14,093	65,317	14,089
5	144,813	20,519	144,734	20,752
6	315,849	30,463	315,771	30,649
7	664,415	45,849	664,364	45,348
8	1,354,207	69,367	1,354,365	71,437
9	2,706,875	105,779	2,707,550	114,430
10	6,165,369	189,320	6,164,589	226,060
Total	11,452,375	283,881	11,452,035	345,803

Example IV – Choice of variance function.

1) Find a buddy (both of you will run a scenario).
2) Make sure the Taylor & Ashe data is used.
3) Make sure the whole triangle is included.
4) Make sure you use the max number of exposure, development, and payment period parameters.
5) Set extrapolation to long term average (weight 1/9), and no offsets.
6) One of you should run this with the Poisson variance function, the other should use Tweedie with p=1.8302.
7) Run bootstrap with 5,000 iterations (more if your laptop is fast).
8) Compare your results with the table on the next slide.

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Example IV – Choice of variance function.

	Poisson			Tweedie $P = 1.8302$		
Year	Reserves	Bias	SEP	Reserves	Bias	SEP
2	93,724	1,299	109,494	94,608	1,373	53,602
3	471,622	6,289	217,957	445,548	3,914	170,265
4	701,228	12,004	264,201	624,935	7,792	197,031
5	1,006,690	21,811	329,609	1,020,618	14,505	293,582
6	1,454,841	37,720	435,118	1,518,453	28,416	422,026
7	2,233,033	51,810	600,267	2,262,267	47,488	630,567
8	3,937,246	106,060	967,900	3,861,552	85,606	1,105,318
9	4,031,013	118,394	1,123,938	4,019,657	103,289	1,262,236
10	4,264,980	184,198	1,924,319	4,266,351	133,610	1,714,389
Total	18,194,376	539,585	3,566,360	18,113,989	425,994	3,422,764

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Example V – Stochastic tail factors.

$$\begin{matrix} (1,1,1) & \cdots & (1,n,n) & \cdots & [1,m,m] \\ & \ddots & & \ddots & \\ \vdots & & \vdots & & \vdots \\ & & \ddots & \ddots & \\ (n,1,n) & \cdots & [n,n,2n-1] & \cdots & [n,m,n+m-1] \end{matrix}$$

Need $_{\beta}\omega_{\ell k}$ and $_{\beta}\kappa_{\ell}$ to extrapolate $\ddot{\beta}_{n+1}, \ldots, \ddot{\beta}_{m}$,
and additional $_{\gamma}\omega_{\ell k}$ and $_{\gamma}\kappa_{\ell}$ to extrapolate $\ddot{\gamma}_{2n}, \ldots, \ddot{\gamma}_{n+m-1}$.

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Example V – Stochastic tail factors.

1) Find a buddy (both of you will run a scenario).
2) Make sure the Xie & Wu data is used.
3) Make sure the whole triangle is included.
4) Make sure you use the max number of exposure, development, and payment period parameters.
5) For ALL payment periods, set the extrapolation weights to long term average (i.e. 1/9), and no offsets.
6) Each of you should do one of the development period extrapolation scenarios indicated on the next slide.
7) Select Split-Linear Bootstrap and run 5,000 iterations.
8) Compare your results with the table on the next slide.

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Example V – Stochastic tail factors.

Leveraging Last Trend

$$_{\beta}\omega_{\ell 9} = 1, \qquad 10 \le \ell \le 14$$

No offsets ($_{\beta}\kappa_{\ell} = 0$)

Last 5 & Offset

$$_{\beta}\omega_{\ell k} = 1/5\,, \qquad 10 \le \ell \le 14$$
$$5 \le k \le 9$$

Offsets

$$_{\gamma}\kappa_{\ell} = 0.0865, \qquad 10 \le \ell \le 14$$

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Example V – Stochastic tail factors.

		Last 1		Last 5	
Year	Reserves	Bias	SEP	Bias	SEP
1	12,032	27,949	109,571	769	9,971
2	22,220	28,407	114,845	829	14,980
3	38,258	27,245	109,422	791	16,073
4	79,650	29,174	117,735	943	19,961
5	160,882	30,704	125,053	985	25,471
6	334,670	33,491	137,447	997	34,724
7	686,768	35,557	148,901	1,034	48,957
8	1,382,369	36,889	162,699	1,180	71,940
9	2,745,449	38,099	185,453	976	107,734
10	6,241,949	41,159	253,964	1,593	192,270
Total	11,704,248	328,674	1,341,998	10,097	318,918

Thank you

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