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Does Stochastic Reserving Work?

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Presentation Overview

- What does it mean to say a method “works”?
- Bias and percentile tests
- Consistency and Robustness
- Conclusions

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Proving a Method Works

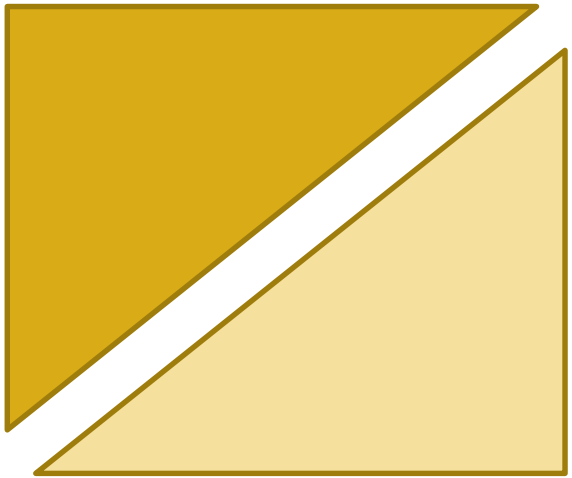
By Testing the Output on Generated Data

Model Testing: Learning from Other Fields

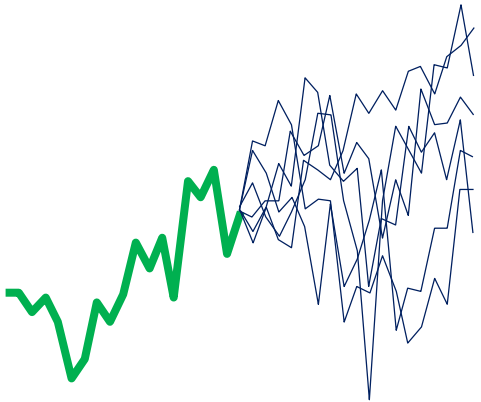
Loss Distribution Fit

Rank	Example
1	26
2	29
3	40
4	48
5	59
6	60
7	69
8	98
9	278
10	293

Stochastic Reserving



Economic Scenarios



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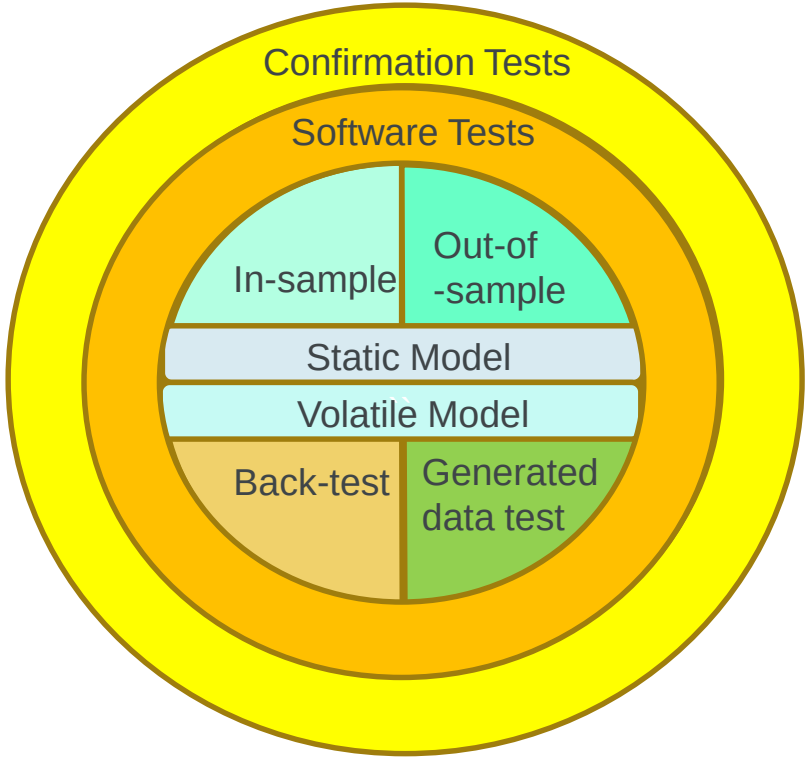
Types of Model Tests



Confirmation tests compare model output to prior (“expert”) expectations.

Software testing seeks to ensure program output conforms to the specification, by identifying and fixing code bugs.

Static model testing compares a fixed model (including fixed parameters) to historic data.

Volatile model testing compares a sequence of model forecasts with subsequent outcomes, testing a way of constructing models rather than a particular model.

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Testing Distribution Fits

Sample data

Rank	Example
1	26
2	29
3	40
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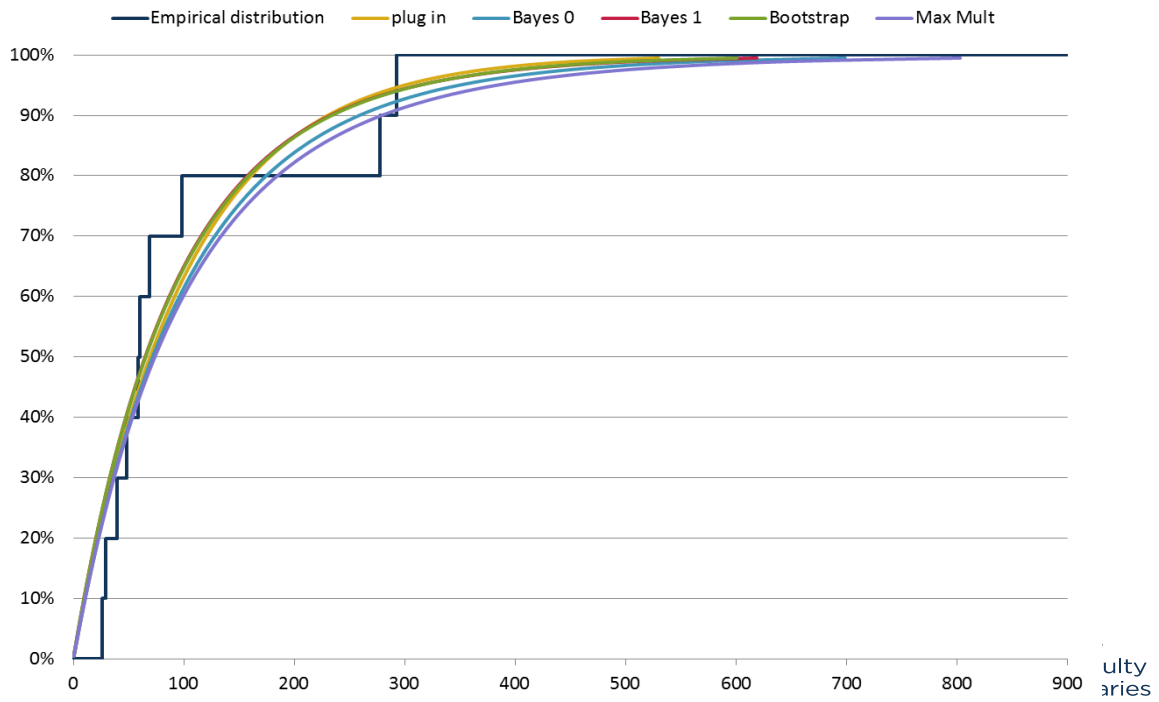
Empirical and Ersatz Distribution Functions



Alternative Stochastic Reserving Methods

"Allow for parameter error"

Bootstrap	Constant Scale	Delay-Varying Scale
Additive	Over-dispersed	ODP with delay-varying scale
Multiplicative		Mack

Bayesian	Constant Scale	Delay-Varying Scale
Additive	Over-dispersed	ODP with delay-varying scale
Multiplicative		

Analytic	Constant Scale	Delay-Varying Scale
Additive	Over-dispersed Poisson	ODP with delay-varying scale
Multiplicative	Over-dispersed negative binomial	Mack

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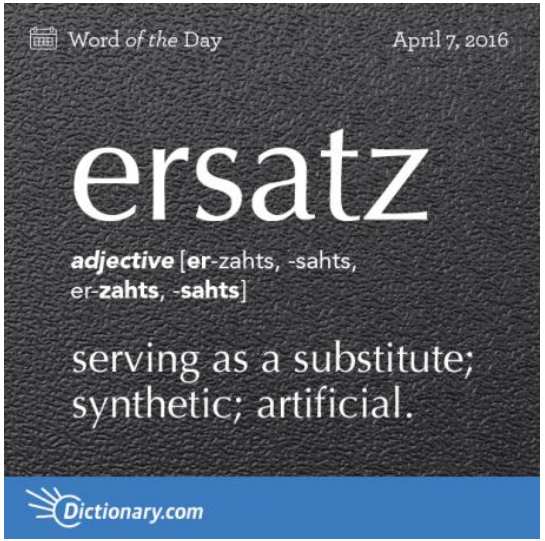

Bias and Percentile Tests

Measuring them from Scenarios

Got History: Need Stochastic Model

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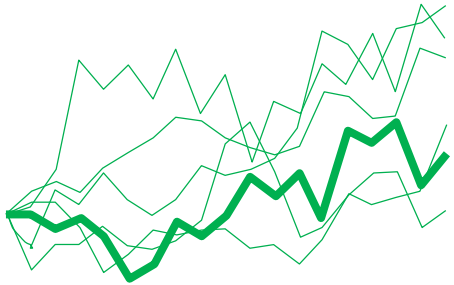
If Data Generation Process Known ….



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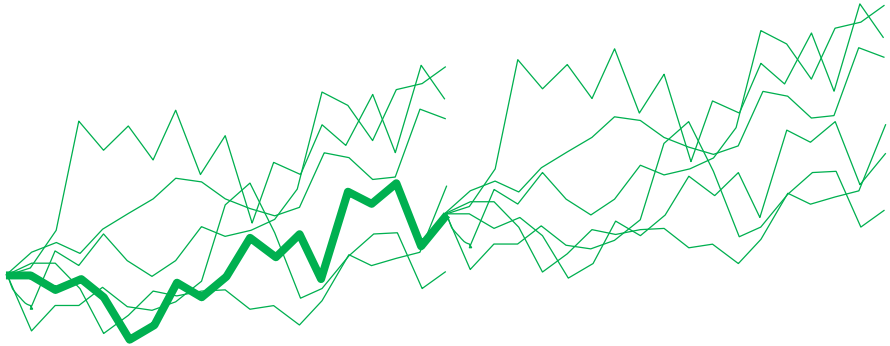
Then we can Generate future Projections:



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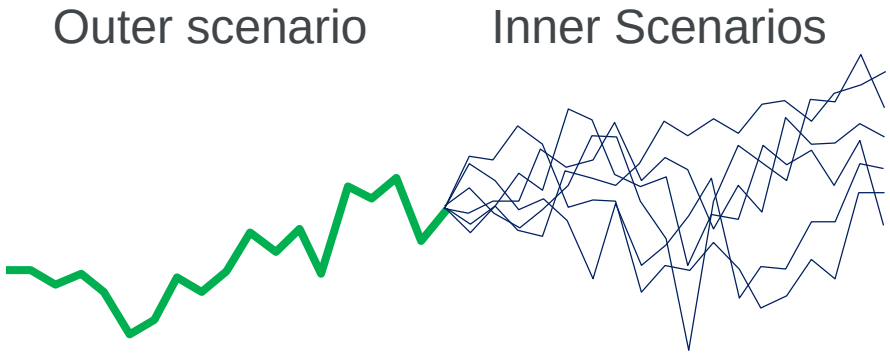
But we have only One History:



So we can only guess the generating process, by statistical fitting.
Our forecast for decisions is an *ersatz model*, which we substitute for the underlying process.

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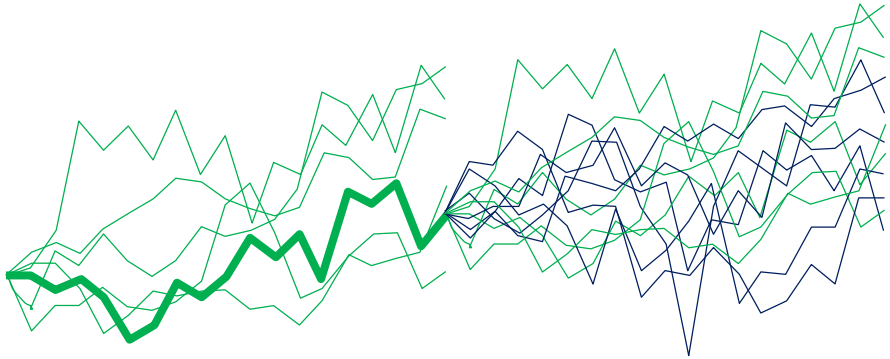
Is the Ersatz Model a Good Substitute?



We want the statistical properties of the ersatz (blue) scenarios to resemble those of the original (green) data generating process, or *reference model.*

We will test the ersatz construction using several reference models.

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Measuring Ersatz Model Bias

Unbiased Mean

- The following are equal:
  - Mean ersatz scenario
  - Mean reference scenario
- Averaged over outer reference scenarios.

Conditional Variance

- The following are equal
  - Conditional ersatz variance
  - Conditional reference variance
- Averaged over outer reference scenarios

Unconditional Variance

- The following are equal
  - Variance of ersatz scenarios
  - Variance of reference scenarios

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Bias Test: Bootstrap Residuals

- Degrees of Freedom calculation in bootstrap residuals
- Taken from England and Verrall (2002)

$$r_{ij}^{adj} = \sqrt{\frac{\frac{1}{2}n(n+1)}{\frac{1}{2}n(n+1) - 2n + 1}} \times r_{ij}^{(P)}$$

- The degrees of freedom adjustment corrects bias in scale parameter estimates.

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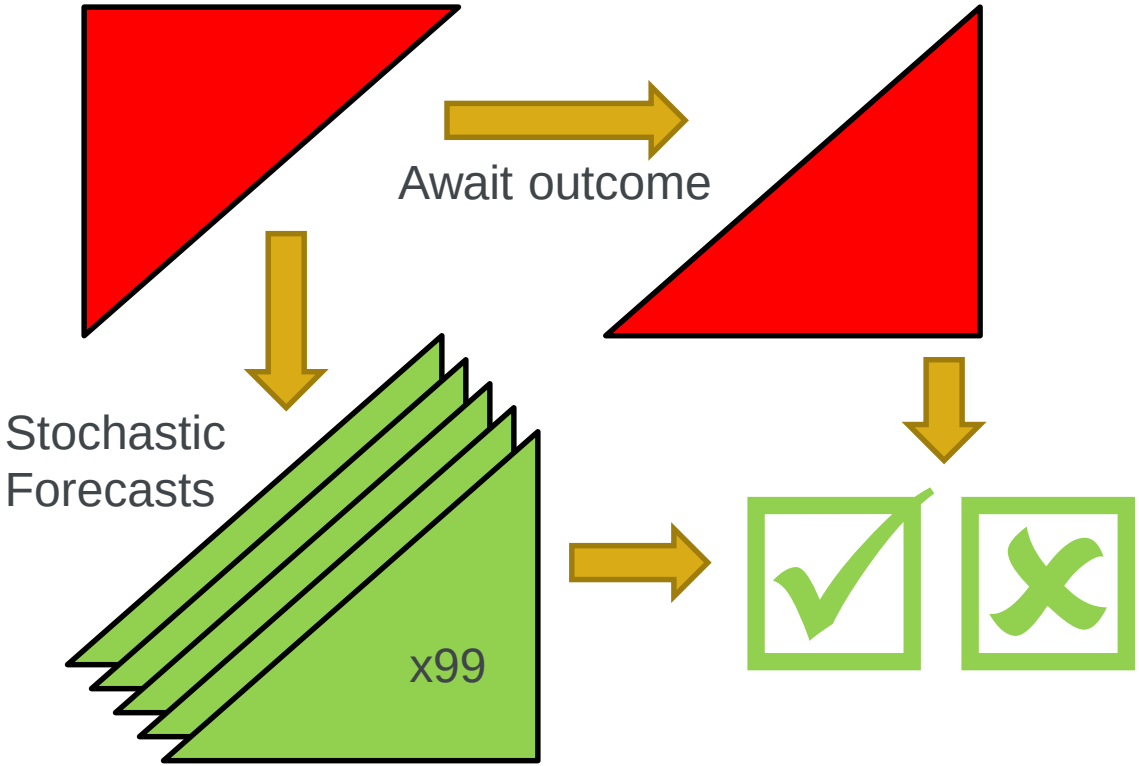
Percentile Testing Stochastic Reserving Methods



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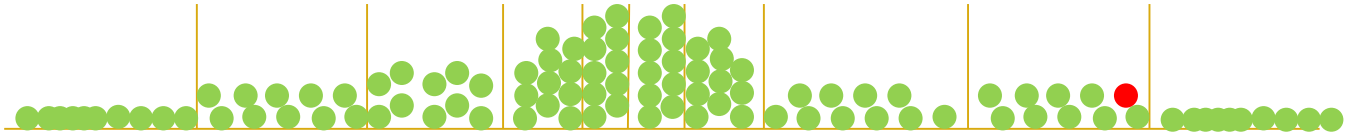
Ranking the Outcomes

- Take 100 future claim scenarios
  - 1 actual outcome ● and 99 from bootstraps ●
- Sort and divide into 10 buckets, each containing 10 observations



aggregate outstanding claims

- Suppose the actual outcome and the bootstrap are independent samples from the same distribution
- Then there is 1-in-10 chance the red lies in each bucket

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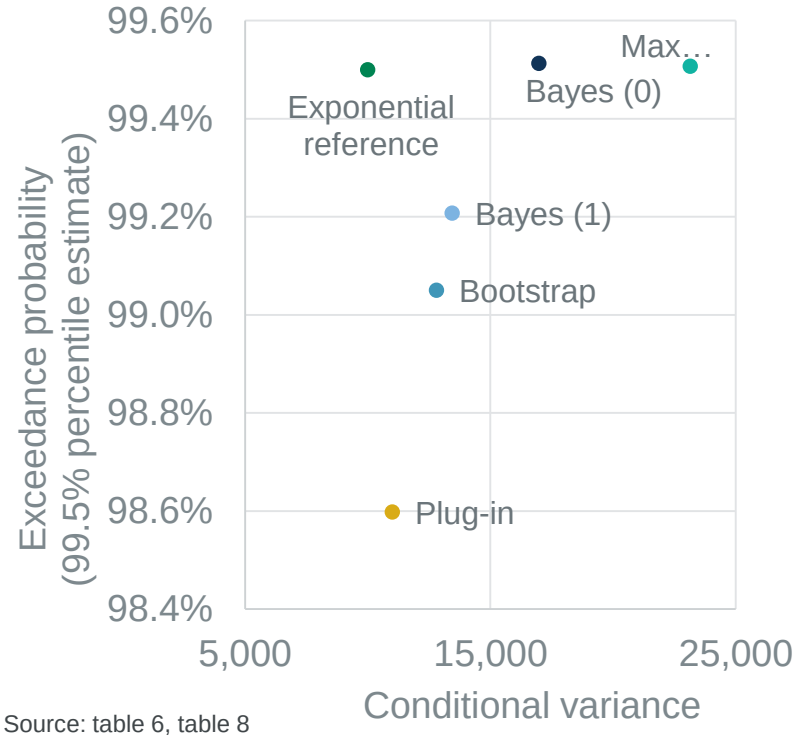
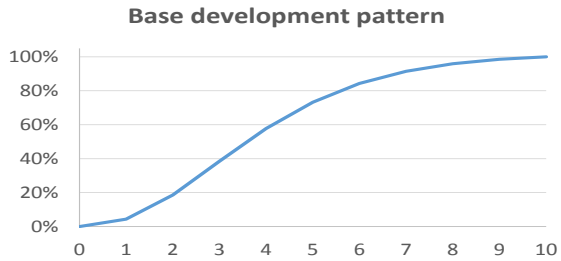
Bias and Percentile Tests for Distribution Fits

- Plug-in method has lowest bias in variance estimate but worst percentile matching
- Bayes(0) and Max Mult are exact percentile matching but have worst bias
- Having both is impossible: if a quantile estimator Q is unbiased – F[E(Q)] = 99.5% - then Q will be exceeded more than 0.5% of the time: E[F[Q]] < 99.5%

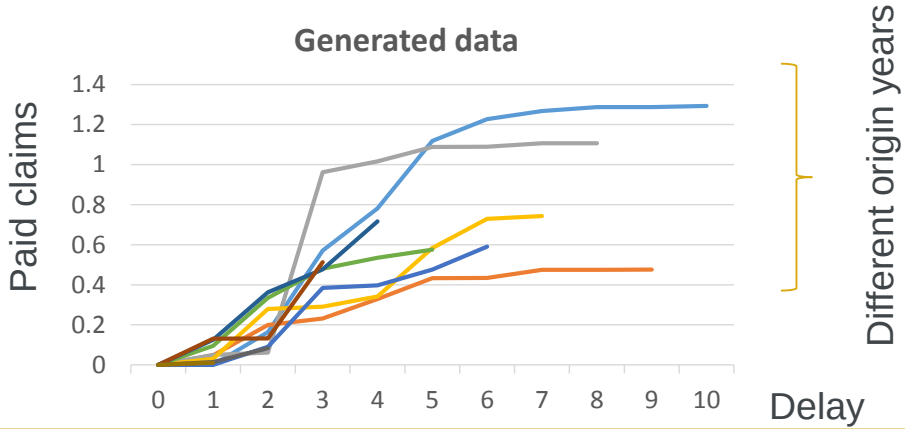


Source: table 6, table 8

Generating Triangles – ODP Model



- SIGNAL: Assume a base development pattern
  - Use the same pattern for all origin years
- NOISE: Incremental claims in each cell generated from a gamma distribution with mean from pattern (with specified gamma vol.)



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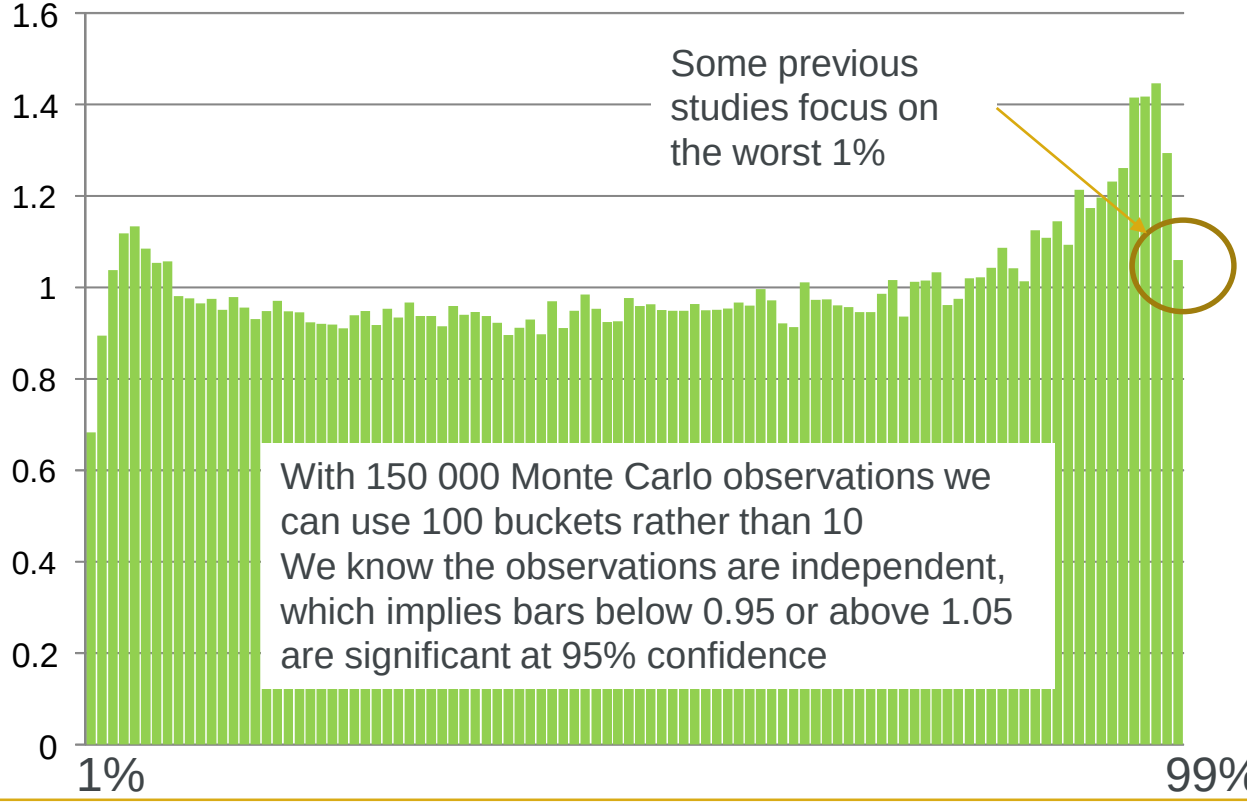
Example Percentile Test (ODP Bootstrap)



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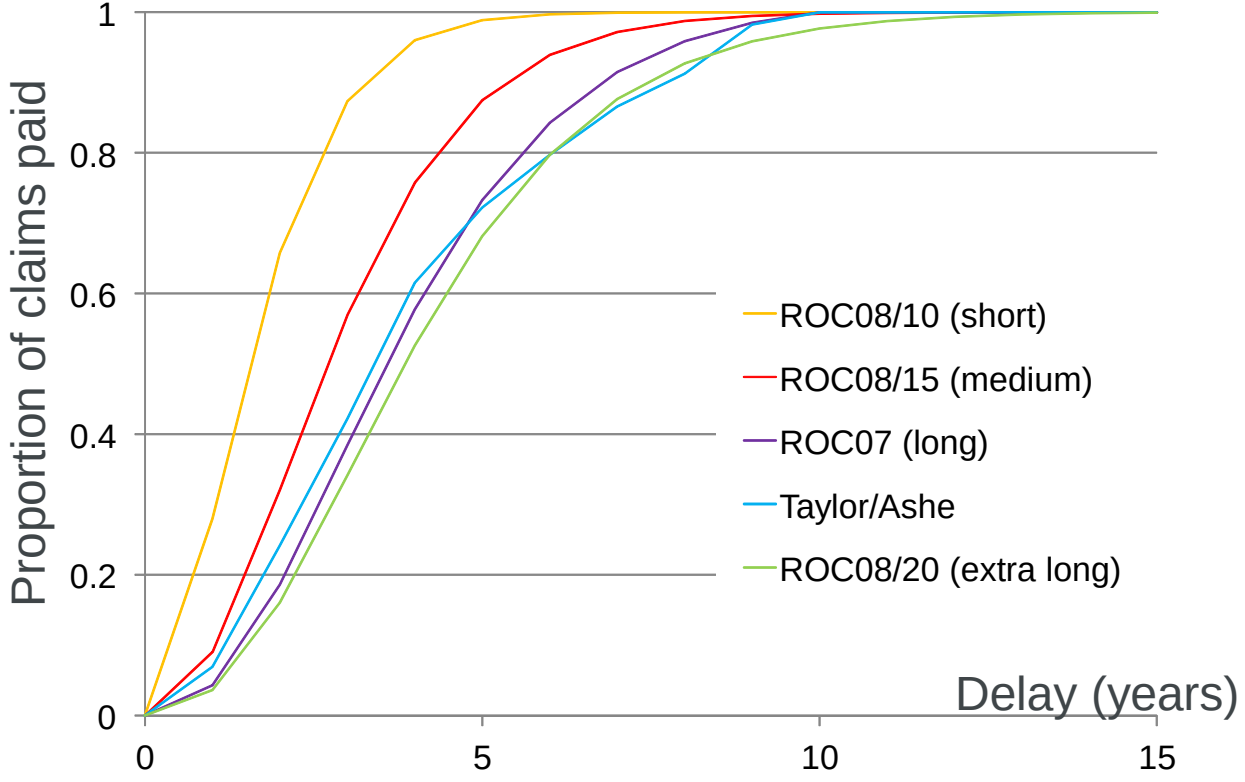
Reference Claims Development Patterns



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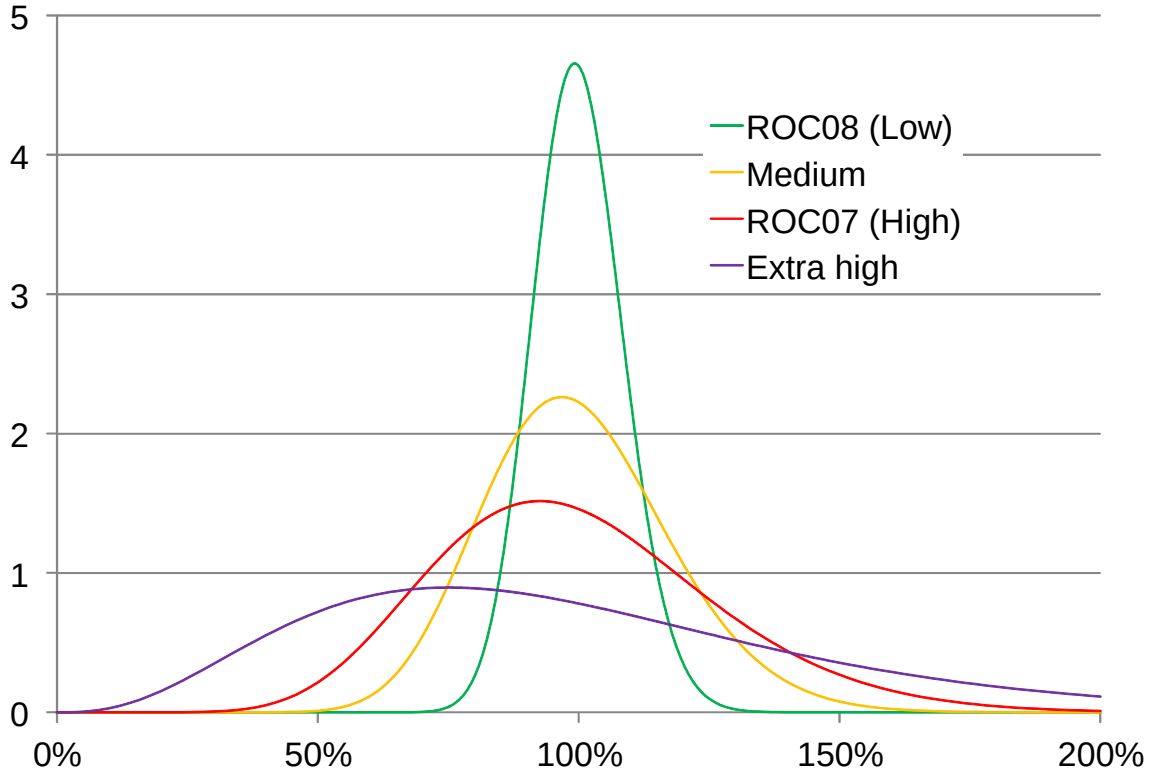
Reference ULR Distribution (each cohort)



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Results for Different Reference Models: Percentile Test: Proportion > Ersatz 99%-ile

ODPB	Development pattern length			
Gamma Volatility	Short	Medium	Long	Extra Long
Low	1.1%	0.7%	0.7%	0.6%
Medium	1.5%	1.1%	0.8%	1.1%
High	1.9%	1.5%	1.1%	1.5%
Extra High	3.0%	2.7%	1.9%	2.7%

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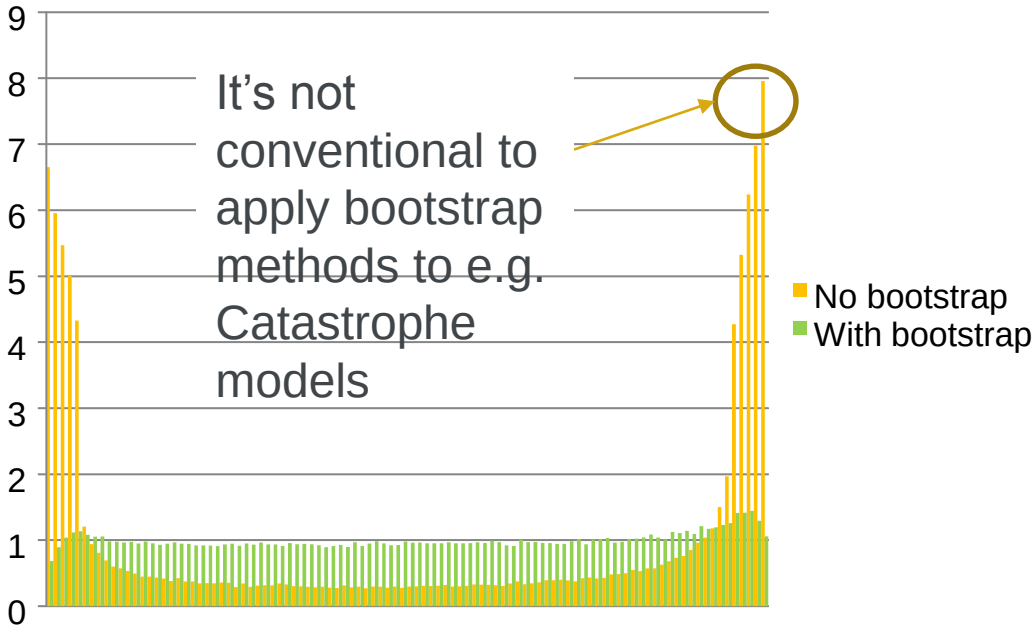
Bootstrapping Helps Percentile Tests - but Hinders Bias Tests



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Consistency and Robustness

Desirable Ersatz Model Properties

Model Assumptions

- Mathematical derivation of stochastic reserving methods relies on strong assumptions; for example:
  - Development pattern is the same for each cohort
  - Different cohorts are independent of each other
- Can we say a method “works” if we can only show it behaves well under strong assumptions?
- Suppose the assumptions are violated: All bets are off? Method cannot be disproved?

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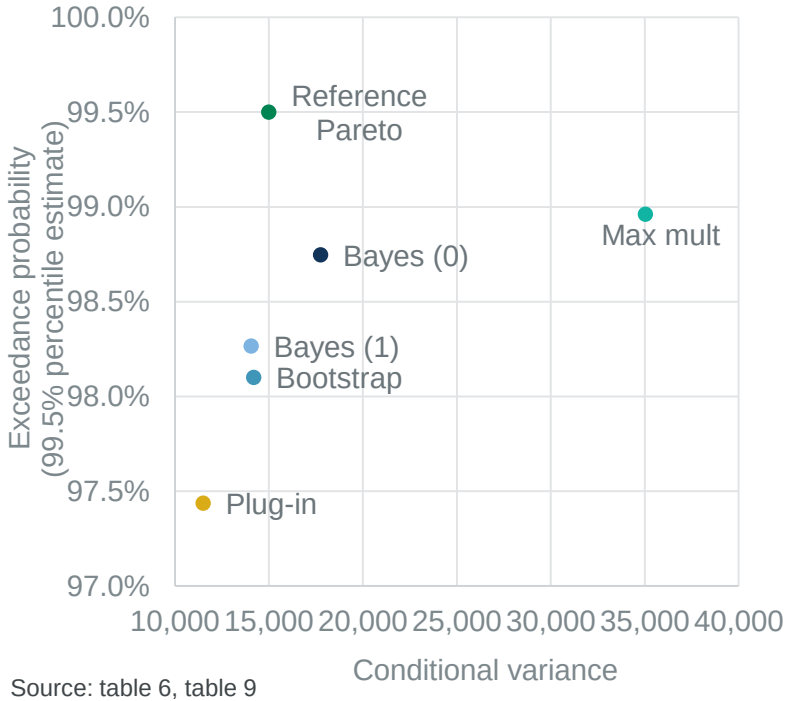
Robustness 1: Impact of Fatter Tail in Ref Model

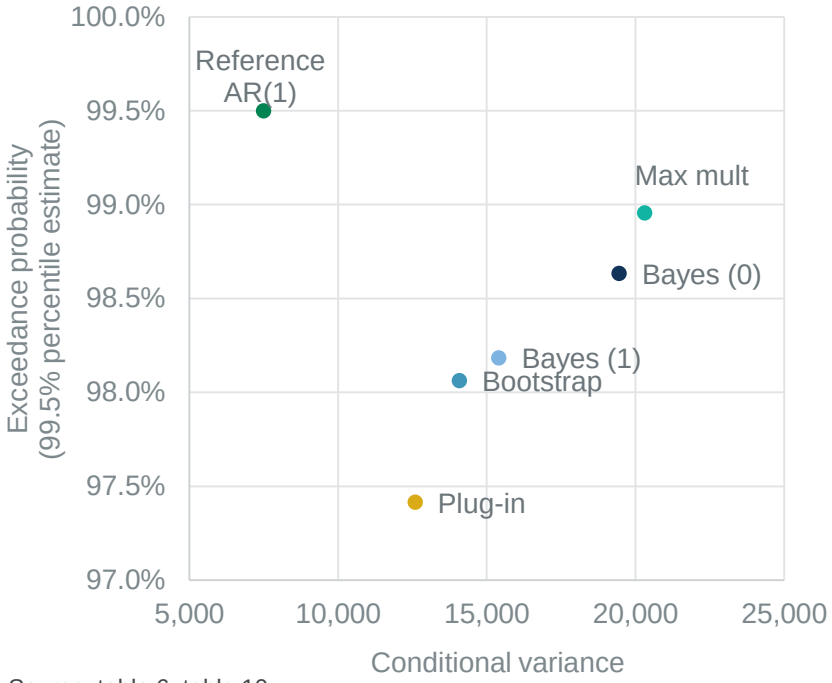
- Pareto (alpha = 6) not exponential
- Again there's a trade off between percentile matching and bias in the variance
- Building ersatz distribution from the maximum rather than sample average provides a best hedge against model error in the tail (according to the percentile test)



Source: table 6, table 9

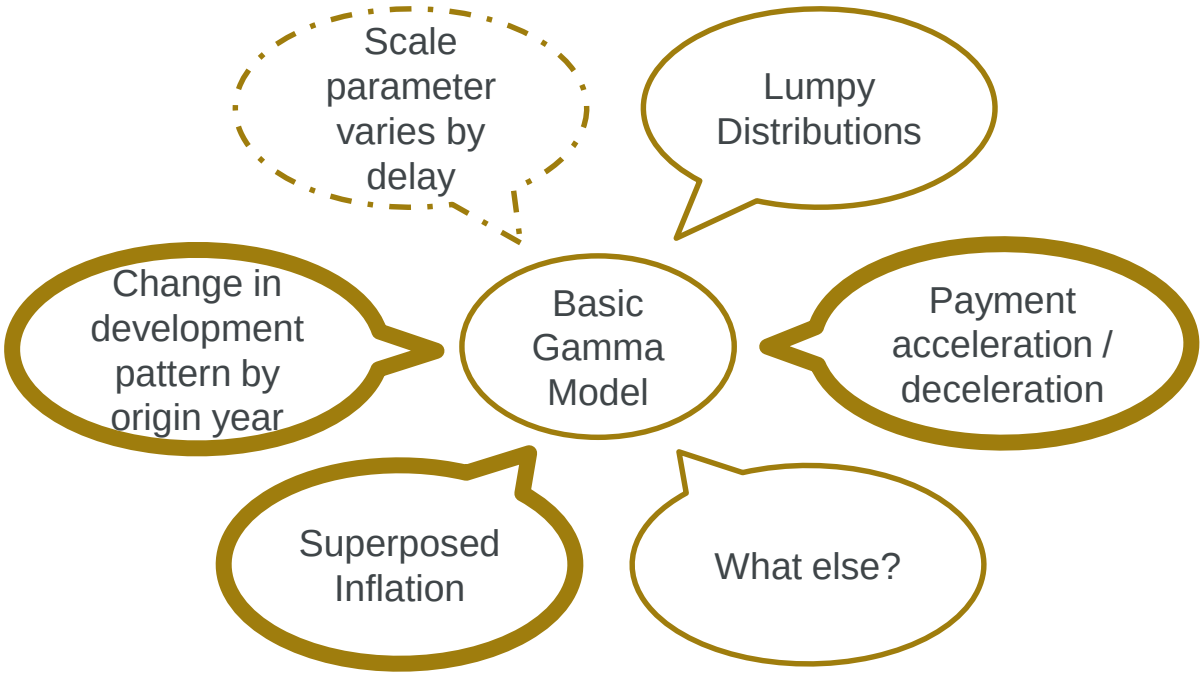
Robustness 2: Auto-correlated Loss Inputs

- AR(1) process with exponential stationary distribution, QA = 0.5
- Fitted ersatz models all i.i.d.
- Again there's a trade off between percentile matching and bias in the variance
- Maximum multiple and Bayes(0) again most robust in the tail to model specification



Source: table 6, table 10

Robustness Tests in Stochastic Reserving



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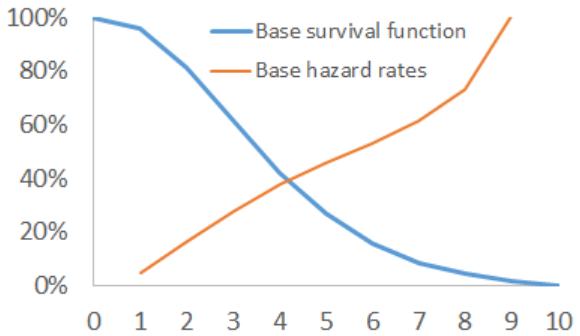
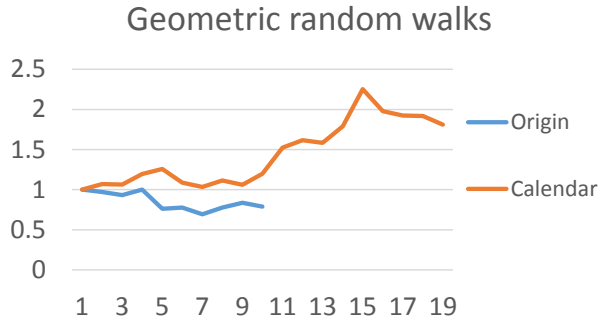
Reference Triangle Models for Robustness Tests

- Express the development pattern as hazard rates (compare force of mortality μ_x = minus log of survival rate p_x)
- Transform these hazard rates for each origin year
  - Multiply by geometric random walks for origin period and calendar (or equivalently, raise survival rates to a power)





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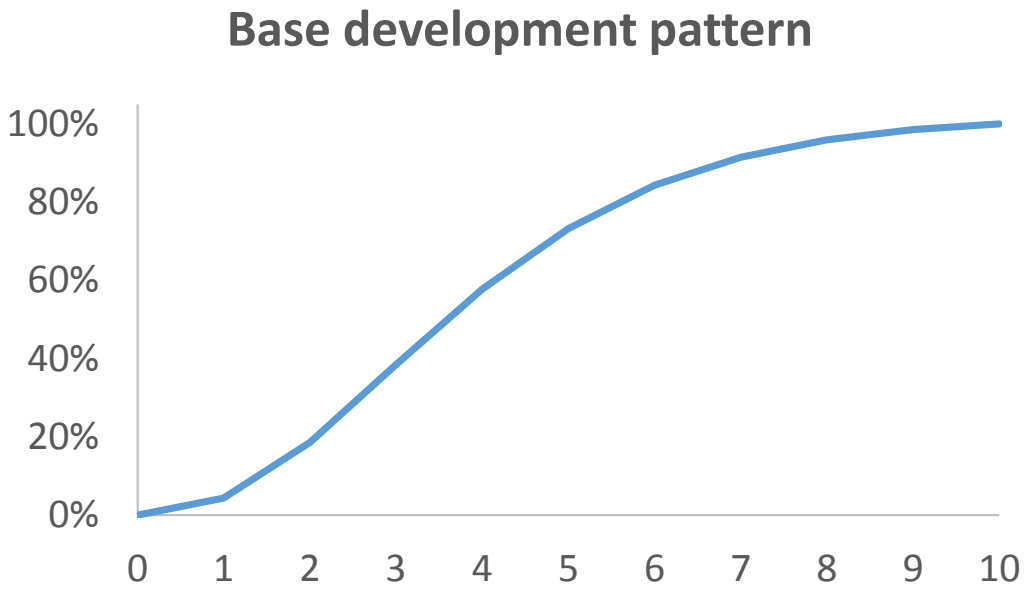
Making Triangles More Realistic



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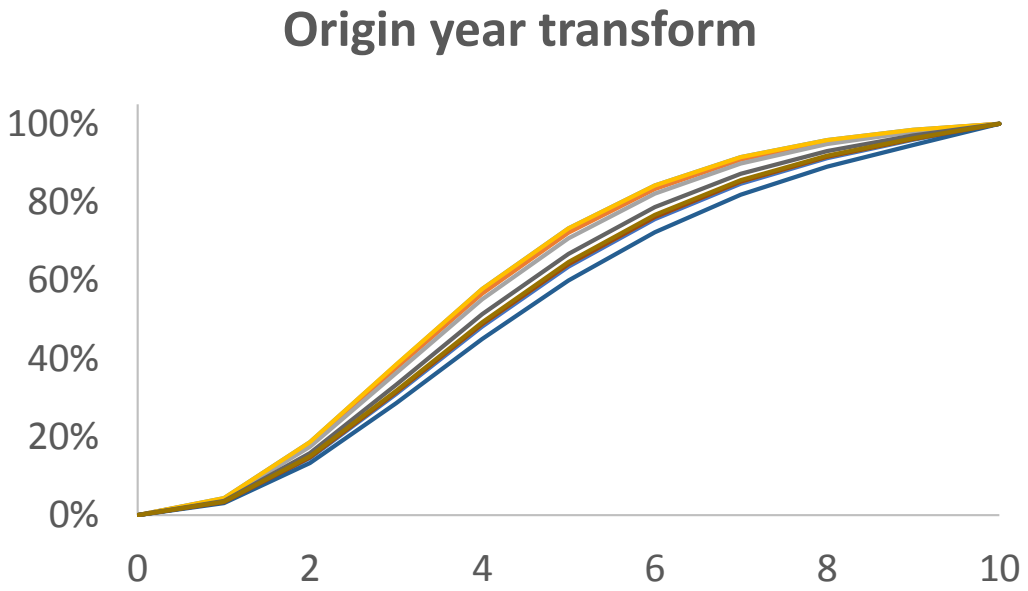

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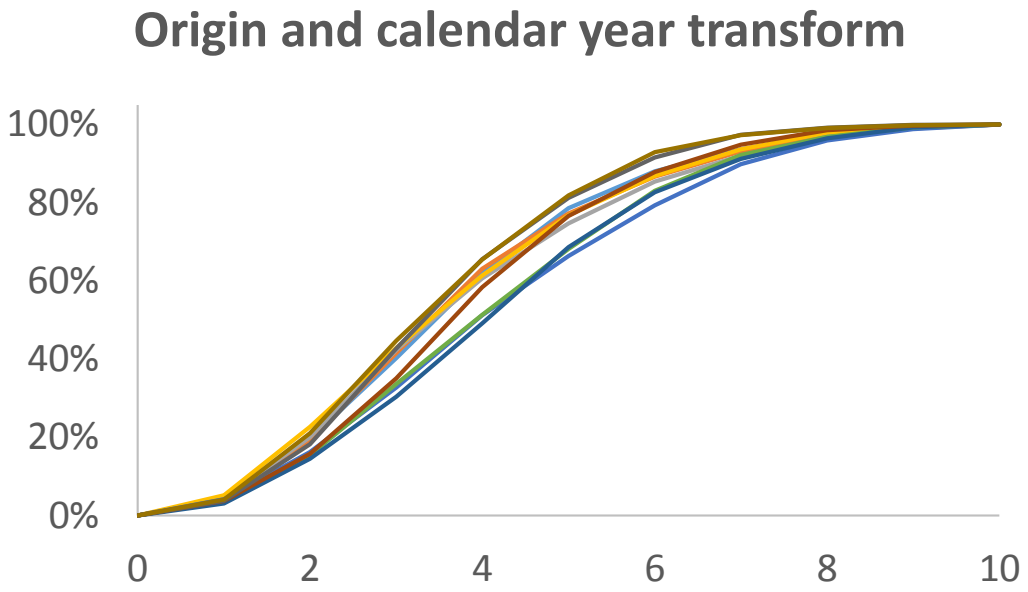

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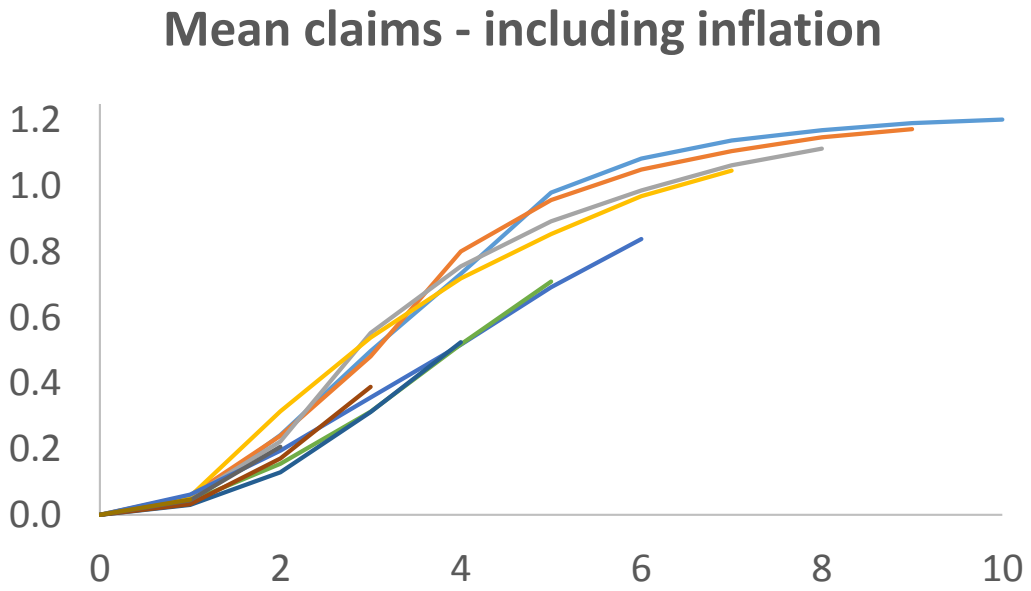

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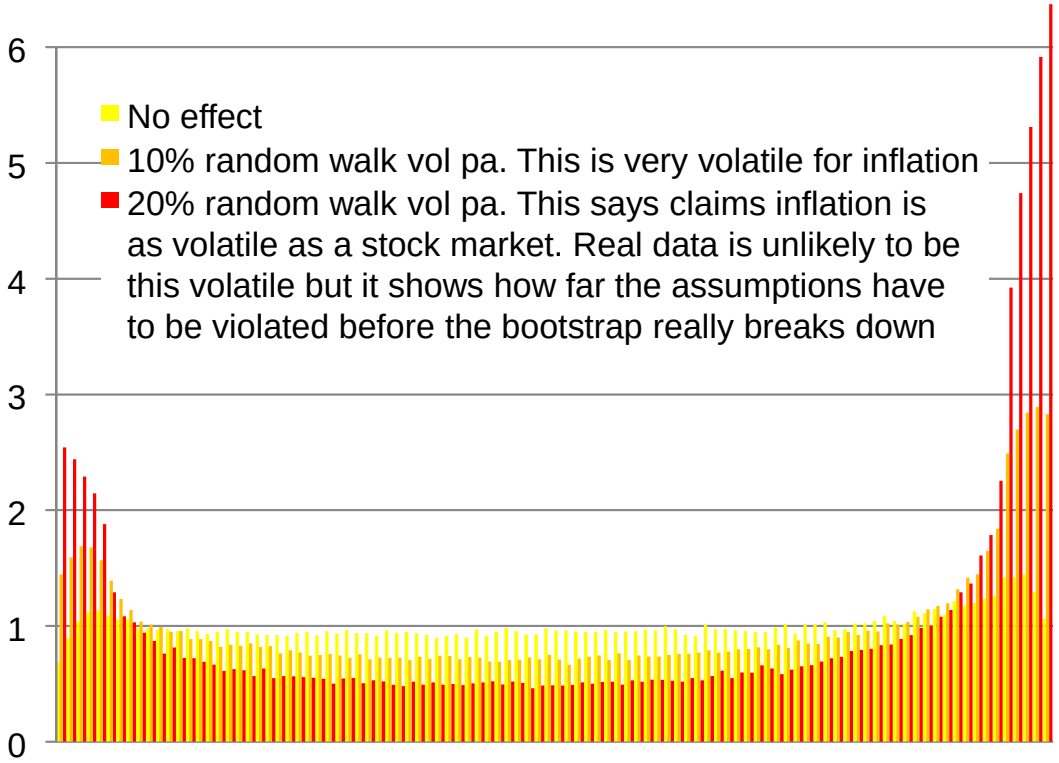
Origin Year and Calendar Year Effects



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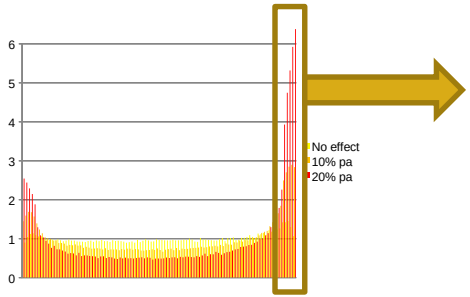

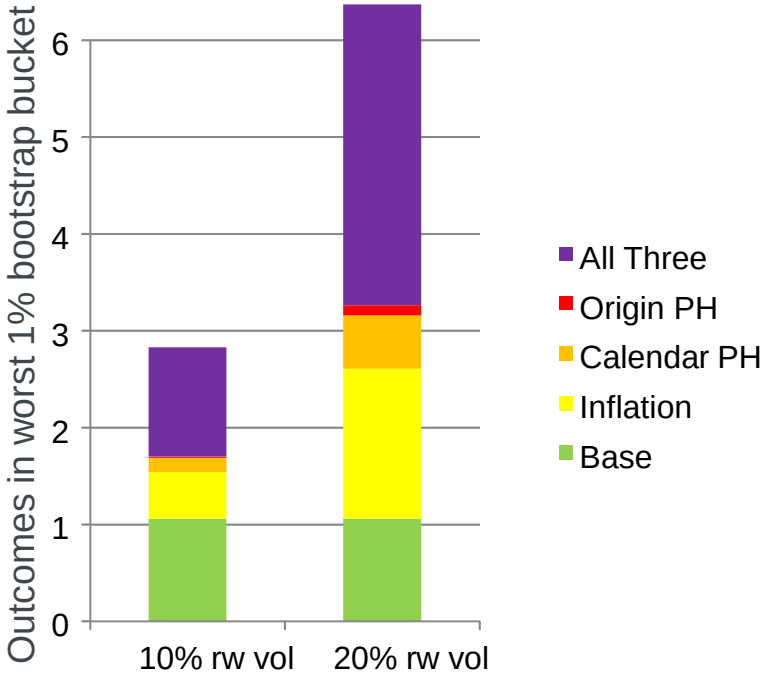

These figures relate to the long development pattern, and high gamma volatility.

This is a form of model reverse stress test. How ugly does the reference model have to be before the ersatz procedure breaks down?

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Conclusions

What have we Learned?

Conclusions

- Testing on generated data increases model test power, allowing us to probe and document model limitations. Do not expect 100% passes.
- Analytical, best estimate, models do well in bias tests. Models “allowing for parameter uncertainty” do better in percentile tests.
- Ersatz models are simplifications of reality; we can test the robustness of these simplifications by using “inconsistent” reference models.
- Different tests may conflict so we need to be clear on the model purpose and select tests accordingly.

Further Reading

- For descriptions of bootstrap, Mack etc the standard reference is England & Verrall (2002). Stochastic Claims Reserving in GI. BAJ 8.
- For Jarvis, Sharpe and Smith (2016) on model testing, with more details on the distribution fit, look at ssrn.com/abstract=2788478
- For moment testing of stochastic reserving methods, see Kevin Chan & Michael Ramyar (2016) “Practical Challenges in Reserve Risk”
- For details of percentile tests on bootstrap, see GIROC 2007/8 and Locke & Smith “What does the Bootstrap Trap” (GIRO 2016).
- Look out for David Hindley’s reserving book in the autumn.

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Questions

Comments

Expressions of individual views by members of the Institute and Faculty of Actuaries and its staff are encouraged.

The views expressed in this presentation are those of the presenter.

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