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Using the Hayne MLE Models: A Practitioner's Guide

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Paper Outline

- Introduction
- Notation
- The Hayne MLE Models
- Practical Issues (Algorithm Enhancements)
- Diagnostics
- Using Multiple Models
- Future Research
- Conclusions

Paper Outline

- Practical Issues (Algorithm Enhancements)
  - Negative Incremental Values
  - Standardized Residuals
  - Using an N-Year Average
  - Missing Values

- Practical Issues (Algorithm Enhancements)

  - Outliers
  - Heteroscedasticity
  - Heteroecthesious Data
  - Parameter Adjustments
  - Tail Extrapolation
  - Incurred Data

The Hayne MLE Models

- Start with a triangle of cumulative claim data:

		d 1	2	3	…	n-1	n
w	**1**	c(1,1)	c(1,2)	c(1,3)	…	c(1,n-1)	c(1,n)
	2	c(2,1)	c(2,2)	c(2,3)	…	c(2,n-1)	
	3	c(3,1)	c(3,2)	c(3,3)	…		
	…	…	…				
	n-1	c(n-1,1)	c(n-1,2)				
	n	c(n,1)					

- For Hayne MLE, we will use the incremental claim data:

		d 1	2	3	…	n-1	n
w	**1**	q(1,1)	q(1,2)	q(1,3)	…	q(1,n-1)	q(1,n)
	2	q(2,1)	q(2,2)	q(2,3)	…	q(2,n-1)	
	3	q(3,1)	q(3,2)	q(3,3)	…		
	…	…	…				
	n-1	q(n-1,1)	q(n-1,2)				
	n	q(n,1)					

The Hayne MLE Models

- We can also use an exposure adjustment – e.g., ultimate claim counts:

w \ *d*	1	2	3	…	n-1	n		u
1	b(1,1)	b(1,2)	b(1,3)	…	b(1,n-1)	b(1,n)	⇨	b(1,u)
2	b(2,1)	b(2,2)	b(2,3)	…	b(2,n-1)		⇨	b(2,u)
3	b(3,1)	b(3,2)	b(3,3)	…			⇨	b(3,u)
…	…	…						…
n-1	b(n-1,1)	b(n-1,2)					⇨	b(n-1,u)
n	b(n,1)						⇨	b(n,u)

- Adjusting for exposures, $A(w,d) = \dfrac{q(w,d)}{b(w,u)}$, we get average claim severity:

w \ *d*	1	2	3	…	n-1	n
1	A(1,1)	A(1,2)	A(1,3)	…	A(1,n-1)	A(1,n)
2	A(2,1)	A(2,2)	A(2,3)	…	A(2,n-1)	
3	A(3,1)	A(3,2)	A(3,3)	…		
…	…	…				
n-1	A(n-1,1)	A(n-1,2)				
n	A(n,1)					

The Hayne MLE Models

- The Hayne MLE formulation is as follows:

$$A(w,d) = M(\boldsymbol{\theta})$$

$$E[A(w,d)] = \mu$$

$$Var[A(w,d)] = \frac{e^{\kappa}(\mu^2)^{\rho}}{N(w)} = e^{\kappa - \ln[N(w)]}(\mu^2)^{\rho}$$

Where: $\boldsymbol{\theta} =$ a parameter vector

$N(w) =$ exposures for year w

$\kappa =$ a variance parameter

$\rho =$ a variance parameter

- Model includes implicit structural heteroscedasticity for both w and d.

The Hayne MLE Models

- Five different model structures are defined:
  - **Berquist-Sherman:**

$$E[A(w,d)] = f(d) \times e^{wG}$$

Where: $f(d) = average\ incremental\ by\ development\ period$

$G = constant\ accident\ period\ trend$

  - **Cape Cod:**

$$E[A(w,d)] = \begin{cases} G(1,1), & w = 1, d = 1 \\ G(1,1) \times G(w), & w > 1, d = 1 \\ G(1,1) \times f(d), & w = 1, d > 1 \\ G(1,1) \times G(w) \times f(d), & w > 1, d > 1 \end{cases}$$

Where: $G(1,1) = constant\ or\ scale$

$G(w) = exposure\ year\ factors$

$f(d) = development\ period\ factors$

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The Hayne MLE Models

- Five different model structures are defined:
  - **Chain Ladder:**

$$E[A(w,d)] = \begin{cases} G(w)\times f(d), & w=1, d<n \\ G(w)\times[1-\sum_{d=1}^{d=n-1} f(d)], & w=1, d=n \\ \dfrac{G(w)\times f(d)}{\sum_{k=1}^{k=n+1-w} f(k)}, & w>1, d<n \\ \dfrac{G(w)\times[1-\sum_{d=1}^{d=n-1} f(d)]}{\sum_{k=1}^{k=n+1-w} f(k)}, & w>1, d=n \end{cases}$$

Where: $G(w) = cumulative\ value\ for\ accident\ period$

$f(d) = development\ period\ factors$

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The Hayne MLE Models

- Five different model structures are defined:
  - **Hoerl Curve:**

$$E[A(w,d)] = e^{G(1)+d\times f(1)+d^2\times f(2)+\ln(d)\times f(3)+w\times G(2)}$$

Where: $G(1) = constant\ level$

$G(2) = constant\ trend\ by\ accident\ period$

$f(1), f(2), f(3) = factors\ for\ development\ lags:\ d,\ d^2,\ \ln(d)$

  - **Wright:**

$$E[A(w,d)] = e^{G(w)+d\times f(1)+d^2\times f(2)+\ln(d)\times f(3)}$$

Where: $G(w) = level\ per\ accident\ period$

$f(1), f(2), f(3) = factors\ for\ development\ lags:\ d,\ d^2,\ \ln(d)$

The Hayne MLE Models

1) Use Maximum Likelihood to solve for the model parameters, including the variance-covariance matrix
2) Sample new parameters using the multi-variate Normal distribution
3) Using sampled parameters, calculate sample mean and standard deviation for each incremental value (whole rectangle)
4) For each incremental value, generate random sample from mean and standard deviation for that cell using the Normal distribution
5) Multiply random sample for each cell times exposures
6) Sum future values to get total unpaid
7) Repeat steps 2) to 6) a significant number of iterations

The Hayne MLE Models

For “Berquist-Sherman” model:

- Here’s a simple example using a 6 x 6 triangle:

Cumulative Data

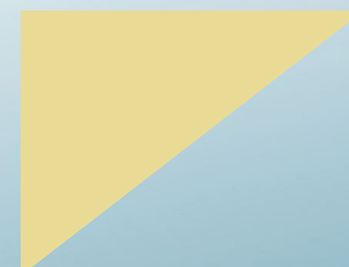
	1	2	3	4	5	6
1	95	150	170	200	215	220
2	110	160	175	205	210	
3	105	165	190	210		
4	120	155	185			
5	130	170				
6	125					

1) Actual Cumulative Data

Incremental Data

	1	2	3	4	5	6
1	95	55	20	30	15	5
2	110	50	15	30	5	
3	105	60	25	20		
4	120	35	30			
5	130	40				
6	125					

2) Actual Incremental Data



The Hayne MLE Models

For “Berquist-Sherman” model :

- Here’s a simple example using a 6 x 6 triangle:

Exposures (Claim Counts)

	1	2	3	4	5	6	u
1	5	9	11	13	14	15	15.0
2	7	11	13	15	16		17.1
3	6	10	12	14			16.1
4	8	12	14				18.8
5	6	10					15.9
6	7						18.1
Factors:		1.625	1.190	1.167	1.071	1.071	

3) “Use” Ultimate Exposures

Average Severity Data

	1	2	3	4	5	6
1	6.33	3.67	1.33	2.00	1.00	0.33
2	6.42	2.92	0.88	1.75	0.29	
3	6.53	3.73	1.56	1.24		
4	6.40	1.87	1.60			
5	8.15	2.51				
6	6.89					

4) “Average” Incremental Data

The Hayne MLE Models

For “Berquist-Sherman” model :

- Here’s a simple example using a 6 x 6 triangle:

Model Parameters from Maximum Likelihood Estimation

	1	2	3	4	5	6
Mean	6.644	2.889	1.310	1.624	0.653	0.277
Std Dev	0.643	0.312	0.196	0.246	0.189	0.180

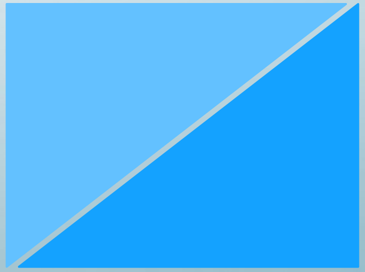
	Trend	K	p	AIC	BIC
Mean	0.005	0.556	0.450	57.71	61.02
Std Dev	0.024	0.439	0.180		

5) Estimate Model Parameters

Fitted Incremental Values

	1	2	3	4	5	6	Future
1	6.68	2.90	1.32	1.63	0.66	0.28	-
2	6.71	2.92	1.32	1.64	0.66	0.28	0.28
3	6.74	2.93	1.33	1.65	0.66	0.28	0.94
4	6.78	2.95	1.34	1.66	0.67	0.28	2.61
5	6.81	2.96	1.34	1.67	0.67	0.28	3.96
6	6.85	2.98	1.35	1.67	0.67	0.29	6.96
							14.75

5) Fitted “Average” Incremental



The Hayne MLE Models

For “Berquist-Sherman” model :

- Here’s a simple example using a 6 x 6 triangle:

Fitted Incremental Variances

	1	2	3	4	5	6	Future
1	0.80	0.55	0.39	0.42	0.28	0.19	-
2	0.75	0.52	0.36	0.40	0.26	0.18	0.18
3	0.78	0.53	0.37	0.41	0.27	0.19	0.33
4	0.72	0.50	0.35	0.38	0.25	0.17	0.49
5	0.78	0.54	0.38	0.42	0.28	0.19	0.65
6	0.74	0.51	0.35	0.39	0.26	0.18	0.80
							1.20

5) Fitted Incremental “Std Dev”

Random Parameters

	1	2	3	4	5	6
Rand	0.2744	0.3944	0.1414	0.6189	0.8761	0.4298
Mean	6.258	2.711	1.067	1.641	0.852	0.245
	Trend	**K**	**p**			
Rand	0.9616	0.1284	0.6877			
Mean	0.038	0.030	0.651			

6) Sample Random Parameters

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The Hayne MLE Models

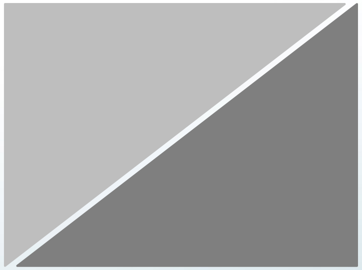
For “Berquist-Sherman” model :

- Here’s a simple example using a 6 x 6 triangle:

Incremental Values from Sample Parameters

	1	2	3	4	5	6	Future
1	6.50	2.82	1.11	1.70	0.88	0.25	-
2	6.75	2.92	1.15	1.77	0.92	0.26	0.26
3	7.01	3.04	1.19	1.84	0.95	0.27	1.23
4	7.27	3.15	1.24	1.91	0.99	0.28	3.18
5	7.55	3.27	1.29	1.98	1.03	0.30	4.59
6	7.84	3.40	1.34	2.06	1.07	0.31	8.17
							17.44

7) Sample “Average” Incremental



Incremental Variances from Sample Parameters

	1	2	3	4	5	6	Future
1	0.89	0.51	0.28	0.37	0.24	0.11	-
2	0.85	0.49	0.27	0.36	0.23	0.10	0.10
3	0.90	0.52	0.28	0.38	0.25	0.11	0.27
4	0.85	0.50	0.27	0.36	0.23	0.10	0.44
5	0.95	0.55	0.30	0.40	0.26	0.12	0.57
6	0.91	0.53	0.29	0.38	0.25	0.11	0.76
							1.09

7) Sample Incremental “Std Dev”

The Hayne MLE Models

For “Berquist-Sherman” model :

- Here’s a simple example using a 6 x 6 triangle:

Sample Random Values

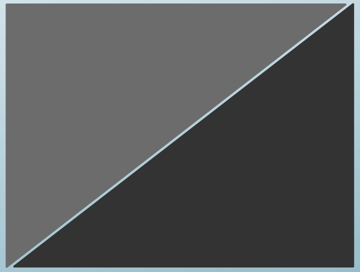
	1	2	3	4	5	6
1	0.9974	0.3210	0.6092	0.6171	0.3584	0.8885
2	0.4619	0.2849	0.4047	0.7240	0.2322	0.1297
3	0.4338	0.3252	0.2019	0.6761	0.6951	0.7265
4	0.4406	0.6977	0.8119	0.2503	0.4591	0.9582
5	0.3158	0.2606	0.4297	0.4608	0.4738	0.6451
6	0.1455	0.5928	0.6287	0.7430	0.8789	0.6591

8) Sample Random Values

Sample Incremental Values

	1	2	3	4	5	6	Future
1	8.97	2.58	1.19	1.81	0.80	0.39	-
2	6.67	2.64	1.09	1.98	0.75	0.15	0.15
3	6.86	2.80	0.96	2.01	1.08	0.34	1.42
4	7.15	3.41	1.48	1.67	0.97	0.46	3.10
5	7.10	2.92	1.24	1.94	1.01	0.34	4.53
6	6.88	3.52	1.43	2.31	1.36	0.35	8.97
							18.16

8) Sample Incremental Values



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The Hayne MLE Models

For "Berquist-Sherman" model :

- Here's a simple example using a 6 x 6 triangle:

Sample Incremental Values x Exposures

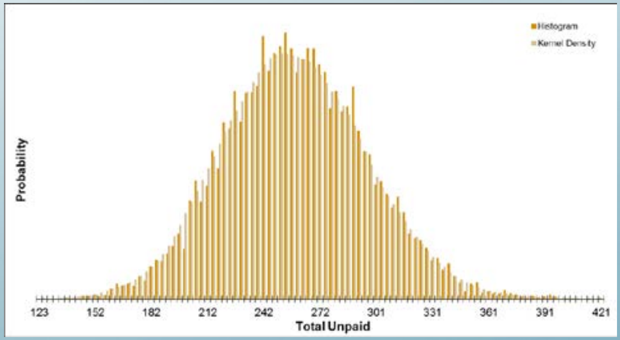
	1	2	3	4	5	6	Unpaid
1	135	39	18	27	12	6	-
2	114	45	19	34	13	3	3
3	110	45	15	32	17	5	22
4	134	64	28	31	18	9	58
5	113	47	20	31	16	5	72
6	125	64	26	42	25	6	163
							318

9) Convert to Unpaid

- Repeat steps 6 – 9 → 10,000 times!

Simulation Results

	Mean	Std Dev	CoV	Min	Max	75.0%	99.0%
1	-	-	-	-	-	-	-
2	4.7	4.7	100.0%	(31.0)	33.0	8.0	17.0
3	15.1	7.4	48.6%	(25.0)	48.0	20.0	33.0
4	48.8	12.3	25.2%	2.0	106.0	57.0	79.0
5	63.1	13.5	21.4%	(2.0)	119.0	72.0	96.0
6	126.0	20.0	15.9%	56.0	224.0	139.0	175.0
TOTAL	257.8	38.5	14.9%	123.0	420.0	283.0	352.0



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  - Standardized Residuals
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  - Outliers
  - Heteroscedasticity
  - Heteroecthesious Data
  - Parameter Adjustments
  - Tail Extrapolation
  - Incurred Data

Tail Extrapolation

- Berquist-Sherman, Cape Cod, Chain Ladder
  - Use Decay of Parameters to Extrapolate
  - E.g., average, linear, logarithmic, power, polynomial
- Hoerl Curve & Wright
  - Continue development parameters

Development Period Parameters (Average Incremental) / **User Selected Parameters**

	12	24	36	48	60	72	84	96	108	120
Mean	620.98	760.69	708.19	553.58	350.01	181.39	70.97	43.88	26.08	15.21
Std Dev	40.50	46.55	43.00	35.49	26.17	17.66	10.39	8.74	7.22	7.35
Decay Ratios:		122.5%	93.1%	78.2%	63.2%	51.8%	39.1%	61.8%	59.3%	158.5%
CoV:	*6.5%*	*6.1%*	*6.1%*	*6.4%*	*7.5%*	*9.7%*	*14.6%*	*19.9%*	*28.1%*	*48.3%*

	Accident Year Trend	K	p	AIC	BIC	Tail Extrapolation Decay Ratio	Periods	Distribution	Implied Tail Factor Parameters / Adjusted	Actual
Mean	0.045	11.217	0.654	647.9	669.5	45.3%	3	Acc Period / Gamma	1.0034	1.0034
Std Dev	0.009	1.094	0.089			13.7%		Dev Period	10	
CoV:	*18.9%*	*9.8%*	*13.6%*					Trend	1	
									11	

Decay Ratio Analysis:

Parameters: User **Curve Type:** Power 3

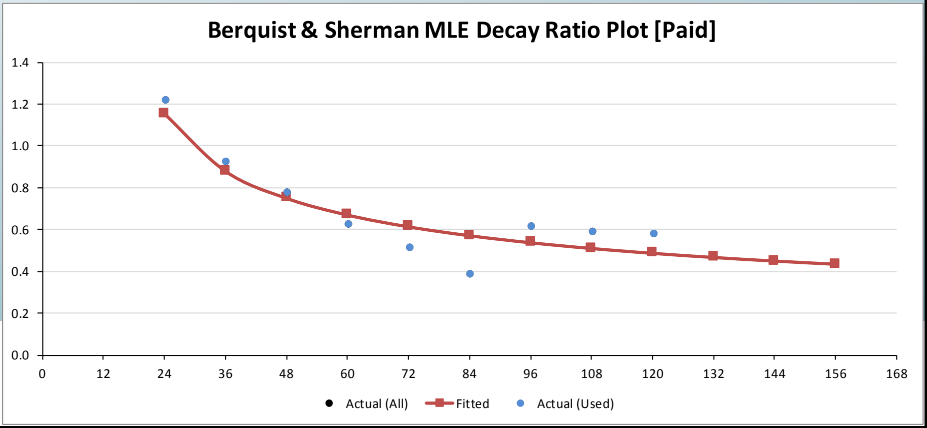
Least Squares Regression Coefficients:

x^a	-0.3915
coefficient	1.1558

Goodness of Fit Statistics:

R^2 Statistic	0.710
Regression Deviation	13.5%
Suggested Decay Parameters:	
Mean	45.3%
Standard Deviation	13.7%

Periods	Decay Ratio	Outliers	Selected Age	Selected Decay Ratio	Incremental Fitted Factors
12-24	1.225	0	1	1.225	1.156
24-36	0.931	0	2	0.931	0.881
36-48	0.782	0	3	0.782	0.752
48-60	0.632	0	4	0.632	0.672
60-72	0.518	0	5	0.518	0.615
72-84	0.391	0	6	0.391	0.573
84-96	0.618	0	7	0.618	0.540
96-108	0.593	0	8	0.593	0.512
108-120	0.585	0	9	0.585	0.489
120-132					0.469
132-144					0.452
144-156					0.437
156-168					0.423
168-180					0.411



Any Final Questions?

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